الرقم :

التاريخ :

Solutions:-

① $Z\{A^k\} = \sum_{k=0}^{\infty} A^k z^{-k} = 1 + Az^{-1} + A^2 z^{-2} + \dots = \frac{1}{1-Az^{-1}} = \frac{z}{z-A}$ [illegible]

② $m(z) = e(z) - z^{-1}e(z) - z^{-1}m(z)$, $(1+z^{-1})m(z) = (1-z^{-1})e(z)$, $\therefore m(z) = \frac{z-1}{z+1}e(z)$,

$e(z) = \sum_{k=0}^{\infty} e(k) z^{-k} = e(0) + e(1)z^{-1} + e(2)z^{-2} + \dots = 1 + z^{-2} + z^{-4} + \dots = \frac{1}{1-z^{-2}} = \frac{z^2}{z^2-1}$

$\therefore m(z) = \frac{z-1}{z+1}\cdot\frac{z^2}{z^2-1} = \frac{1}{1+2z^{-1}+z^{-2}}$ long division gives $m(z) = 1 - 2z^{-1} + 3z^{-2} - 4z^{-3} + \dots$

$\therefore m(0)=1$, $m(1)=-2$, $m(2)=3$, and so on.

③ $\frac{x(z)}{z} = \frac{1}{(z-5)(z-1)} = \frac{A}{(z-5)} + \frac{B}{z-1}$, $A = 0.25$, $B = -0.25$, $x(k) = 0.25(5^k - 1)$

$x(0)=0$, $x(1)=1$, $x(2)=6$, $x(3)=31$, $x(4)=156$.

or: $1-6z^{-1}+5z^{-2} \,)\, z^{-1}$ $\therefore x(z) = z^{-1} + 6z^{-2} + 31z^{-3} + 156z^{-4}$

④ $X(z) - 3z^{-1}X(z) + 2z^{-2}X(z) = 1 + z^{-1}$, $\therefore X(z) = \frac{1+z^{-1}}{1-3z^{-1}+2z^{-2}}$ by long division,

$X(z) = 1 + 4z^{-1} + 10z^{-2} + 22z^{-3}$, $\therefore X(0)=1$, $X(1)=4$, $X(2)=10$, $X(3)=22$.

or: by partial fraction $\frac{z(z+1)}{(z-2)(z-1)} = X(z)$, $\therefore \frac{X(z)}{z} = \frac{z+1}{(z-2)(z-1)} = \frac{A}{z-2} + \frac{B}{z-1}$

$\therefore A=3$, $B=-2$, $\therefore X(k) = 3(2)^k - 2$ $\therefore X(0)=1$, $X(1)=4$, $X(2)=10$, $X(3)=22$.

[illegible] $X(\infty) = \lim_{z\to1}(z-1)X(z) = \lim_{z\to1}(z-1)\cdot\frac{(z+1)z}{(z-1)(z-2)} = \lim_{z\to1}\frac{z(z+1)}{(z-2)} = -2$? incorrect,

unstable pole at $z=2$, $\therefore X(\infty) \to \infty$

⑤ $Z\{c(k+1)\} = zC(z) - zc(0) = zC(z)$ $\because c(0)=0$.

$Z\{c(k+1)\} = \frac{z}{(z-1)(z-0.3)} = \frac{1}{0.7}\left\{\frac{z}{z-1} - \frac{z}{z-0.3}\right\}$

$\therefore c(k+1) = \frac{1}{0.7}\{(1)^k - (0.3)^k\}$ $k=0,1,2,\dots$

$\therefore c(k) = \frac{1}{0.7}\{1 - (0.3)^{k-1}\}$ $k=1,2,3,\dots$

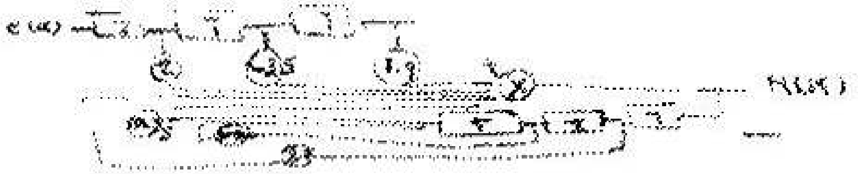
$c(10) = \frac{1}{0.7}\{1 - (0.3)^9\}$,

$c(0) = \lim_{z\to\infty} C(z) = \lim_{z\to\infty}\frac{z^{-2}}{1 - 1.3/z + 0.3/z^2} = 0$.

⑥ a) $\frac{M(z)}{X(z)} = \frac{1}{1-z^{-1}}$ $\therefore M(k) = X(k) + M(k-1)$, b) $e(\infty) = \lim_{z\to1}(z-1)\cdot e(z) = 1$.

c) $e(k) = 1^k$, $e(\infty) = 1$.

⑦ $M(k) = 2e(k-1) - 2.5e(k-2) + 1.4e(k-3) + 2.7M(k-1) - M(k-2) + 0.95M(k-3)$



* Mapping s-plane into z-plane.

Since $Z = e^{TS}$, and $s = \alpha + j\omega$ then $|Z| = e^{\alpha T}$, $\angle Z = \omega T$ {ω varies from $-\infty$ to ∞}
then the left half s-plane where $\alpha < 0$ gives $|Z|$ ranging between 0 and 1 for all ω
the Imaginary axis where $\alpha = 0$ gives $|Z| = 1$ for all ω thus (Unit Circle)
while when $\alpha > 0$ that gives $|Z| > 1$ for all ω.

Problem: 1- Given $X(z) = \frac{2Z^3 + Z}{(Z-1)^2 (Z-2)}$ it is required to:
- Find eigen values. (Roots of characteristics equation)
- Find $X(0)$, $X(\infty)$.
- Apply P.F.E Method to find $X(K)$. What Conclusion can you extract?
- find initial and final value of $X(K)$ Compare your results
- If $X(z) = \frac{Y(z)}{U(z)}$ then find difference equation
- Draw the simulation diagram of the given system

2- Use Z-transform to solve the following difference equation:

$$X(K+2) - X(K+1) + 0.25\, X(K) = u(K+2)$$

where $X(0) = 1$, $X(1) = 2$, and the input function is given by $u(K) = 1$ for $K = 0, 1, 2, \ldots$

"Discrete State-Space Mathematical Representation"

* Discrete state-space Equations:- for SISO the discrete s.s. Equations are:

$\underline{X}(K+1) = G\,\underline{X}(K) + H\,\underline{u}(K)$ State Equation --- (1)

$y(K) = C\,\underline{X}(K) + D\,u(K)$ o/p Equation --- (2) for n^{th} order:

with $G = (n \times n)$, $H = (n \times 1)$, $C = (1 \times n)$, $D = (1 \times 1)$

$\Downarrow$ dynamic Matrix, $\Downarrow$ i/p matrix, $\Downarrow$ o/p matrix, $\Downarrow$ Direct matrix

* Kinds of Discrete s.s. Equations Mathematical Representations:

For Pulse Transfer Function $G(z) = \dfrac{Y(z)}{U(z)} = \dfrac{b_0 z^n + b_1 z^{n-1} + \dots b_n}{z^n + a_1 z^{n-1} \dots + a_n} = \dfrac{b_0 + b_1 z^{-1} + \dots + b_n z^{-n}}{1 + a_1 z^{-1} + \dots + a_n z^{-n}}$

There are many kinds of s.s. representation, two of them are as follows:

1. Direct programming method which gives the Controllable Canonical form as:

"assuming order = 3" then:

$$\begin{bmatrix} x_1(K+1) \\ x_2(K+1) \\ x_3(K+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1(K) \\ x_2(K) \\ x_3(K) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(K)$$

$$y(K) = \begin{bmatrix} b_3 - a_3 b_0 & b_2 - a_2 b_0 & b_1 - a_1 b_0 \end{bmatrix} \begin{bmatrix} x_1(K) \\ x_2(K) \\ x_3(K) \end{bmatrix} + b_0 u(K)$$

2- Partial-Fraction Expansion method which gives the diagonal form as below:-

for $G(z) = b_0 + \dfrac{c_1}{(z-p_1)} + \dfrac{c_2}{(z-p_2)} + \dfrac{c_3}{(z-p_3)}$ then:

$$\begin{bmatrix} x_1(K+1) \\ x_2(K+1) \\ x_3(K+1) \end{bmatrix} = \begin{bmatrix} p_1 & 0 & 0 \\ 0 & p_2 & 0 \\ 0 & 0 & p_3 \end{bmatrix} \begin{bmatrix} x_1(K) \\ x_2(K) \\ x_3(K) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u(K)$$

$$y(K) = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} x_1(K) \\ x_2(K) \\ x_3(K) \end{bmatrix} + b_0 u(K)$$

* Solution of Linear Time Invariant discrete s-s Equation.

1. Homogenous form: $\underline{X}(K+1) = G\,\underline{X}(K)$ "take Z-transform of both sides"

$z\,\underline{X}(z) - z\,\underline{X}(0) = G\,\underline{X}(z) \Rightarrow (zI - G)\,\underline{X}(z) = z\,\underline{X}(0)$

Hence; $\underline{X}(z) = (zI - G)^{-1} z\,\underline{X}(0)$ then $\underline{X}(K) = z^{-1}\left[(zI-G)^{-1} z\right]$

with $(zI - G)^{-1}$ is the state-transition matrix $= \psi(K) = G^K$

2. Non-homogenous Form:-

for $\underline{x}(k+1) = G\underline{x}(k) + Hu(k) \Rightarrow z\underline{x}(z) - z\underline{x}(0) = G\underline{x}(z) + Hu(z)$

then; $(zI - G)\underline{x}(z) = z\underline{x}(0) + Hu(z)$

$\therefore \underline{x}(z) = (zI-G)^{-1} z\underline{x}(0) + (zI-G)^{-1} Hu(z)$ taking z^{-1} of both sides.

$\underline{x}(k) = z^{-1}\left[(zI-G)^{-1}z\right]\underline{x}(0) + z^{-1}\left[(zI-G)^{-1}Hu(z)\right]$ with $G^k = \psi(k) = z^{-1}\left[(zI-G)^{-1}z\right]$.

Ex:- for the 2nd order discrete-time system described by $\underline{x}(k+1) = G\underline{x}(k) + H\underline{u}(k)$ and $y(k) = c\underline{x}(k)$ with $G = \begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix}$, $H = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $c = [1 \;\; 0]$, find state-transition matrix $\psi(k) = G^k = z^{-1}\left[(zI-G)^{-1}z\right]$, then obtain $\underline{x}(k)$ and $y(k)$ for $u(k) = 1$ with $\underline{x}(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

Solution: $(zI - G) = \begin{pmatrix} z & 0 \\ 0 & z \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -0.16 & -1 \end{pmatrix} = \begin{pmatrix} z & -1 \\ +0.16 & z+1 \end{pmatrix}$

$$(zI-G)^{-1} = \frac{\begin{pmatrix} z+1 & 1 \\ -0.16 & z \end{pmatrix}}{z(z+1)+0.16} = \frac{\begin{pmatrix} z+1 & 1 \\ -0.16 & z \end{pmatrix}}{(z+0.8)(z+0.2)} = \begin{bmatrix} \frac{4/3}{(z+0.2)} - \frac{1}{3}\frac{1}{(z+0.8)} & \frac{5/3}{(z+0.2)} - 5/3\cdot\frac{1}{(z+0.8)} \\ \frac{-0.8/3}{(z+0.2)} + \frac{0.8/3}{(z+0.8)} & \frac{-1/3}{(z+0.2)} + \frac{4/3}{(z+0.8)} \end{bmatrix}$$

$$\text{then } z^{-1}\left[(zI-G)^{-1}z\right] = \begin{bmatrix} 4/3(-0.2)^k - \frac{1}{3}(-0.8)^k & 5/3(-0.2)^k - 5/3(-0.8)^k \\ -0.8/3(-0.2)^k + 0.8/3(-0.8)^k & -1/3(-0.2)^k + 4/3(-0.8)^k \end{bmatrix} = \psi(k) = G^k.$$

then, $\underline{x}(k) = G^k\underline{x}(0) + z^{-1}\left[(zI-G)^{-1}H\,u(k)\right] = G^k\begin{bmatrix} 1 \\ -1 \end{bmatrix} + z^{-1}\left[(zI-G)^{-1}\begin{bmatrix} 1 \\ 1 \end{bmatrix}(1)\right]$.

and $y(k) = c\,\underline{x}(k) = [1 \;\; 0]\,\underline{x}(k)$. "Simplify to obtain Solutions".

(Problems)

1. Write down the cannonical and diagonal discrete s.s. equations for the following pulse transfer functions: $G(z) = \frac{2z^2+9z+20}{z^3+6z^2+11z+6}$, $G(z) = \frac{2z^2+6z+5}{z^3+4z^2+5z+2}$ check your answers by appling $Y(z)/U(z) = C(zI-G)^{-1}H + D$.

2. Discretization of continuous-time state equation $\dot{x} = A\underline{x} + Bu$ is evaluated by: $G(T) = e^{AT}$, $H(T) = \left(\int_0^T e^{At}dt\right)B$ and $y(kT) = C\underline{x}(kT) + Du(kT)$ thus obtain the discrete s.s. form for $\dot{\underline{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}\underline{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u$, $y = [1 \;\; 0]\,x(t)$ for $T = 1$ sec.
Ans: $G(T) = \begin{bmatrix} 1 & 0.432 \\ 0 & 0.135 \end{bmatrix}$, $H(T) = \begin{bmatrix} 0.284 \\ 0.432 \end{bmatrix}$, $c = [1 \;\; 0]$,

3. prove that $\psi(0) = I$ and $\psi(-k) = \psi^{-1}(k)$.