


# Theory of machinery




Chapter eight

Static balancing

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Static balancing



Theory of machinery

- Imbalance is inherited in any rotating machinery due to the unsymmetry in the material or the shape of the machine
- The main problem of imbalance is the generation of unbalanced centrifugal forces rotates with the machine which produce shaking and vibration issue in the system.
- Unless the vibration is required, the unbalanced machine will suffer from excessive fatigue stresses.
- To eliminate unbalance, we perform a process called balancing
- Balancing can be static or dynamic
- Static balance is sometimes called **single plane balancing**.
- dynamic balance is sometimes called **multi plane balancing**.

## Static balancing



- ❑ In static balancing the main condition is:  $\sum F - ma = 0$
- ❑ In dynamic balancing the main conditions are:  $\sum F = 0$  and  $\sum M = 0$ .
- ❑ static balance is applied when the inertia is distributed radially rather than axially (i.e. has a large radius compared to the thickness). For example, single slim gear or pulley or fly wheel or single turbine blade disc can be balanced using static balancing.
- ❑ When the ratio between the radius and the thickness become small. or when there are multi discs on the same shaft, dynamic balancing is applied

## Static balancing



### Procedures for static balancing

1. Represent the mass distribution as a concentrated point mass ( $m$ ) located at the center of gravity of that mass (C.G).
2. Measure the distances between the concentrated masses (i.e. C.Gs) to the center of rotation. Call this distance  $R$ .
3. Assume angular velocity ( $\omega$ ).
4. Assume that there is a balance mass ( $m_b$ ) located from the center of rotation by  $R_b$ .
5. Apply the static balance condition :  $\sum F - ma = 0$

$$\left. \begin{aligned} -m_1 \vec{R}_1 \omega^2 - m_2 \vec{R}_2 \omega^2 - \dots - m_n \vec{R}_n \omega^2 - m_b \vec{R}_b \omega^2 &= 0 \\ \Rightarrow -m_1 \vec{R}_1 - m_2 \vec{R}_2 - \dots - m_n \vec{R}_n - m_b \vec{R}_b &= 0 \end{aligned} \right\} \text{---Eq.1}$$

## Static balancing



### Procedures for static balancing

6. Rearrange Eq.1:  $m_b \vec{R}_b = -\left(m_1 \vec{R}_1 + m_2 \vec{R}_2 + \dots + m_n \vec{R}_n\right) \text{--- Eq.2}$

7. Resolve E.2 to its rectangular components

$$\left. \begin{aligned} m_b \vec{R}_{b,x} &= -\left(m_1 \vec{R}_{1,x} + m_2 \vec{R}_{2,x} + \dots + m_n \vec{R}_{n,x}\right) \\ m_b \vec{R}_{b,y} &= -\left(m_1 \vec{R}_{1,y} + m_2 \vec{R}_{2,y} + \dots + m_n \vec{R}_{n,y}\right) \end{aligned} \right\} \text{--- Eq.3}$$

8. Drive expressions for  $R_b$  and  $\theta_b$ :

$$\theta_b = \tan^{-1} \left[ \frac{m_b \vec{R}_{b,y}}{m_b \vec{R}_{b,x}} \right] = \tan^{-1} \left[ \frac{-\left(m_1 \vec{R}_{1,y} + m_2 \vec{R}_{2,y} + \dots + m_n \vec{R}_{n,y}\right)}{-\left(m_1 \vec{R}_{1,x} + m_2 \vec{R}_{2,x} + \dots + m_n \vec{R}_{n,x}\right)} \right] \text{--- Eq.4}$$

$$R_b = \sqrt{\left(\vec{R}_{b,x}\right)^2 + \left(\vec{R}_{b,y}\right)^2} \Rightarrow m_b R_b = \sqrt{\left(m_b \vec{R}_{b,x}\right)^2 + \left(m_b \vec{R}_{b,y}\right)^2} \text{--- Eq.5}$$

## Static balancing



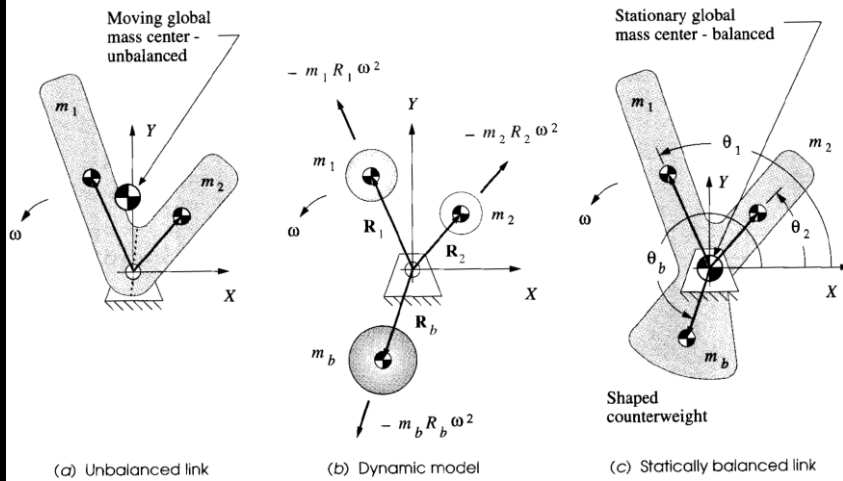
### Procedures for static balancing

As you can see, we can find the angle  $\theta_b$  directly from Eq.4 but to find the values of  $m_b$  and  $R_b$  we have single equation (5). So, the product of distance and mass  $m_b$  and  $R_b$  can be found and according to that you can design the balancing system

## Static balancing



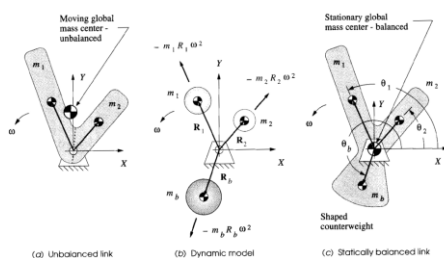
### Procedures for static balancing



## Static balancing



### Example



$$m_1 = 1.2 \text{ kg}$$

$$m_2 = 1.8 \text{ kg}$$

$$\omega = 40 \text{ rad/sec}$$

$$R_1 = 1.135 \text{ m @ } \angle 113.4^\circ$$

$$R_2 = 0.822 \text{ m @ } \angle 48.8^\circ$$

## Static balancing



### Example

**Solution:**

- 1 Resolve the position vectors into  $xy$  components in the arbitrary coordinate system associated with the freeze-frame position of the linkage chosen for analysis.

$$\begin{aligned} R_1 &= 1.135 @ \angle 113.4^\circ; & R_{1x} &= -0.451, & R_{1y} &= 1.042 \\ R_2 &= 0.822 @ \angle 48.8^\circ; & R_{2x} &= +0.541, & R_{2y} &= 0.618 \end{aligned} \quad (a)$$

- 2 Solve equations 12.2c.

$$\begin{aligned} m_b R_{b_x} &= -m_1 R_{1_x} - m_2 R_{2_x} = -(1.2)(-0.451) - (1.8)(0.541) = -0.433 \\ m_b R_{b_y} &= -m_1 R_{1_y} - m_2 R_{2_y} = -(1.2)(1.042) - (1.8)(0.618) = -2.363 \end{aligned} \quad (b)$$

- 3 Solve equations 12.2d and 12.2e.

$$\begin{aligned} \theta_b &= \arctan \frac{-2.363}{-0.433} = 259.6^\circ \\ m_b R_b &= \sqrt{(-0.433)^2 + (-2.363)^2} = 2.402 \text{ kg} \cdot \text{m} \end{aligned} \quad (c)$$

## Static balancing



### Example

- 4 This mass-radius product of 2.402 kg-m can be obtained with a variety of shapes appended to the assembly. Figure 12-1c shows a particular shape whose  $CG$  is at a radius of  $R_b = 0.806$  m at the required angle of  $259.6^\circ$ . The mass required for this counterweight design is then:

$$m_b = \frac{2.402 \text{ kg} \cdot \text{m}}{0.806 \text{ m}} = 2.980 \text{ kg} \quad (d)$$

at a chosen  $CG$  radius of:

$$R_b = 0.806 \text{ m} \quad (e)$$