Automatic Control

Chapter six

PID controllers-Example

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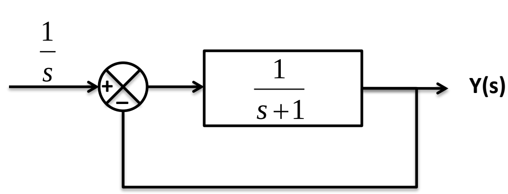
PID controllers

Automatic Control

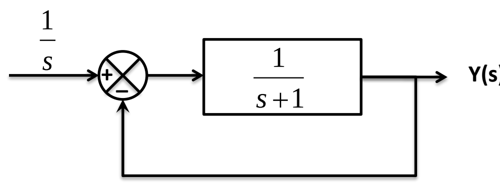
❑Consider the following 1st order system subjected to unit step input (r(t) =1 or R(s) = 1/s) where y(t) is the output



Automatic Control

PID controllers

❑To find the steady state error



$$e_{ss} = \frac{R}{1+k_p} = \frac{1}{1+k_p}; k_p = \lim_{s\to 0} G(s) = \lim_{s\to 0}\left[\frac{1}{s+1}\right] = 1$$

$$\Rightarrow e_{ss} = \frac{1}{1+1} = 0.5$$

❑As you can see, the steady state error equal 0.5 or 50% which means that tahe signal will eventually reach <u>0.5</u> instead of <u>1.0</u>

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PID controllers

❑To find the response, we have to find M(s) and then the Laplace inverse for the function Y(s) = R(s) M(s).

$$M(s) = \frac{G}{1+GH} = \frac{G}{1+G} = \frac{1/s+1}{1+\frac{1}{s+s}} = \frac{1}{s+2}$$

$$Y(s) = R(s)M(s) = \left(\frac{1}{s}\right)\left(\frac{1}{s+2}\right) = \frac{1}{s^2+2s}$$

$$y(t) = \ell^{-1}[Y(s)] = \frac{1}{2} - \frac{1}{2e^{2t}}$$

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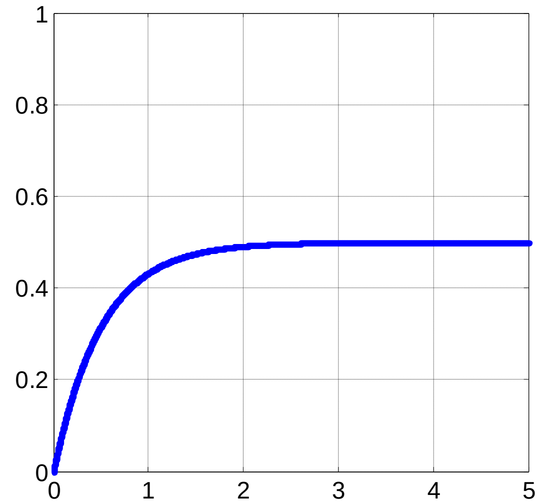
PID controllers

❑The following shows the response (Y=y(t)). As you can see, the response reach the value of 0.5 with e_{ss} equal 50%



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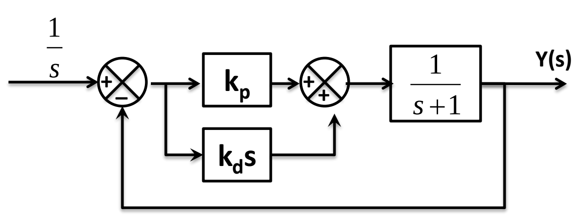
PID controllers

❑Adding PD-controller



$$e_{ss} = \frac{R}{1+\bar{k}_p} = \frac{1}{1+\bar{k}_p}; \bar{k}_p = \lim_{s\to 0} G(s)G_c(s) = \lim_{s\to 0}\left[\frac{1}{s+1}(k_p + k_d s)\right] = k_p + 0$$

$$\Rightarrow e_{ss} = \frac{1}{1+k_p}$$

❑Note that e_{ss} is independent of the differential controller and because of that the differential controller can not work alone

Automatic Control

PID controllers

❑Add PD-controller

$$G(s)=\frac{k_p+k_d s}{s+1}$$

$$M(s)=\frac{G}{1+GH}=\frac{G}{1+G}=\frac{\frac{k_p+k_d s}{s+a}}{1+\frac{k_p+k_d s}{s+a}}=1-\frac{s+1}{k_p+s+k_d s+1}$$

$$Y(s)=R(s)M(s)=\left(\frac{1}{s}\right)\left(1-\frac{s+1}{k_p+s+k_d s+1}\right)=\frac{1}{s}-\frac{s+1}{k_p s+s^2+k_d s^2+s}$$

$$y(t)=\ell^{-1}[Y(s)]=\frac{k_p}{k_p+1}+\frac{k_d-k_p}{e^{\left[\frac{(k_p+1)}{(k_d+1)}t\right]}(k_p+1)(k_d+1)}$$

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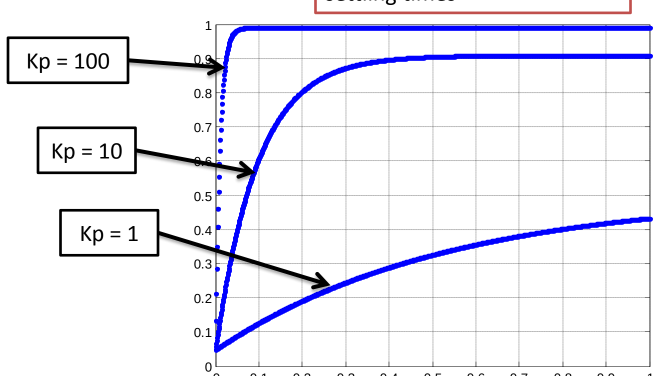
PID controllers

❑Add PD controller

❑Gc = K_p + K_d(s)

❑For Kp=1, 10 and 100 and kd = 0.05

❑As you can see, increasing the preoperational effect decreases the steady state error and both rising and settling times



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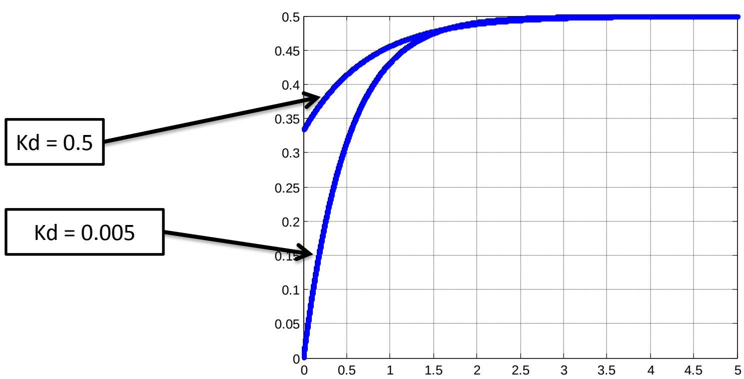
PID controllers

- Add PD controller
- $G_c = K_p + K_d(s)$
- For Kd=0.5,and 0.005and kp = 100

- As you can see, decreasing the differential effect decreases both rising and settling times but does not improve e_{ss}.



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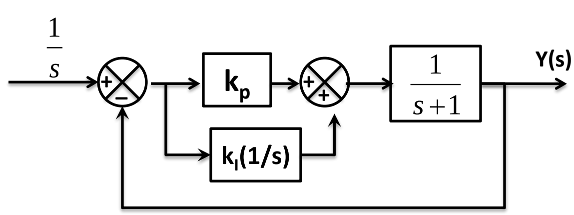
PID controllers

- Adding PI-controller



$$e_{ss} = \frac{R}{1+\bar{k}_p} = \frac{1}{1+\bar{k}_p}; \bar{k}_p = \lim_{s\to 0} G(s)G_c(s) = \lim_{s\to 0}\left[\frac{1}{s+1}\left(k_p + \frac{k_I}{s}\right)\right] = k_p + \frac{k_I}{0} = \infty$$

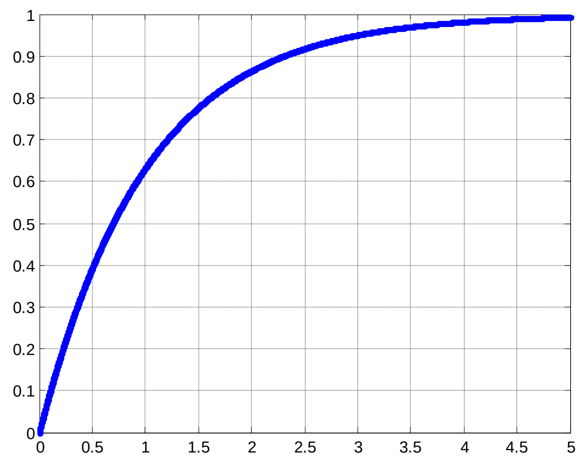
$$\Rightarrow e_{ss} = \frac{1}{1+\infty} = 0$$

- Note that e_{ss} eventually goes to zero because of adding integrator controller
- In the next example you will notice that the response eventually reach the reference input

Automatic Control

PID controllers

- Add PI controller
- $Gc = K_p + K_I(1/s)$
- For Kp=1 and kI = 1



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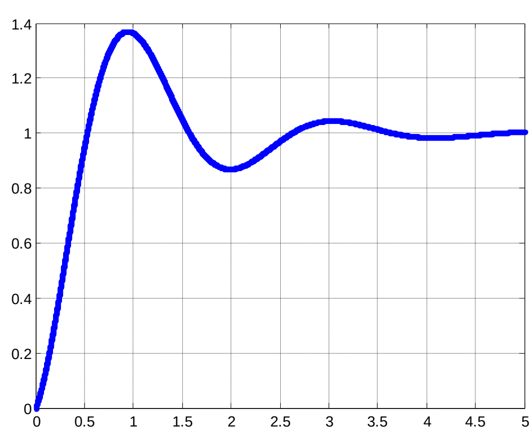
PID controllers

- Add PI controller
- $Gc = K_p + K_I(1/s)$

- For Kp=1 and kI = 10



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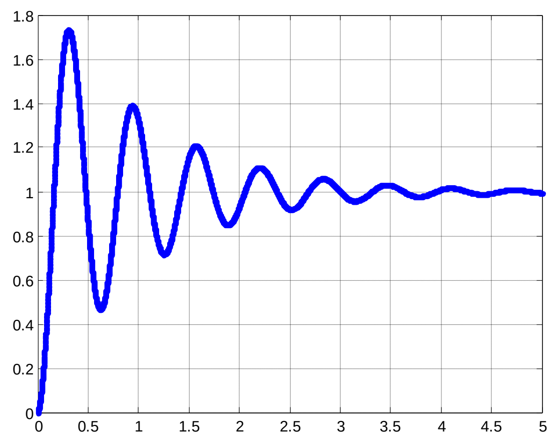
PID controllers

- ❑Add PI controller
- ❑Gc = $K_p + K_I(1/s)$
- ❑For Kp=1 and kI = 100



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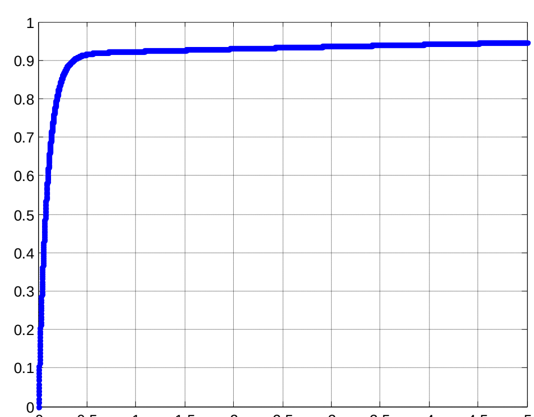
PID controllers

- ❑Add PI controller
- ❑Gc = $K_p + K_I(1/s)$
- ❑For Kp=10 and kI = 1



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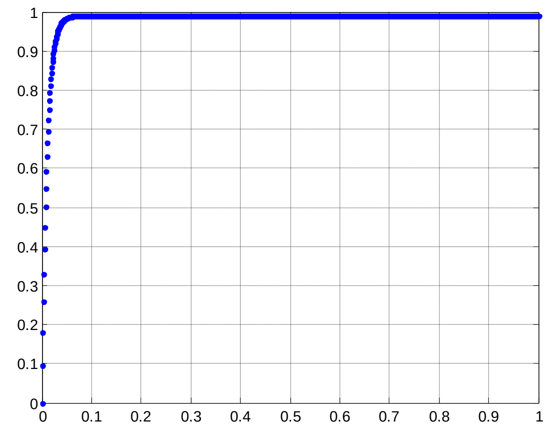
PID controllers

- ❑Add PI controller
- ❑Gc = $K_p + K_I(1/s)$
- ❑For Kp=100 and kI = 1



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PID controllers

- ❑Add PI controller
- ❑Notes:
- ❑Adding the integrator controller reduce the e_{ss}.
- ❑Increasing the integrator, increases the fluctuating in the response signal
- ❑As in PD controller, the proportional effect is the same (increasing the proportional effect reduce both settling and rise times)

- ❑You can now try the PID combination.
- ❑The following is a Matlab program can help you

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PID controllers

❑Matlab program

```
clear
clc
% define the s-domaine
syms s x
% define the time span
ti=input('enter the vaule of time span starting point =  ');
to=input('enter the vaule of time span enfing point =  ');
T=[ti:0.001:to];

% define system transfer function
G=(1/(s+1));

%define the PID controller
% put the value 0 if the controller is not involved
kp=input('enter the vaule of Kp = ');
kd=input('enter the vaule of Kd = ');
ki=input('enter the vaule of KI = ');

% define the new system tarnsfer function
Gc=kp+(kd*s)+(ki/s);
G=G*Gc;

% define the input in s-domain
R=1/s;

```

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PID controllers

❑Matlab program

```
%calculate the system transfer function M(s)
M=G/(1+G);
Y=M*R;
Y=simplify(Y);

%Laplace Inverse
y=ilaplace(Y,s);
pretty(y)

%poltting the responce
y=subs(y,s,T);
plot(T,y,'.','markers',12)
grid
```