6-links mechanism full force analysis

Example 1:

Consider the mechanism sown in Fig.1:

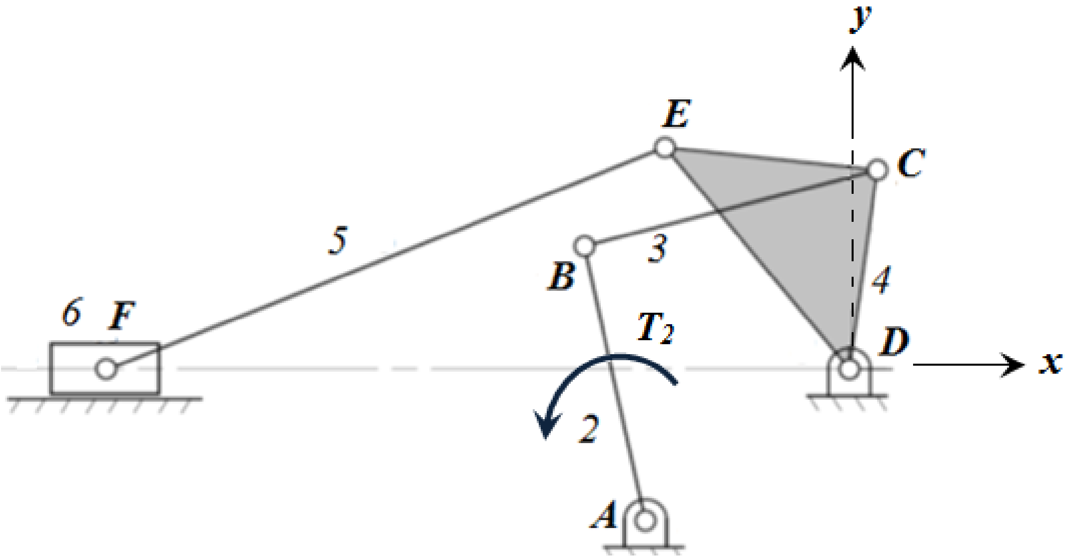


Fig.1. a 6-bar and slider mechanism

If we add an electrical motor of torque equal T_2 at link 2, perform a force analysis to find the force produced at link 6 and all the reaction forces at the joints A, B, C, D, E and F. assume no friction between the slider and the ground.

Solution:

Assumptions:

a. Link 2 is the input and so θ_2, ω_2, α_2 and T_2 are knowns.
b. Links 2, 3 and 5 are massless.
c. In force analysis, the forces to the right and the CCW moments are positives and the left forces and the CW moments are negatives.

Solution plan:

1. Name all the vectors and fixed geometry parameters.
2. Draw the loop closure equations.
3. Perform position analysis to determine the position variables of all links
4. Perform velocity analysis to determine the velocity variables of all links
5. Perform acceleration analysis to determine the acceleration variables of all links
6. Draw free body diagram for each link.

7. Perform static analysis for links 2, 3 and 5 and dynamic analysis for links 4 and 6.

Loop closure equations

The links vectors are drawn in Fig.2 and note the angle β (<EDC) added to the system description.

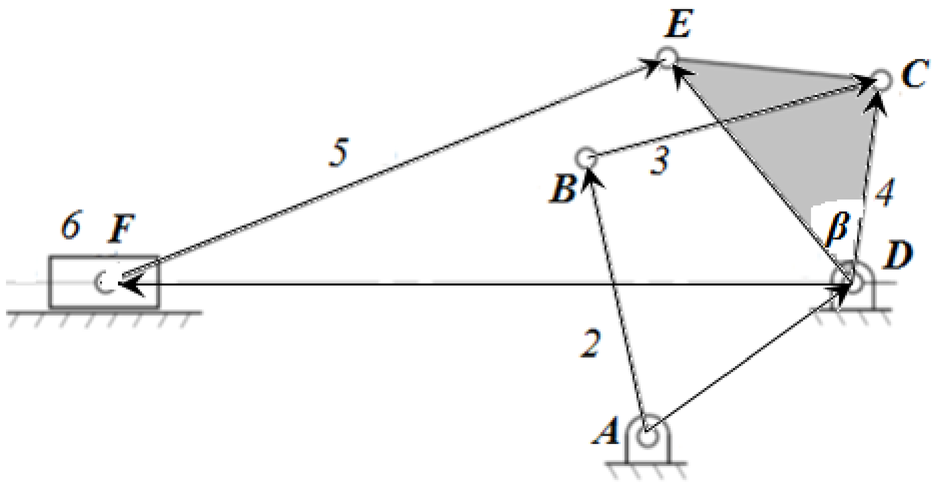


Fig.2. the proposed vectors loop closures

To simplify the drawing, Fig 3 show the two loops which are under investigation.

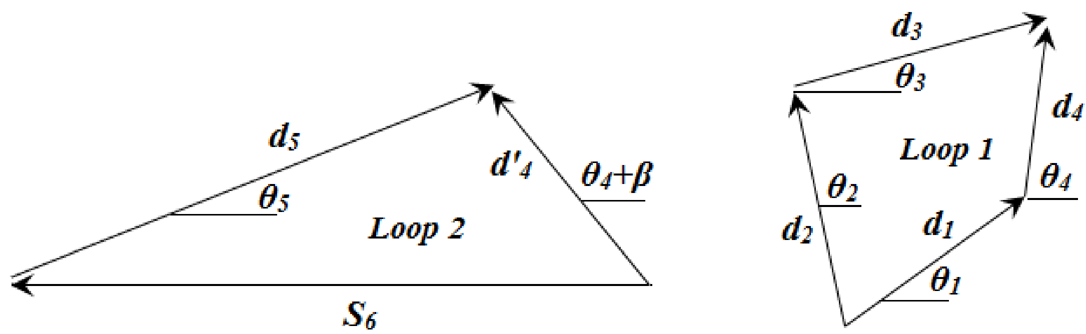


Fig.3. dividing the mechanism into two loops

Loop closure equation for loop 1:

$$d_2 U_{\theta 2} + d_3 U_{\theta 3} = d_1 U_{\theta 1} + d_4 U_{\theta 4} \quad (1)$$

Loop closure equation for loop 2:

$$d_4' U_{\theta 4+\beta} = S_6 U_{180} + d_5 U_{\theta 5} \quad (2)$$

Position analysis:

Loop1:

Loop 1 is for a four bar mechanism and so the angel θ_4 can be found as:

$$\theta_{4-1,2} = 2\tan^{-1}(\Phi_{1,2}) \tag{3}$$

Where:

$$\Phi_{1,2} = \frac{-b \pm \sqrt{b^2 - (c+a)^2}}{c-a}$$

And

$$a = 2d_1d_4\cos(\theta_1) - 2d_2d_4\cos(\theta_2)$$

$$b = 2d_1d_4\sin(\theta_1) - 2d_2d_4\sin(\theta_2)$$

$$c = d_1^2 + d_2^2 + d_4^2 - d_3^2 - 2d_1d_2\cos(\theta_1 - \theta_2)$$

And θ_3 can be found as

$$\theta_{3-1,2} = \tan^{-1}\left[\frac{d_1\sin(\theta_1) + d_4\sin(\theta_{4-1,2}) - d_2\sin(\theta_2)}{d_1\cos(\theta_1) + d_4\cos(\theta_{4-1,2}) - d_2\cos(\theta_2)}\right] \tag{4}$$

Note: as we assume the angels C.C.W, then the positive angles are considered for the further analysis.

Loop 2:

Loop 2 is for slider crank mechanism.

To find S_6, rearrange the equation as shown in Eq.5:

$$d_4' U_{\theta 4+\beta} - S_6 U_{180} = d_5 U_{\theta 5} \tag{5}$$

Then, square Eq.5 to eliminate θ_5 from the equation:

$$d_4'^2 - 2d_4' S_6\cos(\theta_4 + \beta - 180) + S_6^2 = d_5^2 \tag{6}$$

Solve Eq.6 to find S_6:

$$S_{6_{1,2}} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \tag{7}$$

Where:

$$A = 1$$

$$B = -2d_4'\cos(\theta_4 + \beta - 180) = 2d_4'\cos(\theta_4 + \beta)$$

$$C = d_4'^2 - d_5^2$$

To find θ_5 , dot product Eq.5 by U_{90} and eliminate S_6:

$$d_4'\cos(\theta_4 + \beta - 90) - 0 = d_5\cos(\theta_5 - 90) = d_5\sin(\theta_5)$$

$$\Rightarrow \theta_5 = \sin^{-1}\left[\frac{d_4'\sin(\theta_4 + \beta)}{d_5}\right] \tag{8}$$

Velocity analysis

Loop1:

For the Four-bar mechanism, the velocities ω3 and ω4 can be found as:

$$\omega_3 = -\frac{d_2\omega_2 \sin(\theta_4 - \theta_2)}{d_3 \sin(\theta_4 - \theta_3)} \tag{8}$$

$$\omega_4 = \frac{d_2\omega_2 \sin(\theta_3 - \theta_2)}{d_4 \sin(\theta_3 - \theta_4)} \tag{9}$$

Loop2:

To find the velocities of links 5 and 6, take the derivative of Eq.5. with respect to time:

$$d_4'\omega_4 \dot{U}_{\theta 4+\beta} - \dot{S}_6 U_{180} = d_5\omega_5 \dot{U}_{\theta 5} \tag{10}$$

To find ω_5, dot product Eq.10 by $\dot{U}_{180}$ and rearrange the terms as shown in Eq.11:

$$\begin{aligned} & d_4'\omega_4 \cos(\theta_4 + \beta - 180) - 0 = d_5\omega_5 \cos(\theta_5 - 180) = -d_5\omega_5 \cos(\theta_5) \\ & \Rightarrow \omega_5 = \frac{d_4'\omega_4 \cos(\theta_4 + \beta)}{d_5 \cos(\theta_5)} \end{aligned} \tag{11}$$

To find $\dot{S}_6$ dot product Eq.10 by $U_{\theta 5}$ and rearrange the terms as shown in Eq.12:

$$\begin{aligned} & d_4'\omega_4 \sin(\theta_5 - \theta_4 - \beta) - \dot{S}_6 \cos(\theta_5 - 180) = 0 \\ & \Rightarrow \dot{S}_6 = \frac{d_4'\omega_4 \sin(\theta_5 - \theta_4 - \beta)}{\cos(\theta_5 - 180)} \end{aligned} \tag{12}$$

Acceleration analysis

Loop1:

For the Four-bar mechanism, the accelerations α_3 and α_4 can be found as:

$$\alpha_3 = \frac{-d_2\alpha_2 \sin(\theta_4 - \theta_2) + d_2\omega_2^2 \cos(\theta_2 - \theta_4) + d_3\omega_3^2 \cos(\theta_3 - \theta_4) - d_4\omega_4^2}{d_3 \sin(\theta_4 - \theta_3)} \tag{13}$$

$$\alpha_4 = \frac{1}{d_4 \sin(\theta_3 - \theta_4)} \begin{bmatrix} d_2\alpha_2 \sin(\theta_3 - \theta_2) - d_2\omega_2^2 \cos(\theta_2 - \theta_3) - d_3\omega_3^2 \\ + d_4\omega_4^2 \cos(\theta_4 - \theta_3) \end{bmatrix} \tag{14}$$

Loop2:

To find the accelerations of links 5 and 6, take the derivative of Eq.10. with respect to time:

$$d_4'\left[\alpha_4 \dot{U}_{\theta_4+\beta} - \omega_4^2 U_{\theta_4+\beta}\right] - \ddot{S}_6 U_{180} = d_5\left[\alpha_5 \dot{U}_{\theta 5} - \omega_5^2 U_{\theta 5}\right] \tag{15}$$

To find ω5, dot product Eq.15 by $\dot{U}_{180}$ and rearrange the terms as shown in Eq.16:

$$\begin{aligned} & d_4'\left[\alpha_4 \cos(\theta_4 + \beta - 180) - \omega_4^2 \sin(\theta_4 + \beta - 180)\right] \\ & = d_5\left[\alpha_5 \cos(\theta_5 - 180) - \omega_5^2 \sin(\theta_5 - 180)\right] \\ & \Rightarrow \alpha_5 = \frac{d_4'\left[\alpha_4 \cos(\theta_4 + \beta - 180) - \omega_4^2 \sin(\theta_4 + \beta - 180)\right] + d_5\omega_5^2 \sin(\theta_5 - 180)}{d_5 \cos(\theta_5 - 180)} \end{aligned} \tag{16}$$

To find $\ddot{S}_6$ dot product Eq.15 by $U_{\theta 5}$ and rearrange the terms as shown in Eq.17:

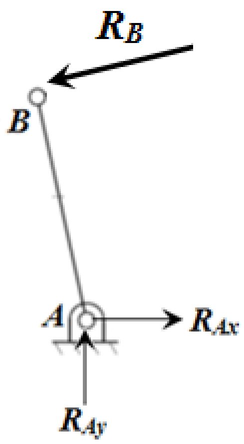
$$d_4'\left[\alpha_4 \sin(\theta_5 - \theta_4 - \beta) - \omega_4^2 \cos(\theta_5 - \theta_4 - \beta)\right] - \ddot{S}_6 \cos[\theta_5 - 180] = -d_5\omega_5^2$$
$$\Rightarrow \ddot{S}_6 = \frac{d_4'\left[\alpha_4 \sin(\theta_5 - \theta_4 - \beta) - \omega_4^2 \cos(\theta_5 - \theta_4 - \beta)\right] + d_5\omega_5^2}{\cos[\theta_5 - 180]} \qquad (17)$$

Force analysis

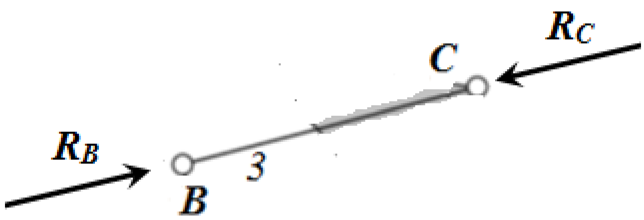
To obtain the force transformed from Link 2 to Link 6 and all the joints reactions, we have to draw a free body diagram for each link and apply Newton's 2nd law.

First, we must name a reference point (i.e. the origin of the Cartesian system) for the position vectors of links masses. Because the only links that have considerable masses are 4 and 6, joint D can be assumed the origin point as shown in Fig.1. and because all joints are pin joints, the number of reactions at each joint can be assumed as 2: one in x-direction and the other in y-direction.

Link 2: neglected mass → static analysis

Newton's 2nd law	**F.B.D**
$\sum F_x = 0 \Rightarrow R_{Ax} - R_B \cos(\theta_3) = 0 --- (18)$ $\sum F_y = 0 \Rightarrow R_{Ay} - R_B \sin(\theta_3) = 0 --- (19)$ $\sum M_A = 0$ $\Rightarrow T_2 + R_B \cos(\theta_3)(d_2)\sin(\theta_2) + R_B \sin(\theta_3)(d_2)\cos(\theta_2) = 0 --- (20)$ **Solve equations 18, 19 and 20 to find R_{Ax}, R_{ay} and R_B**	

Link 3: neglected mass → static analysis

Newton's 2nd law	**F.B.D**
$R_B - R_C = 0 --- (21)$ As you can see from Eq.21 $R_B = R_C$ And so, R_C is now known	

Link 4: dynamic analysis

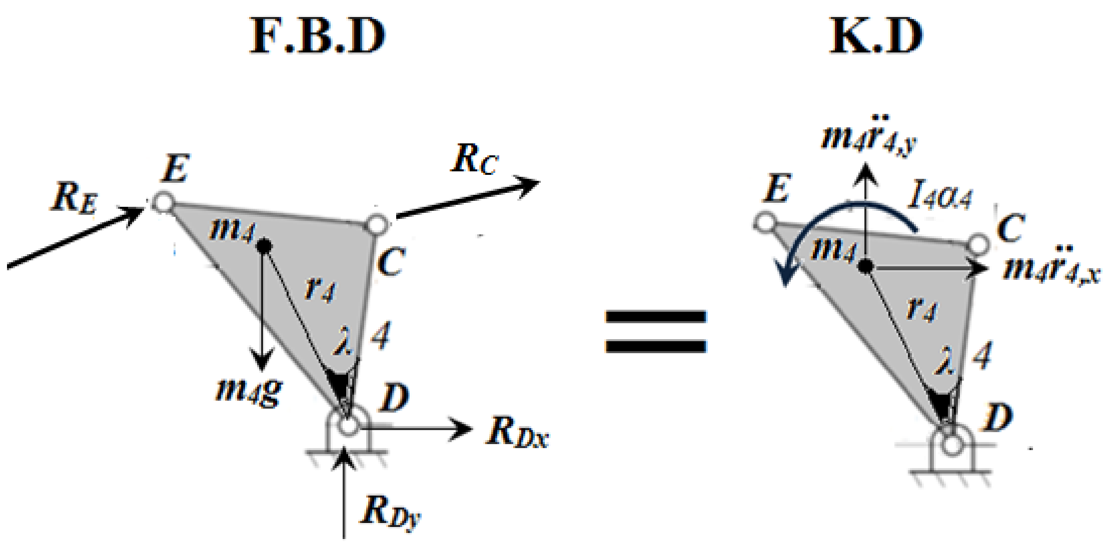


Fig.4. free body diagram and kinetic diagram of link 4

From Fig.4:-

$$R_C\cos(\theta_3)+R_{Dx}+R_E\cos(\theta_5)=m_4\ddot{r}_{4,x}\ ---(22)$$

$$R_C\sin(\theta_3)+R_{Dy}+R_E\sin(\theta_5)-m_4g=m_4\ddot{r}_{4,y}\ ---(23)$$

$$-R_C\cos(\theta_3)(d_4\sin(\theta_4))+R_C\sin(\theta_3)(d_4\cos(\theta_4))-R_{Ex}(d'_4\sin(180-\theta_4-\beta_4))$$
$$-R_{Ey}(d'_4\cos(180-\theta_4-\beta_4))+m_4g(r_4\cos(180-\theta_4-\lambda))=I_4\alpha_4-m_4\ddot{r}_{4,x}(r_4\sin(180-\theta_4-\lambda))$$
$$-m_4\ddot{r}_{4,y}(r_4\cos(180-\theta_4-\lambda))---(24)$$

where:

$$\vec{\mathbf{r}}_4=r_4U_{\theta_4+\lambda}$$

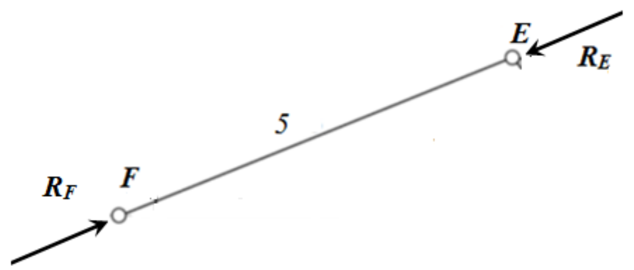
$$\ddot{\vec{\mathbf{r}}}_2=r_4\alpha_4\dot{U}_{\theta_4+\lambda}-r_4\omega_4^2U_{\theta_4+\lambda}$$

$$\ddot{\mathbf{r}}_{4,x}=-r_4\alpha_4\sin(\theta_4+\lambda)-r_4\omega_4^2\cos(\theta_4+\lambda)$$

$$\ddot{\mathbf{r}}_{4,y}=r_4\alpha_4\cos(\theta_4+\lambda)-r_4\omega_4^2\sin(\theta_4+\lambda)$$

Solve equations 22, 23 and 24 to find R_{Dx}, R_{Dy} and R_E.

Link 5: neglected mass → static analysis

Newton's 2nd law	F.B.D
$R_E=R_F\ ---(25)$ As you can see from Eq.25 $R_E = R_F$ And so, R_F is now known	

Link 6: dynamic analysis

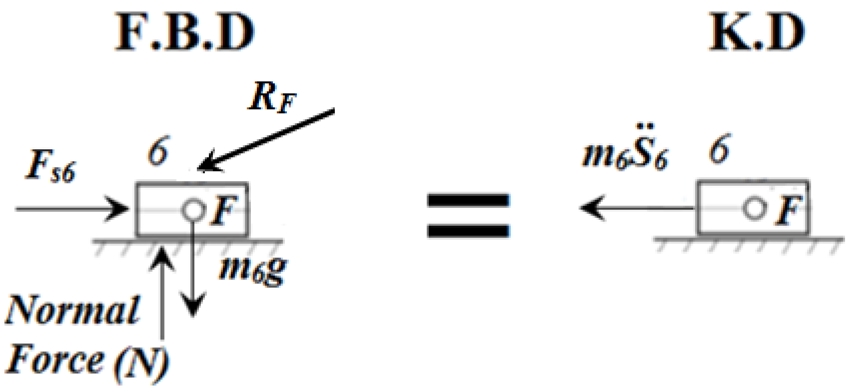


Fig.5. free body diagram and kinetic diagram of link 6

$$R_F\cos(\theta_5) - F_{S6} = m_6\ddot{S}_6 \; --- (26)$$

$$N - R_F\sin(\theta_5) - m_6 g = 0 \; --- (27)$$

Finally, solve Eq.26 to find F_{S6} and solve Eq.27 to find the normal force N.

Summary.

This mechanism is used to transfer the power form the crank (link2) to the slider (link6). The process is done through the reaction forces (R_B, R_C, R_E and R_F) which is shown from the equation where we found that R_B equal R_C and R_E equal R_F.

Equations 18-27 show that we need to perform position, velocity and acceleration analysis before deriving the static and dynamic equilibrium equations.

Finally, we can say that the output (F_{S6}) depends on the geometry of the mechanism and the input variables: θ_2, ω_2, α_2 and T_2.