Image Processing

Chapter(3)

Part 1:Relationships between pixels

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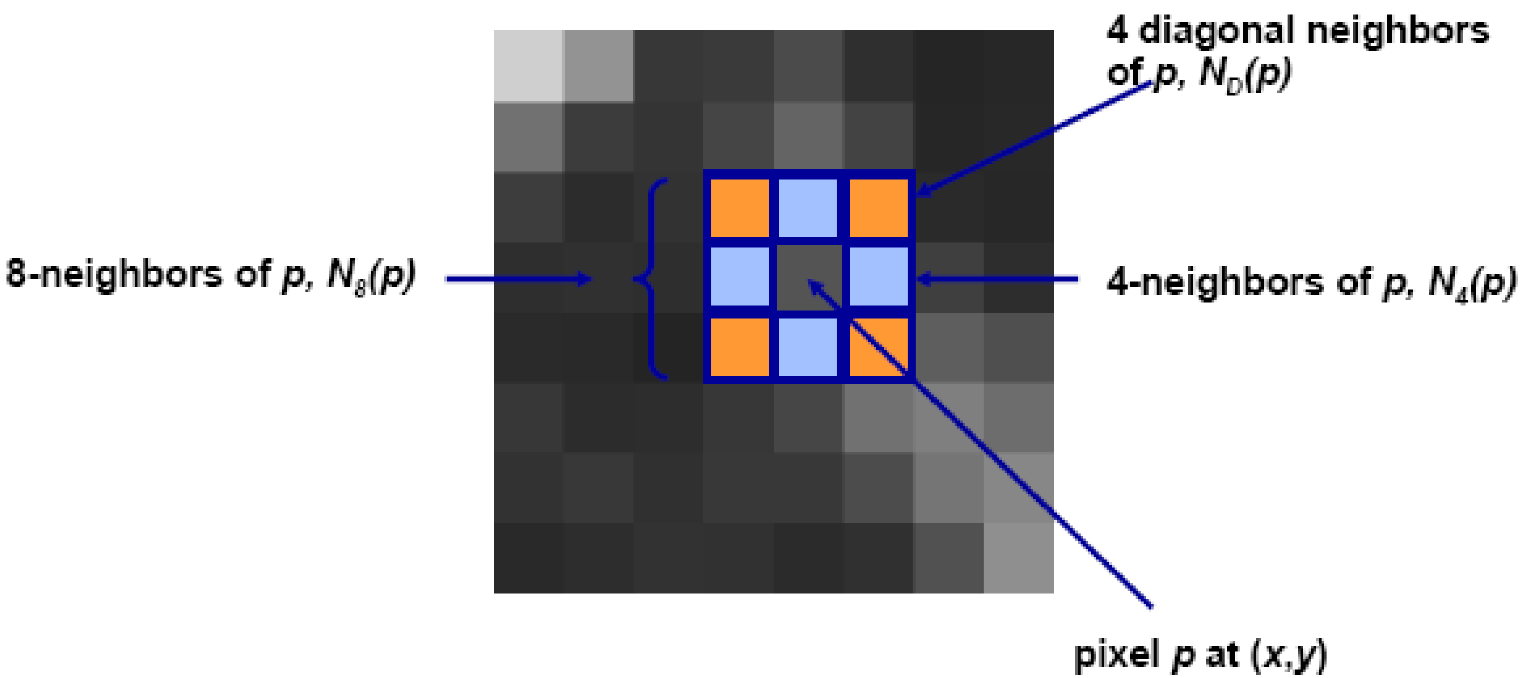
Neighbors of a pixel



❖some of the points in N_D (p) and N_8 (p) fall outside the image if (x,y) is on the border of the image.

Connectivity

- Two pixels are connected if:

– They are neighbors (i.e. adjacent in some sense -- e.g. N4(p), N8(p), …)

– Their gray levels satisfy a specified criterion of similarity (e.g. equality, …)

- V is the set of gray-level values used to define adjacency (e.g. V={1} for adjacency of pixels of value 1)

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Adjacency and Connectivity

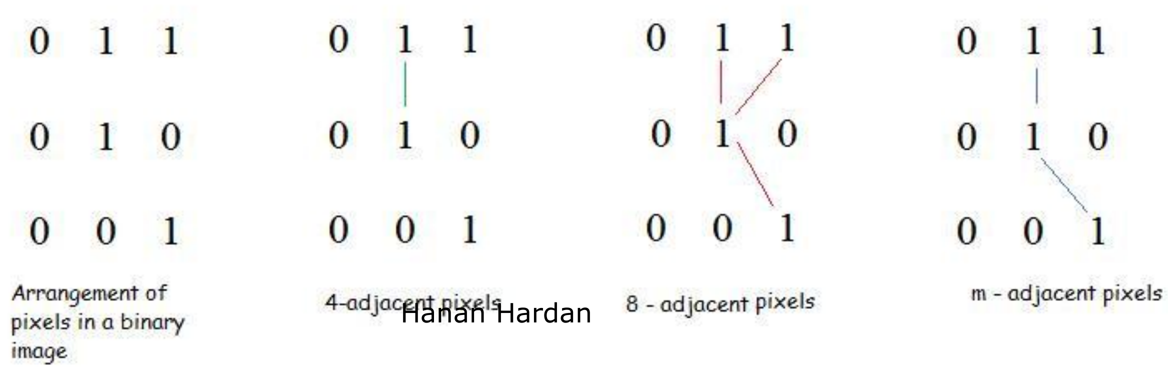
- Let V: a set of intensity values used to define adjacency and connectivity.
- In a binary image, $V = \{1\}$, if we are referring to adjacency of pixels with value 1.
- In a gray-scale image, the idea is the same, but V typically contains more elements, for example, $V = \{180, 181, 182, \ldots, 200\}$
- If the possible intensity values 0 – 255, V set can be any subset of these 256 values

Adjacency

V: set of gray level values (L), (V is a subset of L.)

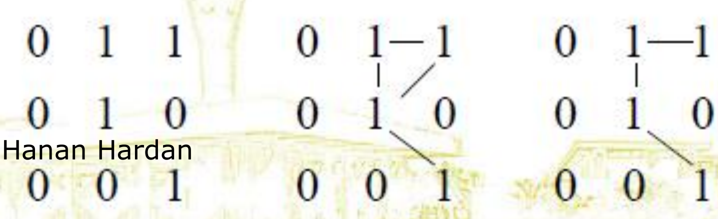
3 types of adjacency

- 4- adjacency: 2 pixels *p* and *q* with values from *V* are 4- adjacent if *q* is in the set $N_4(p)$
- 8- adjacency: 2 pixels *p* and *q* with values from *V* are 8- adjacent if *q* is in the set $N_8(p)$
- *m*- adjacency: 2 pixels *p* and *q* with values from *V* are *m*-adjacent if
  1. *q* is in $N_4(p)$, *or*
  2. *q* is in $N_D(p)$ *and* the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V



Types of Adjacency

- In this example, we can note that to connect between two pixels (finding a path between two pixels):
  - In 8-adjacency way, you can find multiple paths between two pixels
  - While, in m-adjacency, you can find only one path between two pixels
- So, m-adjacency has eliminated the multiple path connection that has been generated by the 8-adjacency.



Types of Adjacency

- Two subsets *S1* and *S2* are adjacent, if some pixel in *S1* is adjacent to some pixel in *S2*. Adjacent means, either 4-, 8- or m-adjacency.

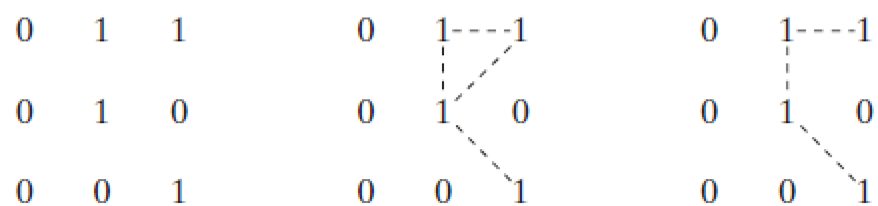
A digital path

- A digital path from pixel p with coordinates (x,y) to pixel q with coordinates (s,t) is a sequence of distinct pixels with coordinates (x0,y0), (x1,y1), ..., (xn,yn), where (x0,y0)= (x,y) and (xn,yn)=(s,t), and pixels (xi,yi) and (xi-1,yi-1) are adjacent for $1 \le i \le n$.

- n is the length of the path
- If $(x_0,y_0) = (x_n, y_n)$, the path is closed.
- We can specify 4-, 8- or m-paths depending on the type of adjacency specified.

A Digital Path

- Return to the previous example:



a b c

FIGURE 2.26 (a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) *m*-adjacency.

In figure (b) the paths between the top right and bottom right pixels are 8-paths. And the path between the same 2 pixels in figure (c) is m-path

Connectivity

- *S*: a subset of pixels in an image.
- Two pixels *p* and *q* are said to be connected in *S* if there exists a path between them consisting entirely of pixels in *S*.
- For any pixel *p* in *S*, the set of pixels that are connected to it in *S* is called a connected component of *S*.

Regions and boundaries

- **Region**

  Let *R* be a subset of pixels in an image, we call *R* a region of the image if *R* is a connected set.

  000000
  010010
  011010
  010110
  000000

- **Boundary**

The *boundary* (also called *border* or *contour*) of a region *R* is the set of pixels in the region that have one or more neighbors that are not in *R*.

Region and Boundary

- If *R* happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns in the image.

Foreground and background

Suppose that the image contains K disjoint regions Rk none of which touches the image border .

Ru : the union of all regions .

$(Ru)^c$: is the complement .

so Ru is called foreground , and $(Ru)^c$: is the background .

Distance measures

If we have 3 pixels: p,q,z respectively

p with (x,y)

q with (s,t)

z with (v,w)

Then:

A. $D(p,q) \geq 0$, $D(p,q) = 0$ iff $p = q$
B. $D(p,q) = D(q,p)$
C. $D(p,z) \leq D(p,q) + D(q,z)$

Distance measures

- Euclidean distance between *p* and *q*:

  $$De(p,q) = [(x-s)^2 + (y-t)^2]^{1/2}$$

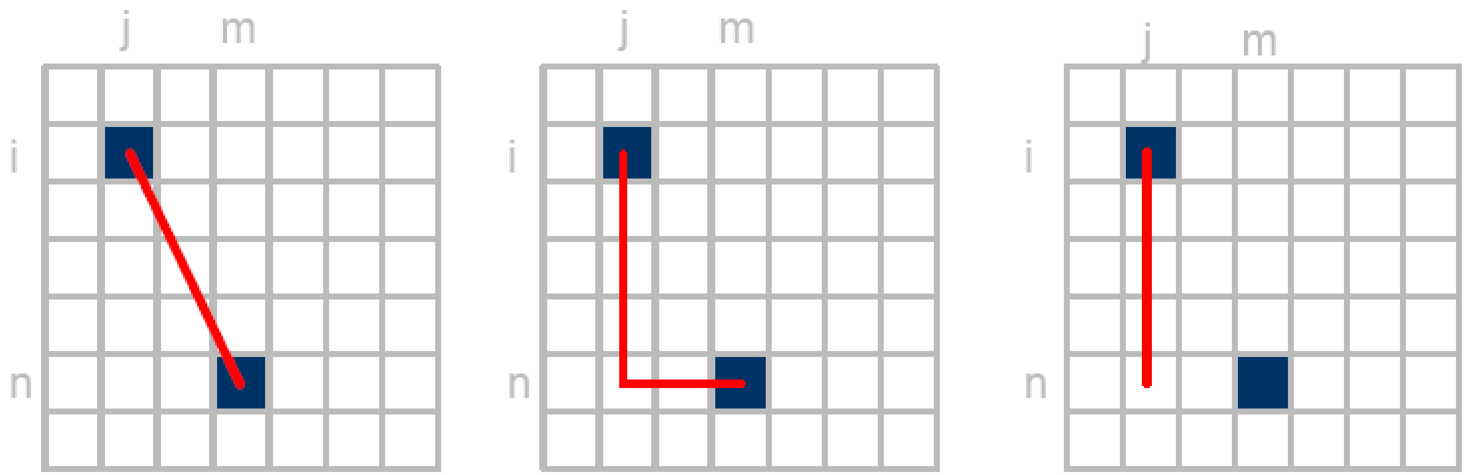
- *D4* distance (also called *city-block distance*):

  $$D4(p,q) = |x-s| + |y-t|$$

- *D8* distance (also called *chessboard distance*) :

  $$D8(p,q) = \max(|x-s|, |y-t|)$$

Distance measures



Euclidean Distance

$= \sqrt{(i-n)^2 + (j-m)^2}$

City Block Distance

$= |i-n| + |j-m|$

Chessboard Distance

$= \max[\,|i-n|, |j-m|\,]$

Example:

Compute the distance between the two pixels using the three distances :

q:(1,1)

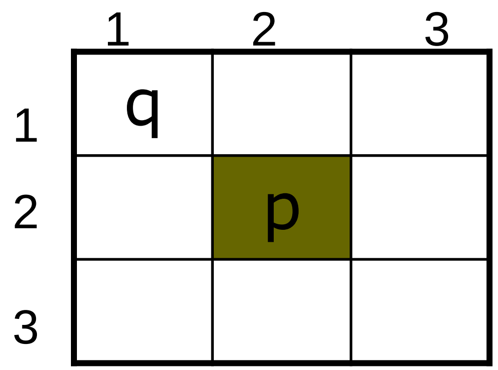
P: (2,2)

Euclidian distance : $((1-2)^2+(1-2)^2)^{1/2}$ = sqrt(2).

D4(City Block distance): |1-2| +|1-2| =2

D8(chessboard distance) : max(|1-2|,|1-2|)= 1

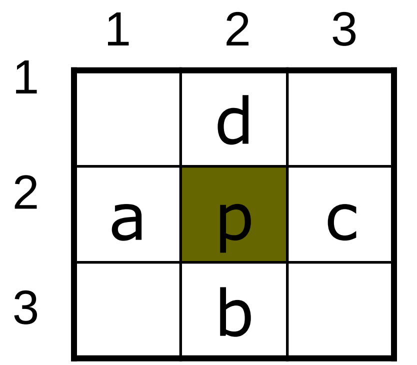
(because it is one of the 8-neighbors)



Distance measures

Example :

Use the city block distance to prove 4-neighbors ?



Pixel A : | 2-2| + |1-2| = 1

Pixel B: | 3-2|+|2-2|= 1

Pixel C: |2-2|+|2-3| =1

Pixel D: |1-2| + |2-2| = 1

Now as a homework try the chessboard distance to proof the 8- neighbors!!!!

Distance Measures

Example:

The pixels with distance $D_4 \le 2$ from (x,y) form the following contours of constant distance.

The pixels with $D_4 = 1$ are the 4-neighbors of (x,y)

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

Distance Measures

Example:

D_8 distance ≤ 2 from (x,y) form the following contours of constant distance.

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

D8 = 1 are the 8-neighbors of (x,y)

Distance Measures

- **Dm distance:**

  is defined as the shortest m-path between the points.

  In this case, the distance between two pixels will depend on the values of the pixels along the path, as well as the values of their neighbors.

Distance Measures

- Example:

  Consider the following arrangement of pixels and assume that p, p_2, and p_4 have value 1 and that p_1 and p_3 can have a value of 0 or 1

  Suppose that we consider the adjacency of pixels values 1 (i.e. $V = \{1\}$)

	p_3	p_4
p_1	p_2	
p		

Distance Measures

- Cont. Example:

  Now, to compute the D_m between points p and p_4

  Here we have 4 cases:

  Case1: If p_1 =0 and p_3 = 0

  The length of the shortest m-path

  (the D_m *distance) is* 2 (p, p_2, p_4)

	0	1
0	1	
1		

Distance Measures

- Cont. Example:

  Case2: If $p_1 = 1$ and $p_3 = 0$

  then, the length of the shortest path will be 3 (p, p_1, p_2, p_4)

	0	1
1	1	
1		

Distance Measures

- Cont. Example:

  Case3: If p_1 =0 and p_3 = 1

  The same applies here, and the shortest – m-path will be 3 (p, p_2, p_3, p_4)

	1	1
0	1	
1		

Distance Measures

- Cont. Example:

  Case4: If p_1 =1 and p_3 = 1

  The length of the shortest m-path will be 4 (p, p_1 , p_2, p_3, p_4)