Theory of machinery

Chapter three

Velocity analysis

By

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Velocity analysis

VECTOR ALGEBRA

•If we assume that the dimension Θ is represented by a unit vector U_{Θ}, then the derivative of the unit vector ($\dot{U}_{\theta}$) can be found as:-

$$\dot{U}_{\theta} = \frac{dU_{\theta}}{dt} = \lim_{\Delta t \to 0} \frac{U_{\theta+\Delta\theta} - U_{\theta}}{\Delta t}$$

Assume a position vector **r** : $\mathbf{r} = rU_{\theta}$

Take derivative with respect to time: $\dot{\mathbf{r}} = (r)(\omega)\dot{U}_{\theta} + \dot{r}U_{\theta}$

But $\dot{r} = 0$ and so:

$$\dot{\mathbf{r}} = (r)(\omega)\dot{U}_{\theta}$$

Velocity analysis

T h e o r y o f m a c h i n e r y

VECTOR ALGEBRA

•According to the previous derivation, if the vector **dU_θ** represent a link then its derivative is found as :

$$\frac{d\left(dU_\theta\right)}{dt} = (d)(\omega)\dot{U}_\theta$$

Note that:- assume $U_{\Theta 1} = \cos(\Theta_1)i+\sin(\Theta_1)j$ and $U_{\Theta 2} = \cos(\Theta_2)i+\sin(\Theta_2)j$:-

$$\dot{U}_{\theta 1} \bullet U_{\theta 1} = 0$$

$$U_{\theta 1} \bullet \dot{U}_{\theta 2} = \sin\left(\theta_1 - \theta_2\right)$$

Velocity analysis

4-BAR MECHANISM

LOOP CLOSURE EQUATION

$$d_2 U_{\theta 2} + d_3 U_{\theta 3} = d_1 U_{\theta 1} + d_4 U_{\theta 4}$$

Derivative

$$d_2 \omega_2 \dot{U}_{\theta 2} + d_3 \omega_3 \dot{U}_{\theta 3} = d_4 \omega_4 \dot{U}_{\theta 4}$$

Dot product both sides by **$U_{\theta 3}$ to eliminate ω_3**

$$d_2 \omega_2 \sin(\theta_3 - \theta_2) + 0 = d_4 \omega_4 \sin(\theta_3 - \theta_4)$$

Solve for ω_4:- $$\omega_4 = \frac{d_2 \omega_2 \sin(\theta_3 - \theta_2)}{d_4 \sin(\theta_3 - \theta_4)}$$

Velocity analysis

Theory of machinery

4-BAR MECHANISM

FIND ω_3

to find ω_3, dot product both sides of derivative equation by $\mathbf{U_{\theta 4}}$ **to eliminate** $\boldsymbol{\omega_4}$:

$$d_2\,\omega_2\,\sin(\theta_4-\theta_2)+d_3\,\omega_3\,\sin(\theta_4-\theta_3)=0$$

solve this equation for ω_3:-

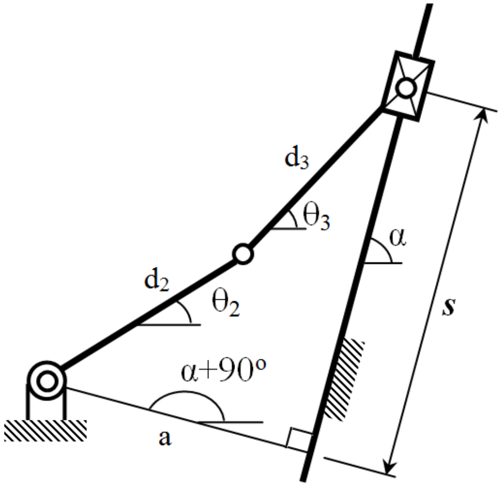
$$\omega_3=-\frac{d_2\omega_2\sin(\theta_4-\theta_2)}{d_3\sin(\theta_4-\theta_3)}$$

Velocity analysis

SLIDER CRANK MECHANISM

LOOP CLOSURE EQUATION

$$d_2 U_{\theta 2} + d_3 U_{\theta 3} + a U_{\alpha+90} = s U_\alpha$$



Derivative

$$d_2 \omega_2 \dot{U}_{\theta 2} + d_3 \omega_3 \dot{U}_{\theta 3} = \dot{S} U_\alpha$$

Dot product both sides by **$U_{\theta 3}$ to eliminate ω_3**

$$d_2 \omega_2 \sin(\theta_3 - \theta_2) + 0 = \dot{S} \cos(\theta_3 - \alpha)$$

Solve for $\dot{S}$:- $\dot{S} = \dfrac{d_2 \omega_2 \sin(\theta_3 - \theta_2)}{\cos(\theta_3 - \alpha)}$

Velocity analysis

Theory of machinery

SLIDER CRANK MECHANISM

FIND ω_3

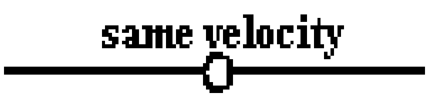
to find ω_3, dot product both sides of derivative equation by $\mathbf{U}^{\cdot}_{\alpha}$ **to eliminate S$^{\cdot}$**:

$$d_2\, \omega_2 \cos(\theta_2-\alpha) + d_3\, \omega_3 \cos(\theta_3-\alpha) = 0$$

solve this equation for ω_3:- $\omega_3 = -\dfrac{d_2 \omega_2 \cos(\theta_2 - \alpha)}{d_3 \cos(\theta_3 - \alpha)}$

Velocity analysis

- Any two bodies in plane motion has a common point at which both bodies has the same velocity . This point is called instant center of velocity
- a normal axis through this point represent an axis of rotation common to the two bodies



- In a mechanism with n links C(№ of instant centers) is found as

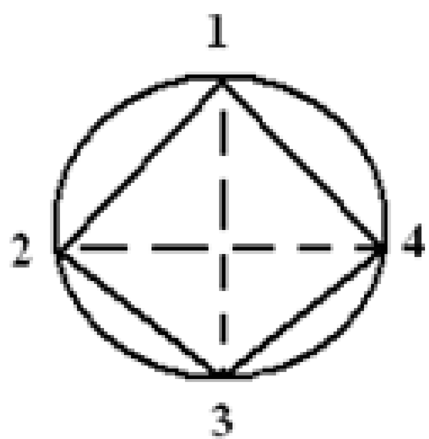
$$C = \frac{n(n-1)}{2}$$

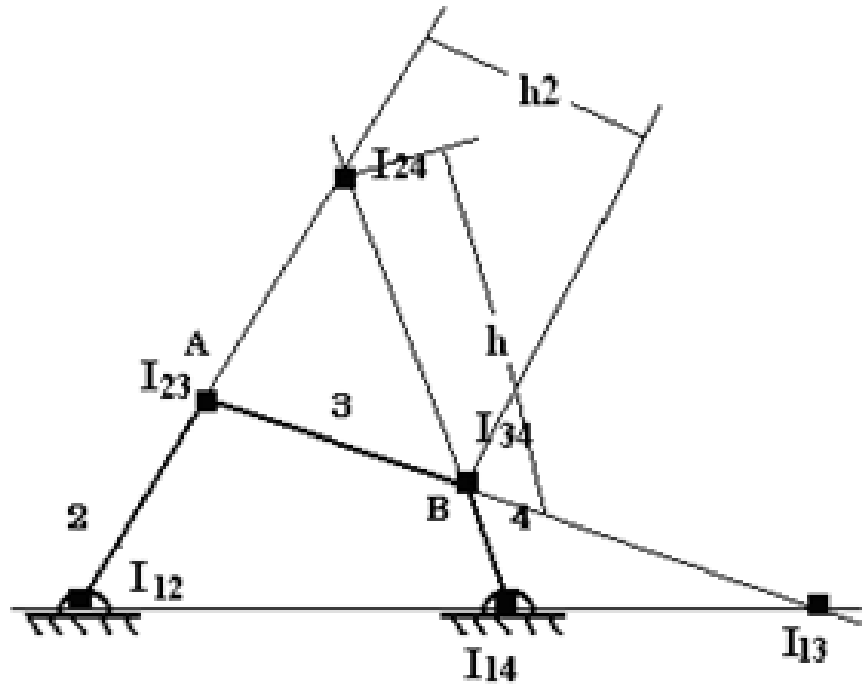
Kennedy's rule : any three bodies have three instant centers of velocity that lie on the same straight line

Velocity analysis

I .C of four bar

$$C = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

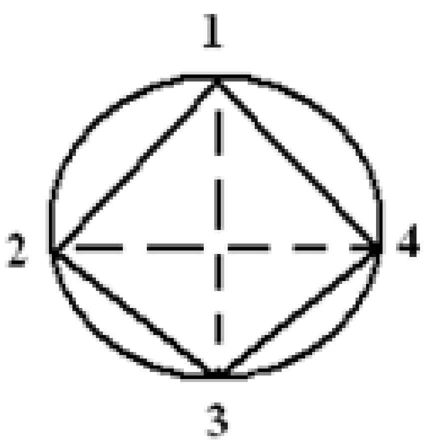


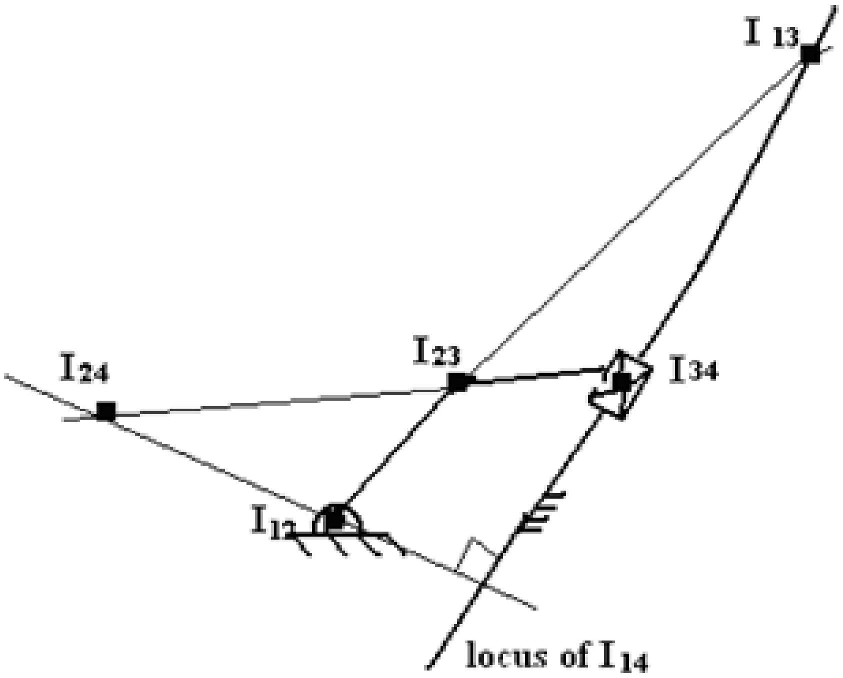


Velocity analysis

I .C of slider crank

$$C = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$





Velocity analysis

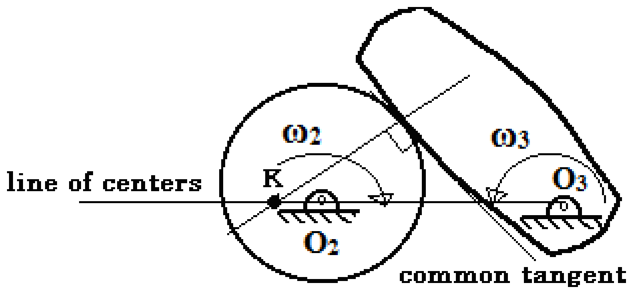
SPEED RATIO

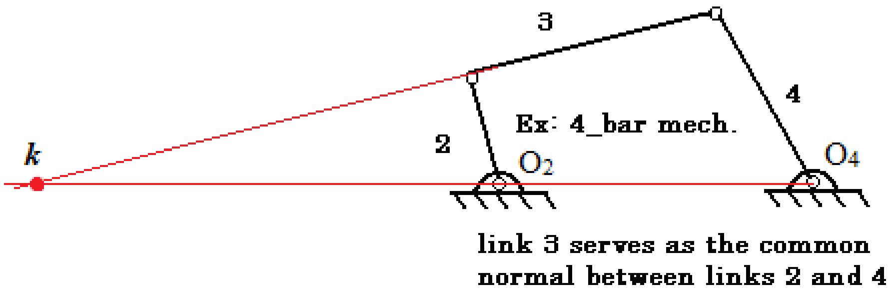
speed ratio is the ratio between motions of rotating links with other rotating or translational links

There are two common cases of finding the speed ratio:

Between two rotating links have a direct contact

Between to links have a common link acts as normal to the two links





Velocity analysis

SPEED RATIO

•Draw line of centers from the rotating axes ($O_2 - O_4$)

•Draw extension line for link number 3 until it intersect the line of center at point **k**.

•The speed ratio: will be found using the following equation

$$\frac{\omega_4}{\omega_2} = \frac{O_2 k}{O_4 k}$$

Where:-

•$\boldsymbol{O_2 k}$ is the straight distance measured between the points $\boldsymbol{O_2}$ and ***k***.

•$\boldsymbol{O_4 k}$ is the straight distance measured between the points $\boldsymbol{O_4}$ and ***k***.

Velocity analysis

SPEED RATIO

First case

- Draw line of centers from the rotating axes ($O_2 - O_3$)
- Draw a tangent from the contact point
- Draw a line start from the tangency point perpendicular to the tangent line and intersect the line of centers at a certain point. Call this point **k**.
- The speed ratio: will be found using the following equation

$$\frac{\omega_3}{\omega_2} = \frac{O_2 k}{O_3 k}$$

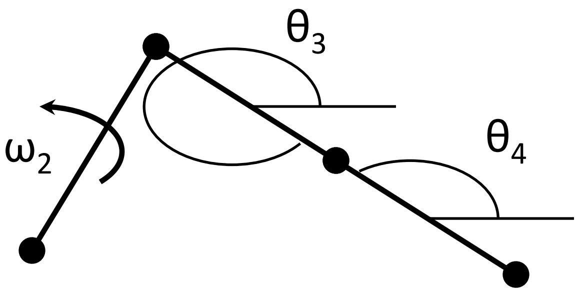
Where:

- ***O_2k*** is the straight distance measured between the points ***O_2*** and ***k***.
- ***O_3k*** is the straight distance measured between the points ***O_3*** and ***k***.

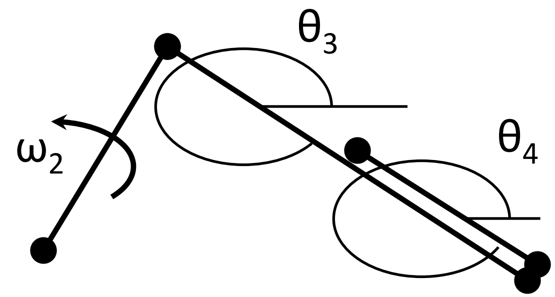
Velocity analysis

4-BAR MECHANISM

SPECIAL CASES



Stable locking position
$\theta_4 = \theta_3 - \pi$



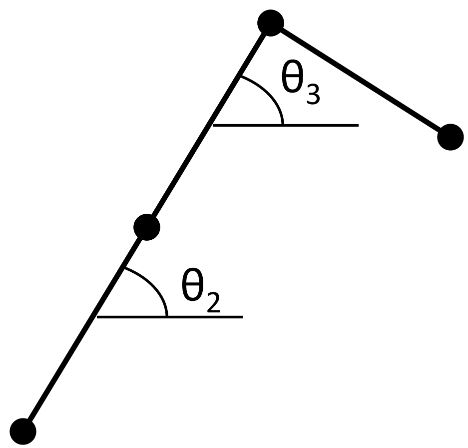
Unstable locking position
$\theta_4 = \theta_3$

- **In locking positions speed is too much before θ_3-θ_4 but it becomes zero after that and motion stops**

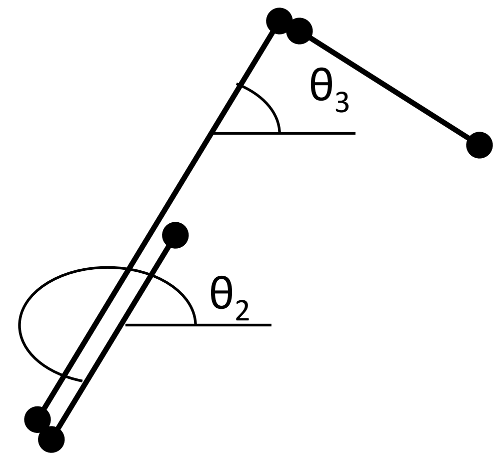
Velocity analysis

4-BAR MECHANISM

SPECIAL CASES



Limit position
$\theta_2 = \theta_3$



Limit position
$\theta_2 = \theta_3 + \pi$

- **In limit positions speed becomes zero when θ3=θ2**