Q2.44

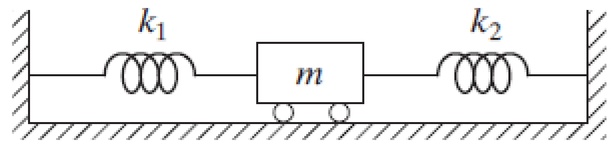
Problem statement: Derive the equation of motion of the system shown in Fig. below using the following methods:

(a) Newton s second law of motion,

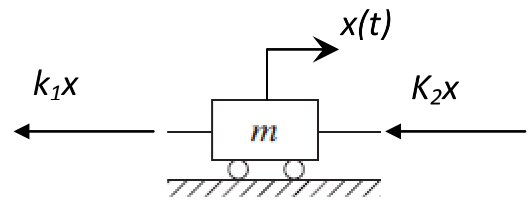
(b) D Alembert s principle

(c) principle of virtual work

(d) principle of conservation of energy



Solution:



a) $m\ddot{x} + (k_1 + k_2)x = 0$

Note that these two springs have equivalent stiffness shows that these springs are in parallel configuration.

b) $F(t) - m\ddot{x} = 0 \Rightarrow -(k_1)x - (k_2)x - m\ddot{x} = 0 \Rightarrow m\ddot{x} + (k_1 + k_2)x = 0$

c) Work done by the mass is due to a virtual distance δv = $\left(m\ddot{x}\right)\delta v = 0$

Work done by the springs = $(k_1 + k_2)x\delta v = 0$

Total virtual work = 0; $-(k_1 + k_2)x\delta v - \left(m\ddot{x}\right)\delta v = 0 \Rightarrow m\ddot{x} + (k_1 + k_2)x = 0$

d) $T = \frac{1}{2}m\dot{x}^2$ $U_1 = \frac{1}{2}k_1x^2$ $U_2 = \frac{1}{2}k_2x^2$ now, T + U_1 + U_2 = constant

so, $\frac{1}{2}m\dot{x}^2 + \frac{1}{2}k_1x^2 + \frac{1}{2}k_2x^2 = Constant \Rightarrow \frac{d}{dt}\left(\frac{1}{2}m\dot{x}^2 + \frac{1}{2}k_1x^2 + \frac{1}{2}k_2x^2\right) = 0$

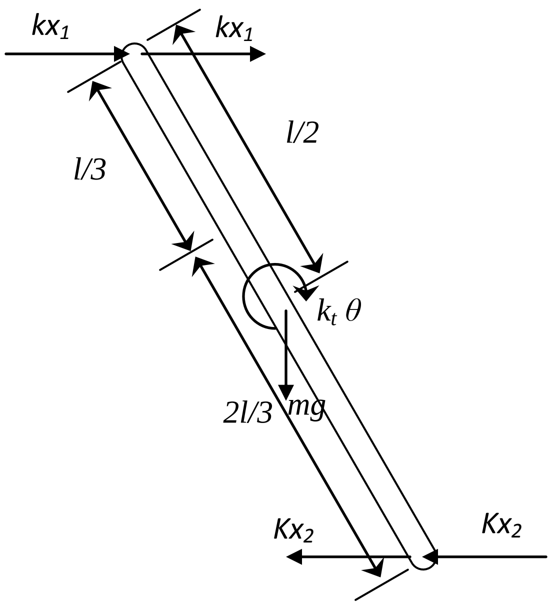
which results in $m\ddot{x} + (k_1 + k_2)x = 0$

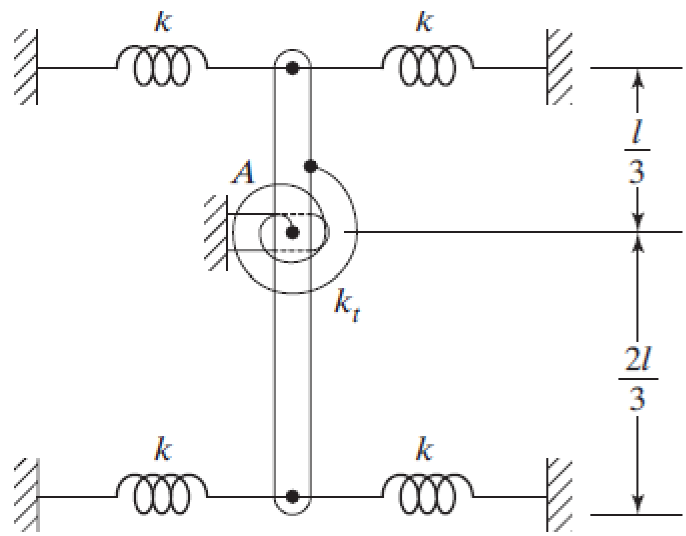
Q2.73

Problem statement: A uniform slender rod of mass *m* and length *l* is hinged at point *A* and is attached to four linear springs and one torsional spring, as shown in Fig. below. Find the natural frequency of the system if k = 2000 N/m, k_t= 1000 N-m/rad, *l* = 5 m and m = 10 kg,

Solution:

F.B.D :





First, assume small angle initial excitation (θ) → sin(θ) ≈ θ and the positive direction is the counterclockwise which in this case, θ is positive and the other moment parts (springs and weight) are negative because they try to rotate the system clockwise.
Second, apply Newton's 2nd law of motion to the system:

$$\sum M_A = J_o \ddot{\theta}$$

$$J_o \ddot{\theta} = -kx_1\left(\frac{l}{3}\right) - kx_1\left(\frac{l}{3}\right) - kx_2\left(\frac{2l}{3}\right) - kx_2\left(\frac{2l}{3}\right) - mg\left(\frac{l}{2} - \frac{l}{3}\right)\theta - k_t\theta$$

Note that sin(θ) in the weight term (mg) is replaced by θ due to the assumption made previously. Note also that the moment produced by the torsional spring equal the torsional stiffness x θ.

Rearrange the terms

$$J_o \ddot{\theta} + kx_1\left(\frac{2l}{3}\right) + kx_2\left(\frac{4l}{3}\right) + mg\left(\frac{l}{6}\right)\theta + k_t\theta = 0$$

Now let us relate the translational distances (x_1 and x_2) to θ :

$$\sin(\theta) \approx \theta = \frac{x_1}{l/3} \Rightarrow x_1 = \theta\left(\frac{l}{3}\right)$$

$$\sin(\theta) \approx \theta = \frac{x_2}{2l/3} \Rightarrow x_2 = \theta\left(\frac{2l}{3}\right)$$

So the mathematical model becomes:

$$J_o\ddot{\theta} + \left\{\left(\frac{10}{9}\right)l^2k + mg\left(\frac{l}{6}\right) + k_t\right\}\theta = 0$$

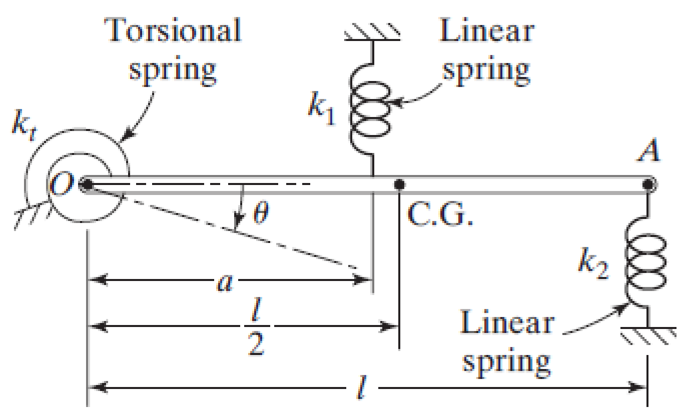
And hence, the natural frequency equal:

$$\omega_n = \sqrt{\left\{\left(\frac{10}{9}\right)l^2k + mg\left(\frac{l}{6}\right) + k_t\right\}/J_o}$$

Note that the weight of the beam affects the mathematical model because when the rod is connected to the springs, its weight does not change the length of any of the springs (i.e. no deflection in the springs even the trosional one)

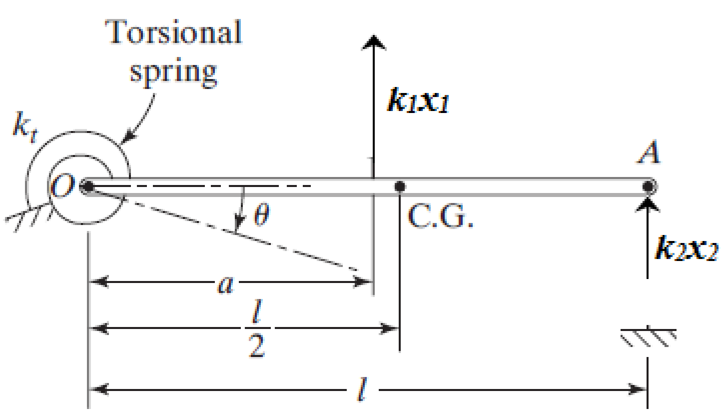
Q2.76

Problem statement: Find the equation of motion of the uniform rigid bar *OA* of length *l* and mass *m* shown in Fig. Also find its natural frequency.



Solution:

F.B.D:



First, assume small angle initial excitation (θ) ⟶ sin(θ) ≈ θ and the positive direction is the clockwise which in this case, θ is positive and the other moment parts (springs) are negative because they try to rotate the system counterclockwise.

Second, the weight of the bar causes a primary deflection (static deflection) and will have no effect on the derivation of mathematical model any more.

Third, apply Newton's 2nd law of motion to the system:

$$\sum M_A = J_o \ddot{\theta}$$

$$J_o \ddot{\theta} = -k_1 x_1 (a) - k_2 x_2 (l) - k_t \theta$$

Now let us relate the translational distances (x_1 and x_2) to θ :

$$\sin(\theta) \approx \theta = \frac{x_1}{a} \Rightarrow x_1 = \theta(a)$$

$$\sin(\theta) \approx \theta = \frac{x_2}{l} \Rightarrow x_2 = \theta(l)$$

So the mathematical model becomes:

$$J_o \ddot{\theta} + (k_1 a^2)\theta + (k_2 l^2)\theta + k_t \theta = 0$$

$$J_o \ddot{\theta} + (k_1 a^2 + k_2 l^2 + k_t)\theta = 0$$

And hence, the natural frequency equal:

$$\omega_n = \sqrt{(k_1 a^2 + k_2 l^2 + k_t)/J_o}$$

You can also try the following questions from the text book:

1. **Q2.40**
2. **Q2.46**
3. **Q2.74**