Karnaugh map methods – II

objectives:

1. Four-variables maps.
2. Simplification using prime Implicants
3. "Don't care" conditions.

(1) Four-variables Karnaugh maps:

wx \ yz	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
00 $\bar{w}\bar{x}$	$\bar{w}\bar{x}\bar{y}\bar{z}$ m_0	$\bar{w}\bar{x}\bar{y}z$ m_1	$\bar{w}\bar{x}yz$ m_3	$\bar{w}\bar{x}y\bar{z}$ m_2
01 $\bar{w}x$	m_4 $\bar{w}x\bar{y}\bar{z}$	m_5 $\bar{w}x\bar{y}z$	m_7 $\bar{w}xyz$	m_6 $\bar{w}xy\bar{z}$
11 wx	m_{12} $wx\bar{y}\bar{z}$	m_{13} $wx\bar{y}z$	m_{15} $wxyz$	m_{14} $wxy\bar{z}$
10 $w\bar{x}$	m_8 $w\bar{x}\bar{y}\bar{z}$	m_9 $w\bar{x}\bar{y}z$	m_{11} $w\bar{x}yz$	m_{10} $w\bar{x}y\bar{z}$

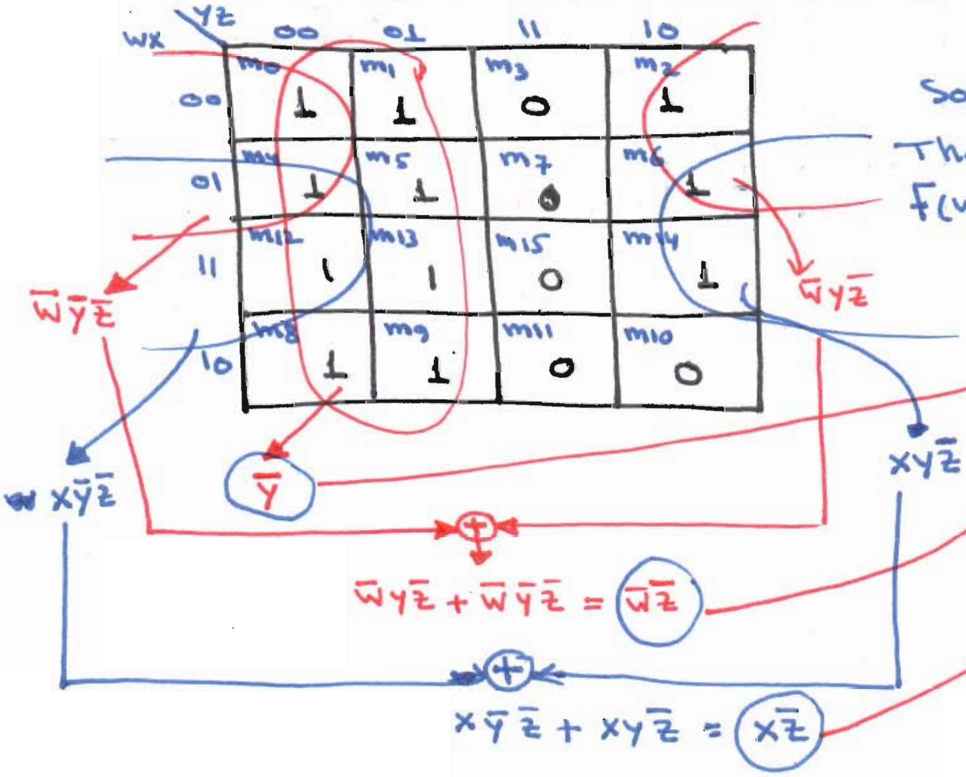
- The rows and columns are numbered in a Gray code sequence.

- One square represent one minterm with four literals.
- two adjacent squares represent one term with 3 literals.
- Four adjacent squares represent one term with 2 literals
- Eight adjacent squares represent one term with 1 literal

Examples:

Example 1: Simplify the boolean function

$$F(w,x,y,z) = \sum(0,1,2,4,5,6,8,9,12,13,14)$$



Solution:

The simplified function

$$F(w,x,y,z) = \bar{y} + \bar{w}\bar{z} + x\bar{z}$$

(2)

Example 2: plot the following 4-variable expression on a Karnaugh map.

$$f(a,b,c,d) = acd + \bar{a}b + \bar{d}$$

cd \ ab	$\bar{a}\bar{b}$	$\bar{a}b$	ab	$a\bar{b}$
$\bar{c}\bar{d}$	1	1	1	1
$\bar{c}d$	0	1	0	0
cd	0	1	1	1
$c\bar{d}$	1	1	1	1

$\bar{a}b$

acd

$\bar{d}$

Example 3:

Simplify the following function:

$$f(a,b,c,d) = \sum m(0,2,3,5,6,7,8,10,11,14,15)$$

cd \ ab	00	01	11	10
00	1			1
01		1		
11	1	1	1	1
10	1	1	1	1

four corner terms combine to give $\bar{b}\bar{d}$

$\bar{a}bd$

c

$$f(a,b,c,d) = c + \bar{b}\bar{d} + \bar{a}bd$$

Example 4: for the following K-map with four variables, obtain the simplified logic expression:

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	0	0
$\bar{A}B$	0	1	1	1
AB	1	1	1	0
$A\bar{B}$	0	0	1	0

$$X = AB\bar{C} + \bar{A}\bar{C}D + \bar{A}BC + ACD$$

Example 5: Use a K-map to simplify

$$Y = \bar{C}(\bar{A}\bar{B}\bar{D} + D) + A\bar{B}C + \bar{D}$$

Solution:

1. multiply out: $Y = \bar{C}\bar{A}\bar{B}\bar{D} + \bar{C}D + A\bar{B}C + \bar{D}$
2. fill the term in K-map:

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	0	1
$\bar{A}B$	1	1	0	1
AB	1	1	0	1
$A\bar{B}$	1	1	1	1

3. Simplify:

$$Y = A\bar{B} + \bar{C} + \bar{D}$$

2 Simplification using prime Implicants

definitions:

- prime Implicant (PI): is a product term obtained by combining the maximum possible number of adjacent squares in the map.
- essential prime Implicant:

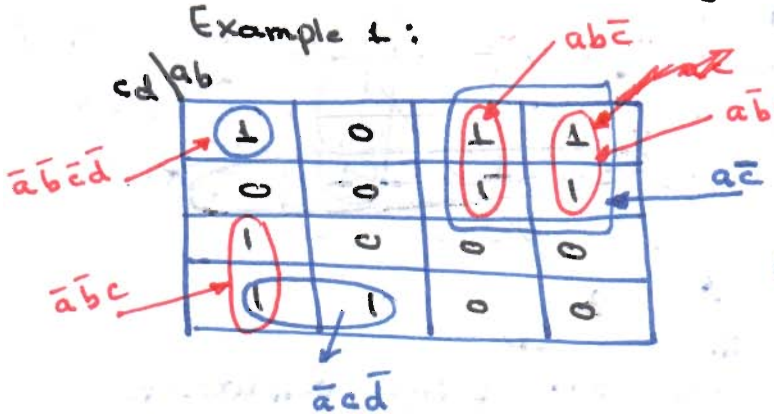
  if a minterm in a square is covered by only one prime implicant, that prime implicant is said to be essential, and it must be included in the final expression.

Note: all of the prime implicants of a function are generally not needed in forming the minimum sum of products.

Procedure for selecting Implicants:

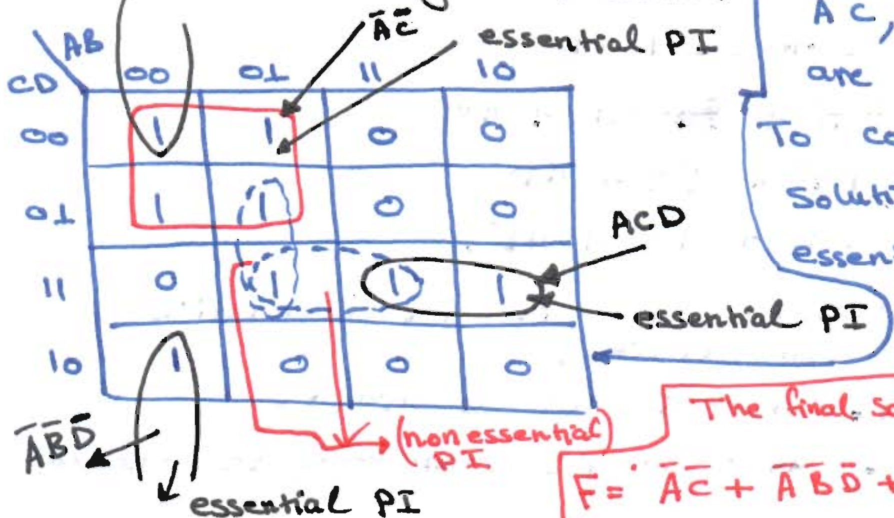
1. find essential prime implicants.
2. find a minimum set of prime implicant which cover the remaining 1's on the map.

Example 1:



- $\bar{a}\bar{b}c$, $\bar{a}c\bar{d}$ and $a\bar{c}$ are prime implicants
- $\bar{a}\bar{b}\bar{c}\bar{d}$, $a\bar{b}c$ and $ab\bar{c}$ are not a prime implicants

Example 2: find the minimum solution for the following expression:



$\bar{A}\bar{C}$, ACD and $\bar{A}\bar{B}\bar{D}$ are essential Prime implicants, To complete the minimum solution, one of the non-essential Prime implicants in needed:

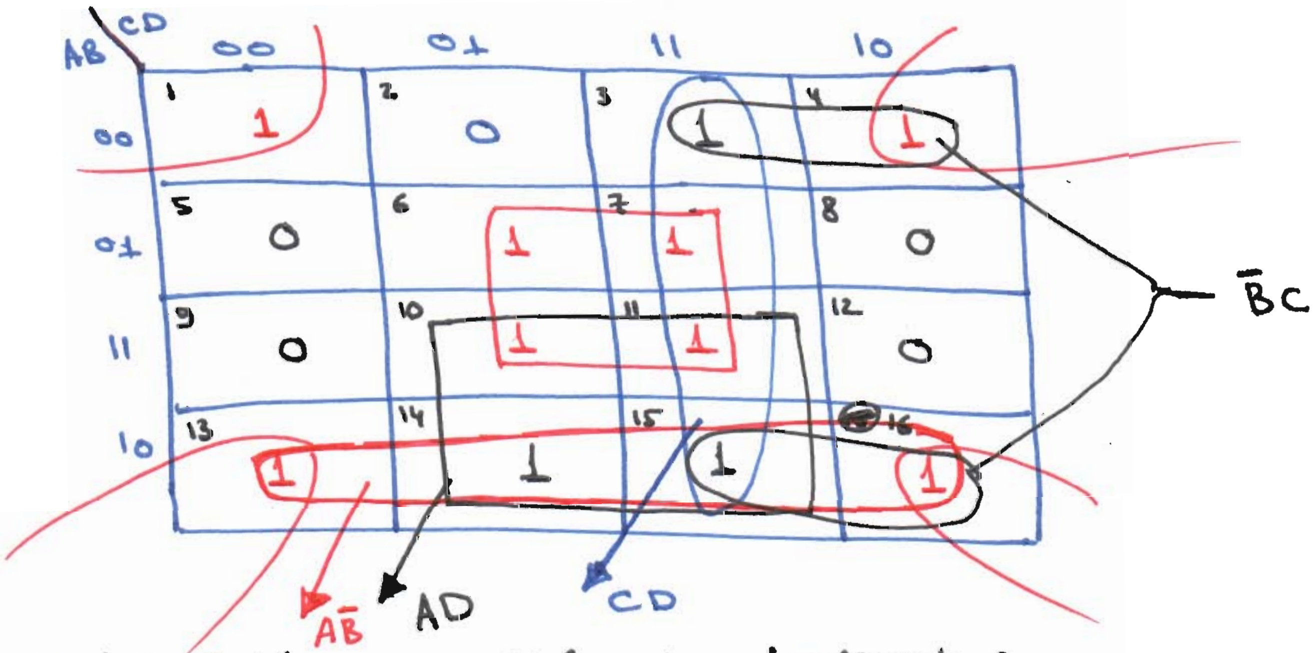
The final solution:

$$F = \bar{A}\bar{C} + \bar{A}\bar{B}\bar{D} + ACD + \begin{Bmatrix} \bar{A}BD \\ \text{or} \\ BCD \end{Bmatrix}$$

Example 3:

$F(A,B,C,D) = \sum m(0,2,3,5,7,8,9,10,11,13,15)$

Simplify using K-map:



- 1, 4, 13, 16 → essential prime implicant :
  $(\bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D}) + (A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D}) = \bar{A}\bar{B}\bar{D} + A\bar{B}\bar{D} = \boxed{\bar{B}\bar{D}}$
- 6, 7, 10, 11 → essential prime implicant : $\boxed{BD}$
- 3, 4, 15, 16 → prime implicant : $\boxed{\bar{B}C}$
- 3, 7, 11, 15 → prime implicant : $\boxed{CD}$
- 10, 11, 14, 15 → prime implicant : $\boxed{AD}$
- 13, 14, 15, 16 → prime implicant : $\boxed{A\bar{B}}$

2 essentials and four prime implicants:

- Square 3 can be covered with either prime implicants CD or $\bar{B}C$
- Square ~~3~~14 → with AD or $A\bar{B}$
- square 15 → with any one of four prime implicants.

final solution : two essential PI with any two prime Implicants that cover minterms 3, 14, 15,

four possible ways :

$$F = BD + \bar{B}\bar{D} + CD + AD \quad (1)$$
$$= BD + \bar{B}\bar{D} + CD + A\bar{B} \quad (2)$$
$$= BD + \bar{B}\bar{D} + \bar{B}C + AD$$
$$= BD + \bar{B}\bar{D} + \bar{B}C + A\bar{B}$$

3. "Don't Care" Conditions:

Some logic circuits can be designed so that there are certain input conditions for which there are no specified output levels: (Can't happened).
A circuit designer is free to make the output for any "don't care condition" either a 0 or a 1 in order to produce the simplest output expression.

Example 1:

A	B	C	Z
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	X
1	0	0	X
1	0	1	1
1	1	0	1
1	1	1	1

X } don't care

AB \ C	$\bar C$	C
$\bar A\bar B$	0	0
$\bar A B$	0	X
AB	1	1
$A\bar B$	X	1

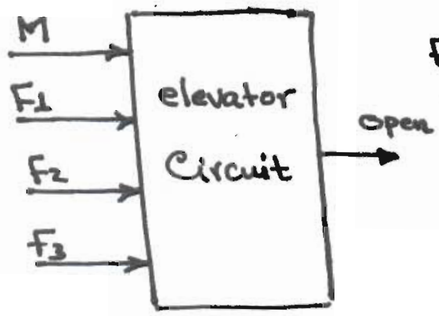
$\Rightarrow$

AB \ C	$\bar C$	C
$\bar A\bar B$	0	0
$\bar A B$	0	0
AB	1	1
$A\bar B$	1	1

$\boxed{Z = A}$

Example 2:

Design a logic circuit that controls an elevator door in a three-story building.



M : Moving signal, (M = 1 → moving), M = 0 → stopped

F1, F2 and F3 → floor indicator signals

M	F1	F2	F3	open
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	X
0	1	0	0	1
0	1	0	1	X
0	1	1	0	X
0	1	1	1	X
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	X
1	1	0	0	0
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

MF_1 \ F_2F_3	$\bar F_2\bar F_3$	$\bar F_2 F_3$	F_2F_3	$F_2\bar F_3$
$\bar M\bar F_1$	0	1	X	1
$\bar M F_1$	1	X	X	X
MF_1	0	X	X	X
$M\bar F_1$	0	0	X	0

⇓

	0	1	1	1
	1	1	1	1
	0	0	0	0
	0	0	0	0

$$\text{open} = \bar M(F_1 + F_2 + F_3)$$

Summary :

- Looping a pair of adjacent 1s in a K-map eleminate the variable that appear in complemented and uncomplemented form.
- Looping a quad (4) of adjacent 1s eleminates the two variables that appear in complemented and uncomplem. form.
- Looping an octet (8) of adjacent 1s eleminates the three variables that appear in complemented and uncom. form.
- Looping groups of Two pairs :

	$\bar{C}$	C
$\bar{A}\bar{B}$	0	0
$\bar{A}B$	1	0
AB	1	0
$A\bar{B}$	0	0

$X = B\bar{C}$

	$\bar{C}$	C
$\bar{A}\bar{B}$	0	0
$\bar{A}B$	1	1
AB	0	0
$A\bar{B}$	0	0

$X = \bar{A}B$

	$\bar{C}$	C
$\bar{A}\bar{B}$	1	0
$\bar{A}B$	0	0
AB	0	0
$A\bar{B}$	1	0

$X = \bar{B}\bar{C}$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	1	1
$\bar{A}B$	0	0	0	0
AB	0	0	0	0
$A\bar{B}$	1	0	0	1

$\bar{A}\bar{B}C$

$X = \bar{A}\bar{B}C + A\bar{B}\bar{D}$

$A\bar{B}\bar{D}$

- Looping Groups of Four (Quads)

	$\bar{C}$	C
$\bar{A}\bar{B}$	0	1
$\bar{A}B$	0	1
AB	0	1
$A\bar{B}$	0	1

$X = C$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	0	0	0
AB	1	1	1	1
$A\bar{B}$	0	0	0	0

$X = AB$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	1	1	0
AB	0	1	1	0
$A\bar{B}$	0	0	0	0

$X = BD$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	0	0	0
AB	1	0	0	1
$A\bar{B}$	1	0	0	1

$X = A\bar{D}$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	0	0	1
$\bar{A}B$	0	0	0	0
AB	0	0	0	0
$A\bar{B}$	1	0	0	1

$X = \bar{B}\bar{D}$

- Looping groups of Eight (octet)

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	1	1	1	1
AB	1	1	1	1
$A\bar{B}$	0	0	0	0

$X = B$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	0	0
$\bar{A}B$	1	1	0	0
AB	1	1	0	0
$A\bar{B}$	1	1	0	0

$X = \bar{C}$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	1	1
$\bar{A}B$	0	0	0	0
AB	0	0	0	0
$A\bar{B}$	1	1	1	1

$X = \bar{B}$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	0	0	1
$\bar{A}B$	1	0	0	1
AB	1	0	0	1
$A\bar{B}$	1	0	0	1

$X = \bar{D}$