Q1: "Choose 3 of the following"

A) Given matrix $A=\begin{bmatrix}2&3\\0&2\end{bmatrix}$, it is required to calculate A^5 using Cayley-Hamilton theorem. (10 Marks)

Sol. $|\lambda I-A|=0$, $\lambda_1=\lambda_2=2$ $\quad f(\lambda_1)=\alpha_0+\alpha_1\lambda \rightarrow (2)^5=32=\alpha_0+\alpha_1\lambda$

$f'(\lambda_1)=5(2)^4=80=\alpha_1$ thus $\alpha_0=-128$

$$(A)^5=\alpha_0 I+\alpha_1 A=\begin{pmatrix}\alpha_0&0\\0&\alpha_0\end{pmatrix}+\begin{pmatrix}2\alpha_1&3\alpha_1\\0&2\alpha_1\end{pmatrix}=\begin{pmatrix}\alpha_0+2\alpha_1&3\alpha_1\\0&\alpha_0+2\alpha_1\end{pmatrix}=\begin{pmatrix}32&240\\0&32\end{pmatrix}.$$

B) Solve the following difference equation using P.F.M and Z-transform.

$x(k)-3x(k-1)+2x(k-2)=e(k)$ with $x(-2)=x(-1)=0$, $e(k)=1$, for $k=0,1$ & $e(k)=0$ for $k\geq 2$. (10Ms)

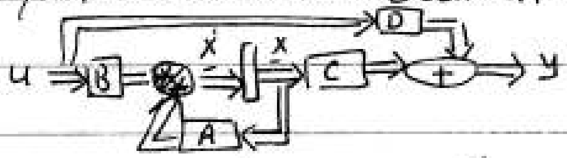
Sol. $X(z)-3z^{-1}X(z)+2z^{-2}X(z)=1+z^{-1}\Rightarrow \therefore X(z)=\frac{1+z^{-1}}{1-3z^{-1}+2z^{-2}}=\frac{z(z+1)}{(z-1)(z-2)}$

$\frac{X(z)}{z}=\frac{z+1}{(z-1)(z-2)}=\frac{A}{(z-2)}+\frac{B}{(z-1)}$ gives $A=3$, $B=-2$

thus $X(z)=\frac{3z}{z-2}-\frac{2z}{z-1}$ $\quad\therefore x(k)=3(2)^k-2$.

C) Given A, B, C, D s.s. matrices it is required to sketch its B.D and derive a formula for T.F. (10 Ms)

Sol. $\dot{\underline{x}}=A\underline{x}+Bu$ $\quad y=Cx+Du$



$sX-AX=Bu\Rightarrow (sI-A)\underline{x}=Bu$

$\therefore x=(sI-A)^{-1}Bu$ $\quad\because y=Cx+Du\Rightarrow y=C(sI-A)^{-1}Bu+Du\Rightarrow \frac{y}{u}=C(sI-A)^{-1}B+D$

D) Solve the given D.E using Laplace transform $y''+y=t$, $y(0)=1$, $y'(0)=-2$ (10 Ms)

Sol. $s^2y-s+2+y=\frac{1}{s^2}\Rightarrow (s^2+1)y=\frac{1}{s^2}+s-2\Rightarrow y=\frac{1}{s^2(s^2+1)}+\frac{(s-2)}{(s^2+1)}$

use P.F.M to obtain $y(s)=\frac{1}{s^2}+\frac{s}{s^2+1}-\frac{3}{s^2+1}$

$\therefore y(t)=t+\cos t-3\sin t$.

Q2: Given $G(s)=\frac{s-1}{(s+1)(s+2)}=\frac{s-1}{s^2+3s+2}$ it is required to:

1. Extract its Cannonical s.s. matricies (5 Ms) Sol. $A=\begin{bmatrix}0&1\\-2&-3\end{bmatrix}$, $B=\begin{bmatrix}0\\1\end{bmatrix}$, $C=[-1\ \ 1]$, $D=0$.
2. Use Sylvester's Criterion to find e^{At} (15 Ms)

Sol. $|\lambda I-A|=0=\begin{vmatrix}\lambda&-1\\2&\lambda+3\end{vmatrix}=\lambda^2+3\lambda+2=0$ $\quad\lambda_1=-1$, $\lambda_2=-2$

$$F_1=\frac{A-\lambda_2 I}{\lambda_1-\lambda_2}=\frac{\begin{pmatrix}0&1\\-2&-3\end{pmatrix}-(-2)\begin{pmatrix}1&0\\0&1\end{pmatrix}}{(-1)-(-2)}=\begin{pmatrix}2&1\\-2&-1\end{pmatrix}$$

$$F_2=\frac{A-\lambda_1 I}{\lambda_2-\lambda_1}=\frac{\begin{pmatrix}0&1\\-2&-3\end{pmatrix}-(-1)\begin{pmatrix}1&0\\0&1\end{pmatrix}}{(-2)-(-1)}=\begin{pmatrix}-1&-1\\2&2\end{pmatrix}$$

$$e^{At}=F_1e^{\lambda_1 t}+F_2e^{\lambda_2 t}=e^{-t}\begin{pmatrix}2&1\\-2&-1\end{pmatrix}+e^{-2t}\begin{pmatrix}-1&-1\\2&2\end{pmatrix}=\begin{pmatrix}2e^{-t}-e^{-2t}&e^{-t}-e^{-2t}\\-2e^{-t}+2e^{-2t}&-e^{-t}+2e^{-2t}\end{pmatrix}.$$

Q3: "Choose 2 of the following"

A) Solve $f(y)=y^{1000}$ using 2 iterations of N-R method start with $y=0.1$, Calculate Erel for last iteration (10 Ms).

sol. $f(y)=y^{1000}$, $y_{new}=y_{old}-\frac{f(y_{old})}{f'(y_{old})}=y_0-\frac{y_0^{1000}}{1000y_0^{999}}=y_0-0.001y_0=0.999y_0$

thus $y_{new}=0.999y_0$

$y_1=0.999*0.1=0.0999$, $y_2=0.999*0.0999=0.0998$.

$Erel\%=\frac{|y_2-y_1|}{|y_2|}*100\%=0.1\%$

B) Apply 2^{nd} Lag. interpolating Poly. for the data given in the table below to estimate $f(-0.25)$. (10 Ms)

x	-1.5	-0.75	0
f(x)	-1.41	-0.9316	0

Sol. $L_0(x)=\left(\frac{x-x_1}{x_0-x_1}\right)\left(\frac{x-x_2}{x_0-x_2}\right)=\left(\frac{0.5}{-0.75}\right)\left(\frac{-0.25}{-1.5}\right)=\left(\frac{-2}{3}\right)\left(\frac{1}{6}\right)=-1/9$

$L_1(x)=\left(\frac{x-x_0}{x_1-x_0}\right)\left(\frac{x-x_2}{x_1-x_2}\right)=\left(\frac{1.25}{0.75}\right)\left(\frac{-0.25}{-0.75}\right)=(5/3)(1/3)=5/9$

$L_2(x)=\left(\frac{x-x_0}{x_2-x_0}\right)\left(\frac{x-x_1}{x_2-x_1}\right)=\left(\frac{1.25}{1.5}\right)\left(\frac{0.5}{0.75}\right)=(5/6)(2/3)=5/9$

$f(-0.25)=L_0(x)f(x_0)+L_1(x)f(x_1)+L_2(x)f(x_2)=-1/9*(-1.41)+5/9*(-0.9316)+5/9*(0)$

$f(-0.25)=0.15667-0.51756=-0.3609$.

C) Find $y(0.3)$ with $h=0.1$ for $\dot{y}=-2y+3e^{-4t}$, with $y(0)=1$ using Euler method (10 Ms).

Sol. $y_{new}=y_{old}+hf(t_0,y_0)$, $y(0.1)=y(0)+0.1f(0,1)=1+0.1(-2+3)=1+0.1=1.1$

$y(0.2)=y(0.1)+0.1f(0.1,1.1)=1.1+0.1[-2.2+3*0.67]=1.081$

$y(0.3)=y(0.2)+0.1f(0.2,1.081)=0.9996$.

Q4: A) Given $Z_1=3+2j$, $Z_2=3-3j$ find Z_1Z_2 & Z_1/Z_2 using both Representations. (15 Ms)

Sol. $Z_1*Z_2=(3+2j)(3-3j)=15-3j$, $Z_1/Z_2=\frac{3+2j}{3-3j}*\frac{3+3j}{3+3j}=1/6+5/6j=0.1667+0.8333j$

$|Z_1|=\sqrt{9+4}=\sqrt{13}=3.6055$, $\theta_1=\tan^{-1}2/3=0.588$

$|Z_2|=\sqrt{9+9}=\sqrt{18}=4.242$, $\theta_2=\tan^{-1}-3/3=\tan^{-1}-1=-0.7854$

$\therefore Z_1=r_1(\cos\theta_1+j\sin\theta_1)=3.6055(\cos0.588+j\sin0.588)$, $Z_2=r_2(\cos\theta_2+j\sin\theta_2)=4.242(\cos-0.7854+j\sin-0.7854)$

$Z_1*Z_2=r_1r_2[\cos(\theta_1+\theta_2)+j\sin(\theta_1+\theta_2)]=15.294[\cos(-0.1974)+j\sin(-0.1974)]$.

$Z_1/Z_2=r_1/r_2[\cos(\theta_1-\theta_2)+j\sin(\theta_1-\theta_2)]=0.85[\cos(1.373)+j\sin(1.373)]$.

Qu: B) Find the constant Fourier coefficients (a_0) for the periodic functions given below: (15 Marks)

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

1) $-\pi \le x \le 0$ $f(x) = x+\pi$, $0 \le x \le \pi$ $f(x) = x+\pi$

$$a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0}(x+\pi)\,dx + \int_{0}^{\pi}(-x+\pi)\,dx\right]$$

$$a_0 = \frac{1}{2\pi}\left[\frac{x^2}{2}+\pi x\Big|_{-\pi}^{0} + -\frac{x^2}{2}+\pi x\Big|_{0}^{\pi}\right] = \frac{1}{2\pi}\left[0-\left(\frac{\pi^2}{2}-\pi^2\right)+\left(-\frac{\pi^2}{2}+\pi^2\right)-0\right]$$

$$a_0 = \frac{1}{2\pi}\left[-\frac{\pi^2}{2}+\pi^2-\frac{\pi^2}{2}+\pi^2\right] = \frac{1}{2\pi}\left\{2\pi^2-\pi^2\right\} = \frac{1}{2\pi}\left[\pi^2\right] = \pi/2$$

2) $-\pi \le x \le 0$ $f(x) = -x-\pi$

$0 \le x \le \pi$ $f(x) = x$

$$a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0}(-x-\pi)\,dx + \int_{0}^{\pi}x\,dx\right] = \frac{1}{2\pi}\left[-\frac{x^2}{2}-\pi x\Big|_{-\pi}^{0} + \frac{x^2}{2}\Big|_{0}^{\pi}\right]$$

$$a_0 = \frac{1}{2\pi}\left[-\left(-\frac{\pi^2}{2}+\pi^2\right)+\frac{\pi^2}{2}\right] = \frac{1}{2\pi}\left[\frac{\pi^2}{2}-\pi^2+\frac{\pi^2}{2}\right] = 0.$$

3) $f(x) =$ 0 if $-\pi \le x \le -0.5\pi$

1 if $-0.5\pi \le x \le 0.5\pi$

0 if $0.5\pi \le x \le \pi$

$$a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{-0.5\pi}0\,dx + \int_{-0.5\pi}^{0.5\pi}dx + \int_{0.5\pi}^{\pi}0\,dx\right] = \frac{1}{2\pi}\left[x\Big|_{-0.5\pi}^{0.5\pi}\right] = \frac{1}{2\pi}\left[0.5\pi+0.5\pi\right]$$

$$a_0 = \frac{1}{2\pi}\left[\pi\right] = \frac{1}{2}.$$