Digital number System

objectives:

1. Understanding decimal, binary, octal and hexadecimal numbers.
2. Counting in decimal, binary, octal and hexadecimal systems.
3. convert a number from one number system (decimal, binary, octal, hexadecimal) to another system
4. Advantage of octal and hexadecimal systems.

1. Understanding decimal, binary, octal and hex numbers:

a. Decimal number Systems

- Decimal numbers are made of decimal digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) : (10-base system)
- The decimal system is a "postional-Value system" in which the value of a digit depends on its position.

Examples

① 453 → 4 hundreds, 5 tens and 3 Units

4 5 3

4 → "most weight" Called "Most significant digit" MSD

3 → Carries the least weight Called "Least Significant digit" LSD

② Number of items that a decimal number represent:

$$9261 = (9\times10^3) + (2\times10^2) + (6\times10^1) + (1\times10^0)$$

③ the decimal fractions:

3267.317 ⇒

$$(3\times10^3)+(2\times10^2)+(6\times10^1)+(7\times10^0)+(3\times10^{-1})+(1\times10^{-2})+(7\times10^{-3})$$

- decimal point used to separate the integer and fractional part of the number.
- formal notation → $(3267.317)_{10}$
- decimal position values of powers of (10)

positional values "weights"	10^4	10^3	10^2	10^1	10^0		10^{-1}	10^{-2}	10^{-3}	10^{-4}
	2	7	7	8	3	•	2	3	4	5
	↑MSD					decimal point				↑LSD

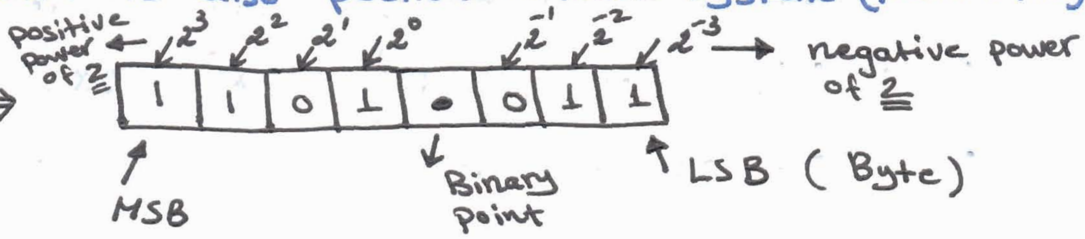
b. Binary numbers

base-2 system (0 or 1).

- We can represent any quantity that can be represented in decimal or other number systems.
- Binary number is also postional-value system (power of 2)

example:

1101.0011 ⇒



Notes:

1. To find the equivalent of the binary numbers in decimal system, We simply take the sum of products of each digit value (0, 1) and its positional value:

example:

$$(1011.101)_2 = (1\times2^3) + (0\times2^2) + (1\times2^1) + (1\times2^0) + (1\times2^{-1}) + (0\times2^{-2}) + (1\times2^{-3}) =$$

$$8 + 0 + 2 + 1 + \frac{1}{2} + 0 + \frac{1}{8} = 11.625_{10}$$

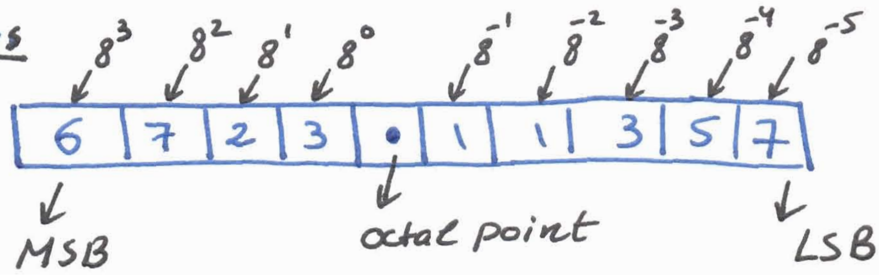
2. in general, any number (decimal, binary, octal or hex) is simply the sum of products of each digit value and its positional value.
3. In binary system, the term binary digit is often called **bit**
4. Binary values at the output of digital system must be converted to decimal values for Presentation to the outside world.
5. decimal values must be converted to binary values befor they are entered into the digital system.
6. Group of 8 bits are called a byte

C. Octal number system

octal number system has a base of 8: (0, 1, 2, 3, 4, 5, 6, 7)

examples

①



② $(4327)_8 \Rightarrow (4\times8^3)+(3\times8^2)+(2\times8^1)+(7\times8^0)$

③ The fractions:

$(372.36)_8=(3\times8^2)+(7\times8^1)+(2\times8^0)+(3\times8^{-1})+(6\times8^{-2})$

Note: octal numbers don't use digits **8** or **9**

d. Hexa decimal number system (16-base)

- Hexa numbers are made of 16 digits, it uses the digits 0 through 9 plus the letters A, B, C, D, E and F.

examples:

1) $(A29)_{16} = (10\times16^2)+(2\times16^1)+(9\times16^0) = (2601)_{10}$

2) The fractions:

$(2C7.38)_{16} = (2\times16^2)+(12\times16^1)+(7\times16^0)+(3\times16^{-1})+(8\times16^{-2})$

Note:

For hex numbers the digits 10, 11, 12, 13, 14, 15 are represented by A, B, C, D, E, F, as shown in the following table:

Table: Counting

zero count →

Hex	Decimal	Binary (weights 2^3 2^2 2^1 2^0)	octal
0	0	0000	0
1	1	0001	1
2	2	0010	2
3	3	0011	3
4	4	0100	4
5	5	0101	5
6	6	0110	6
7	7	0111	7
8	8	1000	10
9	9	1001	11
A	10	1010	12
B	11	1011	13
C	12	1100	14
D	13	1101	15
E	14	1110	16
F	15	1111	17

2. Counting in decimal, binary, octal and hexadecimal systems.

- **Decimal counting.**

1. start with 0 in the units position and take each digit in progression until reach **9**
2. add 1 to the next higer position and start over 0 in the first position
3. Continue process until the count 99.
4. add 1 to the third position and start over with 0_s in the first position.

Notes:

1) The largest number that can be represented using 8 bits is

$$2^N - 1 = 2^8 - 1 = 255_{10} = 11111111_2$$

2) counting in hexadecimal:

N Hex digit positions, we can count from decimal 0 to $16^N - 1$, for a total of 16^N different values.

3) For any number system,

r - base / radix, $\cdots a_4 a_3 a_2 a_1 a_0 . a_{-1} a_{-2} a_{-3} \cdots$

The general representation of number systems, in which the number of radix r can be written as

$$N_r = \cdots + a_4 \cdot r^4 + a_3 \cdot r^3 + a_2 \cdot r^2 + a_1 \cdot r^1 + a_0 \cdot r^0 + a_{-1} \cdot r^{-1} + a_{-2} \cdot r^{-2} + \cdots$$

Binary $r = 2$
octal $r = 8$
Hex $r = 16$
decimal $r = 10$

4) Counting in Binary system: (counting range)

Using N bits, we can represent decimal numbers ranging from 0 to $2^N - 1$, a total of 2^N different numbers.

examples:

1) for N = 4 bits ⇒

We can count from 0000 to 1111_2 (see table) which is 0_{10} to 15_{10} (16 different numbers)

2) How many bits are needed to represent decimal values ranging from 0 to 12 500?

answer:
with 13 bits, we can count from 0 to $2^{13} - 1 = 8191$ (Not enough)
with 14 bits, we can count from 0 to $2^{14} - 1 = 16{,}383$ (okay)