

Unit Vectors Analysis

Assume Unit Vectors:

$$U_{\theta_1} = \cos(\theta_1) i + \sin(\theta_1) j$$

$$U_{\theta_2} = \cos(\theta_2) i + \sin(\theta_2) j$$

Now:

$$\begin{aligned} U_{\theta_1} \cdot U_{\theta_1} &= \cos(\theta_1) \cos(\theta_1) + \sin(\theta_1) \sin(\theta_1) \\ &= \cos^2(\theta_1) + \sin^2(\theta_1) = 1 \end{aligned}$$

$$\begin{aligned} U_{\theta_2} \cdot U_{\theta_1} &= \cos(\theta_2) \cos(\theta_1) + \sin(\theta_2) \sin(\theta_1) \\ &= \cos(\theta_2 - \theta_1) \end{aligned}$$

Derive with respect to time.

$$\dot{U}_{\theta_1} = -\sin(\theta_1) i + \cos(\theta_1) j$$

$$\dot{U}_{\theta_2} = -\sin(\theta_2) i + \cos(\theta_2) j$$

Now:-

$$U_{\theta_1} \cdot \dot{U}_{\theta_1} = -\sin(\theta_1) \cos(\theta_1) + \sin(\theta_1) \cos(\theta_1) = 0$$

$$\begin{aligned} \dot{U}_{\theta_1} \cdot U_{\theta_2} &= -\sin(\theta_1) \cos(\theta_2) + \cos(\theta_1) \sin(\theta_2) \\ &= \sin(\theta_2 - \theta_1) \end{aligned}$$

$$\begin{aligned} \dot{U}_{\theta_1} \cdot \dot{U}_{\theta_2} &= \sin(\theta_1) \sin(\theta_2) + \cos(\theta_1) \cos(\theta_2) \\ &= \cos(\theta_1 - \theta_2) \end{aligned}$$