Statics

Chapter Four

4.3. two and three forces members

By

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Two and three forces members

Two – forces member

Fig. 1.a shows a member loaded by two forces (i.e. two forces member). To assume that this member is under equilibrium:

1. The summation of forces must equal zero. Therefore, $F_1=F_2$.
2. The summation of moments produced by these forces, therefore, the lines of action for both forces must be the same.

Fig 1.b illustrates the condition of equilibrium for two forces members

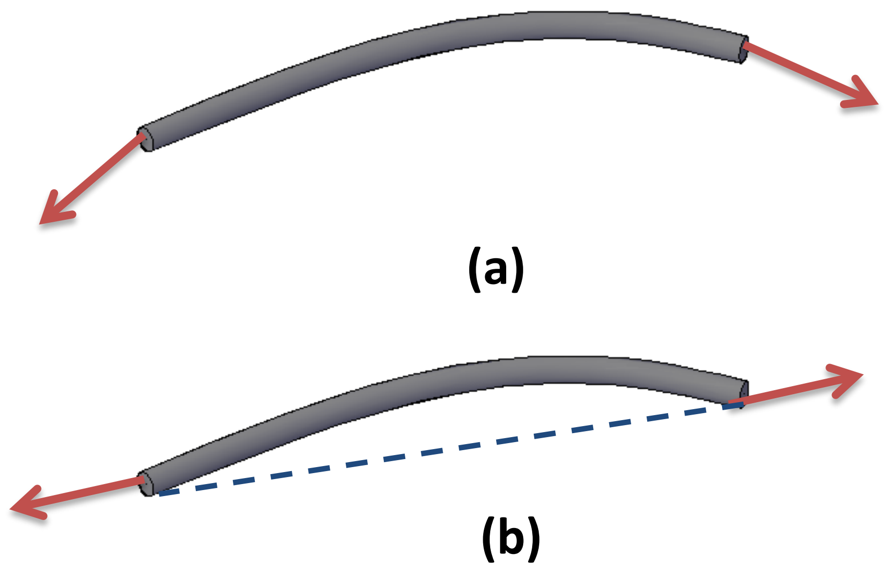


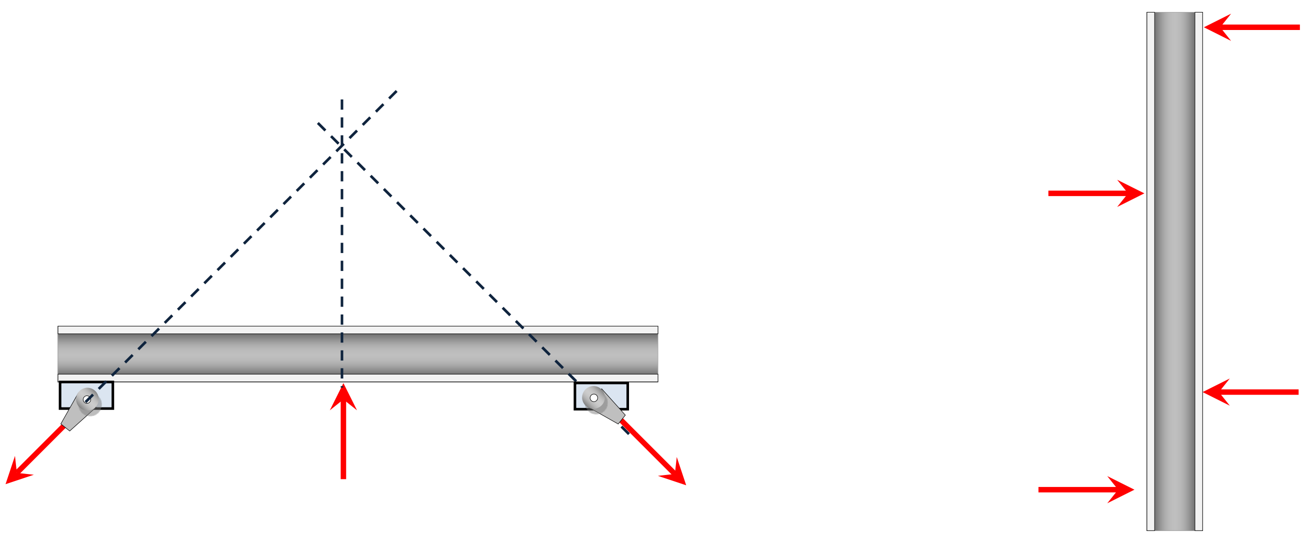
Fig.1

Two and three forces members

S t a t i c s

Three – forces member

Similar to the two forces member, three – forces member is under equilibrium if the summation of the forces equal zero. However, to satisfy the equilibrium moment condition, the three forces must be either concurrent or parallel forces. Fig.2 illustrates two examples of concurrent and parallel forces.



(a) (b)

Fig.2

Two and three forces members

Example [1]

Fig.3 shows a member under the action of 100N. Find the reaction forces at the supporting connections

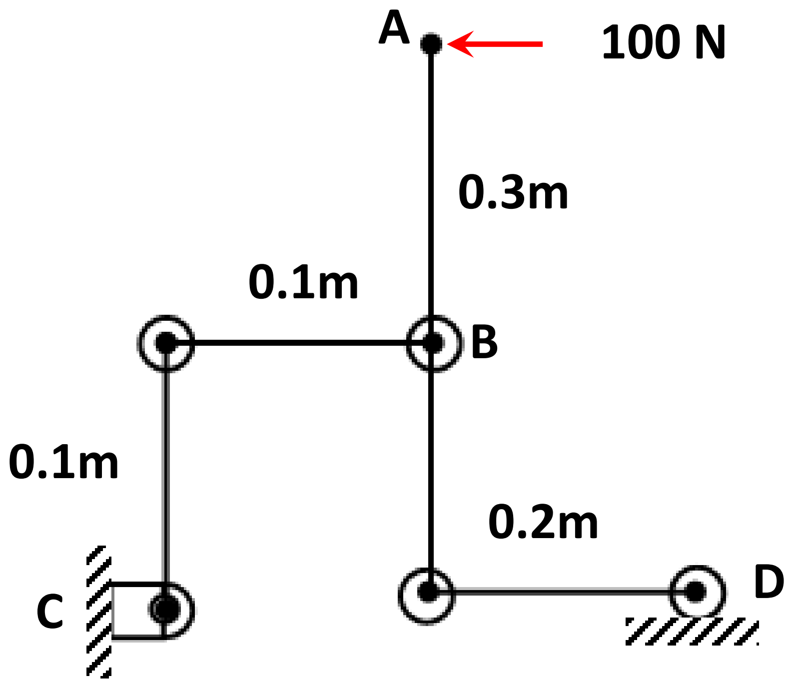


Fig.3

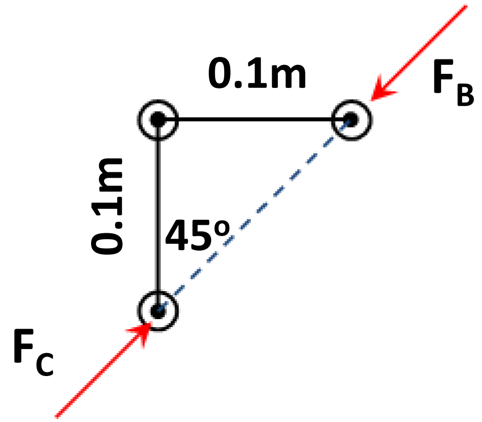
Two and three forces members

Statics

Example [1]

Solution

Let us draw F.B.D for the small member. This member is a two forces (remember, the two forces – member forces are identical). Now let us draw the F.B.D:



You can note that we can calculate the angels of the reaction forces from the member geometry (the line of action of these forces is the same)

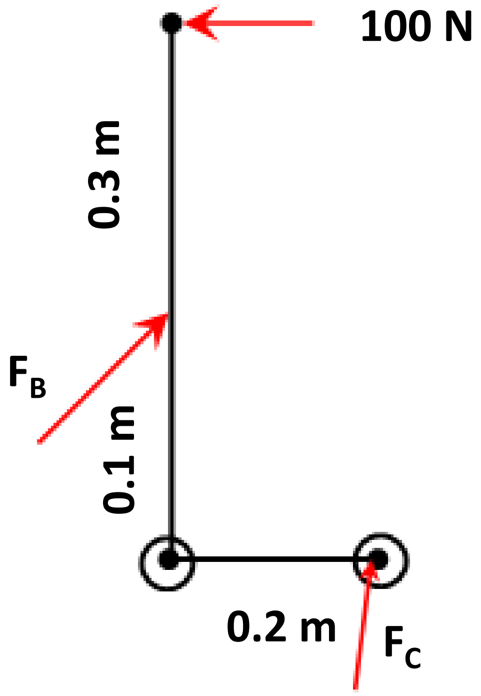
Two and three forces members

Example [1]

Solution

Now , let us draw the F.B.D for the other member

As you can see there is something is missing which is the direction of the reaction force F_C.



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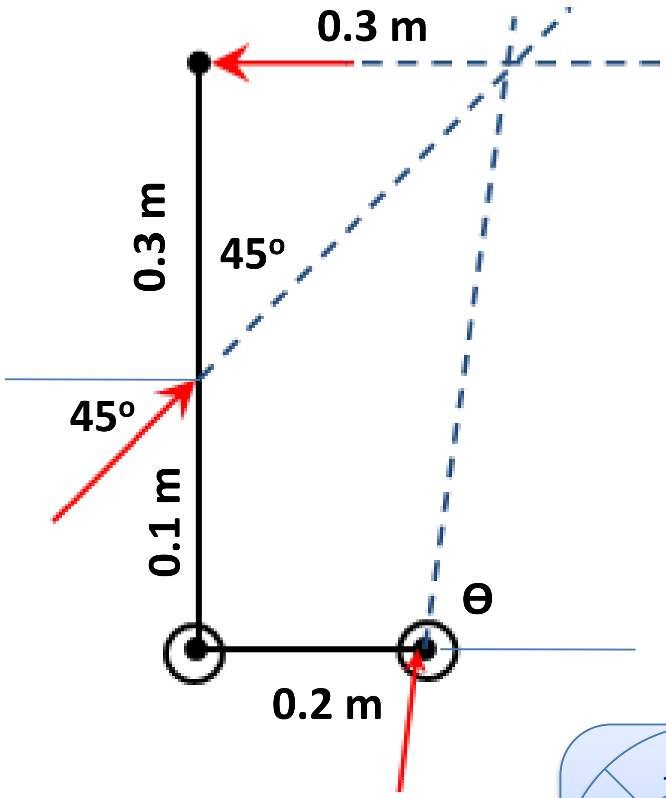
Two and three forces members

Example [1]

Solution

To find the reaction force Fc we can refer to the 2nd equilibrium equations which implies that these are either parallel or concurrent. As you can see these forces can not be parallel and so it must be concurrent as seen in Fig. 4. the angle ϴ can be found as:

$$\theta = \tan^{-1}\left(\frac{0.4}{0.1}\right) = 76^o$$



Two and three forces members

Example [1]

Solution

Now using the force equilibrium equations (∑Fx = ∑Fy = 0)

$$\xrightarrow{+} \sum F_x = 0 \Rightarrow F_B \cos(45) - F_C \cos(76) + 100 = 0 - - - (1)$$

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Note that you can resolve this problem by resolving the unknown reactions forces to their rectangular components and then use the ∑Fx = ∑Fy = ∑M = 0 equations

Substitute Eq.3 into Eq.1 to have

$$(1.37)F_C \cos(45) - F_C \cos(76) + 100 = 0 \Rightarrow F_C = -137N$$

Substitute Fc in Eq.3 to find F_B:

$$F_B = 188N$$

Two and three forces members

Example [2]

Fig.5 shows a member under the action of 100N. Find the reaction forces at the supporting connections

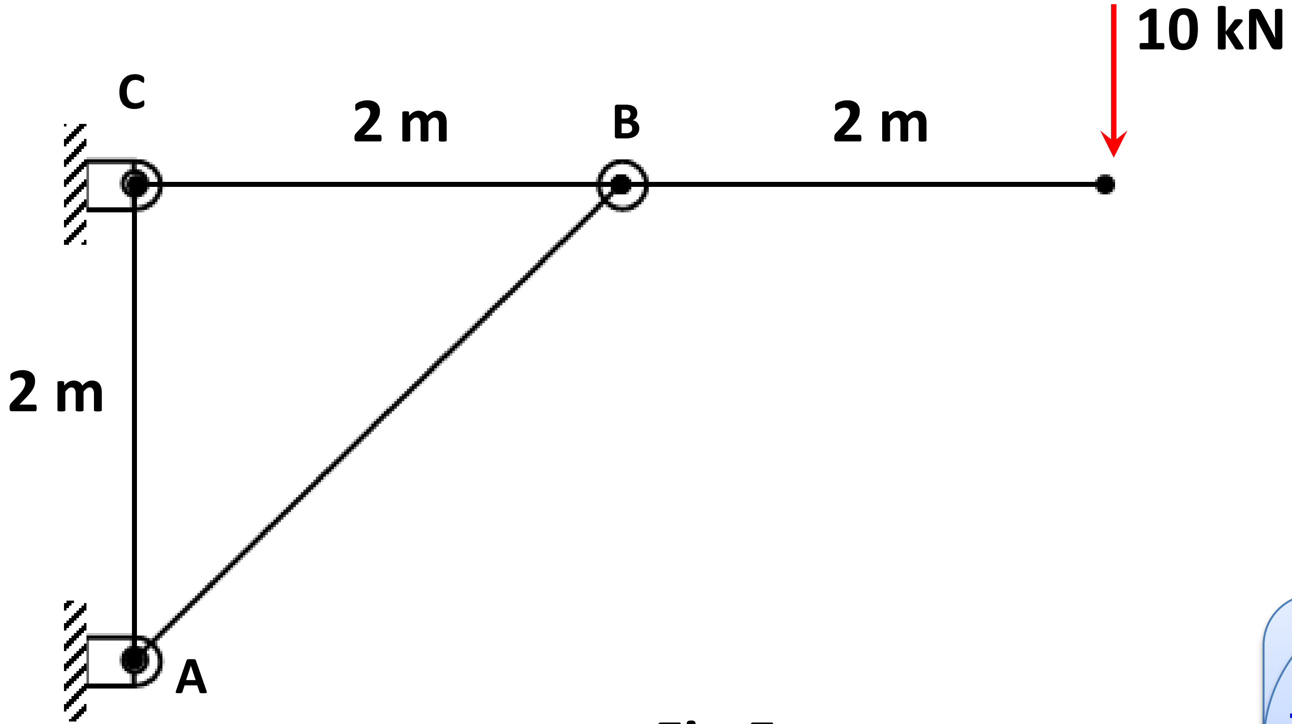

Fig.5

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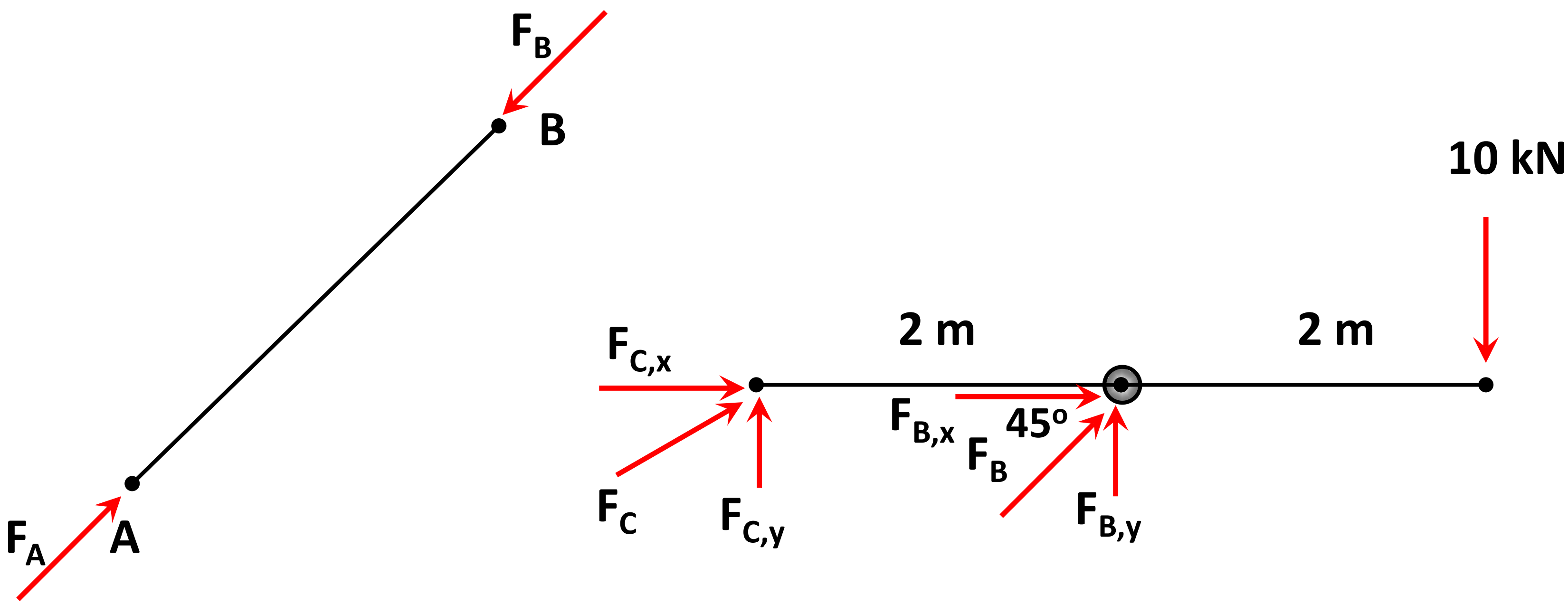
Equations of equilibrium

Example [1]

Solution

Let us draw F.B.D for the this system. Let us draw F.B.D for the member AB. This member is a two forces. And the F.B.D for the member CB as shown in the figure below . Note the resolving of the reaction force at point C.



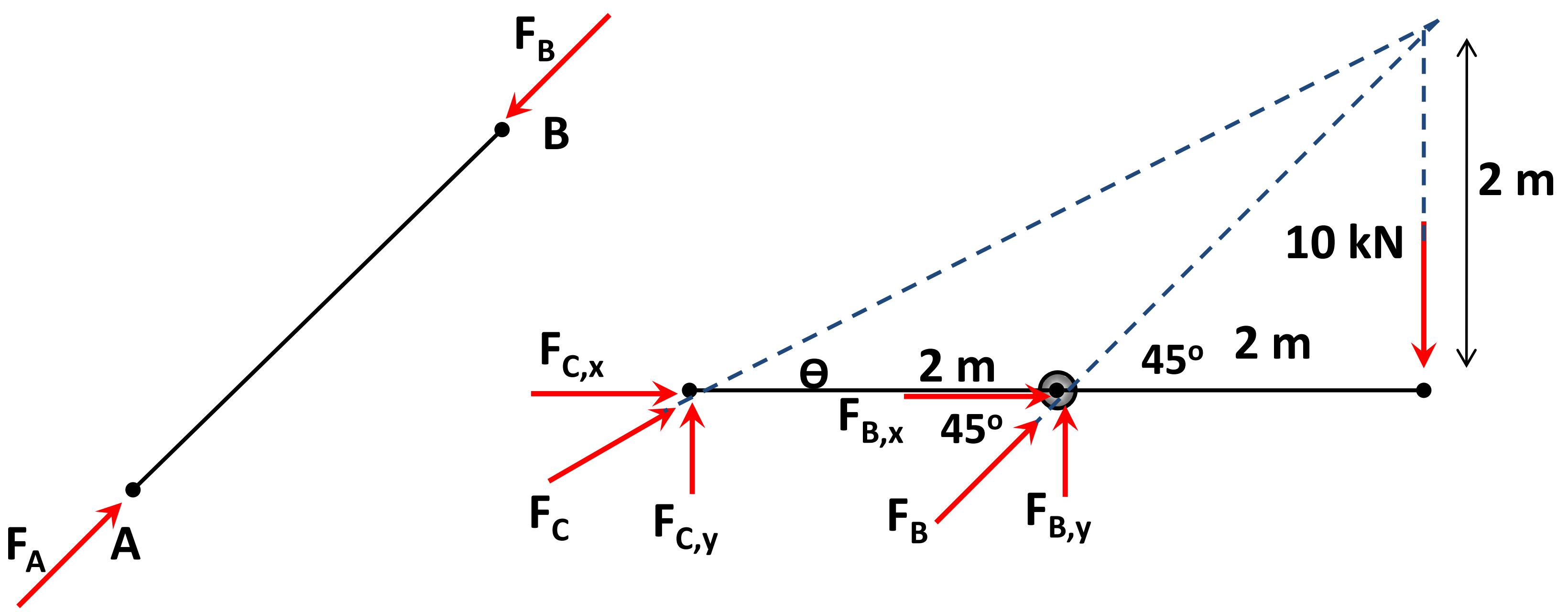
Equations of equilibrium

Example [1]

Solution

As in the previous example, we have to find the angle of Fc. Again, we will use the equilibrium condition fro concurrent forces situation. The angle ϴ can be found as:

$$\theta = \tan^{-1}\left(\frac{2}{4}\right) = 26.6^{o}$$



Equations of equilibrium

Example [1]

Solution

To find the reaction forces, we can use the equilibrium moment equation

First, we can apply the moment equation at point C:

$$\sum M_C = 0 \Rightarrow F_{By}(2) - (10)(4) = 0 \Rightarrow F_{By} = 20kN$$

Then , apply the moment equation at point B:

$$\sum M_B = 0 \Rightarrow -F_{Cy}(2) - (10)(2) = 0 \Rightarrow F_{Cy} - 10kN$$

The negative value of the force mean that we must reverse the assumed sense for the component

Equations of equilibrium

Example [1]

Solution

The figure below show an intermediate stage of the solution where the reaction vertical components were found. To find the horizontal components, we can will use the triangle functions (sine and cosine) because the using of momentum equation will yield a trivial solution.

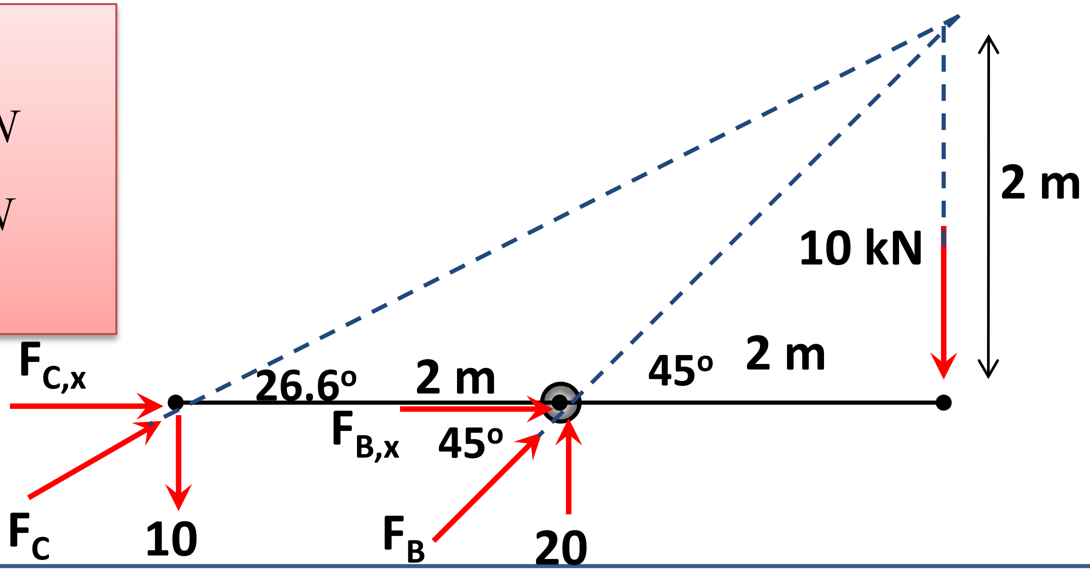
$$F_{Cy} = F_C \sin(26.6) \Rightarrow F_C = 22.33kN; F_{Cx} = F_C \cos(26.6) = 20kN$$

$$F_{By} = F_B \sin(45) \Rightarrow F_B = 28.28kN; F_{Bx} = F_B \cos(45) = 20kN$$

Verify:

$$F_B = \sqrt{20^2 + 20^2} \cong 28.28kN$$

$$F_C = \sqrt{20^2 + 10^2} \cong 22.33kN$$



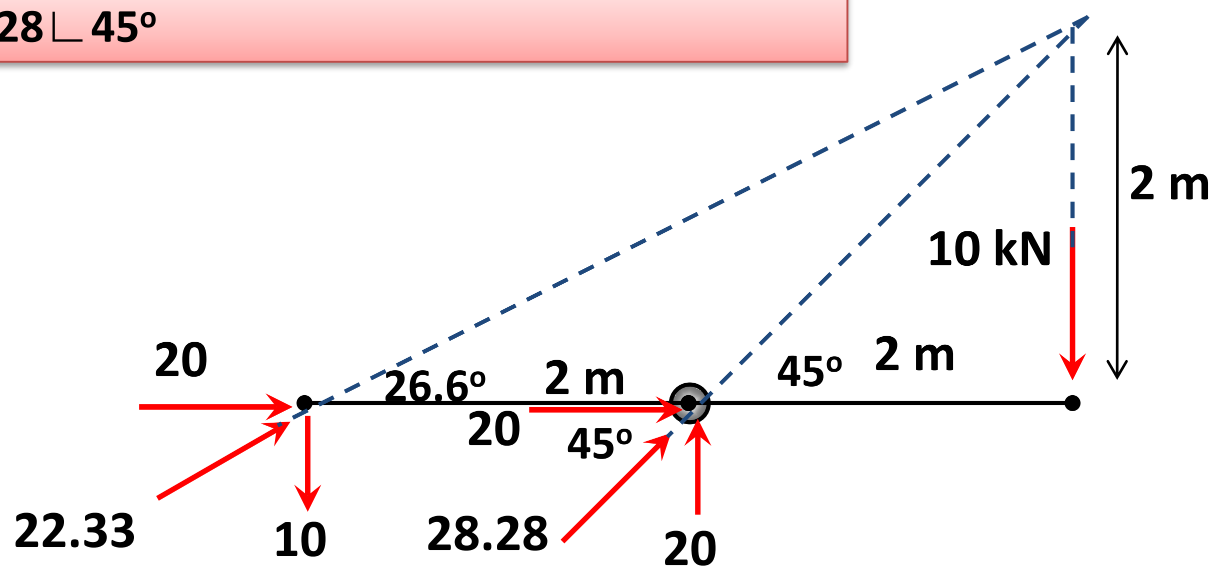
Equations of equilibrium

Example [1]

Solution

the figure below illustrates the final solution

Note that $F_A = F_B = 28.28 \llcorner 45^o$



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Chapter Four

4.4 equilibrium in 3D

By

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equilibrium in 3D

Supported reactions

As in two dimensional problems, :

- if the support restricts the body from a translational monition, a reaction force is developed in this connection.
- if the support restricts the body from a rotational motion, a reaction moment is developed in this connection

equilibrium in 3D

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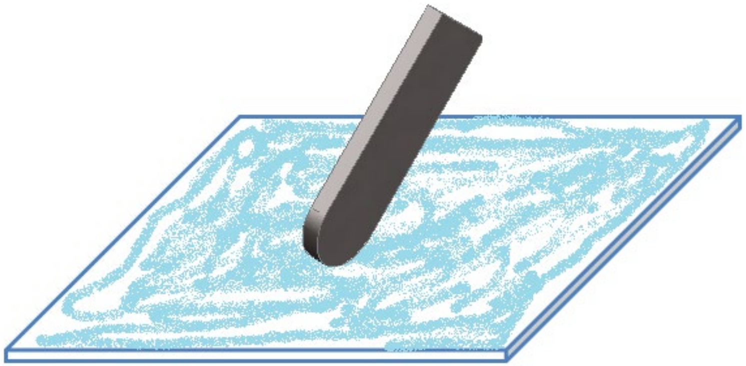
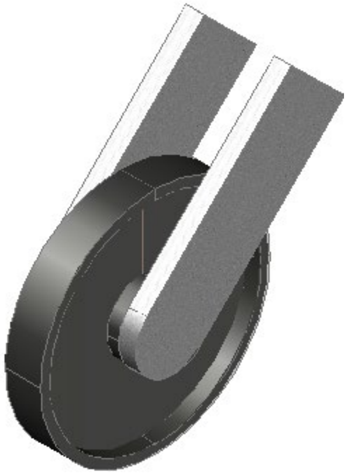
3D connections

Single reaction supports

Both smooth surface and 3D roller supports have a single reaction force acts on the surface of the connection



Smooth surface support



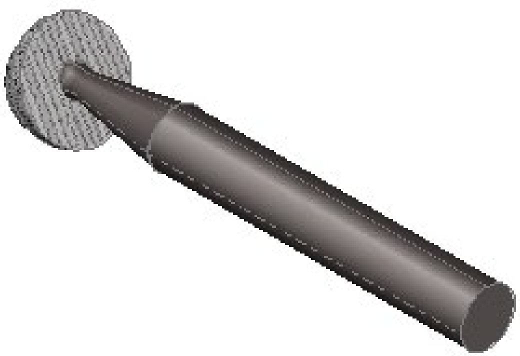
Roller support

equilibrium in 3D

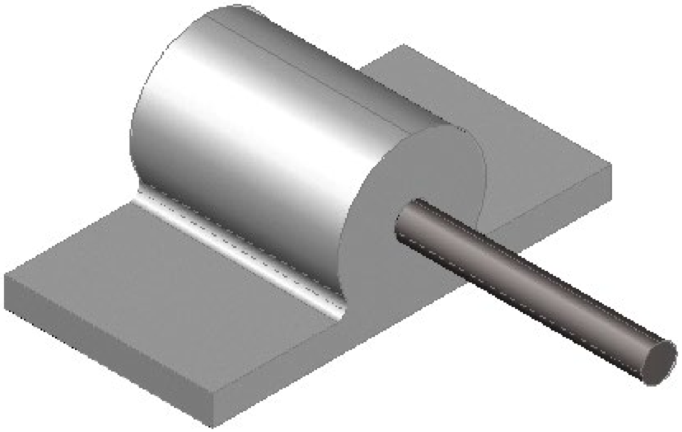
3D connections

Three and four reaction supports

Ball and socket connection one of the famous 3 reaction forces connection and the single journal bearing is a famous example on 4 reactions supporting. The reactions developed in journal bearing are : two forces and two couple moments



Ball and socket connection



Single journal bearing

equilibrium in 3D

3D connections

Five and six reactions support

This category include:

- Single journal bearing with beam shaft: two forces + three couple moments
- single thrust bearing: three forces + two couple moments
- Single smooth pen: three forces + two couple moments
- Single hinge : three forces + two couple moments
- Fixed support (6 reactions): three forces + three couple moments

For more connection reactions, refer to Engineering Mechanics, Statics, 12th edition, R. C. Hibbeler, 2010, pp 238-239

equilibrium in 3D

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Equation of equilibrium

we learn before that we can analyze a 3D problem using vector or scalar approaches

Vector analysis

In vector analysis the equation of equilibrium are written as:

$$\sum \mathbf{F} = 0 \text{ and } \sum \mathbf{M} = 0$$

where F and M are the vectors of the force and moment respectively

Scalar analysis

In scalar analysis the equation of equilibrium are written as:

$$\sum \mathbf{F} = \sum F_x \mathbf{i} + \sum F_y \mathbf{j} + \sum F_z \mathbf{k} = 0$$

$$\sum \mathbf{M} = \sum M_x \mathbf{i} + \sum M_y \mathbf{j} + \sum M_z \mathbf{k} = 0$$

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

equilibrium in 3D

S t a t i c s

Constrains and statical determinacy

The concept of supporting a body under equilibrium is important as the satisfaction of the equilibrium equation itself.
In some cases, body may has more supports than what it needs to be in equilibrium situation while in other cases, the body will not have enough supports to keep it under equilibrium condition. In addition, in other cases, the arrangement of the supports allow the body to move.

Statically determinacy

We can say that the system is statically indeterment if the number of the unknown forces and moments are more than the equilibrium equations.
Remember: algebraically, you need a number of equation equal the number of unknowns you desire to solve.

equilibrium in 3D

Redundant constraints

Assume that we have the beam shown in Fig.1-a

The F.B.D for this beam is shown in Fig.1-b.
As you can see, the number of unknowns is 5: Ax, Ay, By, Cy and Dy and as you know, there are three equilibrium equations to use so you will failed to find all the unknown reactions using the equilibrium equations only because the number of unknowns is larger than the number of equations.

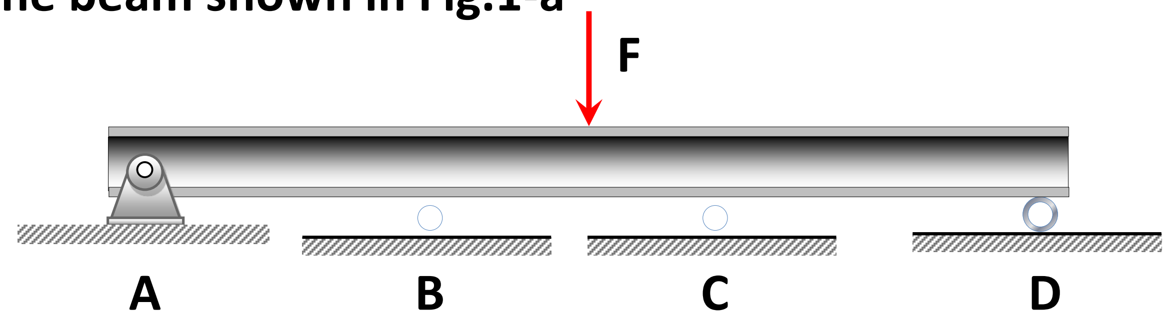


Fig.1-a

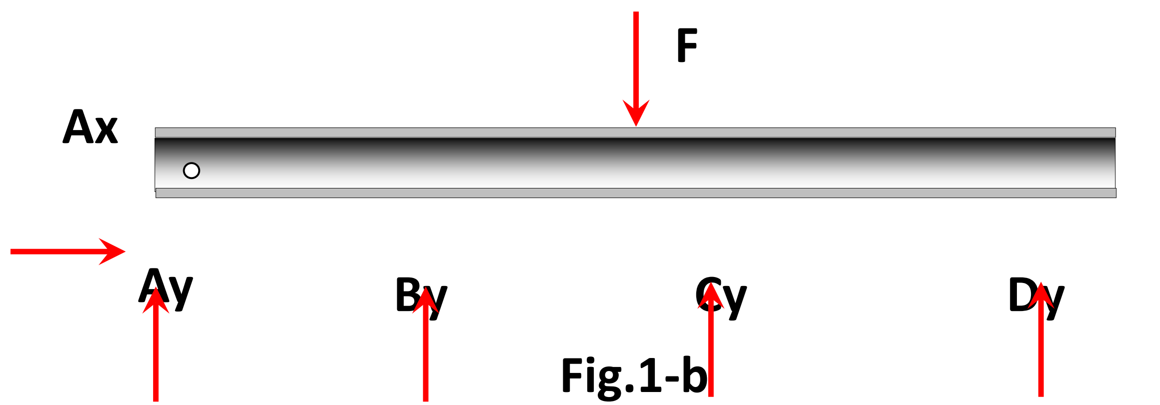


Fig.1-b

Such cases are called statically indeterment

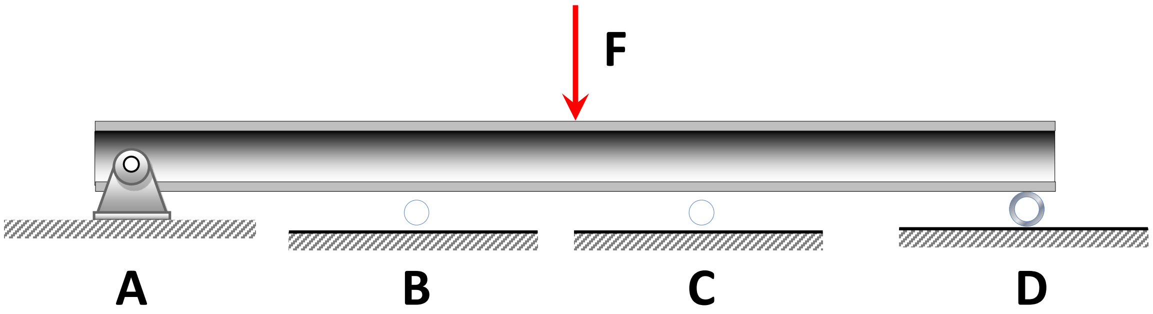
S t a t i c s

equilibrium in 3D

Redundant constraints

If we go back to the same case, you can find that this system can be under equilibrium with only one roller support and so, there are two extra supports used to keep the body under equilibrium.

The case where there are extra supports to keep the body under equilibrium is called redundant constraints



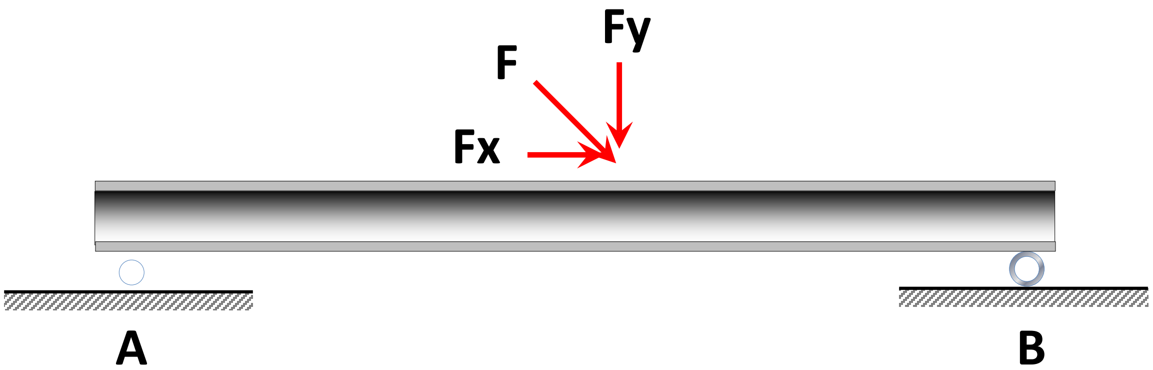
equilibrium in 3D

Improper constraints

The equality between the number of unknowns and the number of equations does not guarantee in all times the equilibrium of a body, for example see the figure below. Such case is called improperly constrained because the system is able to move.

To explain that, let us go back to the figure. As you can see, we resolve the force to its rectangular components. You can note that the horizontal component (Fx) tends to move this system in the horizontal direction and so $\sum F_x = 0$ is not valid any more.

In general, improper constraints must be avoided in engineering practice because it shows instability in the system.



equilibrium in 3D

Example [1]

The figure show a rod supporting the 600 N box.
Find the tension forces developed in the cables B and C

Solution

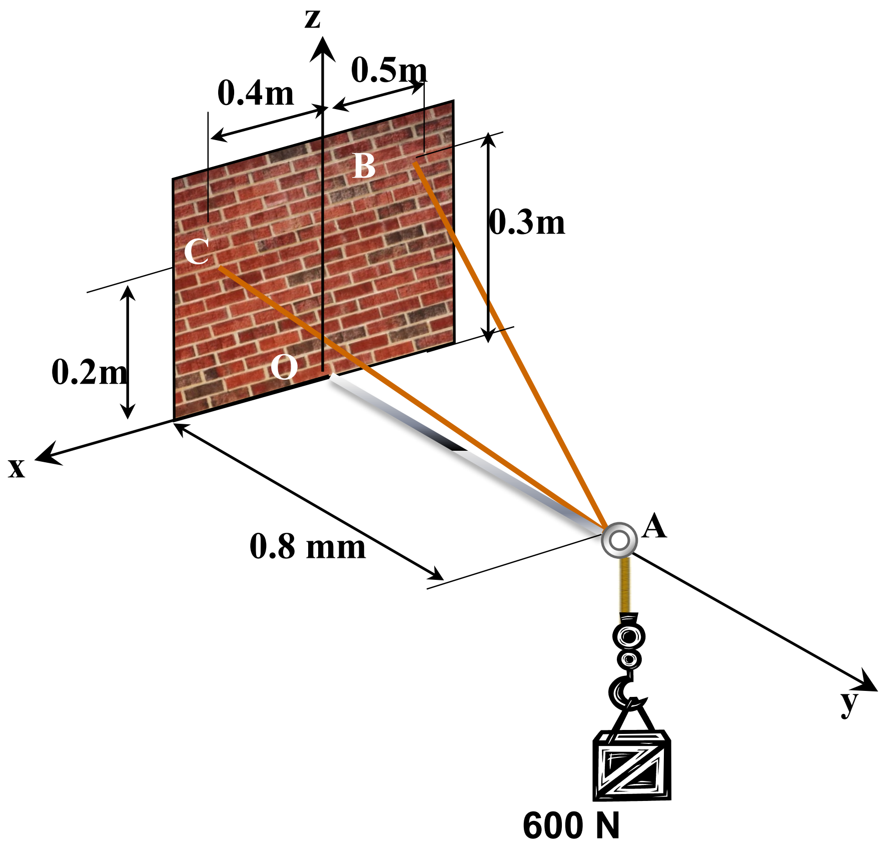
We can use vector analysis

$$\overrightarrow{F_{AB}} = F_{AB}\left(\frac{\overrightarrow{r_{AB}}}{r_{AB}}\right) = F_{AB}\left(\frac{-0.5i - 0.8j + 0.3k}{\sqrt{0.5^2 + 0.8^2 + 0.3^2}}\right)$$

$$\overrightarrow{F_{AB}} \cong F_{AB}(-0.5i - 0.8j + 0.3k)$$

$$\overrightarrow{F_{AC}} = F_{AC}\left(\frac{\overrightarrow{r_{AC}}}{r_{AC}}\right) = F_{AC}\left(\frac{0.4i - 0.8j + 0.2k}{\sqrt{0.4^2 + 0.8^2 + 0.2^2}}\right)$$

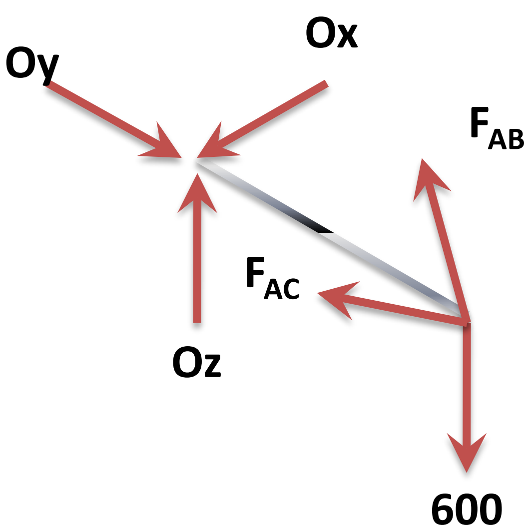
$$\overrightarrow{F_{AC}} \cong F_{AC}(0.436i - 0.873j + 0.218k)$$



equilibrium in 3D

Example [1]

Solution



Let us now present the F.B.D for the connection rod
To eliminate the reactions at point O, we can apply the moment equation at this point:

$$\sum M_O = 0 \Rightarrow r_{OA} x \left(F_{AB} + F_{AC} + w\right) = 0$$

$$But : r_{OA} = 0.8 j$$

$$Then : (0.8 j) x \left[F_{AB}(-0.5i - 0.8j + 0.3k) + F_{AC}(0.436i - 0.873j + 0.218k) - 600k\right] = 0$$

Rearrange the above equation and collect the similar Cartesian terms

$$\sum M_O = (0.24F_{AB} + 0.1744F_{AC} - 480)i - (0.4F_{AB} - 0.3488F_{AC})k = 0$$

equilibrium in 3D

Example [1]

Solution

From the concept of equilibrium in 3D:

$$\sum M_x = 0 \Rightarrow 0.24F_{AB} + 0.1744F_{AC} - 480 = 0 - - - (1)$$

$$\sum M_y = 0 \Rightarrow 0 = 0$$

$$\sum M_z = 0 \Rightarrow 0.4F_{AB} - 0.3488F_{AC} = 0 - - - (2)$$

Solve equation 1 and 2 to find F_{AB} and F_{AC}

Try this by your self.