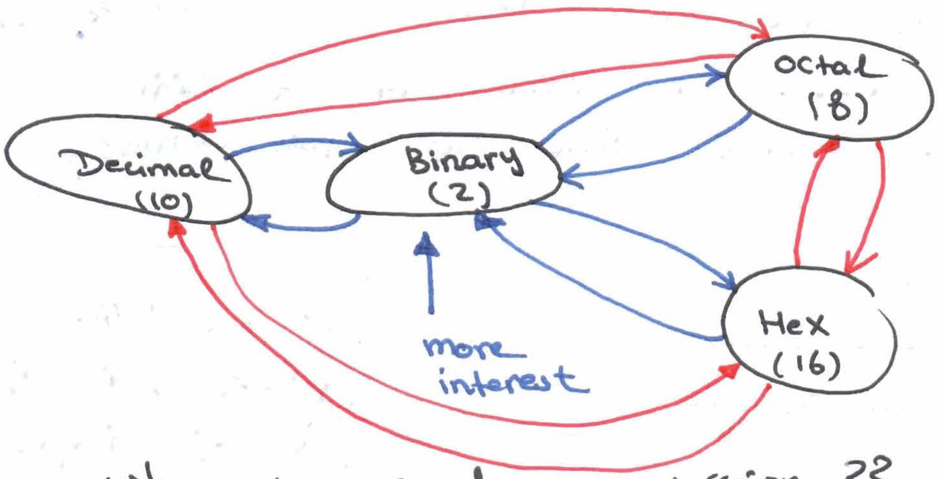
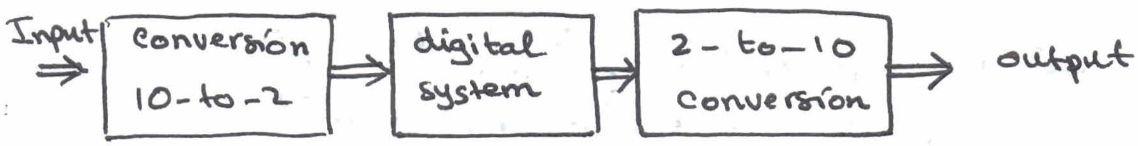
3. Convert a number from one number system to another.

Conversion between number bases:



- why we need conversion ??

real world (for presentation and input) → we need decimal systems: for example: input and input in digital calculator we use 10-based, inside calculator → it's binary.



a. Binary to decimal conversions:

- rule: any binary number can be converted to it's decimal equivalent simply by summing together the weights of the various positions in the binary number which contains a 1.

example 1:

Convert 11011_2 to its decimal equivalent:

$$\begin{array}{ccccccccc} 1 & & 1 & & 0 & & 1 & & 1 \\ \uparrow & + & \downarrow & + & \downarrow & + & \downarrow & & \downarrow \\ 2^4 & & +2^3 & & 0 & & 2^1 & & 2^0 \end{array} = 16 + 8 + 2 + 1 = 27_{10}$$

example 2:

Convert 10110101_2 to decimal equivalent:

$$2^7 + 0 + 2^5 + 2^4 + 0 + 2^2 + 0 + 2^0 = 181_{10}$$

b. Decimal to binary conversions:

There are two ways to convert a decimal number to its equivalent binary representation

1. The reverse of the binary-to- decimal conversion process (optional).

The decimal number is simply expressed as a sum of powers of 2 and then 1s and 0s are written in the appropriate bit positions.

examples:

Convert 45_{10} to binary number

$$45_{10} = 32 + 8 + 4 + 1 = 2^5 + 0 + 2^3 + 2^2 + 0 + 2^0$$

example 2:

$$= 101101_{(2)}$$

convert 76_{10} to binary number:

$$76_{10} = 64 + 8 + 4 = 2^6 + 2^3 + 2^2 = 1001100_2$$

2. Repeated Division:

repeating division the decimal number by 2 and writting down the remainder after each division until a quotient of 0 is obtained.

* Note: the binary result is obtained by writting the first remainder as the LSB and the last remainder as the MSB.

example: Convert 25_{10} to binary number:

$\frac{25}{2} = 12$ + remainder of 1 (LSB)

$\frac{12}{2} = 6$ + remainder of 0

$\frac{6}{2} = 3$ + remainder of 0

$\frac{3}{2} = 1$ + remainder of 1

$\frac{1}{2} = 0$ + remainder of 1 (MSB)

$$25_{10} \Leftrightarrow 11001_2$$

example 2:

Convert 13_{10} to binary:

		Quotient integer		remainder	
$\frac{13}{2}$	=	6	+	1	(a_0)
$\frac{6}{2}$	=	3	+	0	(a_1)
$\frac{3}{2}$	=	1	+	1	(a_2)
$\frac{1}{2}$	=	0	+	1	(a_3)

Answer $(13)_{10} = (a_3 a_2 a_1 a_0) = (1101)_2$.

General Rule 1:

Conversion from decimal to other base:

- Divide decimal number by the base (2, 8, 16, ...)
- The remainder is the lowest-order digit
- Repeat first two steps until no divisor remains.

General Rule 2:

Deimal fraction conversion to another base

- multiply decimal number by the base (2, 8, ..)
- The integer is the highest - order digit
- Repeat first two steps until fraction becomes zero.

Examples:

1) convert 0.625_{10} to binary number.

		Integer		fraction	coefficient
0.625×2	=	1	+	0.25	$a_{-1} = 1$
0.250×2	=	0	+	0.50	$a_{-2} = 0$
0.500×2	=	1	+	0 (stop)	$a_{-3} = 1$

Correct order

Answer

$(0.625)_{10} = (0.a_{-1}a_{-2}a_{-3})_2 = (0.101)_2$

c. octal-to-decimal :

To convert multiply each octal digit by its positional weight :

example1:

$$372_{(8)} = (3 \times 8^2) + (7 \times 8^1) + (2 \times 8^0) = (3 \times 64) + 56 + 2$$
$$= 250_{10}$$

example 2 :

$$24.6_8 = (2 \times 8^1) + (4 \times 8^0) + (6 \times 8^{-1}) = 20.75_{10}$$

d. decimal to octal :

repeated division by 8 :

example1: convert 266_{10} to octal number.

$\frac{266}{8} = 33$ + remainder of 2 (LSD)

$\frac{33}{8} = 4$ + remainder of 1

$\frac{4}{8} = 0$ + remainder of 4

$$266 = 412_{(8)}$$

example 2 :

convert 0.35_{10} to octal :

	integer	fraction	coefficient
$0.35 \times 8 =$	2	.80	2
$0.8 \times 8 =$	6	.40	6
$0.4 \times 8 =$	3	.20	3
$0.2 \times 8 =$	1	.60	1
$0.6 \times 8 =$	4	.80	4

repeated

"stop"

$$0.35_{(10)} = 0.26314_{(8)}.$$

e. He* - be - decimal

examples:

- Convert $356_{(16)}$ to decimal:

$$356_{(16)} = (3 \times 16^2) + (5 \times 16^1) + (6 \times 16^0) = 3 \times 256 + 80 + 6$$
$$= 854_{(10)}$$

- Convert $2AF_{(16)}$ to decimal:

$$2AF_{(16)} = (2 \times 16^2) + (10 \times 16^1) + (15 \times 16^0) = 512 + 160 + 15$$
$$= 687_{(10)}$$

f. decimal-to-hex: (using repeated division by 16)

examples:

1. Convert 423_{16} to hex:

$$\frac{423}{16} = 26 + \text{remainder of } 7 \ (LSB)$$

$$\frac{26}{16} = 1 + \text{remainder of } 10$$

$$\frac{1}{16} = 0 + \text{remainder of } 1$$

$$423_{(10)} = 1A7_{(16)}$$

g. Hex-to-binary:

each hex digit is converted to its bur- bit binary equivalent:

example1: Convert to binary numbers

1. $9F2_{(16)}$ =

$$9 \rightarrow 1001 \qquad F \rightarrow 1111 \qquad 2 \rightarrow 0010$$

$$9F2_{(16)} = 100111110010_{(2)}$$

example 2 =

2. $BA6_{16} = (\underline{1011}\ \underline{1010}\ \underline{0110})_2$

h. Binary-to-hex.

The binary number are grouped into groups of four bits, and each group is converted to its equivalent hex digit.

Zero are added as needed to complete a four-bit group.

Examples: convert $1110100110_{(2)}$ to hex number

$$\underline{00\,11} \quad \underline{1010} \quad \underline{0110}$$

added zero

0011	1010	0110
↓	↓	↓
3	A	6

$\Rightarrow \; 1110100110_{(2)} = 3A6_{(16)}$

Example 2: Convert $101011111_2 = 15F_{(16)}$

i- octal to binary conversion:

Conversion each octal digit to it three bit binary equivalent.

Conversion table:

octal digit	0	1	2	3	4	5	6	7
Binary equivalent	000	001	010	011	100	101	110	111

using this table, we can convert any octal number to binary by individually converting each digit:

exampLL: convert $472_{(8)}$ to binary number

4	7	2
↓	↓	↓
100	111	010

The octal number 472 is equivalent to binary 100 111 010.

example 2: Convert $5431_{(8)}$ to binary number:

$5431_8 \leftrightarrow 101\ 100\ 011\ 001 \Rightarrow 5431_{(8)} = 101100011001_{(2)}$

J. Binary to octal conversion:

The bits of the binary number are grouped into groups of 3 bits starting at the LSB, then each group is converted to its octal equivalent (see table).

examples: convert $11010110_{(2)}$ to octal number

added 0 → 011 010 110

011	010	110
↓	↓	↓
3	2	6

$\Rightarrow \; 11010110_{(2)} = 326_{(8)}$

Note: zero was placed to the left of the MSB to produce groups of 3 bits.

General example:

Convert 177_{10} to its eight-bit binary equivalent by first converting to octal.

Solution:

$\frac{177}{8} = 22$ + remainder of 1 (LSB)

$\frac{22}{8} = 2$ + remainder of 6

$\frac{2}{8} = 0$ + remainder of 2 (MSB)

Thus $177_{10} = 261_{(8)}$ → Now we can quickly convert this octal number to its binary equivalent ~~0~~10110001 → to get eight bit representation

So: $177_{(10)} \longrightarrow 10110001_{(2)}$

Important Note: This method of decimal-to-octal-to-binary conversion is often quicker than going directly from decimal to binary, especially for large numbers.

4. Advantage of octal and hexadecimal systems:

1. Hexa and octal numbers are used as a "short hand" way to represent strings of bits.
2. Error prone to write the binary number, in Hex and octal less error.
3. The octal and hexadecimal number systems are both used (in memory addressing & microprocessor technology).