Curve fitting and Interpolation

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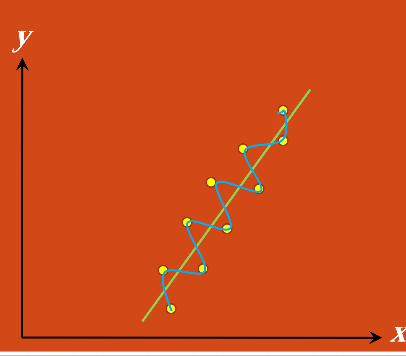
Curve fitting and Interpolation

Curve fitting

***What is* Curve fitting**

- Curve fitting is the process where a function is used to describe a set of points pairs: (x_1,y_1), (x_2,y_2),…. (x_n,y_n).
- When we have the case of precise points, the fitting is called **interpolation polynomial.**
- When we have the case of imprecise points, the fitting is called **regression.**



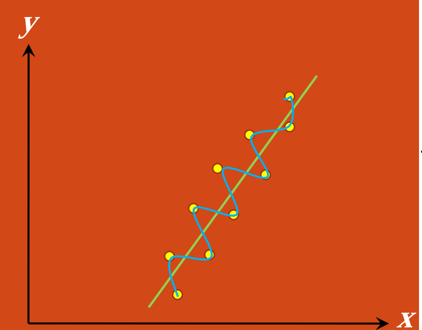
Curve fitting and Interpolation

Interpolation

Methods of interpolation

❖Is used to estimate the value between the points and we use a polynomial: $F(x)=a_0+a_1x+a_2x^2+\ldots+a_nx^n$ for (n+1) data points $(x_0,f(x_0))$, $x_1,f(x_1)$, …., $x_n,f(x_n)$.

First method is Newton divided difference interpolating polynomial.



Curve fitting and Interpolation

Interpolation

Newton divided difference interpolating polynomial

❖General formula

$$F_n(x)=b_0+b_1(x-x_0)+\ldots+b_n(x-x_0)(x-x_1)\ldots(x-x_{n-1})$$
$$b_0=F(x_0)$$
$$b_1=F[x_1,x_0]$$
$$b_n=F[x_n,x_{n-1},\ldots,x_1,x_0]$$

❖Where the bracket functions F[x] are the finite divided difference:

$$F[x_i,x_j]=\frac{F(x_i)-F(x_j)}{x_i-x_j}$$
$$F[x_i,x_j,x_k]=\frac{F[x_i,x_j]-F[x_j,x_k]}{x_i-x_k}$$
$$F[x_n,x_{n-1},\ldots,x_1,x_0]=\frac{F[x_n,x_{n-1},\ldots,x_1]-F[x_{n-1},x_{n-2},\ldots,x_0]}{x_n-x_0}$$

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Interpolation formulas

$$F_1(x)=F(x_0)+\frac{F(x_1)-F(x_0)}{x_1-x_0}(x-x_0)$$

Linear interpolation

$$F_2(x)=b_0+b_1(x-x_0)+b_2(x-x_0)(x-x_1)$$

$$\Downarrow$$

$$F_2(x)=a_0+a_1x+a_2x^2$$

$$a_0=b_0-b_1x_0+b_2x_0x_1$$

$$a_1=b_1-b_2x_0+b_2x_1$$

$$a_0=b_2$$

Quadratic interpolation

$$b_0=F(x_0)$$

$$b_1=\frac{F(x_1)-F(x_0)}{x_1-x_0}$$

$$b_1=\frac{\frac{F(x_2)-F(x_1)}{x_2-x_1}-\frac{F(x_1)-F(x_0)}{x_1-x_0}}{x_2-x_0}$$

Curve fitting and Interpolation

Linear Interpolation

Example

Find the natural logarithmic at 2 (*ln* (2)) if you now that *ln(1)* = 0 and *ln*(5)= 1.6094. use the linear interpolation Newton formula. In hence your calculations by reducing the interval to be between x=1 and x=3. the true value of *ln* (2) = 0.6931

Solution:

$$F_1(x)=F(x_0)+\frac{F(x_1)-F(x_0)}{x_1-x_0}(x-x_0)$$

$$F_1(2)=F(1)+\frac{F(5)-F(1)}{5-1}(2-1)$$

$$F_1(2)=0+\frac{1.6094-0}{5-1}(2-1)=0.401225$$

$$\varepsilon_a=42.11\%$$

$$F_1(2)=F(1)+\frac{F(3)-F(1)}{3-1}(2-1)$$

$$F_1(2)=0+\frac{1.0986-0}{3-1}(2-1)=0.5493$$

$$\varepsilon_a=20.75\%$$

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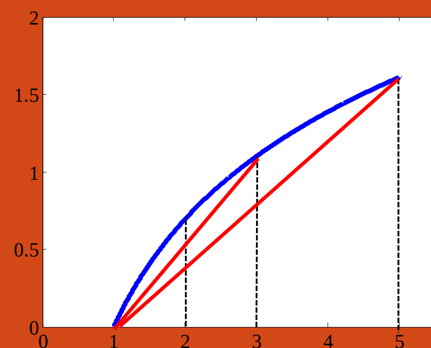
Linear Interpolation

Example

Graphical representation



Curve fitting and Interpolation

Quadratic Interpolation

Example

Find the quadratic interpolation polynomial for the following data points

i	x	F(x)
0	1	0
1	4	1.386294
2	6	1.791759

Solution:

$$b_0 = 0$$

$$b_1 = \frac{F(4) - F(1)}{4-1} = \frac{1.386294 - 0}{4-1} = 0.4620981$$

$$b_1 = \frac{\frac{1.791759 - 1.386294}{6-4} - \frac{1.386294 - 0}{4-1}}{6-1} = -0.0518731$$

$$F_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

$$F_2(x) = 0 + 0.4620981(x-1) - 0.0518731(x-1)(x-4)$$

$$F_2(x) = 0.7215x - 0.6696 - 0.0519x^2$$

Curve fitting and Interpolation

Interpolation

Lagrange interpolating polynomial

❖**General formula**

$$F_n(x)=\sum_{i=0}^{n} L_i(x)F(x_i)$$

$$L_i(x)=\prod_{\substack{j=0\\ j\neq i}}^{n}\frac{x-x_j}{x_i-x_j}$$

$$F_1(x)=\frac{x-x_1}{x_0-x_1}F(x_0)+\frac{x-x_0}{x_1-x_0}F(x_1)$$

$$F_2(x)=\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}F(x_0)+\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}F(x_1)+\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}F(x_2)$$

Curve fitting and Interpolation

Quadratic Interpolation

Lagrange interpolating polynomial

Find the Lagrange 1st and 2nd order interpolation polynomials for the following data points

i	x	F(x)
0	1	0
1	4	1.386294
2	6	1.791759

Solution:

$$x_0=1, x_1=4, x_2=6$$

$$F_1(x)=\frac{x-4}{1-4}0+\frac{x-1}{4-1}1.386294=0.4621x-0.4621$$

$$F_2(x)=\frac{(x-4)(x-6)}{(1-4)(1-6)}0+\frac{(x-1)(x-6)}{(4-1)(4-6)}1.386294+\frac{(x-1)(x-4)}{(6-1)(6-4)}1.791759$$

$$F_2(x)=-0.0519x^2+\ 0.7215x-0.6696$$