Boolean Algebra - I

objectives:

1. Binary operators and their representations
2. Relationships between Boolean expressions, Truth tables and logic Circuits.
3. Logic Gates' Postulates, Laws and properties.

① Binary Operators and their representations:

Boolean Algebra is the basic mathematics needed for Logic design of digital systems, Boolean algebra uses Boolean (logical) variables with two values (0 or 1).
"Two-Valued Boolean Algebra".

- **Basic operations:**

The basic operations of Boolean algebra are AND, OR, and NOT (complement).

1. NOT operation (NOT Gate):

$\overline{1} = 0$; $\overline{0} = 1$.

- The not operator is also called the complement or the inverse:

$\overline{X}$ is the complement of X.

⇒ Not Gate representation

Truth table:

1 - High
0 - Low

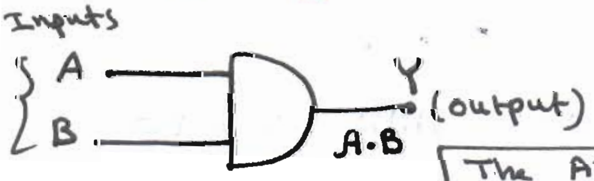
X (input)	$\overline{X}$ (output)
0	1
1	0

} • output is opposite of input

Truth table: Truth table describes inputs and outputs in terms of 1_s and 0_s rather than physical (voltage) levels.

2. AND operation (AND Gate).

- the output is 1 only if all inputs are 1, if any of the input is (0), then the output is 0.

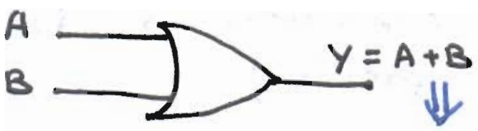


The truth table of 2-inputs, 1-output AND Gate:

Inputs A	Inputs B	output A·B
0	0	0
0	1	0
1	0	0
1	1	1

The AND operation is referred to as Logical multiplication

3. OR operation (OR Gate)



⇓

$y = \underline{A}$ OR $\underline{B}$

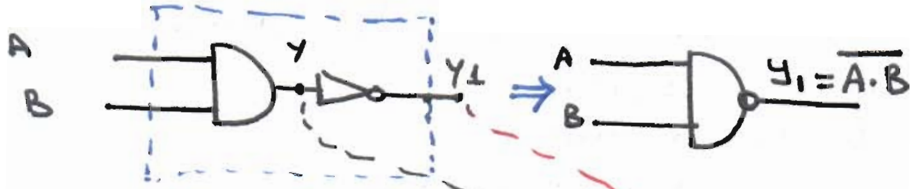
* the output is 1 if
  - A is 1 or if
  - B is 1

Truth table of OR Gate (2-inputs and 1-output)

Inputs A	Inputs B	output $Y = A+B$
0	0	0
0	1	1
1	0	1
1	1	1

- OR operation is sometimes referred to as "inclusive OR" or Logical addition.

4. NAND Gate: (Not AND Gate)

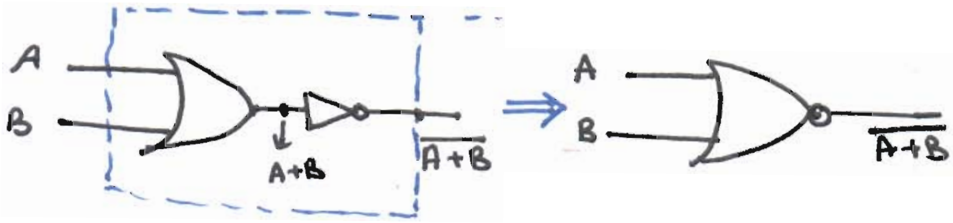


Truth table:

⇒ 2-Inputs NAND = $\overline{\text{2-Inputs AND}}$

Inputs A	Inputs B	output Y	output Y1
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

5. NOR Gate (Not OR):

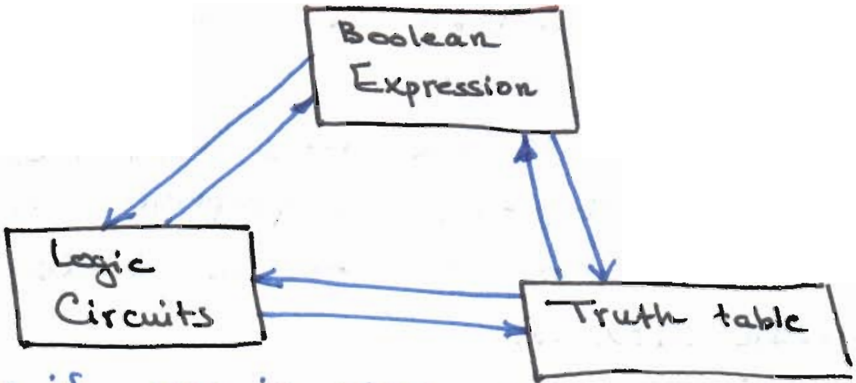


2- Inputs NOR = $\overline{\text{2-inputs OR}}$

Truth table:

Inputs A	Inputs B	A+B	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

② Relationships between Boolean expression, Truth tables and logic circuits.



* if one is given, we can get the other.

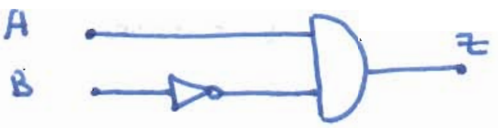
Example : Boolean Expression ⟶ logic circuit

To draw a circuit from a boolean expression :

- From the left, make an input line for each variable.
- Next, put a NOT gate in for each variable, that appears negated in the expression.
- Still working, from left to right.

Examples:

① $z = A \cdot \overline{B}$

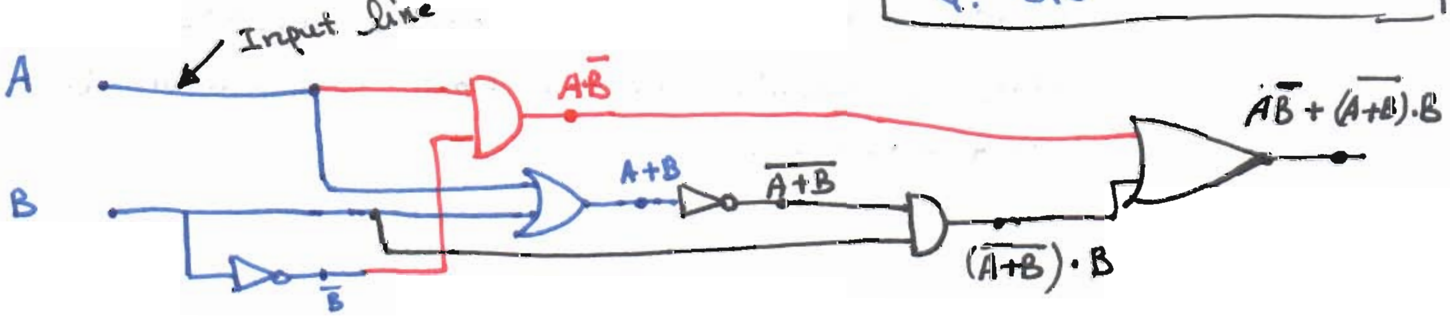


⇒ this is a logic circuit.

precedence of operators:
1. parentheses.
2. Not
3. AND
4. OR

② Example 2:

$A \cdot \overline{B} + \overline{(A+B)} \cdot B$



3. Logic Gates' postulates, laws and properties.

postulates are used to deduce the rules, theorems and properties.

1. postulates of Boolean Algebra :

P1 : $A + 0 = A \Longrightarrow A \cdot 1 = A$

P2 : $A + \bar{A} = 1 \Longrightarrow A \cdot \bar{A} = 0$

P3 : $A + B = B + A \Longrightarrow A \cdot B = B \cdot A$

P4 : $A \cdot (B + C) = A \cdot B + AC \Rightarrow A + BC = (A+B) \cdot (A+C)$

Duality priciple

Duality priciple states that every algebraic expression is deducible if the operators and the identity elements are interchanged.

Identity elements :	0	for	or
	1	for	and

2. Boolean Algebra Theorems :

There are six theorems of Boolean Algebra

T1: Idempotent Laws :

a) $A + A = A \xrightarrow{\text{duality}}$ b) $A \cdot A = A$

T2: operations with 0 and 1

a) $A + 1 = 1 \xrightarrow{\text{duality}}$ b) $A \cdot 0 = 0$

T3: Involution Law :

$\bar{\bar{A}} = A$

T4: Associative Laws :

Proof

a) $A + (B + C) = (A + B) + C \longrightarrow$ b) $A \cdot (B \cdot C) = (A \cdot B) \cdot C$

T5: De Morgan Laws (Inversion Law)

a) $\overline{A + B} = \bar{A} \cdot \bar{B} \longrightarrow$ b) $\overline{AB} = \bar{A} + \bar{B}$

T6 : Absorption Laws :

a) $A + AB = A \longrightarrow$ b) $A(A + B) = A$

first way : Theorems of Boolean Algebra can be shown to hold by means of Truth table.

Example1

A	B	AB	$A + AB$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

$\Rightarrow$ $A + AB = A$

Example 2: verify the DeMorgan's Law using a truth table.

x	y	x'	y'	$x+y$	$\overline{x+y}$	$\bar{x}\cdot\bar{y}$	xy	$\overline{xy}$	$\bar{x}+\bar{y}$
0	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	0	0	1	1
1	0	0	1	1	0	0	0	1	1
1	1	0	0	1	0	0	1	0	0

$$\overline{x+y} = \bar{x}\cdot\bar{y}$$
$$\overline{x\cdot y} = \bar{x}+\bar{y}$$

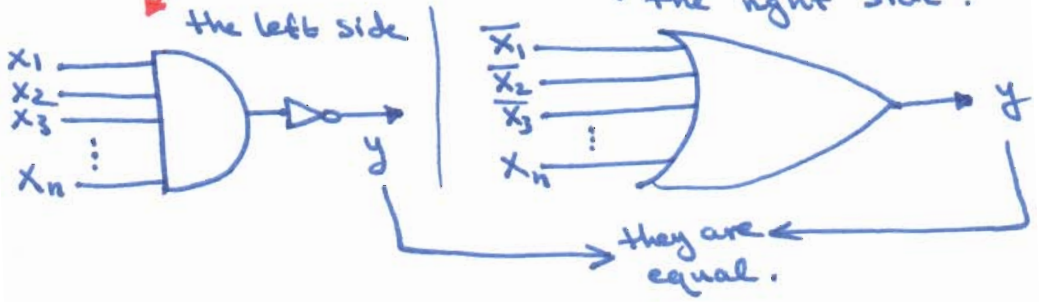
Some details: ⇒

- The duality principle is formed by replacing AND with OR, OR with AND, 0 with 1, 1 with 0, variables and complements are left unchanged.
- DeMorgan's Laws:

  allow us to convert between types of gates; we can generaliz them to n variables:

$$\overline{(x_1 + x_2 + x_3 + \cdots + x_n)} = \bar{x}_1 \cdot \bar{x}_2 \cdot \bar{x}_3 \cdot \ldots \cdot \bar{x}_n$$
$$\overline{(x_1 x_2 x_3 \ldots x_n)} = \bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \cdots + \bar{x}_n$$

the right side.



Second way: algebraically using the basic theorems.

Examples:

a) $A + AB = A$ b) $A(A+B) = A$

proof (a):

$A + AB = A\cdot 1 + AB = A(1+B) = A\cdot 1 = A$

Proof (b):

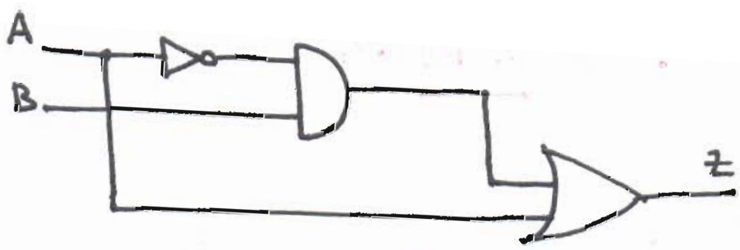
$A(A+B) = A \longrightarrow$ by duality.

Examples:

1) $(A\overline{B} + D)E + 1 = 1$

2) $(A\overline{B} + D) \cdot \overline{(A\overline{B}+D)} = 0$

3) An input A is inverted and applied to an AND Gate. The other input is B. The output of the AND gate is applied to an OR gate. A is the second input to OR gate.
Draw the Logic Circuit and the truth table.



$Z = \overline{A}B + A$

truth table

A	B	Z
0	0	0
0	1	1
1	0	1
1	1	1

OR Gate

4) Proof the following Boolean Expression
"Theorems"

a) $X + XY = X$

Proof: $X + XY = X \cdot 1 + XY = X(1+Y) = X \cdot 1 = X$

b) $X(X+Y) = X$

proof:

$X(X+Y) = X \cdot X + X \cdot Y = X + XY = X(1+Y) = X \cdot 1 = X$

c) $X\overline{Y} + Y = X + Y$

$X\overline{Y} + Y = Y + X\overline{Y} = (Y+X)(Y+\overline{Y}) = (Y+X) \cdot 1 = Y+X$

home work:

Proof the following theorems:

1. $XY + X\overline{Y} = X$
2. $(X+Y)(X+\overline{Y}) = X$
3. $(X+\overline{Y})Y = XY.$