Hydraulic machines

Chapter One

Introduction to hydraulic machines

Velocity triangles

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$$V_r = V - u$$

or

$$V = V_r + u$$

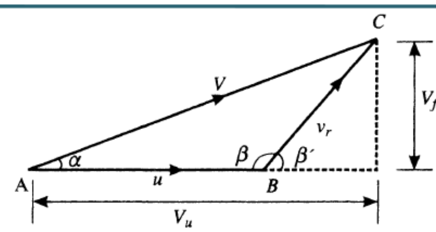

Fig. 1.8 *Velocity triangle,* $\beta > \pi/2$

V is absolute velocity; m/s.
V_r is relative velocity; m/s.
u is peripheral velocity of the wheel; m/s
α is guide vane angle; degree.
β is blade angle; degree.
V_f is velocity of flow; m/s.
V_u is velocity of whirl or velocity of swirl

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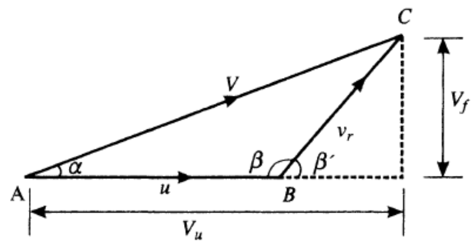

Fig. 1.8 *Velocity triangle, $\beta > \pi/2$*

Table 1.1 *Relationship for V_f and V_u for $\beta \geq \pi/2$*

Parameter	For $\beta > \frac{\pi}{2}$ (Let $\beta' = (180-\beta)$)	For $\beta = \pi/2$
$V_f =$	$v_r \sin \beta' = V \sin \alpha$ (1.22-a)	$v_r = V \sin \alpha$ (1.22-b)
$V_u =$	$V \cos \alpha = u + v_r \cos \beta'$ (1.23-a)	$V \cos \alpha = u$ (1.23-b)

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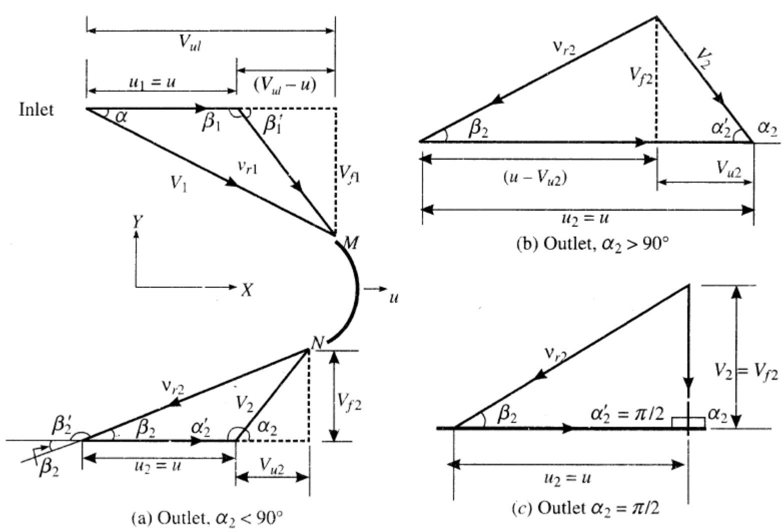

Fig. 1.11 *Velocity triangles for jet impingement on a moving vane for different outlet angle conditions*

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V_1 = Absolute velocity of water at entry in to the vane
v_{r1} = Relative velocity of water with respect to the vane at entry
u = Velocity of the vane. In this case $u = u_1 = u_2$
α_1 = *Guide vane angle* at the entry. It is the angle made by the absolute velocity vector $\vec{V}$ with the direction of the velocity of vane u at the inlet
α_2 = *Guide vane angle* at the exit. It is the angle made by the absolute velocity vector $\vec{V}$ with the direction of the velocity of vane u at the exit
β_1 = *Blade angle* at the inlet. It is the angle made by the relative velocity vector $\vec{v}$ with the negative direction of the velocity of vane u at the inlet.
β_2 = *Blade angle* at the outlet. It is the angle made by the relative velocity vector $\vec{v}$ with the negative direction of the velocity of vane u at the outlet.
V_{u1} = *Velocity of whirl*, component of the absolute velocity V_1 in the direction of $u = V_1 \cos \alpha_1$
V_{f1} = *Velocity of flow*, component of the absolute velocity V_1 in the direction normal to that of $u = V_1 \sin \alpha_1$

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Table 1.2 *Summary of force and power in various situations of the jet impingement in Case G_1 and Case G_2*

Value of α_2	Single vane (Case G_1)	Series of vanes (Case G_2)
$\alpha_2 < 90°$	$F_x = \rho(Av_{r1})(V_{u1} + V_{u2})$ $P = \rho(Av_{r1})(V_{u1} + V_{u2})u$ $\eta = \frac{2(V_{u1} + V_{u2})v_{r1}u}{V_1^3}$ Also, when there is no energy loss $\eta = \frac{v_{r1}}{V_1}\left[1 - \left(\frac{V_2}{V_1}\right)^2\right]$	$F_x = \rho(AV_1)(V_{u1} + V_{u2})$ $P = \rho(AV_1)(V_{u1} + V_{u2})u$ $\eta = \frac{2(V_{u1} + V_{u2})u}{V_1^2}$ Also, when there is no energy loss $\eta = \left[1 - \left(\frac{V_2}{V_1}\right)^2\right]$

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$\alpha_2 = 90°$	$F_x = \rho(Av_{r1})\, V_{u1}$ $P = \rho(Av_{r1})\, V_{u1}\, u$ $\eta = \dfrac{2V_{u1}v_{r1}u}{V_1^3}$ *Also, when there is no energy loss* $\eta = \dfrac{v_{r1}}{V_1}\left[1-\left(\dfrac{V_2}{V_1}\right)^2\right]$	$F_x = \rho\,(AV_1)\, V_{u1}$ $P = \rho(AV_1)\, V_{u1}u$ $\eta = \dfrac{2V_{u1}u}{V_1^2}$ *Also, when there is no energy loss* $\eta = \left[1-\left(\dfrac{V_2}{V_1}\right)^2\right]$
$\alpha_2 > 90°$	$F_x = \rho(Av_{r1})\,(V_{u1} - V_{u2})$ $P = \rho(Av_{r1})\,(V_{u1} - V_{u2})u$ $\eta = \dfrac{2\,(V_{u1} - V_{u2})\,v_{r1}u}{V_1^3}$ Also, when there is no energy loss $\eta = \dfrac{v_{r1}}{V_1}\left[1-\left(\dfrac{V_2}{V_1}\right)^2\right]$	$F_x = \rho(AV_1)\,(V_{u1} - V_{u2})$ $P = \rho(AV_1)\,(V_{u1} - V_{u2})u$ $\eta = \dfrac{2(V_{u1} - V_{u2})u}{V_1^2}$ Also, when there is no energy loss $\eta = \left[1-\left(\dfrac{V_2}{V_1}\right)^2\right]$

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***EXAMPLE 1.11

A free jet of water with an initial velocity of 40 m/s impinges on a series of curved vanes moving at 20 m/s. The direction of the jet from the nozzle is at 20° with the direction of motion of the vanes. Assuming the outlet relative velocity is 95% of the relative velocity at the inlet, compute (i) the angle made by the tangent to the relative velocity at its tips with the direction of motion of vanes, (ii) power transmitted per unit weight of water, and (iii) hydraulic efficiency of the system. Assume the absolute velocity of water at the outlet is normal to the direction of motion of vane.

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Solution

Figure 1.21 shows the inlet and outlet velocity triangles. Given Data:

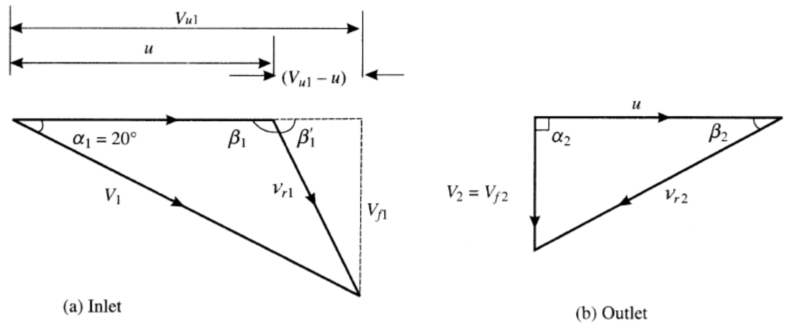

Fig. 1.21 *Velocity triangles of Example 1.11*

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For the inlet velocity triangle: V_1 = Absolute velocity of jet = 40 m/s, u = 20 m/s and $\alpha_1 = 20°$.

For the outlet velocity triangle: V_2 = Absolute velocity of jet = V_{f2}, $v_{r2} = 0.95 v_{r1}$, and $\alpha_2 = 90°$,

Flow velocity $V_{f1} = V_1 \sin \alpha_1 = 40 \times \sin 20° = 13.68$ m/s

Velocity of whirl $V_{u1} = V_1 \cos \alpha_1 = 40 \times \cos 20° = 37.59$ m/s

$$\tan \beta_1' = \frac{V_{f1}}{(V_{u1} - u)} = \frac{13.68}{(37.59 - 20.0)} = 0.778$$

$\beta_1' = 37.87°$ = Angle made by the tangent to the relative velocity with the direction of motion of vanes at the inlet.

$$v_{r1} = \frac{V_{f1}}{\sin \beta_1'} = \frac{13.68}{\sin 37.87°} = 22.28 \text{ m/s}$$

$v_{r2} = 0.95\, v_{r1} = 0.95 \times 22.28 = 21.166$ m/s

Also velocity of blades = u = 20 m/s

From the outlet velocity triangle:

$\alpha_2 = 90°$, $V_{f2} = V_2$ and $V_{u2} = 0$.

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Also $V_2 = \sqrt{v_{r2}^2 - u^2} = \sqrt{(21.166)^2 - (20)^2} = 6.928$ m/s

$$\tan \beta_2 = \frac{V_{f2}}{u} = \frac{V_2}{u} = \frac{6.928}{20} = 0.346$$

$\beta_2 = 19{\cdot}106° =$ Angle made by the tangent to the relative velocity with the direction of motion of vanes at the outlet.

Since a series of vanes are used, the vanes capture all of the discharge from the nozzle. Hence, the full discharge Q is used in the calculation of the power.

Power per unit weight of water $P_1 = \frac{\rho Q(V_{u1} + V_{u2})u}{\rho g Q} = \frac{(V_{u1} + V_{u2})u}{g}$

$$= \frac{(37.59 + 0) \times 20}{9.81} = 76.636 \text{ m}$$

Kinetic Energy per unit weight of water = Energy head

$$= H_{E1} = \frac{V_1^2}{2g} = \frac{(40)^2}{2g} = 81.549 \text{ m}$$

Efficiency of the system $= \eta = \frac{P_1}{H_{E1}} = \frac{76.636}{81.549} = 0.9397 = 94\% = 94.0\%$

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Impingement of Free Jets

Self study examples:

1. 1.9
2. 1.10