Advanced Measurement Systems & Sensors
(0640732)

Lecture (3)
Sensor Characteristics
(Part Two)

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3. Computation of Stimulus:
The main objective of sensing is to determine a value of the input stimulus \( s \) from the value of the sensor output signal \( S \). This can be done by two methods.

1. From the inverted transfer function \( \{ s = F(S) \} \) or its approximation, or
2. From a direct transfer function \( \{ S = f(s) \} \) by use of the iterative computation.

Computation from Linear Piecewise Approximation:
Consider triangles \( \triangle p_1p_2p_3 \) and \( \triangle p_1p_5p_2 \): Both triangles are similar, a linear equation is used for computing the unknown stimulus \( s_x \) from the measured value \( n_x \):

\[
s_x = s_i + \frac{n_x - n_i}{n_{i+1} - n_i} (s_{i+1} - s_i)
\]

<table>
<thead>
<tr>
<th>Knot</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>...</th>
<th>i</th>
<th>...</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>( n_0 )</td>
<td>( n_1 )</td>
<td>( n_2 )</td>
<td>...</td>
<td>( n_i )</td>
<td>...</td>
<td>( n_k )</td>
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<tr>
<td>Input</td>
<td>( s_0 )</td>
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<td>...</td>
<td>( s_i )</td>
<td>...</td>
<td>( s_k )</td>
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Example: Assume the thermistor is used to measure temperature from 0°C to +60°C. The output count from the ADC can be modeled by a nonlinear function \( n(T) \) of temperature:

\[
n_x = N_0 \frac{R_0 e^{\beta(T^{-1}-T_0^{-1})}}{R_1 + R_0 e^{\beta(T^{-1}-T_0^{-1})}}
\]

Where;
- \( T \) is the measured temperature,
- \( T_0 \) is the reference temperature,
- \( R_0 \) is the resistance of the thermistor at \( T_0 \).

The inverted transfer function which enables us to compute analytically the input temperature is:

\[
T_x = \left( \frac{1}{T_0} + \frac{1}{\beta} \ln \left( \frac{n_x}{N_0 - n_x} \frac{R_1}{R_0} \right) \right)^{-1}
\]
Sensor Calibration:

- Calibrate the sensor at two temperatures \( T_{c1} \) and \( T_{c2} \) in order to find out values of the constants \( R_0 \) and \( \beta \).
- Select two calibrating temperatures in the middle of the operating range as \( T_0 = T_{c1} = 293.15 \text{ K} \) and \( T_{c2} = 313.15 \text{ K} \), which correspond to 20°C and 40°C.
- The thermistor sequentially is immersed into a liquid bath at these two temperatures and the ADC counts are registered as \( n_{c1} = 1863 \) and \( n_{c2} = 1078 \).
- By substituting these pairs into Equ. 1, we find the values of \( R_0 = 8.350 \text{ k}\Omega \) and \( \beta = 3.895 \text{ K} \).
- The second equation can be used for computing temperature from any reasonable ADC count.

**Example:** A thermistor is used to measure some unknown temperature and receive counts \( n_x = 1505 \).

\[
t_x = t_1 + \frac{n_x - n_1}{n_2 - n_i}(t_2 - t_1) = 20 + \frac{1505 - 1863}{1078 - 1863}(40 - 20) = 29.12
\]

To check how far this computed temperature 29.12°C deviates from that computed from a “true” temperature, we plug the same \( n_x = 1505 \) into second equation and compute \( t_x = 28.22 \text{C} \).
Iterative Computation of Stimulus (Newton Method):
Numerical iterative methods for finding roots of this typically nonlinear equation can be used for calculating the unknown stimulus \( s \) without the knowledge of the inverse transfer function.
If a sensor transfer function is \( f(s) \), the Newton method prescribes computing for any measured output value \( S \) the following sequence of the stimuli values which after several steps converges to the sought input \( s \).

\[
    s_{i+1} = s_i - \frac{f(s_i) - S}{f'(s_i)}
\]

**Example:** If a 3rd degree polynomial with coefficients \( a=1.5, b=5, c=25, d=1 \) is used to illustrate the Newton method.

\[
f(s) = as^3 + bs^2 + cs + d
\]

\[
s_{i+1} = s_i - \frac{as_i^3 + bs_i^2 + cs_i + d - S}{3as_i^2 + 2bs_i + c} = \frac{2as_i^3 + bs_i^2 - d + S}{3as_i^2 + 2bs_i + c}
\]
If the measured sensor’s response is \( S = 22 \) and our initial guess of the stimulus is \( s_0 = 2 \), then:

\[
\begin{align*}
    s_1 &= \frac{2 \cdot 1.5 \cdot 2^3 + 5 \cdot 2^2 - 1 + 22}{3 \cdot 1.5 \cdot 2^2 + 2 \cdot 5 \cdot 2 + 25} = 1.032 \\
    s_2 &= \frac{2 \cdot 1.5 \cdot 1.032^3 + 5 \cdot 1.032^2 - 1 + 22}{3 \cdot 1.5 \cdot 1.032^2 + 2 \cdot 5 \cdot 1.032 + 25} = 0.738 \\
    s_3 &= \frac{2 \cdot 1.5 \cdot 0.738^3 + 5 \cdot 0.738^2 - 1 + 22}{3 \cdot 1.5 \cdot 0.738^2 + 2 \cdot 5 \cdot 0.738 + 25} = 0.716 \\
    s_4 &= \frac{2 \cdot 1.5 \cdot 0.716^3 + 5 \cdot 0.716^2 - 1 + 22}{3 \cdot 1.5 \cdot 0.716^2 + 2 \cdot 5 \cdot 0.716 + 25} = 0.716
\end{align*}
\]

At step 4, the Newton algorithm stops and the stimulus value is deemed to be \( s = 0.716 \).

To check accuracy of this solution, plug this number into polynomial Equa and obtain 
\( f(s) = S = 22.014 \), which is within the resolution error (0.06%) of the actually measured response \( S = 22 \).
4. Full Scale Input (Span):
Full scale input represents the highest possible input value, which can be applied to the sensor without causing unacceptably large inaccuracy. A decibel scale is used with nonlinear response characteristic, represents low level signals with high resolution while compressing the high level numbers.

5. Full-Scale Output:
Full-scale output is the algebraic difference between the electrical output signals measured with maximum input stimulus and the lowest input stimulus applied.
6. **Accuracy:**
A very important characteristic of a sensor is accuracy, which really means inaccuracy. Inaccuracy is measured as a highest deviation of a value represented by the sensor from the ideal or true value of a stimulus at its input.

- The deviation is a difference between the value, which is computed from the output voltage, and the actual input value.
- All runs of the real transfer functions must fall within the limits of a specified accuracy. These permissive limits differ from the ideal transfer function line by ±Δ. The real functions deviate from the ideal by ±δ, where δ≤Δ.
Inaccuracy rating may be represented:

- Directly in terms of measured value (Δ): It is used when error is independent on the input signal magnitude. For example, it can be stated as 0.15°C for a temperature sensor.
- In % of the input full scale: It is useful for a sensor with a linear transfer function.
- In % of the measured signal: It is useful for a sensor with a highly nonlinear transfer function.
- In terms of the output signal: It is useful for sensors with a digital output format so the error can be expressed, for example, in units of LSB.

Absolute & Relative Errors:
Accuracy is the capacity of a measuring instrument to give RESULTS close to the TRUE VALUE of the measured quantity.

\[
\text{ABSOLUTE ERROR} = \text{RESULT} - \text{TRUE VALUE} \\
\text{RELATIVE ERROR} = \frac{\text{ABSOLUTE ERROR}}{\text{TRUE VALUE}}
\]
7. Calibration Error:

- It is inaccuracy permitted by a manufacturer when a sensor is calibrated in the factory.
- This error is of a systematic nature and is added to all possible real transfer functions. It shifts the accuracy of transduction for each stimulus point by a constant.
- This error is not necessarily uniform over the range and may change depending on the type of error in calibration.

**Example:** Two-point calibration of a real linear transfer function. Determine the slope \( b \) and the intercept \( a \) of the function using two stimuli, \( s_1 \) & \( s_2 \).

\[
\delta_a = a_1 - a = \frac{\Delta}{s_2 - s_1}
\]

\[
\delta_b = -\frac{\Delta}{s_2 - s_1}
\]
8. **Hysteresis:**
A hysteresis error is a deviation of the sensor’s output at a specified point of the input signal when it is approached from the opposite directions.

**Example:** A displacement sensor;
- When the object moves from left to right at a certain point produces voltage, which differs by 20 mV from that when the object moves from right to left.
- If sensitivity of the sensor is 10 mV/mm, the hysteresis error is 2 mm.
9. **Nonlinearity**: 

- Nonlinearity error is specified for sensors whose transfer function may be approximated by a straight line.
- A nonlinearity is a maximum deviation (L) of a real transfer function from the approximation straight line.
- There are several ways to specify nonlinearity, depending how the line is superimposed on the transfer function;
  1. Use terminal points to determine output values at the smallest and highest stimulus values and to draw a straight line through these two points.
  2. Use best straight line, which is a line midway between two parallel straight lines closest together and enveloping all output values on a real transfer function.
10. Saturation:
The sensor exhibits a span-end nonlinearity or saturation if further increase in stimulus does not produce a desirable output.
11. Repeatability:
Repeatability error is caused by the inability of a sensor to represent the same value under presumably identical conditions. (The same output signal S1 corresponds to two different input Signals).
It is usually represented as % of FS:

\[ \delta_r = \frac{\Delta}{FS} \times 100\% \]

12. Dead Band:
It is insensitivity of a sensor in a specific range of the input signals. In that range, the output may remain near a certain value (often zero) over an entire dead band zone.
13. **Resolution:**
Resolution describes smallest increments of stimulus, which can be sensed. The resolution of digital output format sensors is given by the number of bits in the data word.

14. **Output Impedance ($Z_{out}$):**
Output impedance is important to know to better interface a sensor with the electronic circuit. The output impedance is connected to the input impedance ($Z_{in}$) of the circuit either in parallel (voltage connection) or in series (current connection).
15. **Sensor Output Format:**

- Output format is a set of the output electrical characteristics that is produced by the sensor.
- The characteristics may include voltage, current, charge, frequency, amplitude, phase, polarity, shape of a signal, time delay, and digital code.

![Sensor Output Format Diagram](image)

16. **Sensor Excitation:**

- Excitation is the electrical signal needed for operation of an active sensor.
- Excitation is specified as a range of voltage and/or current.
17. Sensor Dynamic Characteristics:
A sensor is characterized with a time-dependent characteristic.

**Zero-order Sensor:**
- It is characterized by a transfer function that is time independent.
- Such a sensor does not incorporate any energy storage devices, like a capacitor.
- A zero-order sensor responds instantaneously. Such a sensor does not need any dynamic characteristics to be specified.

\[ y(t) = k \cdot x(t) \Rightarrow \frac{Y(s)}{X(s)} = k \]

- Zero-order is the desirable response of a sensor
  - No delays
  - Infinite bandwidth
  - The sensor only changes the amplitude of the input signal
- Zero-order systems do not include energy-storing elements
- Example of a zero-order sensor
  - A potentiometer used to measure linear and rotary displacements
    - This model would not work for fast-varying displacements
17. Sensor Dynamic Characteristics:

First-order Sensor:

- A sensor that incorporates one energy storage component is specified by a first-order differential equation.
- An example: a temperature sensor where energy storage is thermal capacity.

Inputs and outputs related by a first-order differential equation

\[
a_1 \frac{dy}{dt} + a_0 y(t) = x(t) \quad \Rightarrow \quad \frac{Y(s)}{X(s)} = \frac{1}{a_1 s + a_0} = \frac{k}{\tau s + 1}
\]

- First-order sensors have one element that stores energy and one that dissipates it.
- Step response
  - \( y(t) = A k (1 - e^{-t/\tau}) \)
    - \(A\) is the amplitude of the step
    - \(k (= 1/a_0)\) is the static gain, which determines the static response
    - \(\tau (= a_1/a_0)\) is the time constant, which determines the dynamic response
- Ramp response
  - \( y(t) = A k t - A k \tau u(t) + A k \tau e^{-t/\tau} \)
- Frequency response
  - Better described by the amplitude and phase shift plots
First-order sensor response:

- Step response
- Ramp response
- Corner frequency $\omega_c = 1/\tau$
- Bandwidth
- Frequency response
A first order system response is as follows:

\[ S = S_m(1 - e^{-t/\tau}) \]

where \( S_m \) is steady-state output, and \( t \) is time.

Substituting \( t = \tau \), we get;

\[ \frac{S}{S_m} = 1 - \frac{1}{e} = 0.6321 \]

This means that after a time equal to one time constant, the response reaches about 63% of its steady-state level. Similarly, it can be shown that after two time constants, the height will be 86.5% and after three time constants it will be 95% of the level that would be reached at infinite time.

As a rule of thumb, a simple formula can be used to establish a connection between the cutoff frequency \((f_c)\) and time constant \((\tau)\) in a first-order sensor:

\[ f_c \approx \frac{0.159}{\tau} \]
Example of a first-order sensor

- A mercury thermometer immersed into a fluid
  - What type of input was applied to the sensor?
  - Parameters
    - C: thermal capacitance of the mercury
    - R: thermal resistance of the glass to heat transfer
    - $\theta_F$: temperature of the fluid
    - $\theta(t)$: temperature of the thermometer
  - The equivalent circuit is an RC network

- Derivation
  - Heat flow through the glass \( \frac{(\theta_F - \theta(t))}{R} \)
  - Temperature of the thermometer rises as \( \frac{d\theta(t)}{dt} = \frac{\theta_F - \theta(t)}{RC} \)
  - Taking the Laplace transform

\[
s \ \theta(s) = \frac{\theta_F(s) - \theta(s)}{RC} \Rightarrow (RCs + 1) \theta(s) = \theta_F(s) \Rightarrow \\
\Rightarrow \theta(s) = \frac{\theta_F(s)}{(RCs + 1)} \Rightarrow \theta(t) = \theta_F \left( 1 - e^{-\frac{t}{RC}} \right)
\]
A second-order Sensor:
It incorporates two energy storage components, and represented by a second-order differential equation. The relationship between the input $s(t)$ and output $S(t)$ is;

$$b_2 \frac{d^2 S(t)}{dt^2} + b_1 \frac{dS(t)}{dt} + b_0 S(t) = s(t)$$

We can express this second-order transfer function as;

$$\frac{Y(s)}{X(s)} = \frac{k \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}$$

with $k = \frac{1}{a_0}$, $\zeta = \frac{a_1}{2\sqrt{a_0 a_1}}$, $\omega_n = \sqrt{\frac{a_0}{a_2}}$

- Where
  - $k$ is the static gain
  - $\zeta$ is known as the damping coefficient
  - $\omega_n$ is known as the natural frequency
Second-order step response

- **Response types**
  - Underdamped ($\zeta<1$)
  - Critically damped ($\zeta=1$)
  - Overdamped ($\zeta>1$)

- **Response parameters**
  - Rise time ($t_r$)
  - Peak overshoot ($M_p$)
  - Time to peak ($t_p$)
  - Settling time ($t_s$)
Example of second-order sensors

- A thermometer covered for protection
  - Adding the heat capacity and thermal resistance of the protection yields a second-order system with two real poles (overdamped)

- Spring-mass-dampen accelerometer
  - The armature suffers an acceleration
    - We will assume that this acceleration is orthogonal to the direction of gravity
  - \( x_0 \) is the displacement of the mass \( M \) with respect to the armature
  - The equilibrium equation is:

\[
M(\ddot{x}_i - \ddot{x}_0) = Kx_0 + B\dot{x}_0
\]

\[
\downarrow
\]

\[
Ms^2x_i(s) = X_0(s)[K + Bs + Ms^2]
\]

\[
\downarrow
\]

\[
\frac{X_0(s)}{s^2x_i(s)} = \frac{M}{K} \frac{K/M}{s^2 + s(B/M) + K/M}
\]
18. Reliability:

- It is the ability of a sensor to perform a required function under stated conditions for a stated period.
- It is expressed in statistical terms as a probability that the device will function without failure over a specified time or a number of uses.
- The procedure for predicting in-service reliability is the MTBF (mean-time between-failure) calculation.
- The qualification tests on sensors are performed at combinations of the worst possible conditions. One approach (suggested by MIL-STD-883) is 1000 h, loaded at maximum temperature. Three goals are behind the test:
  1. to establish MTBF;
  2. to identify first failure points that can then be strengthened by design changes;
  3. to identify the overall system practical life time.
References: