Statics

Chapter Four

4.1. condition for equilibrium +FBD

By

Laith Batarseh

S t a t i c s

condition for equilibrium +FBD

In this lecture

- Introduce the conditions for rigid body equilibrium
- Learn how to draw FBD (reactions analysis)

condition for equilibrium +FBD

Internal, external forces and F.B.D

Internal forces are the forces caused by the interaction between body particles

External forces are the forces act on the outer surface of the body

Free – body diagram (F.B.D) shows only the external forces

Internal forces are opposite forces cancel each other

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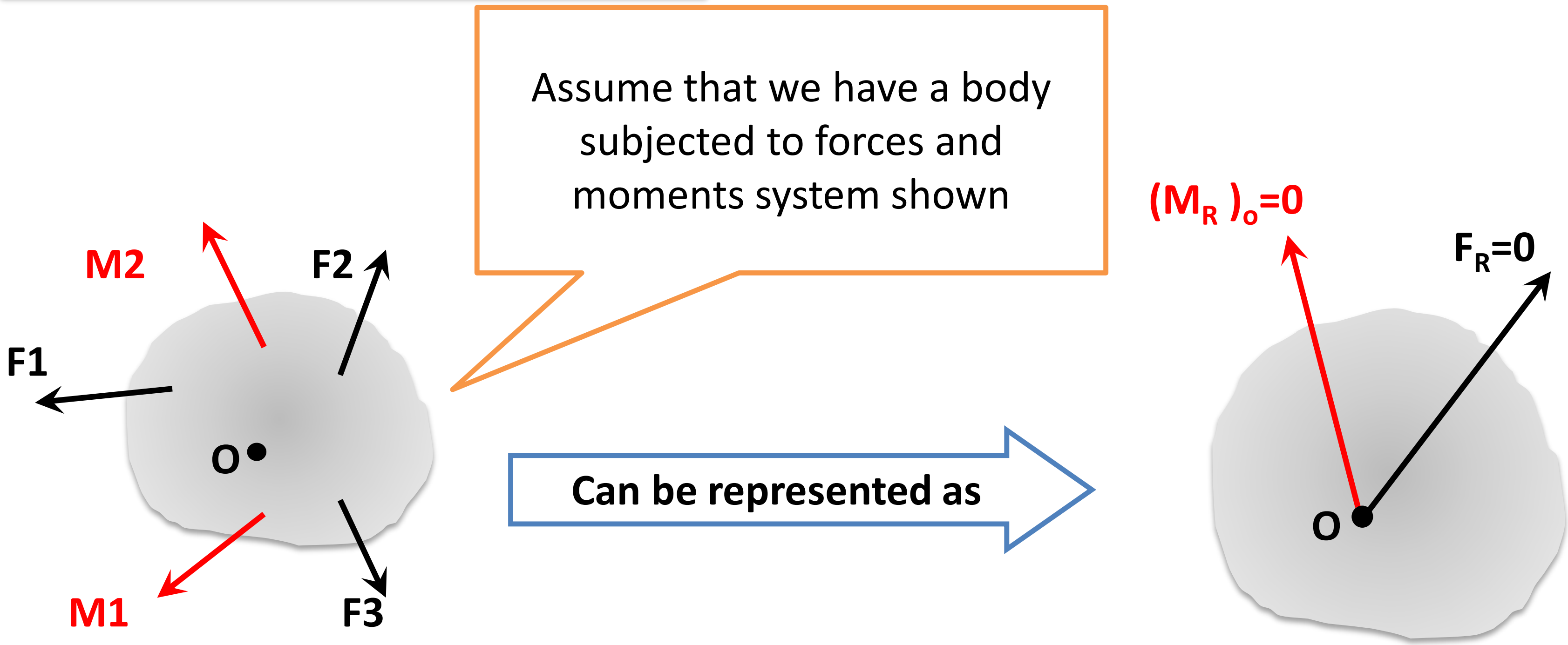
Equilibrium conditions



$$F_R = \sum F = 0 \text{ and } (M_R)_o = \sum M_o = 0$$

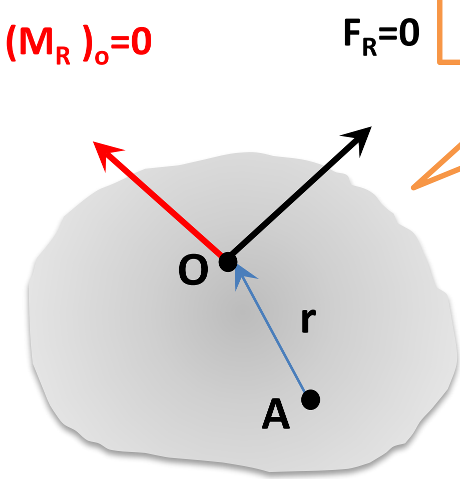
condition for equilibrium +FBD

Equilibrium conditions

$$F_R = \sum F = 0 \quad \text{and} \quad (M_R)_o = \sum M_o = 0$$

These equations are su … body

Assume that we have a body subjected to resultant force and moment system shown



$$M_A = r \times F_R + M_{R,O} = 0$$

Fixed, ≠ 0

Must = 0

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F.B.D

To apply the equilibrium conditions successfully, all the external forces acting on the body must be specified

Free body diagram is a way to represent these forces graphically.

F.B.D is a sketch of the outlined shape of the body isolated from its surrounding (i.e. connections)

All the forces and couple moments generated from the surrounding on the body must be drawn in the F.B.D.

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Support reactions

Reaction is a resistance force developed in the supporting points due to the application of external load (force)

The type of reaction in support depends on the type of motion prevented by the support.

Prevent translational motion →	**Reaction force**
Prevent rotational motion →	**Reaction moment**

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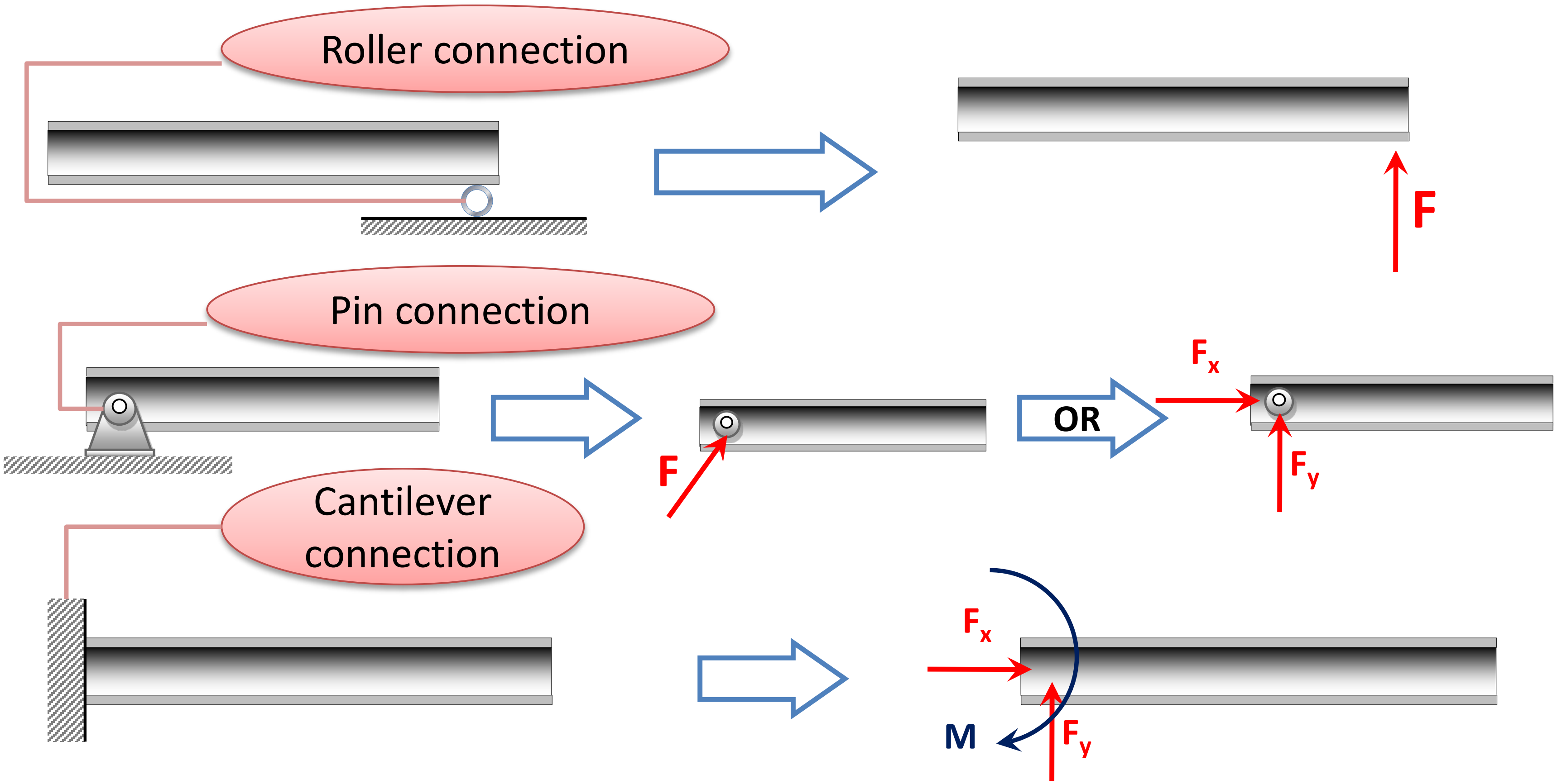
Examples

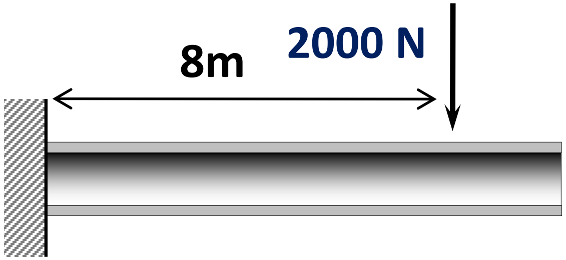


For more connection reactions, refer to Engineering Mechanics, Statics, 12th edition, R. C. Hibbeler, 2010, pp 202-203

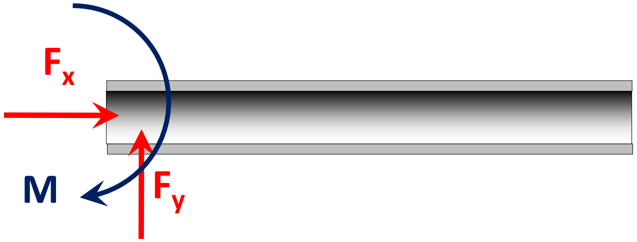
condition for equilibrium +FBD

Example [1]

Draw the F.B.D for the body shown in the figure



Solution



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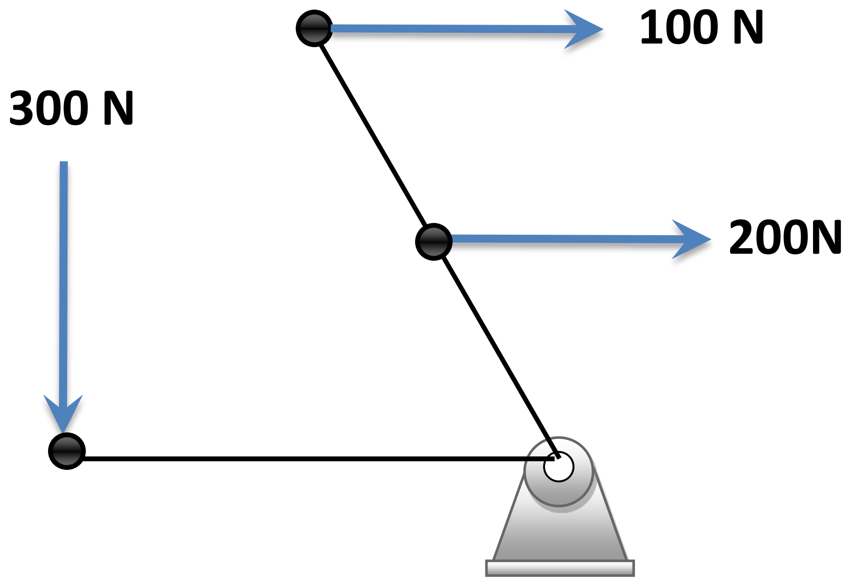
Example [2]

Draw the F.B.D for the body shown in the figure and find the reactions that make this system under equilibrium



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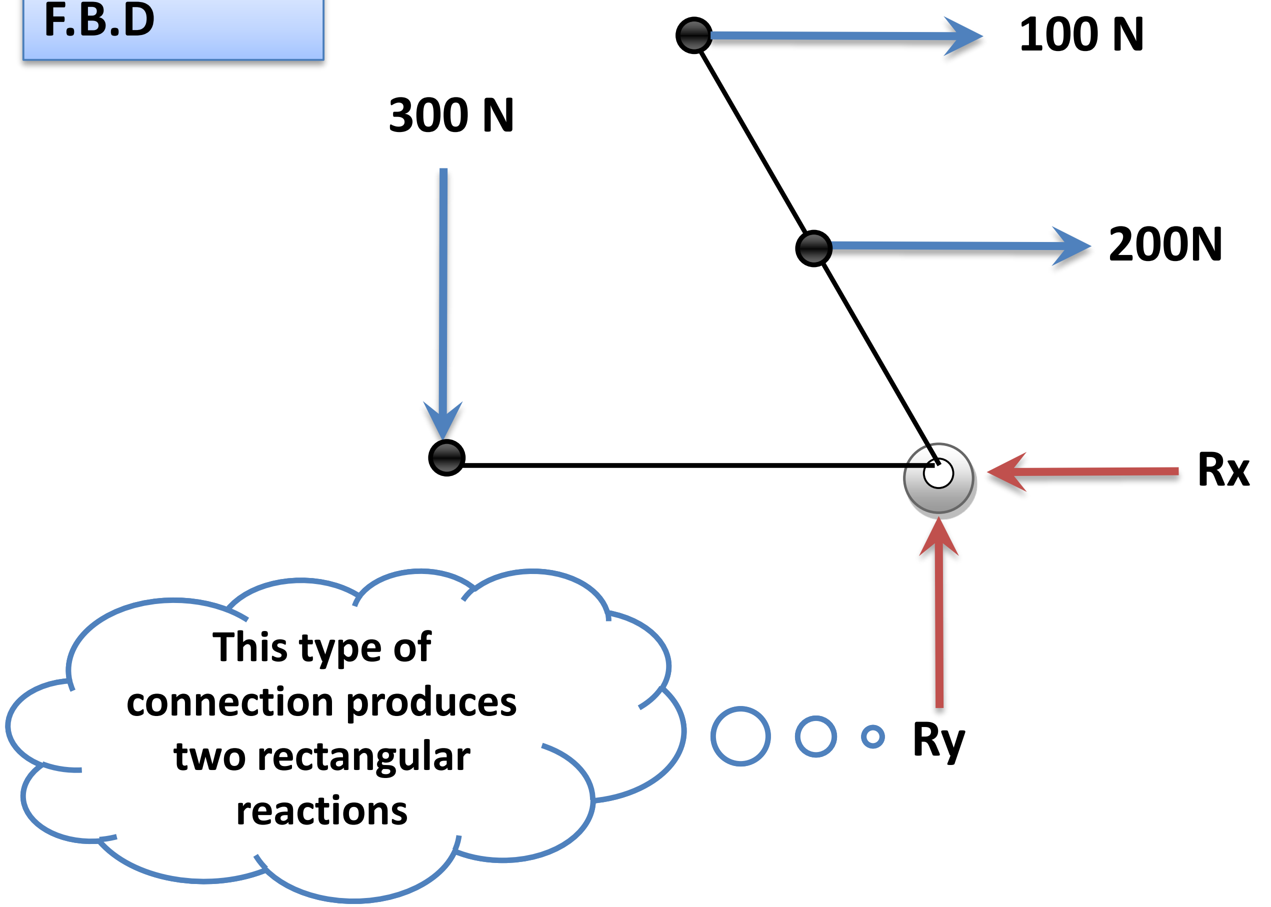
Example [2]

Solution

F.B.D



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Example [2]

Solution

Equilibrium conditions

$$F_x = 0;\ 100 + 200 - R_x = 0.\ \text{so, } R_x = -300\ \text{N}$$
$$F_y = 0;\ -300 + R_y = 0.\ \text{so, } R_y = 300\ \text{N}$$

As you see, the assumed directions are correct. In other cases where the sign associated with the reaction is opposite to the assumed direction then, the assumed direction must be reversed

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4.2. equations of equilibrium

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Equations of equilibrium

In this lecture

- ❑Learn how to use equilibrium equations to find unknown reactions or forces

Equations of equilibrium

Equations of equilibrium

as said previously, the body will be under equilibrium if the summation of both resultant forces and moments equal zero

Mathematically, $\sum F = \sum M = 0$

In planer forces (2D problems), the rectangular notation for forces is very useful

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M_o = 0$$

Equations of equilibrium

Alternative set of equilibrium equations

In some cases we can use only the equilibrium of forces equations or the moment equation only

The main criteria that determine the number of equations used to solve a problem is the number of the unknowns

Algebraically, to solve a certain number of unknowns, you need the same number of equations. For example, you can solve 4 equations with 4 unknowns

Equations of equilibrium

Statics

Analysis procedures

Establish F.B.D with all the acting forces and couple moments beside the reaction forces and moments shown clearly.

Use the equilibrium equations to find the unknown reactions, forces or moments required. These equations are $\sum F_x = \sum F_y = \sum M = 0$

If the assumed force or moment show an opposite sense (i.e. different sign from the originally assumed) the sense of force or moment must be reversed

It is preferred to use equilibrium moment equation at the reaction that have two unknowns to eliminate them

Equations of equilibrium

Example [1]

Draw the F.B.D for the body shown in Fig. 1 and find the reactions at both points A and B that make this system under equilibrium

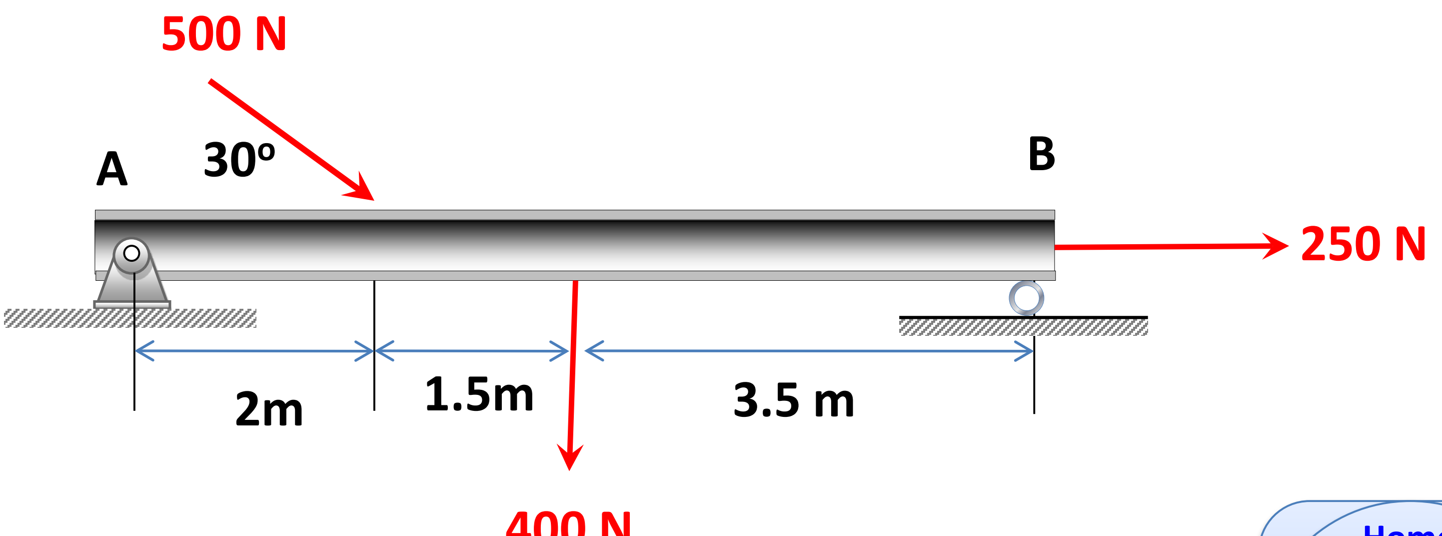

Fig.1

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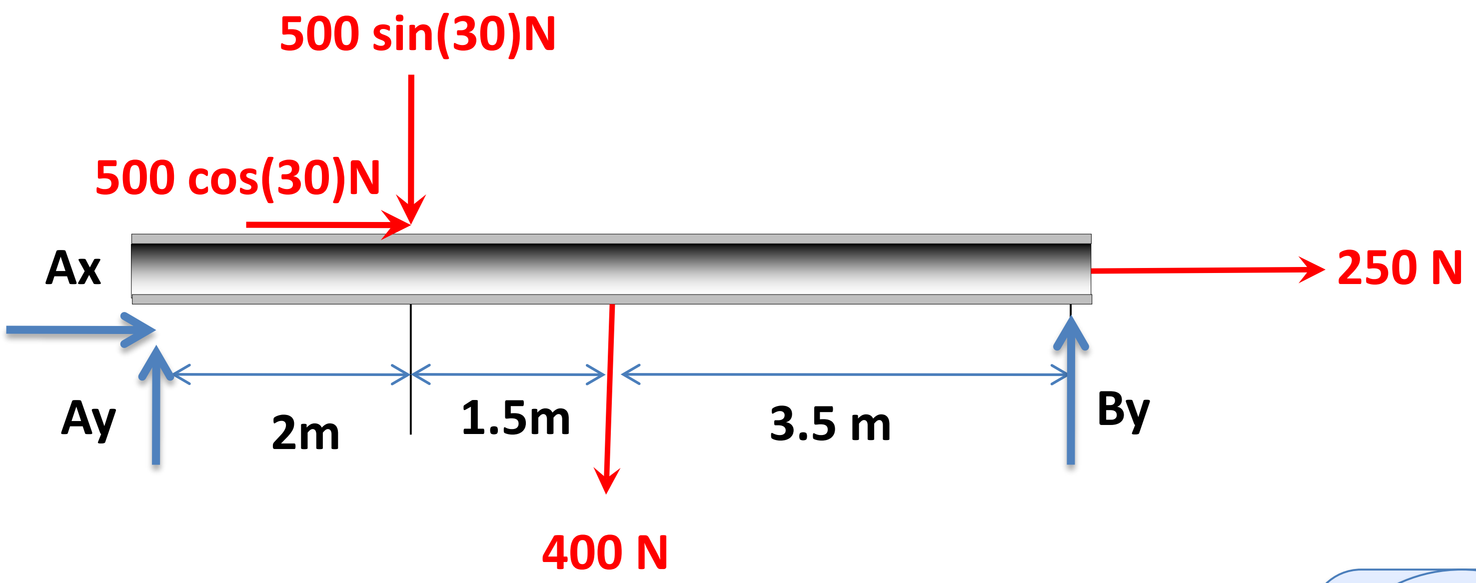
Equations of equilibrium

Example [1]

Solution

F.B.D



Equations of equilibrium

Example [1]

Solution

Equilibrium equations

$$\overset{\rightarrow}{+}\sum Fx = 0 \Rightarrow Ax + 500\cos(30) + 250 = 0 \Rightarrow Ax = -683N$$

$$+\uparrow \sum Fy = 0 \Rightarrow Ay + By - 500\sin(30) = 0 \Rightarrow Ay + By = 250N \text{ - - - } (1)$$

$$\sum M_A = 0 \Rightarrow -500\sin 30(2) - 400(3.5) + By(7) = 0 \Rightarrow By = 271N$$

Back to Eq.1

$$Ay + 271 - 500\sin(30) = 0 \Rightarrow Ay = 379N$$

Equations of equilibrium

Example [2]

Draw the F.B.D for the body shown in Fig.2 and find the reactions at the supporting points that make this system under equilibrium

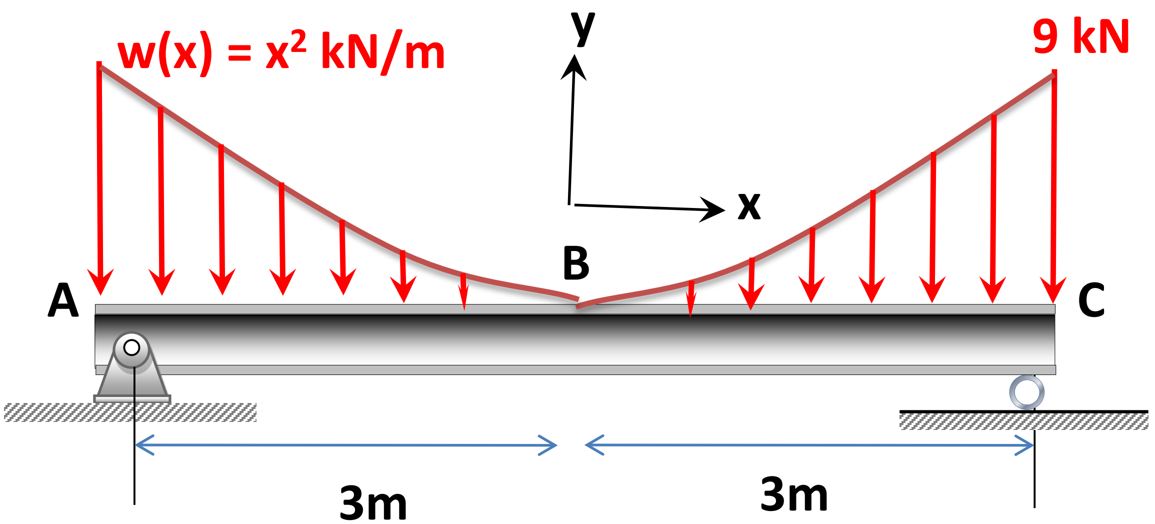


Fig.2

Equations of equilibrium

Example[2]

Solution

- First, We divide the distributed load w(x) into two identical regions: A→B and B→C. then, we must find the resultant force F_R for each region. By using the method of reduction distrusted loads that we learn previously, F_R can be found for region A→B as:

$$F_R = \int_0^3 x^2 .dx = \left. \frac{x^3}{3} \right|_0^3 = \frac{27}{3} = 9\,kN$$

- the location of F_R (x’) is found as:

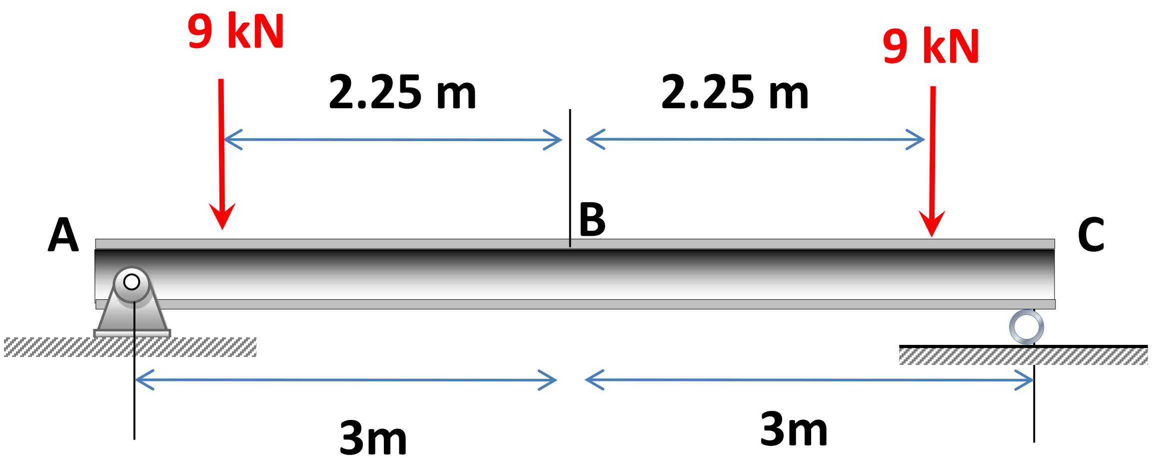
$$x' = \frac{\int_0^3 x\left(x^2\right).dx}{\int_0^3 \left(x^2\right).dx} = \frac{\left. \frac{x^4}{4} \right|_0^3}{9} = 2.25m$$

Equations of equilibrium

Example[2]

Solution

❑Due to the symmetry between the two regions (A→B and B→C), the reduction process of the distributed load acting on region A→B can be ,simply, duplicated to find the magnitude and the location of the resultant force acting on region B→C. now we can represent these forces on the beam:



Equations of equilibrium

Example[2]

Solution

❑After reducing the distributed loads, the time now is appropriate to draw F.B.D. The F.B.D is shown in Fig.3

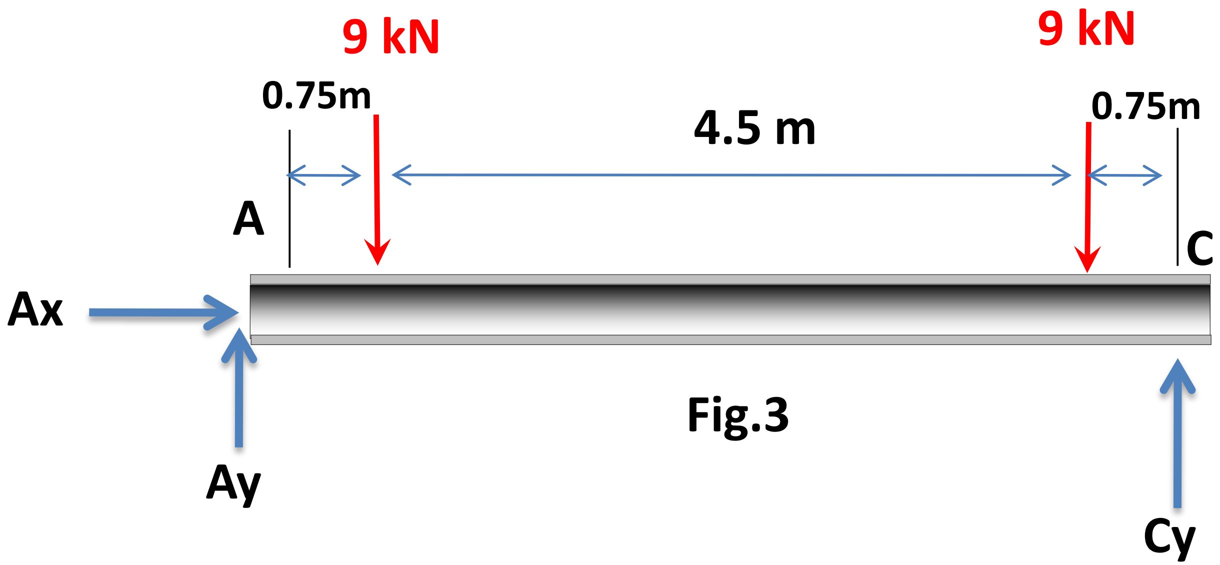


Fig.3

❑Again the direction of the reaction were assumed randomly.

Equations of equilibrium

Example[2]

Solution

- As you can see, there is no force acting on this body at x-direction. So, Ax = 0 and the summation of forces at y-direction yield to:

$$By + Ay = 9+9 = 18\ kN \text{ ---- } (1)$$

- applying the equilibrium equation about a certain point for example point A to find the reaction By:

$$\sum M_A = 0 \rightarrow (9)(4.50+0.75)+(9)(0.75) - By\ (6) \text{ --- } (2)$$

- Rearrange Eq.2 and solve for By: By = 9kN↑ **Ans**
- . Back to Eq.2, substitute the value of By to find Ay:

$$Ay + 9 = 18 \rightarrow Ay = 9N \ \textbf{Ans}$$

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Equations of equilibrium

Example[2]

Solution

You can note:

- Because of the symmetry in this problem, the reactions were identical. Such note can be generalized on other symmetrical system.

- You can use the momentum equation again to find Ay instead of using the force equation. This procedure can be used also to verify the solution