Theory of machinery

Chapter seven

Gears

By

Laith Batarseh

Gears

Gears are very important in power transmission between a drive rotor and driven rotor

What are the functions of gears?

1- Transmit motion and torque (power) between shafts

2- Maintain constant speed ratios between power transmission shafts

What is gear ? In general , a gear is a circular disk with teeth along the circumference

Gears

Gear types

Gears are divided into four main types depending on the relation between the tooth axis and the gear axis this relation provide different form of transmission and these types are

Rack and pinion gear

- ❑Rack is gear that has infinite radius.
- ❑This type is used to transform rotational torque into axial force

Gears

Gear types

Spur gear

- ❑Axis of the gear transmits motion between two parallel shafts.
- ❑The teeth have straight line shape

Helical gear

- ❑The tooth axis is apart of helix about the gear axis.
- ❑This type can transmit the power between two parallel or none parallel shafts

Gears

Gear types

Bevel gear

- The tooth axis is apart of cone about the gear axis.
- This type can transmit the power between two intersecting shafts

Worm gear

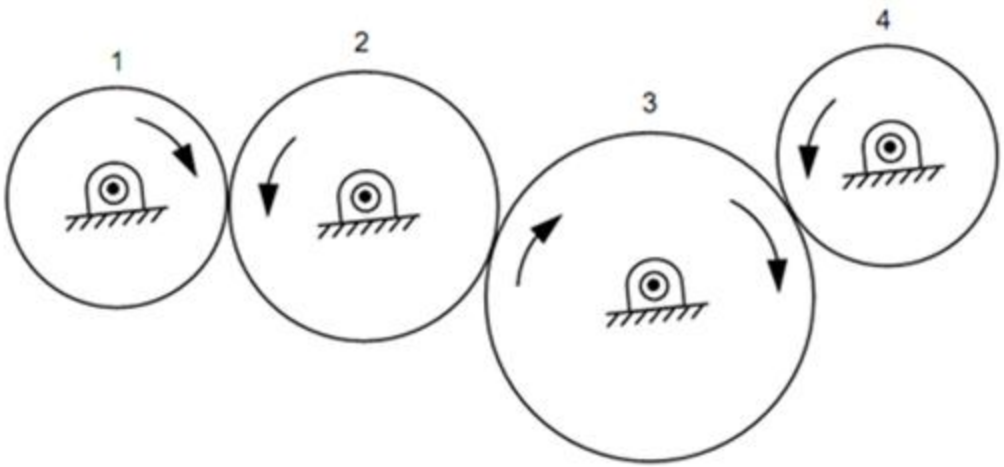
- Is a special case of helical gear and used to transmit power from high speed shaft to low speed shaft with different ratios

Gears

Gear concepts

gear train

- Gear train is a **sequence** of consecutive meshed gears such the one shown below

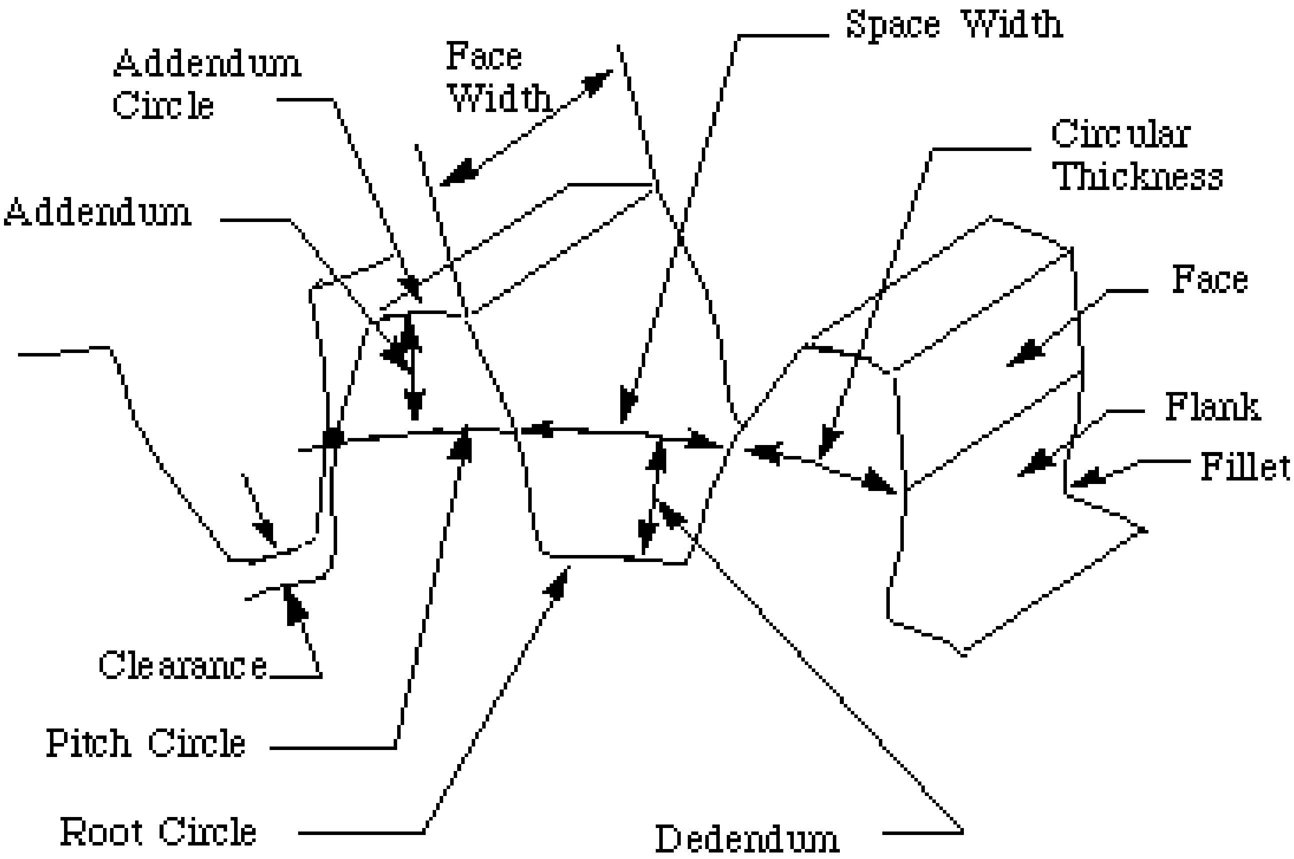


- When gears are meshed in gear train, one of the gears is **drive** (input) and the others are **driven.** However, one of the driven gears is called **output**
- In gear train, the gear which have the largest number of teeth is called **gear** and the gear which have the lest number of teeth is called **pinion**

Gears

Theory of machinery

Gear parameters



Gears

Torque, gear ratio & Efficiency

the power of rotating disc can be given as $P = \omega . T$

Where:

P: power; Watt (W)

T: torque; N.m .

ω: angular speed; rad/s

In an ideal gear train, the input and output powers are the same so;

$$P = \omega_{in} T_{in} = \omega_{out} T_{out} \Rightarrow \frac{T_{out}}{T_{in}} = \frac{\omega_{in}}{\omega_{out}} = GR$$

Where:

GR: gear ratio

T_{in}: input torque (i.e. driver gear torque); N.m.

T_{out}: output torque (i.e. driven gear torque); N.m.

ω_{in}: driver gear angular velocity; rad/s or RPM

ω_{out}: driven gear angular velocity; rad/s or RPM

Gears

Torque, gear ratio & Efficiency

Gear ratio is defined as the ratio between the input speed (driver) and the output gear (driven). As its shown from GR, the relation between the speed and torque is revere (i.e. the pinion have a higher speed but lesser torque and the gear have a lesser speed but higher torque)

There are three cases for the gear ratio:

1. GR>1 when the pinion is the driver
2. GR=1 when both gears have the same size
3. GR<1 when the gear is the driver

Efficiency

the main function of gear train is to transmit power between two or more shafts. But, because of the friction between gears teeth some of the input power is dissipated in form of heat.
Efficiency of system means how much we get from the input power. In other words, more efficient gear train means less power loss due to friction.

Gears

Torque, gear ratio & Efficiency

Mathematically, the efficiency of gear train can be given as

$$\eta = \frac{Power\ \ out}{Power\ \ In} = \frac{2\pi \times \omega_{out} T_{out} \times 60}{2\pi \times \omega_{in} T_{in} \times 60} = \frac{\omega_{out} T_{out}}{\omega_{in} T_{in}}$$

Where:

ω_{in} is the angular speed of the input gear; RPM or Rad/s

ω_{out} is the angular speed of the output gear; RPM or Rad/s

T_{in} is the torque of the input gear; RPM or Rad/s

T_{out} is the torque of the output gear; RPM or Rad/s

Gears

Gear concepts

Example [1]

A gear box has an input speed of 1500 rev/min clockwise and an output speed of 300 rev/min anticlockwise. The input power is 20 kW and the efficiency is 70%. Determine the following.

i. The gear ratio; ii. The input torque.; iii. The output power.; iv. The output torque; v. The holding torque.

Solution :

$$G.R \ or \ VR = \frac{Input \ \ speed}{Output \ \ speed} = \frac{\omega_1}{\omega_2} = \frac{1500}{300} = 5$$

$$Input \ \ Power = \frac{2\pi \times \omega_1 T_1}{60} \quad \Rightarrow \quad T_1 = \frac{60 \times Input \ \ Power}{2\pi \times \omega_1}$$

Gears

Gear concepts

Example [1]

$$\therefore\ Input\ \ torque = T_1 = \frac{60\times 20000}{2\pi\times 1500} = 127.3\,N\,m$$

$$(Negative - clockwise)$$

$$\eta = 0.7 = \frac{Output\ \ power}{Inpu\ \ power}$$

$$Power\ \ Output = 0.7\times 20 = 14kW$$

$$\therefore\ Output\ \ torque = T_2 = \frac{60\times 14000}{2\pi\times 300} = 445.6\,N\,m$$

$$(Positive\ - unticlockwise)$$

Gears

Gear concepts

Example [1]

$$\Rightarrow \qquad T_1 + T_2 + T_3 = 0$$

$$-127.3 + 445.6 + T_3 = 0$$

$$T_3 = 127.3 - 445.6 = -318.3\, N\, m$$

$$-Clockwise$$

Gears

Gear concepts

Velocity ratio, m_v

Velocity ratio is defined as the ratio between the velocity of the output gear and the velocity of the input gear. However, there is a proportional relation between the number of gear teeth and its diameter. Also, there is a reverse relation between the size of gear and its speed (i.e. the pinion rotates faster than the gear). This relation is given in as:

$$m_v = \pm \frac{\omega_{out}}{\omega_{in}} \pm \frac{D_{in}}{D_{out}} = \pm \frac{N_{in}}{N_{out}}$$

Where: D is the gear diameter and N is the number of teeth. For more than two gear train, velocity ratio can be given as:

$$m_v = \pm \left(\frac{N_1}{N_2}\right)\left(\frac{N_2}{N_3}\right)\left(\frac{N_3}{N_4}\right)\cdots\left(\frac{N_{n-1}}{N_n}\right)$$

Gears

Gear concepts

Torque ratio, m_T

as in the speed ratio, we can define a torque ratio which will be the opposite of the speed ratio or:

$$m_T = \pm \frac{T_{out}}{T_{in}} = \pm \frac{D_{out}}{d_{in}} = \pm \frac{N_{out}}{N_{in}}$$

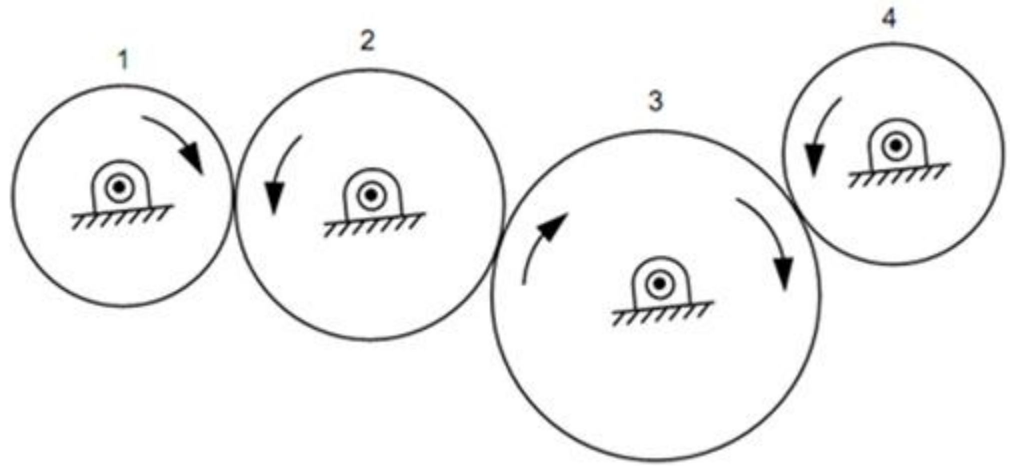
And for more than two gears train:

$$m_T = \pm \left(\frac{N_n}{N_{n-1}} \right) \left(\frac{N_{n-1}}{N_{n-2}} \right) \ldots \left(\frac{N_2}{N_1} \right)$$

Gears

Gear concepts

Simple gear train



$$\frac{\omega_4}{\omega_1} = -\frac{\omega_2}{\omega_1} x - \frac{\omega_3}{\omega_2} x - \frac{\omega_4}{\omega_3} = -\frac{N_1}{N_2} x - \frac{N_2}{N_3} x - \frac{N_3}{N_4} = -\frac{N_1}{N_4}$$

The negative sign means change in the direction of rotation. As its noticed here: for simple gear train, if the number of gears is even, the direction is reversed between the input and the output and if the number of gears is odd the direction of the input is the same direction of the input.

Gears

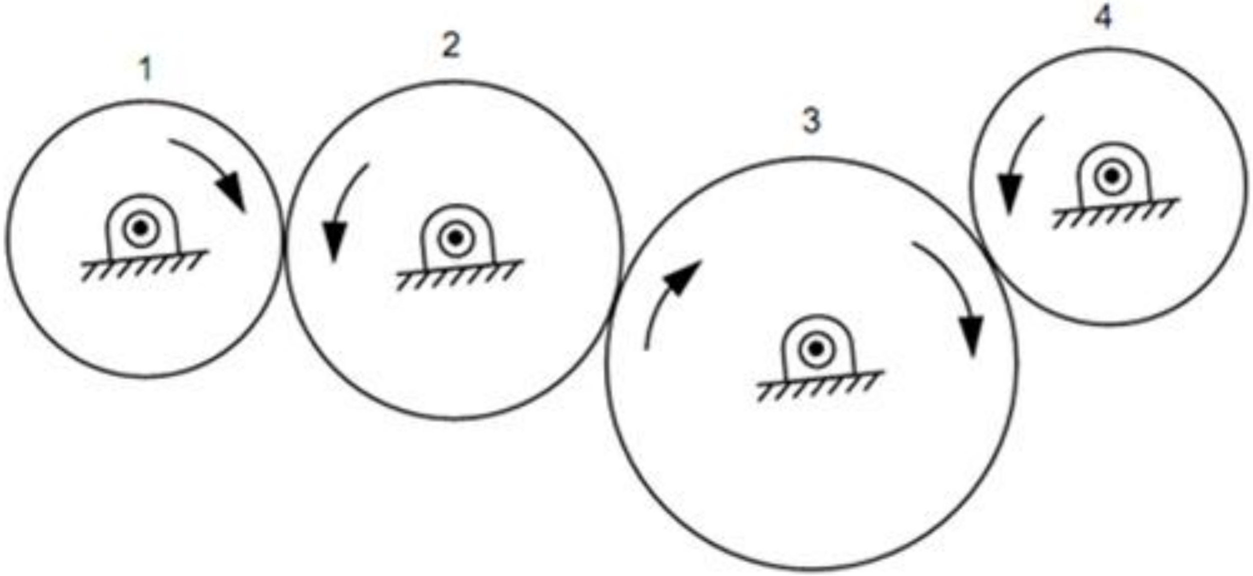
Gear concepts

Example: Simple gear train

Consider the simple gear train shown in the figure. If ω_1 **= 500 RPM C.W,**

$N_1 = 30T$, $N_2=50T$, $N_3=70T$, $N_4=15T$

Find ω_4**?**



Gears

Gear concepts

Solution

$$\frac{\omega_4}{\omega_1} = -\frac{\omega_2}{\omega_1} x - \frac{\omega_3}{\omega_2} x - \frac{\omega_4}{\omega_3} = -\frac{N_1}{N_2} x - \frac{N_2}{N_3} x - \frac{N_3}{N_4} = -\frac{N_1}{N_4}$$

$$\Rightarrow \omega_4 = \omega_1 \left[-\frac{N_1}{N_4} \right] = -500 \frac{30}{15} = -1000 RPM \; = 1000 RPM \left(C.C.W \right)$$

The negative sign means change in the direction of rotation. Therefore, if the input is

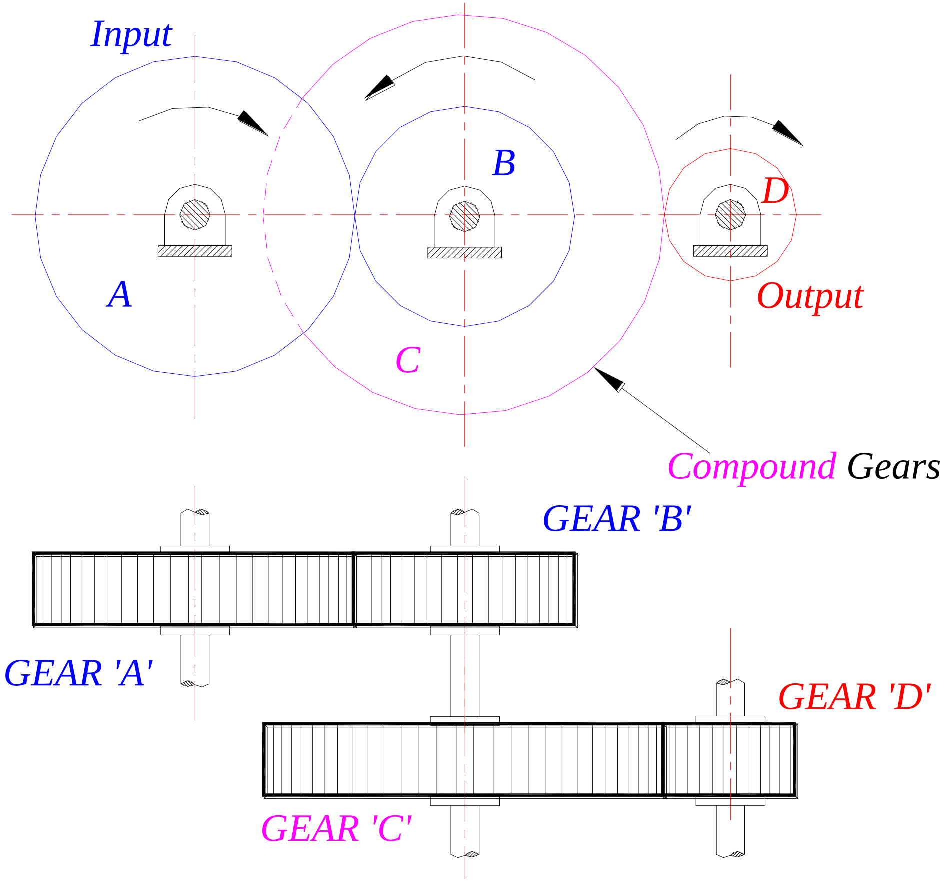
Gears

T h e o r y o f m a c h i n e r y

Gear concepts

Compound Gear train

Compound gears are simply a chain of simple gear trains with the input of the second being the output of the first. A chain of two pairs is shown below. Gear B is the output of the first pair and gear C is the input of the second pair. Gears B and C are locked to the same shaft and revolve at the same speed.

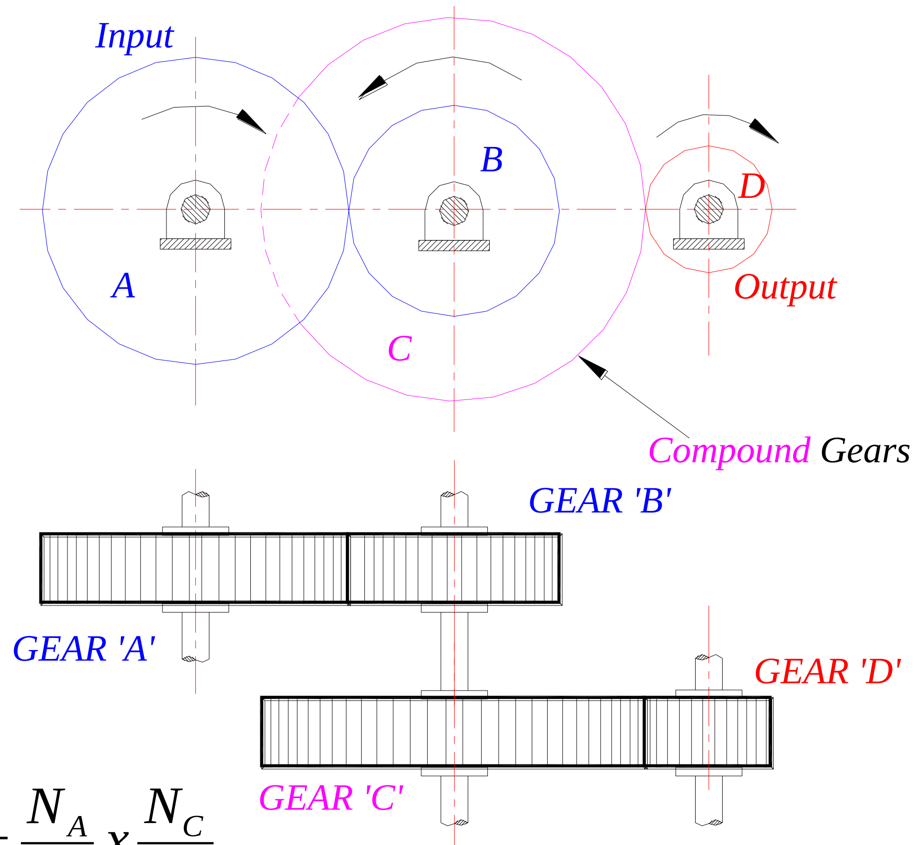


Gears

Gear concepts

Compound

For large velocities ratios, compound gear train arrangement is preferred.



The velocity of each tooth on A and B are the same so:

$\omega_A t_A = \omega_B t_B$

-as they are simple gears.

Likewise for C and D,

$\omega_C t_C = \omega_{D\ tD}$.

$$\frac{\omega_D}{\omega_A} = -\frac{\omega_B}{\omega_A} x \frac{\omega_C}{\omega_B} x - \frac{\omega_D}{\omega_C} = \frac{\omega_B}{\omega_A} x \frac{\omega_D}{\omega_C} = \frac{N_A}{N_B} x \frac{N_C}{N_D}$$

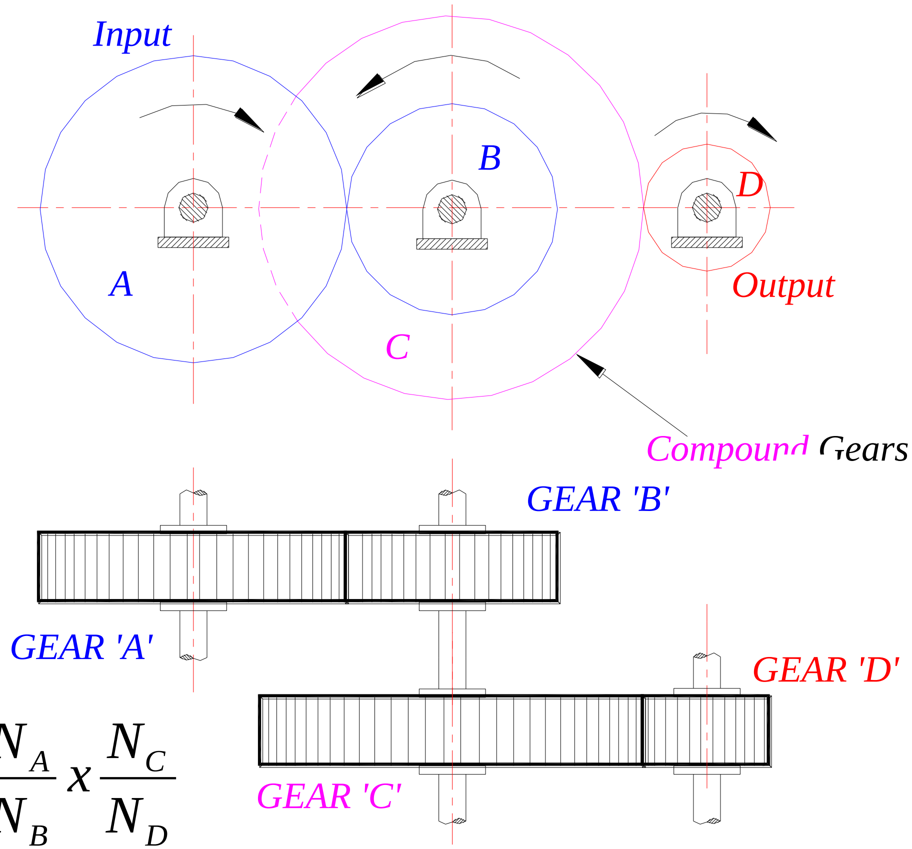
Gears

Gear concepts

Compound Gear train
Example

Take:

ω_A = 500 RPM
N_A= 30
N_B=50
N_C=75
N_D=15
Find ω_D?



$$\frac{\omega_D}{\omega_A} = -\frac{\omega_B}{\omega_A} x \frac{\omega_C}{\omega_B} x - \frac{\omega_D}{\omega_C} = \frac{\omega_B}{\omega_A} x \frac{\omega_D}{\omega_C} = \frac{N_A}{N_B} x \frac{N_C}{N_D}$$

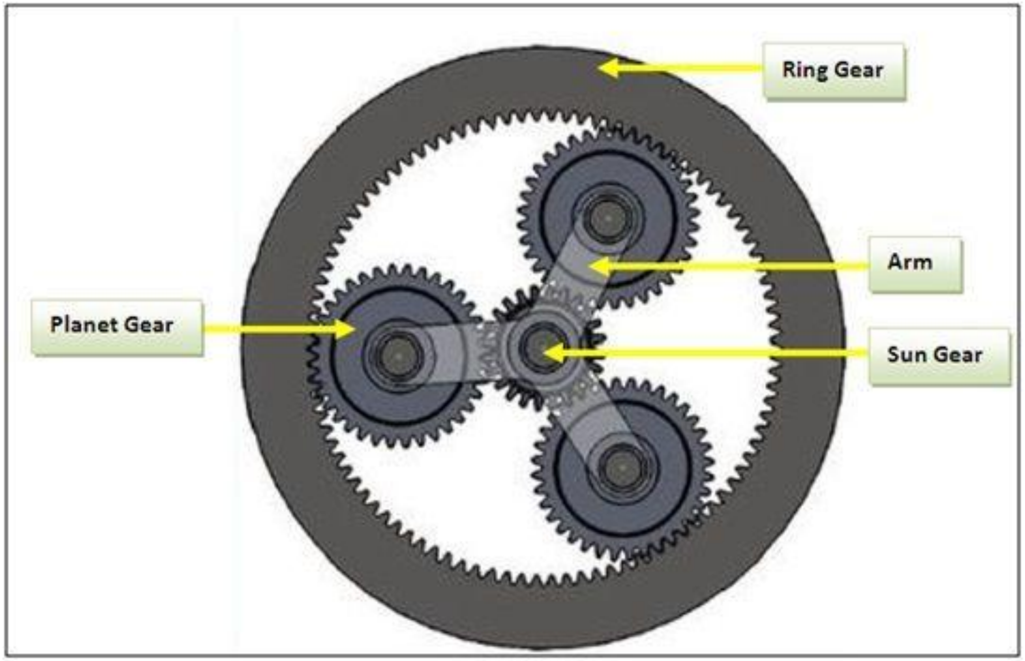
$$\Rightarrow \omega_D = \omega_A \left[\frac{N_A}{N_B} x \frac{N_C}{N_D} \right] = -500 \left[\frac{30}{50} x \frac{75}{15} \right] = 1500 RPM$$

Gears

Epicyclic or planetary gear train

Some gears experience planetary motion , it revolves about its own axis and its axis revolves about fixed axis (sun gear) .The planet gear is held in its orbit by an arm called the planet arm . the mobility of this set of gears is

$$M = 3(4-1) -2(3) -1 = 2 \text{ (two inputs)}$$



Gears

Epicyclic or planetary gear train

Speed ratio

To find the speed we must take the speed of arm and this can be done by observed the whole motion from the arm point view and for this process defined e which called **the train value** as observed by the arm

$$e = \frac{\omega_{out} - \omega_{arm}}{\omega_{in} - \omega_{arm}}$$

Gears

Epicyclic or planetary gear train

Example (problem 9.26):

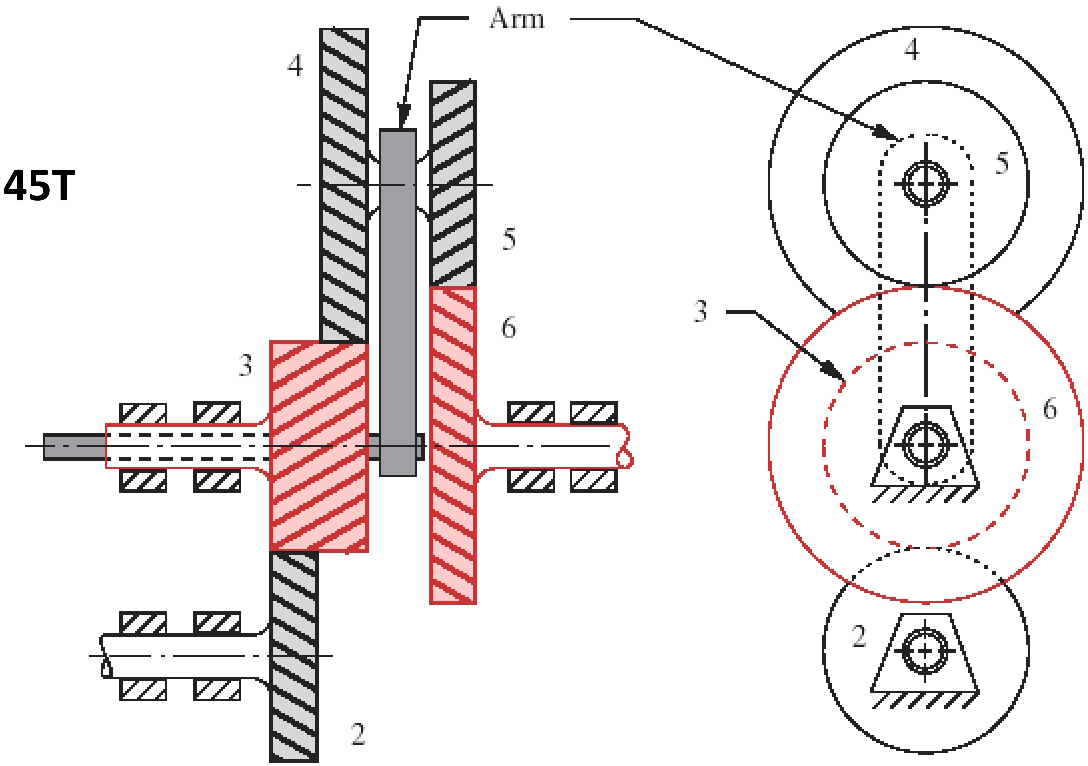
Find ω_2

If

$N_2 = 50T$, $N_3 = 25T$, $N_4 = 45T$

, $N_5 = 30T$, $N_6 = 40T$,

$\omega_6 = 20$, $\omega_{arm} = -50$



Gears

Epicyclic or planetary gear train

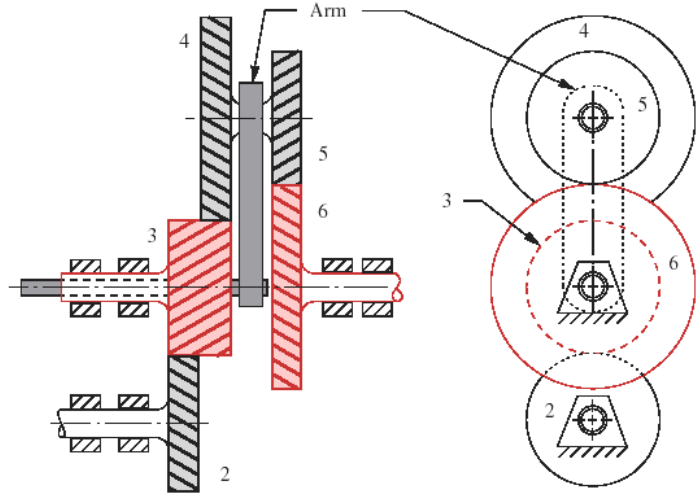
Solution

Let 2 to be input and 6 output

$$e = \frac{\omega_6 - \omega_{arm}}{\omega_2 - \omega_{arm}} = \frac{N_2 N_3 N_5}{N_3 N_4 N_6} = -\frac{N_2 N_5}{N_4 N_6}$$

$$\Rightarrow \frac{\omega_6 - \omega_{arm}}{\omega_2 - \omega_{arm}} = -\frac{(50)(30)}{(45)(40)} = -\frac{5}{6} = \frac{20+50}{\omega_2 + 50}$$

$$\Rightarrow \omega_2 = -134 RPM$$



Gears

Gear operation

The goal is to have constant speed ratio , it can be observed that is the motion transmit between gears teeth is a cam mechanism so, to guaranty the constant speed ratio, the intersection between the line of action and the line of center (k) is held constant in space ,therefore the tooth profile must guaranty this requirement. This requires the line of action to be stationary in space and the tooth profile which guaranty this can be constructed by involute profile .

The involute profile is the resultant of the straight line motion of the point of contact along the common normal or line of action and the negative of the rotating motion of the observer attached to the gear at the base circle

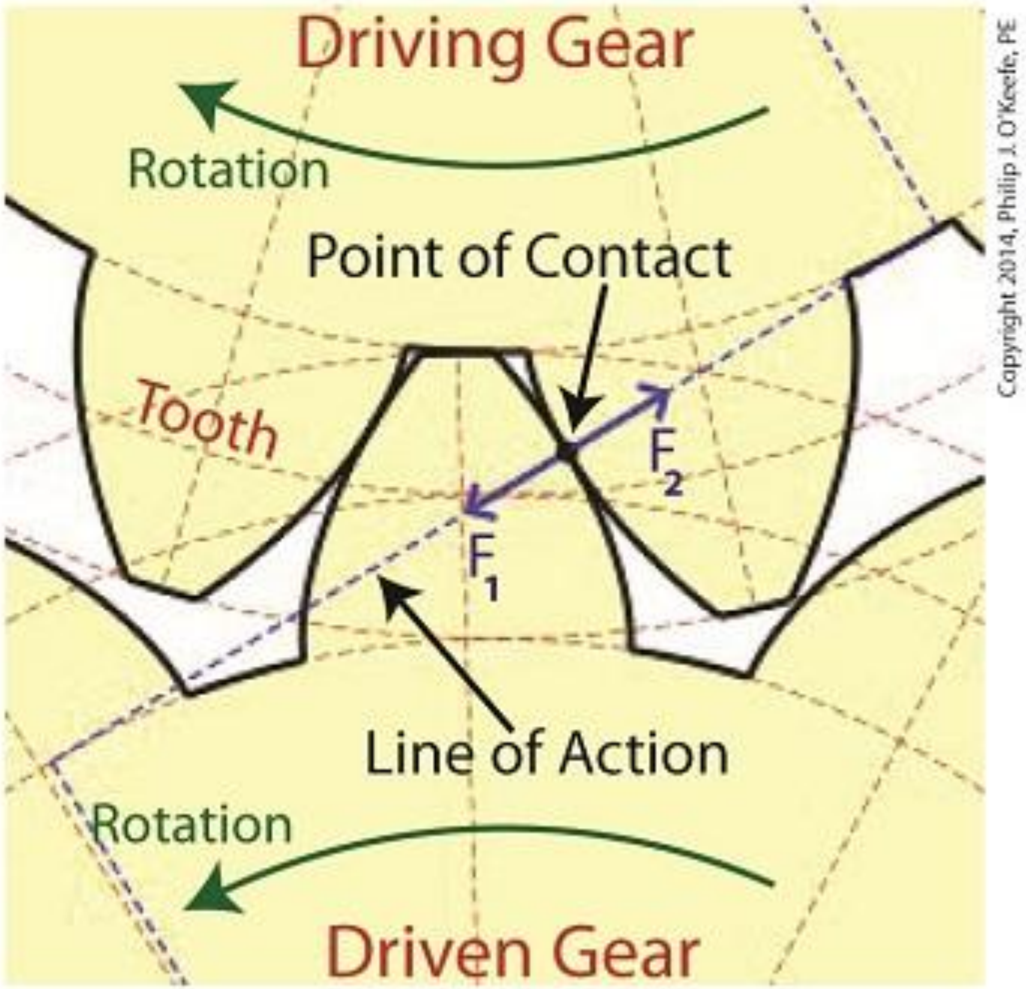
Gears

Gear operation

line of action

Line of action is the line connected between two point sin the space:

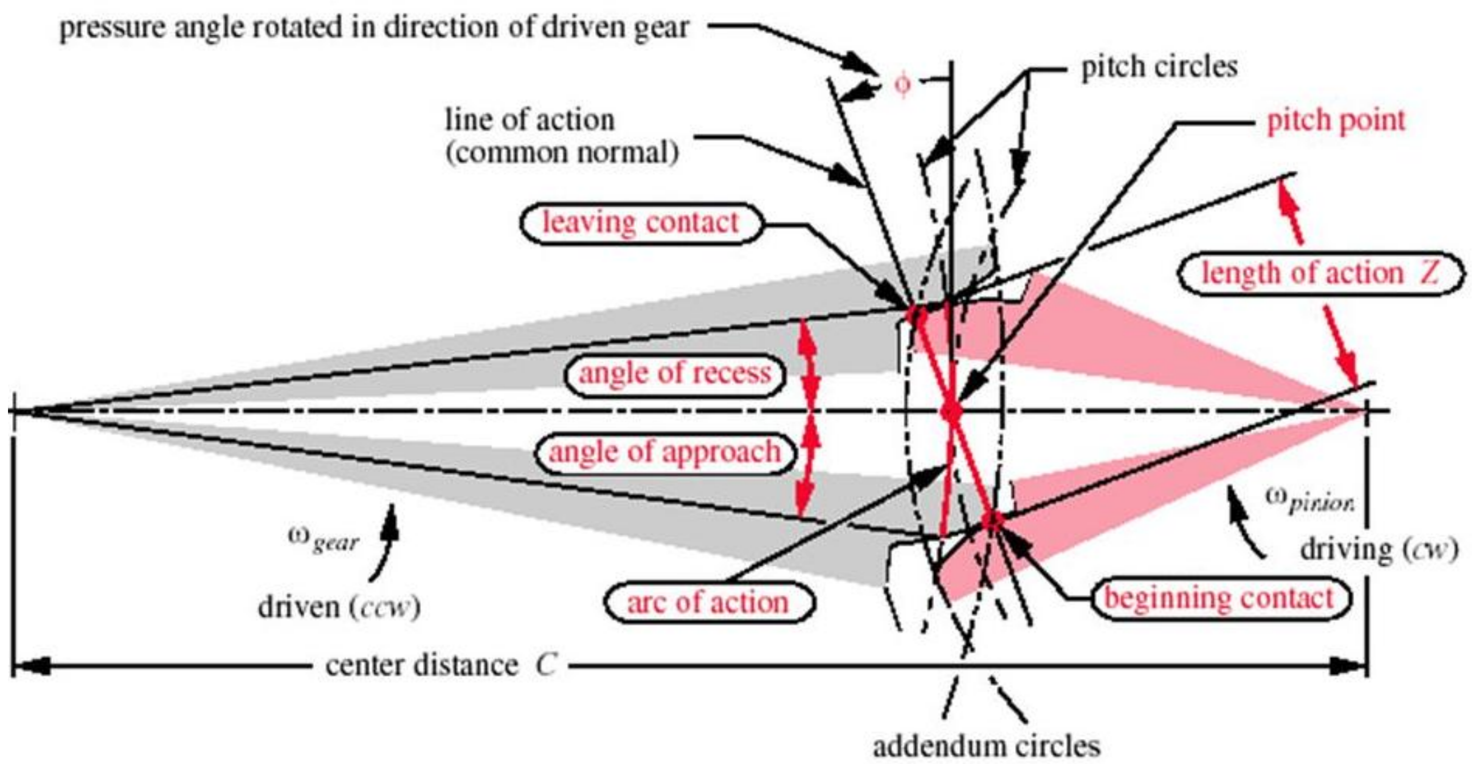
1. point of beginning of contact
2. the point of leaving contact.



Gears

Gear concepts

Length of line of action (Z)



Gears

Gear concepts

Length of line of action (Z)

$$Z=\sqrt{\left(r_p+a_p\right)^2-\left(r_p\cos\phi\right)^2}+\sqrt{\left(r_g+a_g\right)^2-\left(r_g\cos\phi\right)^2}-C\sin\phi$$

Where:

Z is the line of action length; m

r_p is the pinion pitch circle radius; m

a_p is the pinion addundum; m

r_g is the gear pitch circle radius; m

a_g is the gear addundum; m

Φ is the pressure angle; degree

C is the distance between the centers of two meshed gears; m

Gears

Gear operation

Pc = circular pitch = distance between two tooth along the pitch circle

$$Pc = \frac{\text{circumference of the pitch circle}}{\text{Number of tooth}} = \frac{\pi d}{N}$$

To find the number of teeth involved in the meshing process, use the following equation. This number must be grater than one to insure continuity in contact.

$$\text{number of teeth involved in meshing} = \frac{Z}{Pc}$$