Theory of machinery

Chapter four

Acceleration analysis

By

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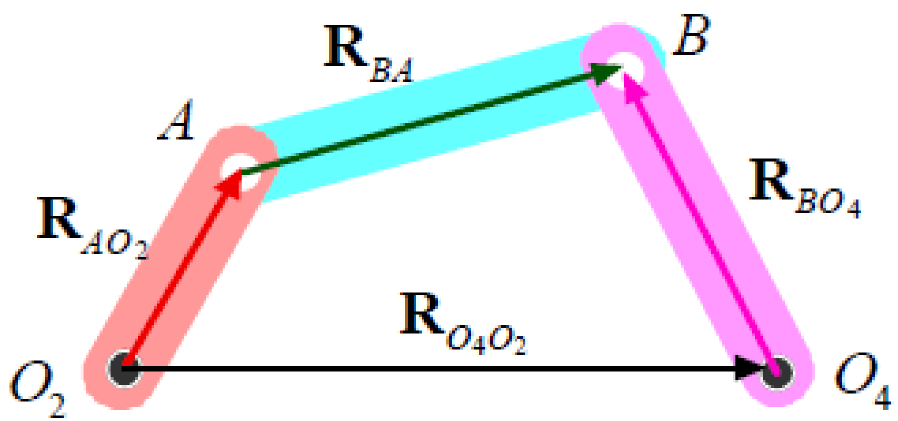
Acceleration analysis

For a known four-bar mechanism, in a given configuration and known velocities, and a given angular acceleration of the crank, α_2 (say CCW), construct the acceleration polygon. Determine α_3 and α_4.



Acceleration analysis

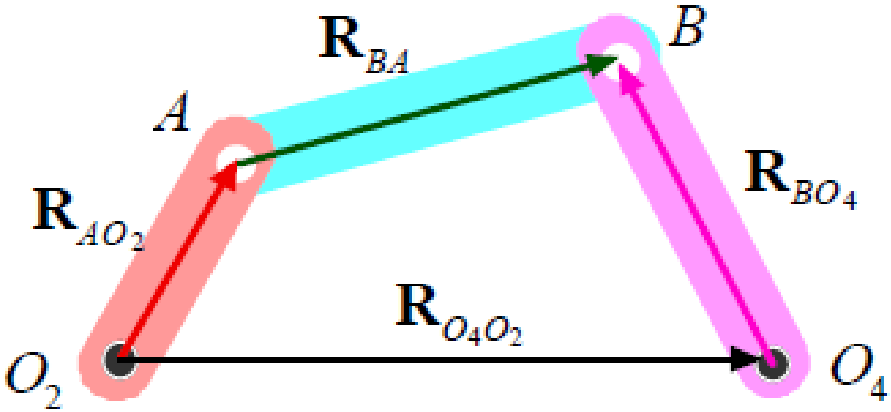
Solution

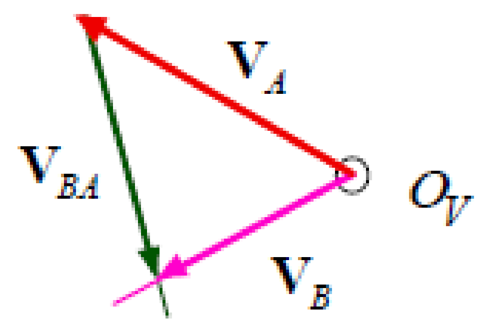
For the position vector loop equation:

$$R_{AO2}+R_{BA} - R_{BO4} - R_{O4O2} = 0 \text{ --- (1)}$$

the velocity equation is

$$V_{AO2}+V_{BA} - V_{BO4} = 0 \text{ --- (2)}$$





Acceleration analysis

Theory of machinery

Solution

The acceleration equation is obtained from the time derivative of the velocity equation as:

$$\mathbf{A}_A + \boldsymbol{A}_{BA} = \boldsymbol{A}_B$$

Since R_{AO2} , R_{BA} , and R_{BO2} are moving vectors with constant lengths, their acceleration vectors have normal and tangential components:

$$\mathbf{A}_A^n + \mathbf{A}_A^t + \mathbf{A}_{BA}^n + \mathbf{A}_{BA}^t - \mathbf{A}_B^n - \mathbf{A}_B^t = \mathbf{0}$$

$$-\omega_2^2 \mathbf{R}_{AO_2} + \alpha_2 \breve{\mathbf{R}}_{AO_2} - \omega_3^2 \mathbf{R}_{BA} + \alpha_3 \breve{\mathbf{R}}_{BA} - (-\omega_4^2 \mathbf{R}_{BO_4}) - \alpha_4 \breve{\mathbf{R}}_{BO_4} = \mathbf{0}$$

Now, **ω2, ω_3, ω_4 and α_2 are known, so the components** A_A^n, A_A^t, A_{BA}^n *and* A_B^n **are known and can be drawn directly**

Acceleration analysis

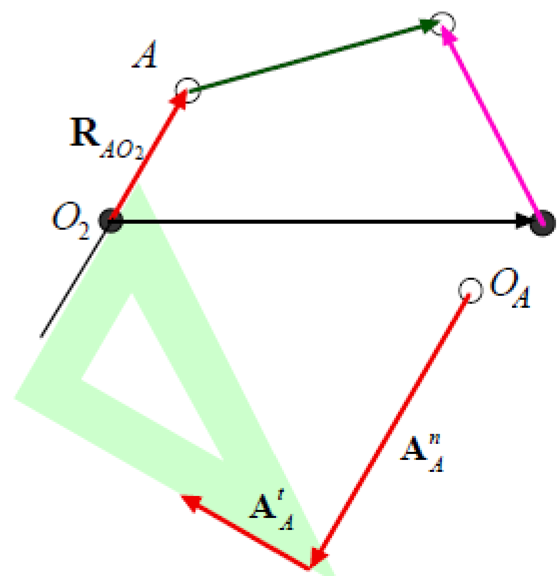
Solution

Rearrange the loop equation to be seem as follow

$$-\omega_2^2\mathbf{R}_{AO_2}+\alpha_2\breve{\mathbf{R}}_{AO_2}-\omega_3^2\mathbf{R}_{BA}-(-\omega_4^2\mathbf{R}_{BO_4})+\alpha_3\breve{\mathbf{R}}_{BA}-\alpha_4\breve{\mathbf{R}}_{BO_4}=0$$

Drawing procedures:

1. Select a point in a convenient position as the reference for zero acceleration. Name this point O_A (origin of accelerations).
2. Compute the magnitude of $\mathbf{A}_A^n$ as $R_{AO_2}\omega_2^2$. From O_V construct vector $\mathbf{A}_A^n$ in the opposite direction of $\mathbf{R}_{AO_2}$.
3. Compute the magnitude of $\mathbf{A}_A^t$ as $R_{AO_2}\alpha_2$. The direction of $\mathbf{A}_A^t$ is determined by rotating $\mathbf{R}_{AO_2}$ 90° in the direction of α_2. Add this vector to $\mathbf{A}_A^n$. Note that the sum of $\mathbf{A}_A^n$ and $\mathbf{A}_A^t$ is $\mathbf{A}_A$.

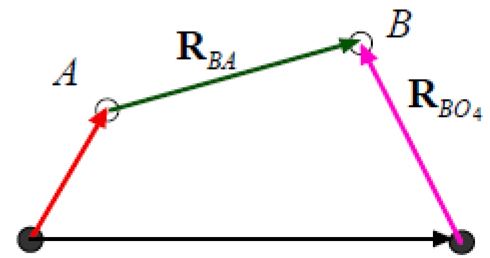


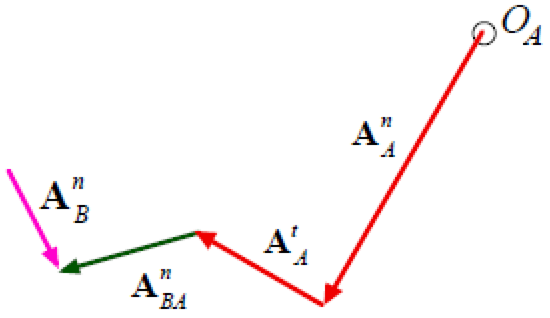
Acceleration analysis

Solution

3. Compute the magnitude of $\mathbf{A}^n_{BA}$ as $R_{BA}\omega_3^2$. Add this vector in the opposite direction of $\mathbf{R}_{BA}$ to the other two vectors.

4. Compute the magnitude of $\mathbf{A}^n_B$ as $R_B\omega_4^2$. Note that $\mathbf{A}^n_B$ is in the opposite direction of $\mathbf{R}_{BO_4}$. Since $\mathbf{A}^n_B$ itself appears with a negative sign in the acceleration equation, it should be added to the other vectors in the diagram as shown; i.e., head-to-tail.



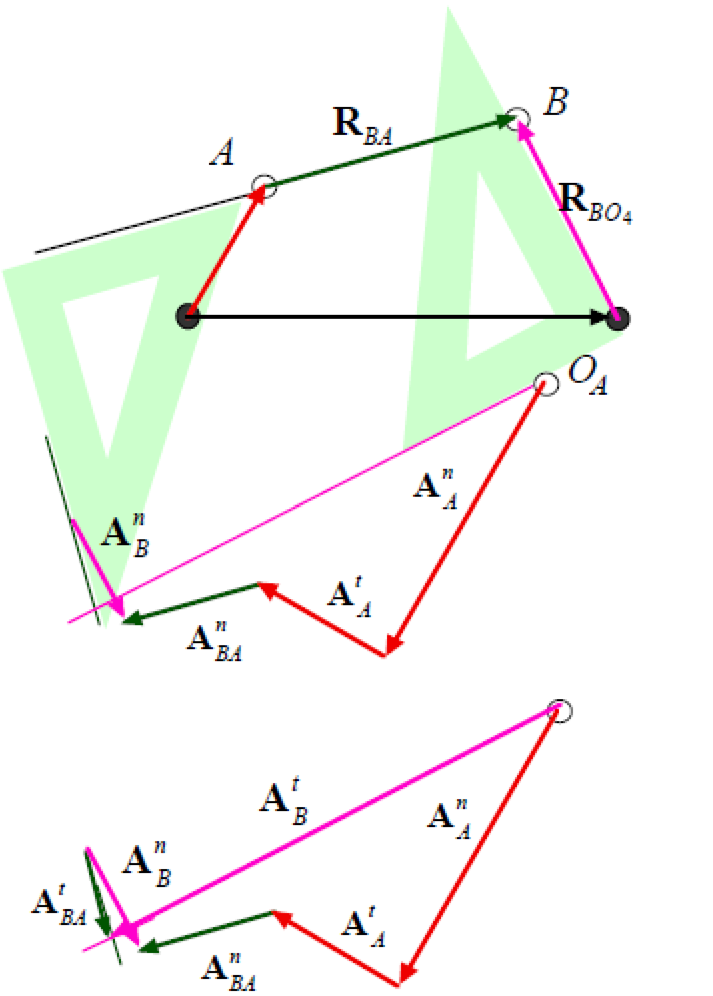


Acceleration analysis

Theory of machinery

Solution

5. Since $\mathbf{A}_{BA}^{t}$ must be perpendicular to $\mathbf{R}_{BA}$, draw a line perpendicular to $\mathbf{R}_{BA}$ in anticipation of adding $\mathbf{A}_{BA}^{t}$ to the diagram.
6. Since $\mathbf{A}_{B}^{t}$ must be perpendicular to $\mathbf{R}_{BO_4}$, draw a line perpendicular to $\mathbf{R}_{BO_4}$ closing (completing) the polygon.
7. Construct vectors $\mathbf{A}_{BA}^{t}$ and $\mathbf{A}_{B}^{t}$ on the polygon.
8. Determine the magnitude of $\mathbf{A}_{BA}^{t}$ from the polygon. Compute α_3 as $\alpha_3 = A_{BA}^{t} / R_{BA}$ (in this diagram it is CW).
9. Determine the magnitude of $\mathbf{A}_{B}^{t}$ from the polygon. Compute α_4 as $\alpha_4 = A_{B}^{t} / R_{BO_4}$ (in this diagram it is CCW).



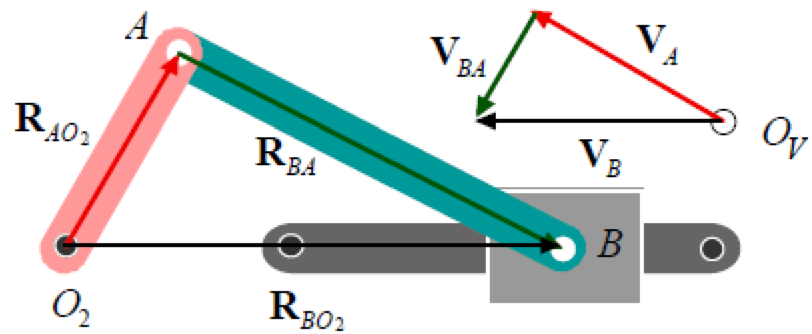
Acceleration analysis

Solution

$$\mathbf{R}_{AO_2} + \mathbf{R}_{BA} - \mathbf{R}_{BO_2} = 0$$

$$\mathbf{V}_{A} + \mathbf{V}_{BA} - \mathbf{V}_{B} = 0$$

$$\mathbf{A}_{A} + \mathbf{A}_{BA} - \mathbf{A}_{B} = 0$$



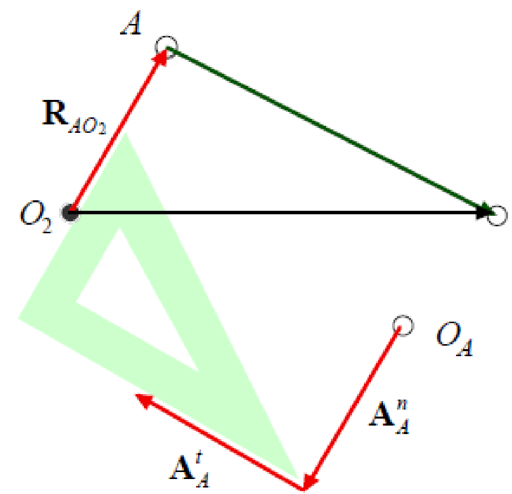
$$\mathbf{A}_{A}^{n} + \mathbf{A}_{A}^{t} + \mathbf{A}_{BA}^{n} + \mathbf{A}_{BA}^{t} - \mathbf{A}_{B}^{s} = 0$$

$$-\omega_2^2 \mathbf{R}_{AO_2} + \alpha_2 \breve{\mathbf{R}}_{AO_2} - \omega_3^2 \mathbf{R}_{BA} + \alpha_3 \breve{\mathbf{R}}_{BA} - \mathbf{A}_{B}^{s} = 0$$

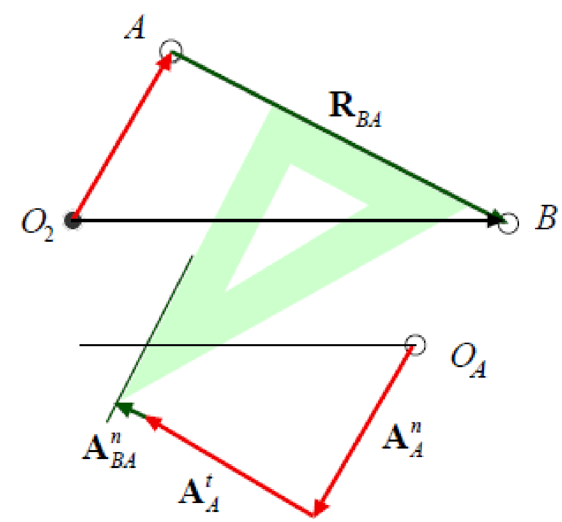
Acceleration analysis

Solution

1. Select a point in a convenient position as the reference for zero acceleration, O_A .
2. Compute $A_A^n = R_{AO_2}\omega_2^2$. From O_V construct $\mathbf{A}_A^n$ in the opposite direction of $\mathbf{R}_{AO_2}$.
3. Compute $A_A^t = R_{AO_2}\alpha_2$. The direction of $\mathbf{A}_A^t$ is determined by rotating $\mathbf{R}_{AO_2}$ 90° in the direction of α_2 . Add this vector to the diagram.



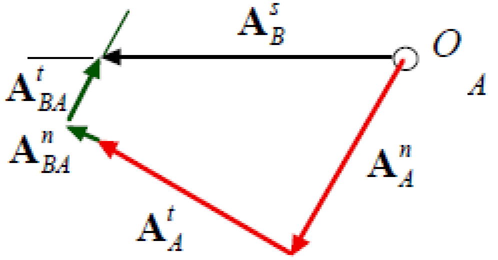
4. Compute $A_{BA}^n = R_{BA}\omega_3^2$. Construct $\mathbf{A}_{BA}^n$ in the opposite direction of $\mathbf{R}_{BA}$.
5. $\mathbf{A}_{BA}^t$ must be perpendicular to $\mathbf{R}_{BA}$. Draw a line perpendicular to $\mathbf{R}_{BA}$ in anticipation of adding $\mathbf{A}_{BA}^t$ to $\mathbf{A}_{BA}^n$.
6. From O_A draw a line parallel to the sliding axis. $\mathbf{A}_B$ must reside on this line.



Acceleration analysis

Solution

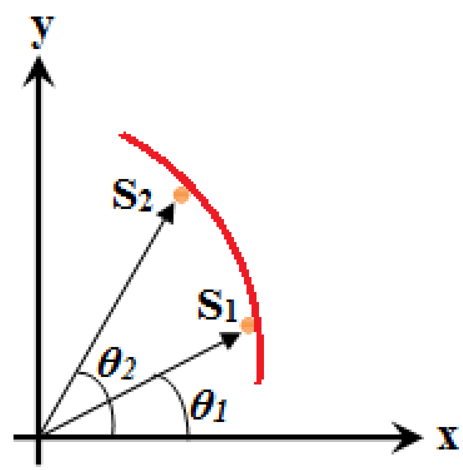
7. Construct vectors $\mathbf{A}_{BA}^{t}$ and $\mathbf{A}_{B}$.
8. Determine the magnitude of $\mathbf{A}_{BA}^{t}$. Compute α_3 as $\alpha_3 = A_{BA}^{t} / R_{BA}$. Determine the direction of α_3 (in this example it is CCW).
9. Determine the magnitude of $\mathbf{A}_{B}$ from the polygon. The direction in this example is to the left.



Theory of machinery

Acceleration analysis

Using vector algebra



$$r(t) = S(t)U_\theta(t)$$

Derive with respect to time

$$\dot{r}(t) = \dot{S}U_\theta + S\omega\dot{U}_\theta$$

Derive another time with respect to time to find the acceleration

$$\ddot{r}(t) = \ddot{S}U_\theta + \dot{S}\omega\dot{U}_\theta + \dot{S}\omega\dot{U}_\theta + S\left(\alpha\dot{U}_\theta + \omega^2\ddot{U}_\theta\right)$$

But $\ddot{U}_\theta(t) = -U_\theta(t)$

Rearrange the terms

$$\ddot{r}(t) = \left(\ddot{S} - S\omega^2\right)U_\theta + \left(2\dot{S}\omega + S\alpha\right)\dot{U}_\theta$$

Acceleration analysis

4-bar mechanism

- For the 4-bar mechanism, the length of links are constant and so:

$$\ddot{S} \text{ and } \dot{S} = 0$$

And the equation for acceleration become

$$\ddot{r}(t) = -\left(S\omega^2\right)U_\theta + \left(S\alpha\right)\dot{U}_\theta = \left(S\alpha\right)\dot{U}_\theta - \left(S\omega^2\right)U_\theta$$

- As shown in pervious chapters we can find the position and velocity analysis to 4-bar mech. And in this section we will find the acceleration analysis by adding new input which is α_2 and new unknown and they are α_3 and α_4

Acceleration analysis

4-bar mechanism

- To apply acceleration analysis on 4-bar mechanism, we derive the loop closure equation

$$\left[d_2\alpha_2\dot{U}_{\theta_2} - d_2\omega_2^2 U_{\theta_2}\right] + \left[d_3\alpha_3\dot{U}_{\theta_3} - d_3\omega_3^2 U_{\theta_3}\right] = \left[d_4\alpha_4\dot{U}_{\theta_4} - d_4\omega_4^2 U_{\theta_4}\right]$$

- Dot product both sides by **$U_{\theta 3}$ to eliminate α_3**

$$d_2\alpha_2 \sin(\theta_3 - \theta_2) - d_2\omega_2^2 \cos(\theta_3 - \theta_2) - d_3\omega_3^2$$
$$= d_4\alpha_4 \sin(\theta_3 - \theta_4) - d_4\omega_4^2 \cos(\theta_3 - \theta_4)$$

- Solve for α_4:

$$\alpha_4 = \frac{d_2\alpha_2 \sin(\theta_3 - \theta_2) - d_2\omega_2^2 \cos(\theta_3 - \theta_2) - d_3\omega_3^2 + d_4\omega_4^2 \cos(\theta_3 - \theta_4)}{d_4 \sin(\theta_3 - \theta_4)}$$

Acceleration analysis

4-bar mechanism

- Dot product both sides by $\mathbf{U_{\theta 4}}$ **to eliminate** $\boldsymbol{\alpha_4}$

$$d_2\alpha_2\sin(\theta_4-\theta_2)-d_2\omega_2^2\cos(\theta_4-\theta_2)+d_3\alpha_3\sin(\theta_4-\theta_3)-d_3\omega_3^2\cos(\theta_4-\theta_3)$$
$$=-d_4\omega_4^2$$

- Solve for α_3:

$$\alpha_3=\frac{-d_2\alpha_2\sin(\theta_4-\theta_2)+d_2\omega_2^2\cos(\theta_4-\theta_2)+d_3\omega_3^2\cos(\theta_4-\theta_3)-d_4\omega_4^2}{d_3\sin(\theta_4-\theta_3)}$$

Acceleration analysis

Slider crank mechanism

- For slider crank mechanism the input is α2 and the outputs will be:

$$\ddot{S} \text{ and } \alpha_3$$

The L.C.E is

$$d_2 U_{\theta 2} + d_3 U_{\theta 3} + a U_{\alpha+90} = S U_\alpha$$

- Derive twice with respect to time

$$\left[d_2 \alpha_2 \dot{U}_{\theta_2} - d_2 \omega_2^2 U_{\theta_2}\right] + \left[d_3 \alpha_3 \dot{U}_{\theta_3} - d_3 \omega_3^2 U_{\theta_3}\right] = \ddot{S} U_\alpha$$

- Dot product both sides by $\mathbf{U_{\theta 3}}$ **to eliminate** $\boldsymbol{\alpha_3}$

$$d_2 \alpha_2 \sin(\theta_3 - \theta_2) - d_2 \omega_2^2 \cos(\theta_3 - \theta_2) - d_3 \omega_3^2 = \ddot{S} \cos(\theta_3 - \alpha)$$

Acceleration analysis

Slider crank mechanism

- Solve for $\ddot{S}$

$$\ddot{S} = \frac{d_2\alpha_2 \sin(\theta_3-\theta_2) - d_2\omega_2^2 \cos(\theta_3-\theta_2) - d_3\omega_3^2}{\cos(\theta_3-\alpha)}$$

- Dot product both sides by **$U^{\bullet}_{\alpha}$ to eliminate** $\ddot{S}$

$$d_2\alpha_2 \cos(\theta_2-\alpha) - d_2\omega_2^2 \sin(\theta_2-\alpha) + d_3\alpha_3 \cos(\theta_3-\alpha) - d_3\omega_3^2 \sin(\theta_3-\alpha) = 0$$

$$\Rightarrow \alpha_3 = \frac{d_2\omega_2^2 \sin(\theta_2-\alpha) + d_3\omega_3^2 \sin(\theta_3-\alpha) - d_2\alpha_2 \cos(\theta_2-\alpha)}{d_3 \cos(\theta_3-\alpha)}$$