Hydraulic machines

Chapter One

Introduction to hydraulic machines

IMPINGEMENT of JETS

By

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Introduction to hydraulic machines

Hydro- turbines

Impulse

Reaction

- ❑Energy extracted at the atmospheric pressure
- ❑Pelton wheel is an example.

- ❑Energy extracted due to difference in pressure
- ❑Frances turbine is an example.

Introduction to hydraulic machines

PRINCIPLES of IMPINGEMENT of JETS

Control Volume

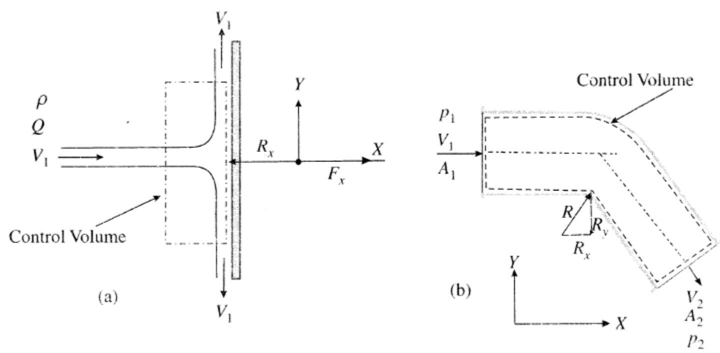

Fig. 1.1 *Examples of use of control volume*

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PRINCIPLES of IMPINGEMENT of JETS

Linear Momentum Equation

$$\sum F_x = F_{px} + F_{sx} + F_{bx} = \frac{\partial}{\partial t}(M_x)_{cv} + M_{x\,out} - M_{x\,in} \qquad (1.1)$$

$$\sum F_x = F_{px} + F_{sx} + F_{bx} = M_{x\,out} - M_{x\,in} = (\rho Q V_x)_{out} - (\rho Q V_x)_{in} \qquad (1.2)$$

Assumptions :

1. **The jet is open to atmospheric pressure**
2. **No friction between the fluid and the plate**

Introduction to hydraulic machines

Impingement of Free Jets

1. Case A: Jet Impingement on a Stationary Plate

$$0 - R_n = ((\rho Q)x(0)) - (\rho QV\sin(\theta))$$
$$\Rightarrow R_n = \rho QV\sin(\theta)$$

Notes:

1. When Θ = 90 degree, R_n is maximum and equal ρQV.

2. Because the plate is stationary, no work is produced

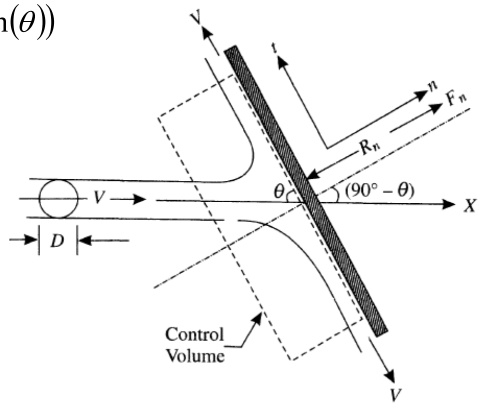

Fig. 1.2 *Jet impingement on an inclined plate*

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Impingement of Free Jets

2. Case B: Jet Impingement on a Moving Plate

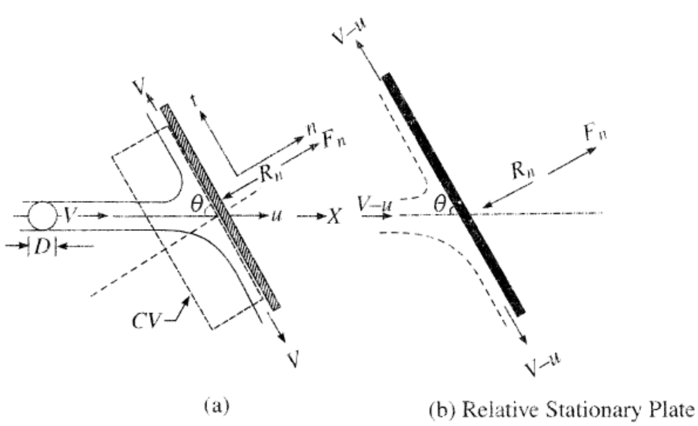

Fig. 1.3 *Jet impingement on a moving plate*

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Impingement of Free Jets

2. Case B: Jet Impingement on a Moving Plate

$$V_r = V - u$$
$$Q_r = AV_r = A(V-u)$$
$$A = \frac{\pi}{4}D^2$$
$$F_n = \rho Q_r V_r \sin(\theta)$$
$$F_n = \rho A V_r^2 \sin(\theta)$$
$$F_n = \rho A (V-u)^2 \sin(\theta)$$
$$Power(P) = F_n\, u \sin(\theta)$$
$$P = \rho A (V-u)^2 u \sin^2(\theta)$$

$$Efficiency\,(\eta) = \frac{Power}{K.E}$$
$$\Rightarrow \eta = \frac{\rho A (V-u)^2 u \sin^2(\theta)}{(1/2)\rho (AV) V^2}$$
$$\Rightarrow \eta = \frac{2(V-u)^2 u \sin^2(\theta)}{V^3}$$

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Impingement of Free Jets

3. Case C: Jet Impingement on a Set of Flat Plates Mounted on a Wheel

$$F_x = \rho Q V_r = \rho A V (V-u)$$
$$P = \rho A V (V-u) u$$
$$\eta = \frac{\rho A V (V-u) u}{(1/2)\rho (AV) V^2}$$
$$\Rightarrow \eta = \frac{2(V-u)u}{V^2}$$

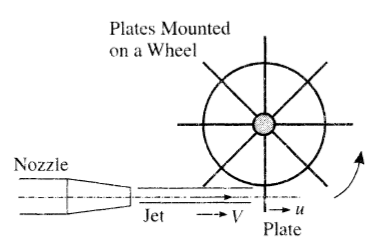

Fig. 1.4 *Jet impingement on plates mounted on a wheel*

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Impingement of Free Jets

4. Case D: Jet Impingement on a Stationary Curved Plate

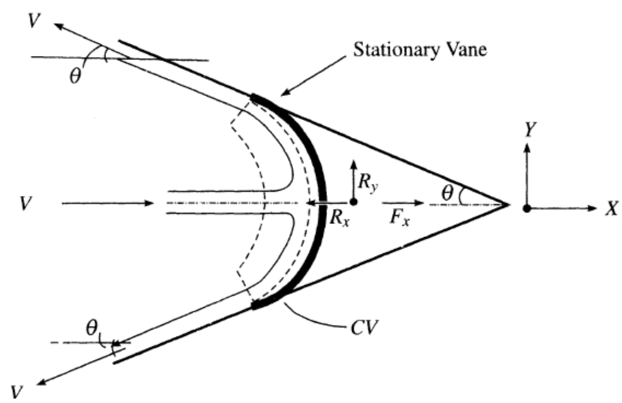

Fig. 1.5 *2D Jet impingement at centre of a stationary symmetric vane*

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Impingement of Free Jets

4. Case D: Jet Impingement on a Stationary Curved Plate

$\sum$(Forces in x-direction) =

$$-R_x = \left(-\rho(AV)V\cos(\theta)\right) - \left(\rho(AV)V\right)$$

$$R_x = \rho AV^2\left(1+\cos(\theta)\right) = F_x$$

$\sum$(Forces in y-direction) =

$$-R_y = \left(\rho(AV)V\sin(\theta)\right) + \left(-\rho(AV)V\sin(\theta)\right) - 0 = 0$$

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Impingement of Free Jets

5. Case E: Jet Impingement on a Single Moving Symmetric Curved Plate

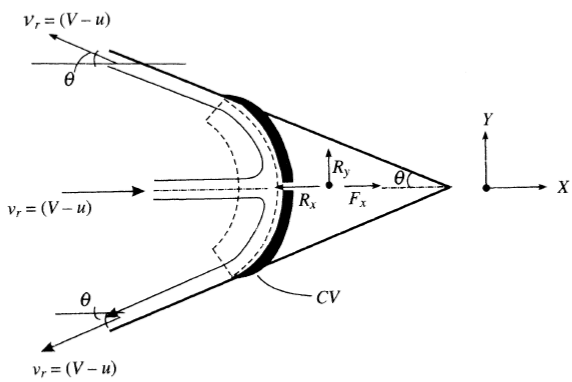

Fig. 1.6 *Jet impingement at centre of a symmetric moving vane—equivalent relative flow*

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Impingement of Free Jets

5. Case E: Jet Impingement on a Single Moving Symmetric Curved Plate

$\sum$ (Forces in x-direction)

$= (\text{Momentum flux in } x\text{-direction})_{out} - (\text{Momentum flux in } x\text{-direction})_{in}$

$$0 - R_x = [-\rho A\,(V - u)\,(V - u)\cos\theta\,] - [\rho A\,(V - u)\,(V - u)]$$

$$R_x = \rho A (V - u)^2\,(1 + \cos\theta)$$

$\sum$ (Forces in y-direction) =

$(\text{Momentum flux in } y\text{-direction})_{out} - (\text{Momentum flux in } y\text{-direction})_{in}$

$$0 - R_y = [\rho A (V - u)(V - u)\sin\theta\,] - [\rho A (V - u)(V - u)\sin\theta] - 0 = 0$$

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Work done by the jet per second = Power transferred to the vane

$$= F_x.u = \rho A(V-u)^2 u\,(1+\cos\theta) \quad (1.12)$$

Kinetic energy supplied by the jet per unit time = $\frac{1}{2}\rho(AV)V^2$

$$\text{Efficiency of the system} = \frac{\text{Power transmitted to the vane}}{\text{Kinetic energy supplied by the jet per unit time}}$$

$$= \eta = \frac{\rho A(V-u)^2 u(1+\cos\theta)}{\frac{1}{2}\rho(AV)V^2} = \frac{2(V-u)^2 u(1+\cos\theta)}{V^3} \quad (1.13)$$

For a given vane (θ = constant) and fixed jet velocity (V = constant), the condition of maximum efficiency of the system can be obtained by putting $\frac{d\eta}{du} = 0$. Hence

$$\frac{d\eta}{du} = \frac{d}{du}\left[\frac{2(V-u)^2 u(1+\cos\theta)}{V^3}\right] = \frac{2(1+\cos\theta)}{V^3}\frac{d}{du}[V^2u+u^3-2Vu^2]$$

Thus for maximum (or minimum) efficiency, $\frac{d\eta}{du} = \frac{d}{du}[V^2u+u^3-2Vu^2] = 0$

$$V^2+3u^2-4Vu=0$$

$$(V-u)\,(V-3u)-0$$

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5. Case E: Jet Impingement on a Single Moving Symmetric Curved Plate

The value of maximum efficiency corresponding to u = V/3

$$\eta_{\max} = \frac{2\left(V-\frac{V}{3}\right)^2\left(\frac{V}{3}\right)(1+\cos\theta)}{V^3} = \frac{8}{27}(1+\cos\theta) \quad (1.14)$$

Further, if θ is varied, maximum most efficiency of $\eta_{\max} = \frac{2\times 8}{27} = \frac{16}{27} = 59.26\%$ is obtained for $\theta = 0°$, that is for a semi-circular vane.

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Impingement of Free Jets

6. Case F: Jet Impingement on a Series of Curved Vanes Mounted on a Wheel

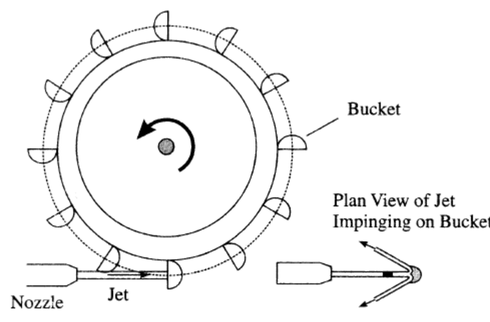

Fig. 1.7 *Schematic sketch of a Pelton wheel*

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Impingement of Free Jets

6. Case F: Jet Impingement on a Series of Curved Vanes Mounted on a Wheel

$\sum$(Forces in x direction) =

(Momentum flux in x-direction)$_{out}$ – (Momentum flux in x-direction)$_{in}$

$$0 - R_x = [-\rho AV(V-u)\cos\theta] - [\rho AV(V-u)]$$

$$R_x = \rho AV(V-u)(1+\cos\theta)$$

$\sum$ (Forces in y-direction) =

(Momentum flux in y-direction)$_{out}$ – (Momentum flux in y-direction)$_{in}$

$$0 - R_y = [\rho AV(V-u)\sin\theta] - [\rho AV(V-u)\sin\theta] - 0 = 0$$

Work done by the jet per second = Power transmitted to the wheel

$$= F_x \cdot u = \rho AV(V-u)\,u\,(1+\cos\theta) \tag{1.16}$$

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Impingement of Free Jets

6. Case F: Jet Impingement on a Series of Curved Vanes Mounted on a Wheel

$$\eta = \frac{\rho AV(V-u)u(1+\cos\theta)}{\frac{1}{2}\rho(AV)V^2} = \frac{2(V-u)\,u\,(1+\cos\theta)}{V^2} \qquad (1.17)$$

(a) For a given vane (θ = constant) and fixed jet velocity (V = constant), the condition of maximum efficiency can be obtained by putting $\frac{d\eta}{du} = 0$.

$$\frac{d\eta}{du} = \frac{d}{du}\left[\frac{2u(V-u)(1+\cos\theta)}{V^2}\right] = \frac{2(1+\cos\theta)}{V^2}\,\frac{d}{du}\,[2Vu - u^2]$$

Thus for maximum efficiency, $\frac{d}{du}[2Vu - u^2] = 0$ which gives the condition

$$u = \frac{V}{2} \qquad (1.18)$$

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Impingement of Free Jets

EXAMPLE 1.1

A free jet issuing from a nozzle of 0.0113 m² cross-sectional area impinges normally on a fixed vertical plate. The force on the plate due to this impingement is found to be 2.54 kN.

(a) Estimate the discharge from the nozzle.

(b) If the plate is allowed to move with a velocity of 3.0 m/s in the direction of the jet, what would be the

(i) Force on the plate and

(ii) power transferred to the plate in this system?

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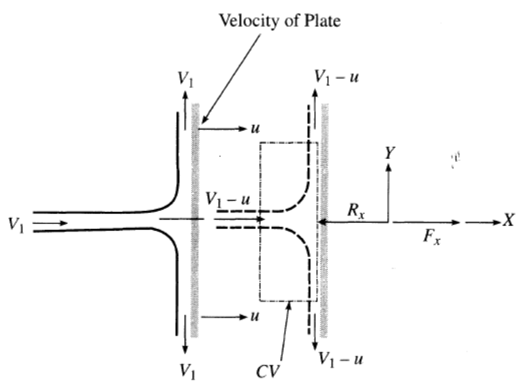
Impingement of Free Jets

EXAMPLE 1.1



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Impingement of Free Jets

EXAMPLE 1.1

Solution:

(a)

$\sum$ (Forces in x-direction) = (Momentum flux in x-direction)$_{out}$ – (Momentum flux in x-direction)$_{in}$

$0 - R_x = \rho Q\,(0 - V)$

$R_x = \rho QV = F_x$ in positive x-direction

Hence, $F_x = \rho QV = \dfrac{\rho Q^2}{A}$.

Here, $F_x = 2.54$ kN, area $A = 0.0113$ m^2 and hence

$$Q = \sqrt{\frac{AF_x}{\rho}} = \sqrt{\frac{0.0113 \times 2.54}{0.998}} = 0.1696 \text{ m}^3/\text{s}$$

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Impingement of Free Jets

EXAMPLE 1.1

Solution:

(b) When the plate is moving

Velocity of jet $V = \frac{Q}{A} = \frac{0.1696}{0.0113} = 15.0$ m/s.

Velocity of the plate $= u = 3.0$ m/s

Relative velocity $v_r = V - u = 15.0 - 3.0 = 12.0$ m/s

(i) Force on the plate in the x-direction $= F_x = \rho Q_r V_r$

$= 0.998 \times (0.0113 \times 12) \times 12 = 1.624$ kN

(ii) Power transferred to the plate $= P = F_x u = 1.624 \times 3.0 = 4.872$ kW

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Impingement of Free Jets

EXAMPLE 1.3:

A two-dimensional free jet of water of thickness B and discharge q per unit width of the jet strikes a stationary, frictionless, plate at angle B to the normal to the plate-Calculate

(a) the force on the plate,

(b) the ratio of discharges in the two streams that move on the plate on either side of the impact zone, and

(c) the force on the plate when B = 15º, B = l0 cm and q = 1.5 m³/s.

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Impingement of Free Jets

EXAMPLE 1.3:

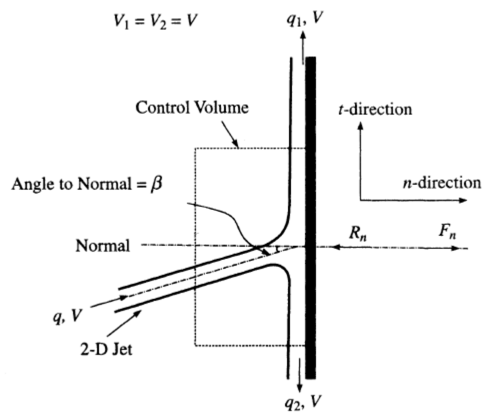

Fig. 1.15 *Impact of 2D Jet, Example 1.3*

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Impingement of Free Jets

EXAMPLE 1.3:

Solution

(a)

Since the pressure is atmospheric throughout the control volume, in the normal direction the linear momentum equation can be written as

$$0 - R_n = \rho q\,(0 - V\cos\beta) \text{ where } V = \text{Velocity of the jet.}$$

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(b) Since there is no friction to the flow in the transverse direction, the velocity of the two streams in the transverse direction (V_1 and V_2) is the same as the velocity of the impinging jet V. Hence by referring to Fig. 1.15 $V = V_1 = V_2$. Since the pressure is atmospheric all over the control volume, linear momentum equation in the transverse direction is

$0 = [\rho q_1 V_1 + (-\rho q_2 V_2)] - \rho q\, V \sin \beta$

$qV \sin \beta = q_1 V_1 - q_2 V_2$

Since $V = V_1 = V_2$, $\quad q \sin \theta = q_1 - q_2$ (i)

Also, by continuity $\quad q = q_1 - q_2$ (ii)

Substituting ($q_2 = q - q_1$) in (i)

$q \sin \beta = q_1 - (q - q_1)$ leading to $q_1 = \dfrac{q}{2}(1 + \sin \beta)$ (iii)

Similarly substituting ($q_1 = q - q_2$) in (i)

$q \sin \beta = q - q_2 - q_2$ leading to $q_2 = \dfrac{q}{2}(1 - \sin \beta)$ (iv)

Thus the ratio $\dfrac{q_1}{q_2} = \dfrac{1 + \sin \beta}{1 - \sin \beta}$

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Impingement of Free Jets

(c) When $\beta = 15°$, $B = 10$ cm and $q = 1.5\ \text{m}^3/\text{s}$,
Considering unit width, velocity of jet $V = q/B = 1.5/(0.10) = 15$ m/s
Normal force on the plate $F_n = \rho q V \cos\beta = 998 \times 1.5 \times (15 \cos 15°) = 21690$ N

The ratio of the discharges $\dfrac{q_1}{q_2} = \dfrac{1 + \sin \beta}{1 - \sin \beta} = \dfrac{1 + \sin 15°}{1 - \sin 15°} = 1.698$

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Impingement of Free Jets

****EXAMPLE 1.6**

A 10 cm diameter jet of water strikes a curved vane with a velocity of 25 m/s. The inlet angle of the vane is zero and the outlet angle is 150° measured with respect to the impinging jet direction. Determine the resultant force on the vane (a) when the vane is stationary, and (b) when the vane is moving in the direction of the jet at 10 m/s velocity.

Solution

Let suffix 1 and 2 denote the inlet and outlet conditions respectively. Figure 1.17(a) is a definition sketch of the problem when the vane is stationary.

(a) *When the jet is stationary:* $u = 0$. Assuming no losses, $V_1 = V_2 = V = 25.0$ m/s. The direction of exit velocity, V_2 is $(180° - 150°) = 30°$ with the negative x-direction.

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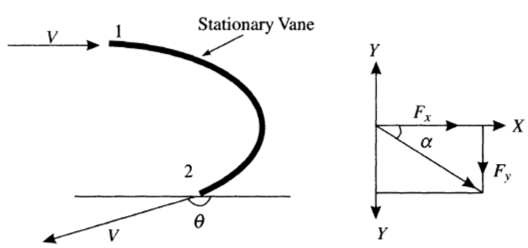
Impingement of Free Jets



(a)

Fig. 1.17(a) *Example 1.6—stationary vane*

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Impingement of Free Jets

Discharge $Q = \frac{\pi}{4} \times (0.10)^2 \times 25 = 0.1963 \text{ m}^3/\text{s}$

By linear momentum equation, the force on the vane in the x-direction is

$$F_x = \rho Q(V - (-V \cos 30°)) = \frac{998}{1000} \times 0.1963 \times (25 + 25 \cos 30°) = 9.14 \text{ kN}$$

By momentum equation in y-direction, force on the plate in y-direction is

$$F_y = \rho Q(0 - V \sin 30°) = \frac{998}{1000} \times 0.1963 \times (0 - 25 \sin 30°) = -2.45 \text{ kN}$$

acting in negative y-direction.

Resultant force $F = \sqrt{F_x^2 + F_y^2} = \sqrt{(9.14)^2 + (2.45)^2} = 9.46$ kN acting at an angle α to x-axis,

where α is given by $\alpha = \tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\frac{(-2.45)}{9.14} = -15°$ to positive x-direction as shown in Fig. 1.17 (a)

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(b) *When the vane is moving away from the jet with velocity u:*
Considering the equivalent flow relative to a stationary vane:
Relative velocity $v_r = V - u = (25 - 10) = 15$ m/s. Assuming no losses, $v_{r1} = v_{r2} = v_r$.

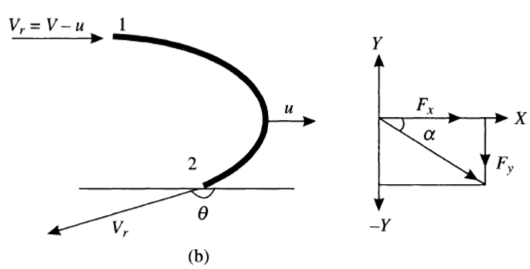

Fig. 1.17(b) *Relative flow, Example 1.6(b)*

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Impingement of Free Jets

The direction of exit relative velocity, v_{r2}, is (180° – 150°) = 30° with the negative x-direction.

Relative discharge $Q_r = A(V - u) = \frac{\pi}{4} \times (0.10)^2 \times (25 - 10) = 0.1178 \text{ m}^3/\text{s}$

The force on the plate in the x-direction is

$$F_x = \rho Q_r[v_r - (-v_r \cos 30°)] = \frac{998}{1000} \times 0.1178 \times (15 + 15 \cos 30°) = 3.29 \text{ kN}$$

By momentum equation in y-direction, force on the plate in y-direction

$$F_y = \rho Q_r(0 - v_r \sin 30°) = \frac{998}{1000} \times 0.1178 \times (0 - 15 \sin 30°)$$

$$= -0.882 \text{ kN acting in negative } y\text{-direction}$$

Resultant force $F = \sqrt{F_x^2 + F_y^2} = \sqrt{(3.291)^2 + (0.882)^2} = 3.407$ kN acting at an angle α to x-axis, where α is given by $\alpha = \tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\frac{-0.882}{3.291} = -15°$ to positive x-direction as shown in Fig. 1.17 (b).

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Impingement of Free Jets

Self study examples:

1. 1.2
2. 1.4
3. 1.7
4. 1.8