Hydraulic machines

Chapter One

Introduction to hydraulic machines

Similarity Analysis

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Similarity laws for Turbines

Table 1.4 *Variables affecting performance of a turbine*

Symbol	Variable	Dimensions
D	Diameter of the runner, (Reference length parameter)	$[L]$
N	Rotational speed (RPM)	$[T^{-1}]$
H	Energy head (= Energy per unit weight)	$[L]$
Q	Discharge through the machine	$[L^3 T^{-1}]$
P	Power developed by rotor	$[ML^2 T^{-3}]$
g	Acceleration due to gravity	$[LT^{-2}]$
ρ	Density of water	$[ML^{-3}]$
μ	Coefficient of dynamic viscosity of water	$[ML^{-1} T^{-2}]$

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Similarity laws for Turbines

Table 1.5 *Ratios for similarity in turbines*

Basic Similarity Ratios	Derived Similarity Ratios
$\frac{N_m D_m}{H_m^{1/2}} = \frac{N_p D_p}{H_p^{1/2}} = \text{Constant} = C_H$	$N_{11} = \left[\frac{ND}{\sqrt{H}}\right]_m = \left[\frac{ND}{\sqrt{H}}\right]_p = \text{Constant}$
$\frac{Q_m}{N_m D_m^3} = \frac{Q_p}{N_p D_p^3} = \text{Constant} = C_Q$	$Q_{11} = \left[\frac{Q}{D^2\sqrt{H}}\right]_m = \left[\frac{Q}{D^2\sqrt{H}}\right]_p = \text{Constant}$
$\frac{P_m}{N_m^3 D_m^5} = \frac{P_p}{N_p^3 D_p^5} = \text{Constant} = C_P$	$P_{11} = \left[\frac{P}{H^{3/2}D^2}\right]_m = \left[\frac{P}{H^{3/2}D^2}\right]_p = \text{Constant}$
	Specific Speed $= N_s$ $\left[\frac{N\sqrt{P}}{H^{5/4}}\right]_m = \left[\frac{N\sqrt{P}}{H^{5/4}}\right]_p = \text{Constant}$

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*EXAMPLE 1.16

A turbine is to operate under a head of 25 m at a speed of 300 rpm. The discharge is 9.0 m^3/s. If the efficiency is 90%, determine the performance of the turbine under a head of 20 m.

Solution

Given: $D_1 = D_2$, $N_1 = 300$ rpm, $H_1 = 25.0$ m, $H_2 = 20.0$ m, $Q_1 = 9.0$ m^3/s, $\eta = 0.9$

$$P = \eta\,\gamma QH = 0.9 \times 9.79 \times 9.0 \times 25 = 1982.5 \text{ kW}$$

Since the diameter is the same, i.e. $D_1 = D_2$, $\frac{N_1}{H_1^{1/2}} = \frac{N_2}{H_2^{1/2}}$

$$N_2 = N_1\sqrt{\frac{H_2}{H_1}} = 300 \times \sqrt{\frac{20}{25}} = 268.3 \text{ rpm}$$

By similar unit relationships $Q_2 = Q_1\sqrt{\frac{H_2}{H_1}} = 9.0 \times \sqrt{\frac{20}{25}} = 8.05$ m^3/s

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****EXAMPLE 1.17**

A 1/5 scale model of a Kaplan turbine is designed to operate at a head of 25 m. The prototype produces 18.50 MW of power under a head of 49 m when operating at a speed of 250 rpm. Find the speed, discharge and power of the model. Assume the efficiency of the model and prototype is the same at a value of 88%.

Solution

Given: Scale ratio $D_r = 1/5$, $H_m = 25$, $H_p = 49$ m, $P_p = 18500$ kW, $N_p = 250$ rpm, $\eta_{0m} = \eta_{0p} = 0.88$

For prototype:

Power $P_p = \eta_0 \gamma Q_p H_p = 18500\text{ kW}$

Discharge $Q_p = \dfrac{P_p}{\eta_0 \gamma H_p} = \dfrac{18500}{0.88 \times 9.79 \times 49} = 43.82\text{ m}^3/\text{s}$

For a 1/5 Scale Model: $\dfrac{D_m}{D_p} = \dfrac{1}{5}$, Head ratio $\dfrac{H_m}{H_p} = \dfrac{25}{49}$

Speed: $\dfrac{N_m D_m}{\sqrt{H_m}} = \dfrac{N_p D_p}{\sqrt{H_p}}$

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$$N_m = N_p \left(\frac{D_p}{D_m}\right)\sqrt{\frac{H_m}{H_p}} = 250 \times \left(\frac{5}{1}\right)\sqrt{\frac{25}{49}} = 892.9\text{ rpm}$$

Hence, speed of model = 852.9 rpm

Discharge: $\dfrac{Q_m}{N_m D_m^3} = \dfrac{Q_p}{N_p D_p^3}$

$$Q_m = Q_p \left(\frac{D_m}{D_p}\right)^3 \left(\frac{N_m}{N_p}\right) = 43.82 \times \left(\frac{1}{5}\right)^3 \left(\frac{892.9}{250}\right) = 1.252\text{ m}^3/\text{s}$$

Model discharge is 1.252 m^3/s

Power developed by the model = $P_m = \eta_0 \gamma Q_m H_m$

$$P_m = 0.88 \times 9.79 \times 1.252 \times 25 = 269.7\text{ kW}$$

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**EXAMPLE 1.22

A reaction turbine has the following double unit values: Peak Specific Power P_{11} = 9.0, Peak Specific Discharge Q_{11} = 1.021, Peak Defined Speed N_{11} = 150. Estimate the specific speed, runner diameter, discharge, efficiency and speed of rotation of a homologous turbine working under a head of 25 m and developing 12 MW of power. Assume the efficiency to be constant for all sizes of turbines.

Solution

Given: At peak, [$P_{11} = 9$, $Q_{11} = 1.021$, $N_{11} = 150$], $H = 25$ m, $P = 12000$ kW

$$\text{Specific speed at peak values} = N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{ND}{\sqrt{H}} \times \frac{\sqrt{P}}{\sqrt{(H^{3/2}D^2)}} = \left(N_{11}\sqrt{P_{11}}\right)_{Peak}$$

$$= 150 \times \sqrt{9} = 450$$

From the definition of the various parameters:

$$Q_{11} = \frac{Q}{D^2\sqrt{H}} \text{ giving } Q = Q_{11}D^2\sqrt{H}$$

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$$Q_{11} = \frac{Q}{D^2\sqrt{H}} \text{ giving } Q = Q_{11}D^2\sqrt{H}$$

$$P_{11} = \frac{P}{D^2H^{3/2}} \text{ giving } P = P_{11}D^2H^{3/2}$$

Power $\quad P = P_{11}\,D^2\,H^{3/2} = 9.00 \times D^2 \times (25)^{3/2} = 12000 \text{ kW}$

$$D^2 = \frac{12000}{9 \times (25)^{3/2}} = 10.67, \text{ giving runner diameter } D = 3.27 \text{ m}$$

Speed $\quad N = \dfrac{N_{11}\sqrt{H}}{D} = \dfrac{150 \times \sqrt{25}}{3.27} = 229.4 \text{ rpm}$

Discharge $\quad Q = Q_{11}D^2\sqrt{H} = 1.021 \times (3.27)^2 \times \sqrt{25} = 54.6 \text{ m}^3/\text{s}$

Efficiency $= \dfrac{P_{11}}{\gamma Q_{11}} = \dfrac{9.0}{9.79 \times 1.021} = 0.90$

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1.7 BASIC FEATURES OF HYDRAULIC TURBINES

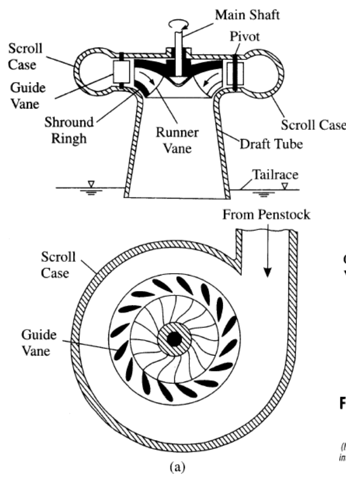

Fig. 1.23(a) *Schematic sketch of a Francis turbine*

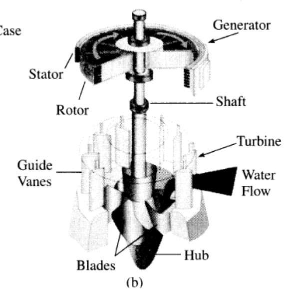

Fig. 1.23(b) *Schematic sketch of a Kaplan turbine and generator set.*

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Classification of hyd. Turbines

- Nature of interaction
  - Impulse turbines
  - Reaction turbines
- Nature of flow
  - Radial flow turb.
  - Tangential flow
  - Mixed flow
  - Axial flow

Classification of hyd. Turbines

- Head
  - High head >400m
  - Medium head 60<H<400m
  - Low head 3<H<60m
- Specific speed; Ns
  - Pelton: Ns=8-30
  - Francis : Ns=40-450
  - Kaplan Ns= 300-900

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Efficiencies of Turbines

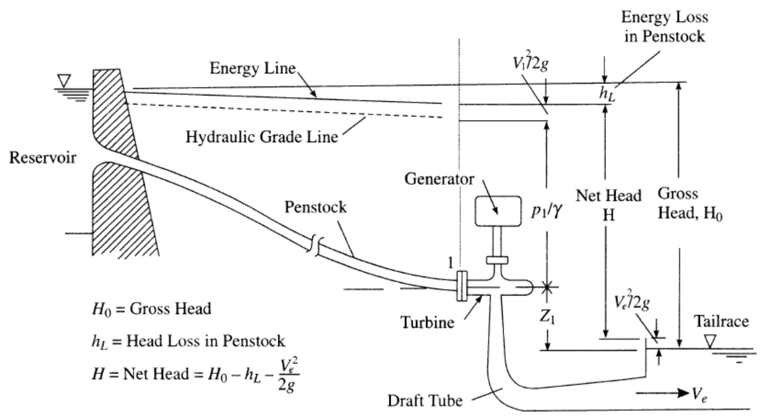

Fig. 1.24 *Schematic layout of a reaction turbine*

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1. Volumetric Efficiency, η_v

It is possible that out of a total discharge of Q supplied to the turbine, some quantity of discharge (Q_L) supplied at the inlet of the turbine may exit the turbine in the form of various leaks without doing any work on the rotor. The volumetric efficiency of the turbine η_v is defined as

$$\eta_v = \frac{Q - Q_L}{Q} = \frac{\text{Discharge doing work on the rotor}}{\text{Discharge supplied to the rotor}} \qquad (1.34)$$

Generally, leakage is a very small percentage of the discharge Q and is of the order of 0.5%.

2. Hydraulic Efficiency, η_h

While the net head H is available to the turbine, the energy head extracted by the turbine runner H_e will be less than H by an amount h_{fr} due fluid friction and form loss at the rotor including entrance and exit losses at the rotor. Thus,

$$H_e = H - h_{fr}$$

The hydraulic efficiency of the turbine is given by

$$\eta_h = \frac{\text{Head extracted by rotor}}{\text{Net head available to the rotor}} = \frac{H - h_{fr}}{H} = \frac{H_e}{H} \qquad (1.35)$$

The discharge acting on the rotor is $(Q - Q_L)$ and as such the power produced by the runner is $\gamma(Q - Q_L)\,H_e$.

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3. *Mechanical Efficiency, η_m*

as $$\eta_m = \frac{\text{Power available at the rotor shaft}}{\text{Power produced by the runner}} = \frac{\gamma(Q - Q_L)(H_e - h_m)}{\gamma(Q - Q_L)H_e} \quad (1.36)$$

Overall Efficiency, η_0

$$\eta_0 = \frac{\text{Power available at the rotor shaft}}{\text{Power supplied by water to the runner}} = \frac{\gamma(Q - Q_L)(H_e - h_m)}{\gamma Q H} \quad (1.37)$$

$$\eta_0 = \frac{(Q - Q_L)}{Q} \times \frac{\gamma(Q - Q_L)H_e}{\gamma(Q - Q_L)H} \times \frac{\gamma(Q - Q_L)(H_e - h_m)}{\gamma(Q - Q_L)H_e}$$

giving $\eta_0 = \eta_v \, \eta_h \, \eta_m$ (1.38)

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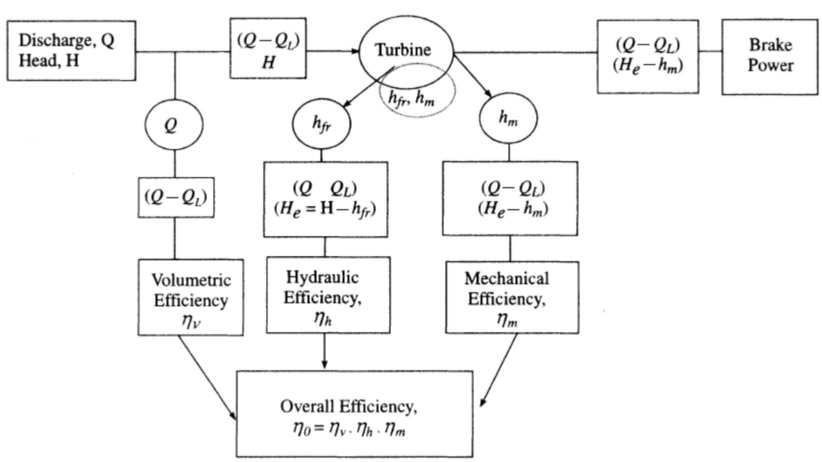

Fig. 1.25 *Schematic representation of turbine losses and efficiencies*

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**EXAMPLE 1.13

In a hydroelectric project, the upstream reservoir water level is 200 m above the tailrace water level of the power plant. When a discharge of 3.0 m^3/s is supplied to the turbine, the frictional losses in the penstock are 20 m and the head utilised in the turbine is 160 m. The leakage loss is estimated at 0.1 m^3/s and the mechanical losses can be taken as 100 kW.

(a) Calculate the (i) volumetric efficiency, (ii) hydraulic efficiency (iii) mechanical efficiency, and (iv) overall efficiency of the system.

(b) If a homologous turbine having a runner of diameter ratio 0.8 were tested under similar conditions, what would be its overall efficiency?

Solution

Given: Gross head = H_0 = 200 m,

Penstock losses = h_f = 20 m,

Net head = H = 200 – 20 = 180 m,

Utilised head = H_e = 160

(i) Volumetric efficiency $\eta_v = \dfrac{Q - Q_L}{Q} = \dfrac{3.0 - 0.1}{3.0} = 0.967 = 96.7\%$

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(ii) Hydraulic efficiency $\eta_h = \dfrac{H_e}{H} = \dfrac{160}{180} = 0.889 = 88.9\%$

Theoretical power extracted = $P_{th} = \gamma(Q - Q_L)\, H_e = 9.79 \times 2.9 \times 160$

= 4542.6 kW

Actual Brake power developed = 4542.6 – 100 = 4442.6 kW

(iii) Mechanical efficiency $\eta_m = \dfrac{4442.6}{4542.6} = 97.8\%$

(iv) Overall efficiency $\eta_0 = \eta_v \eta_h \eta_m = 0.967 \times 0.889 \times 0.978 = 84.1\%$

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**EXAMPLE 1.14

In a hydroelectric project, the available discharge is 60 m^3/s with a net head of 35 m. Assuming a turbine efficiency of 92% and rotational speed of 300 rpm, determine the least number of turbines of the same size and having a specific speed of 275.

Solution

Given: Power potential $P = \eta_0\, \gamma Q\, H = 0.92 \times 9.79 \times 60 \times 35 = 16212$ kW

For a specific speed of 250, power produced per machine P_1 is given by:

$$N_s = \frac{N\sqrt{P_1}}{H^{5/4}} = 275 = \frac{300\sqrt{P_1}}{(35)^{5/4}}$$

and $$P_1 = \left(\frac{N_s}{N}\right)^2 H^{5/2} = \left(\frac{275}{300}\right)^2 \times (35)^{5/2} = 6090 \text{ kW}$$

Number of turbines required $n \approx (16212)/6090 = 3$

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**EXAMPLE 1.23

A turbine is to operate under a head of 150 m and uses 130 m^3/s of water. It has a speed of 140 rpm and the overall efficiency is 92%. Calculate the specific speed (a) in dimensional form N_s using SI units, and (b) in nondimensional form, S_p (revolutions). Use unit weight of water = 9.79 kN/m^3.

Solution

Power $P = \eta_0\, \gamma QH = 0.92 \times 9.79 \times 130 \times 150 = 175633$ kW

Specific speed in *SI* units $N_s = \dfrac{N\sqrt{P}}{H^{5/4}} = \dfrac{140 \times \sqrt{175633}}{(150)^{5/4}} = 111.8$

Specific speed in nondimensional form

$$S_p = \frac{N\sqrt{P}}{(gH)^{5/4}} \times \frac{1}{\rho^{1/2}} = \frac{N\sqrt{P}}{(H)^{5/4}} \times \frac{1}{g^{3/4}} \times \frac{1}{\gamma^{1/2}}$$

In the above $g = 9.81$ m/s^2 and take given $\gamma = 9790$ N/m^3. Further the speed N is in *rps*, P is in *watts* and H in *metres*. Thus

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$$S_p = \frac{N(\text{in rps})\sqrt{P(\text{in Watts})}}{[H(\text{in m})]^{5/4}} \times \frac{1}{g^{3/4}} \times \frac{1}{\gamma^{1/2}}$$

$$= \frac{\left(\frac{140}{60}\right) \times \sqrt{175633 \times 1000}}{(150)^{5/4}} \times \frac{1}{(9.81)^{3/4}} \times \frac{1}{(9790)^{1/2}}$$

$S_p = 0.1074$ (revolutions)

Alternately: Using the conversion equation Eq. (1.50)

$S_p = 9.61 \times 10^{-4}\, N_s$

$= 9.61 \times 10^{-4} \times 111.8 = 0.1074$ revolutions

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Self study examples:

1.18 – 1.22