

Karnaugh Map Methods

①

objectives:

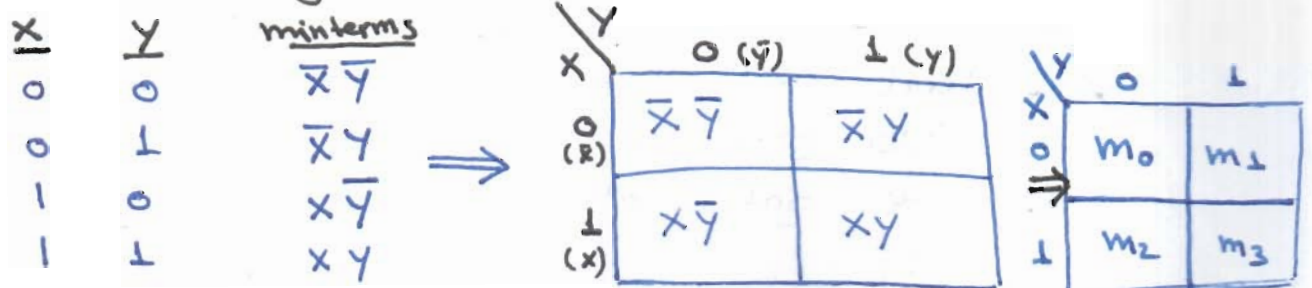
1. Karnaugh Map definition
2. Karnaugh map construction.
 - Two-variables k-maps.
 - Three-variables k-maps.
 - four variables k-maps
3. Summary.

① Karnaugh map definition:

- The Karnaugh map (k-map) is a graphical tool used to simplify (minimize) the logic functions, so that it can be implemented with minimum number of gates (minimum number of product terms and minimum number of literals).
- The k-map is used to convert a truth table to its corresponding logic circuit.

② Karnaugh map construction (2-variables)

- A two-variable function has four possible minterms, we can re-arrange these minterms into a Karnaugh map:



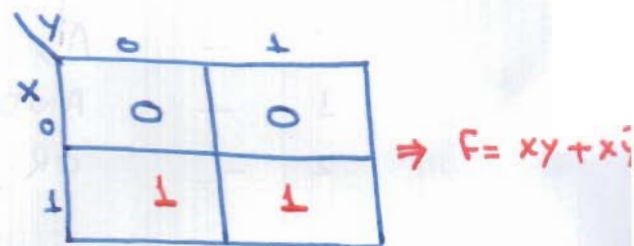
example:

$$F = xy + x\bar{y} \Rightarrow$$

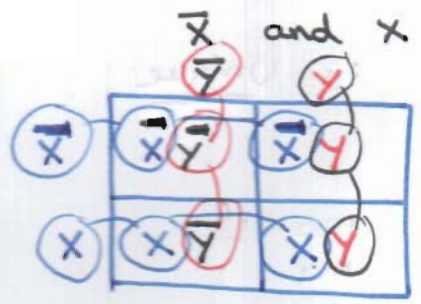
Karnaugh map will be:

Note: We can easily from the k-map see which

minterms contain common literal



- Minterms on the left and right sides contain \bar{y} and y respectively.
- Minterms in the top and bottom rows contain \bar{x} and x respectively.



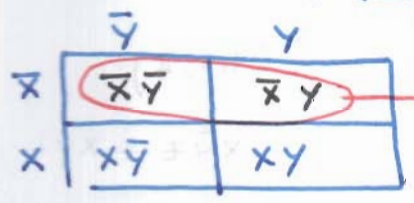
Karnaugh map simplification:

1. The k-map squares labeled so that horizontally adjacent square differ only in one variable

Note: each case in the truth table corresponds to a square in the k-map

example: Imagine a two-variable sum of minterms, both of these minterms appear in the top row of a karnaugh map, which means that they

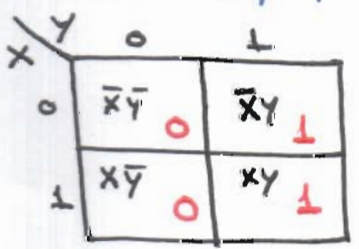
both contain the literal \bar{x}



$$F = \bar{x}\bar{y} + \bar{x}y = \bar{x}(\bar{y} + y) = \bar{x}$$

example 2:

$F = \bar{x}y + xy$ - minimize it using k-map.



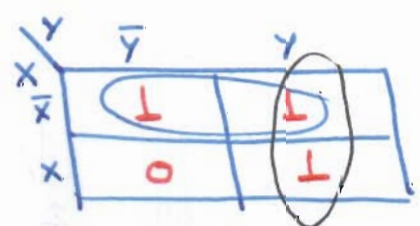
- both minterms appear in the right-side where y is uncomplemented
- Thus, we can reduce $\bar{x}y + xy$ just to y

$$F = \bar{x}y + xy = y(x + \bar{x}) = y$$

example 3:

$$z = \bar{x}\bar{y} + \bar{x}y + xy$$

- We have $\bar{x}\bar{y} + \bar{x}y$ in the top row, corresponding to \bar{x}



- There's also $\bar{x}y + xy$ in the right side corresponding to y . \Rightarrow The result $\Rightarrow z = \bar{x} + y$.

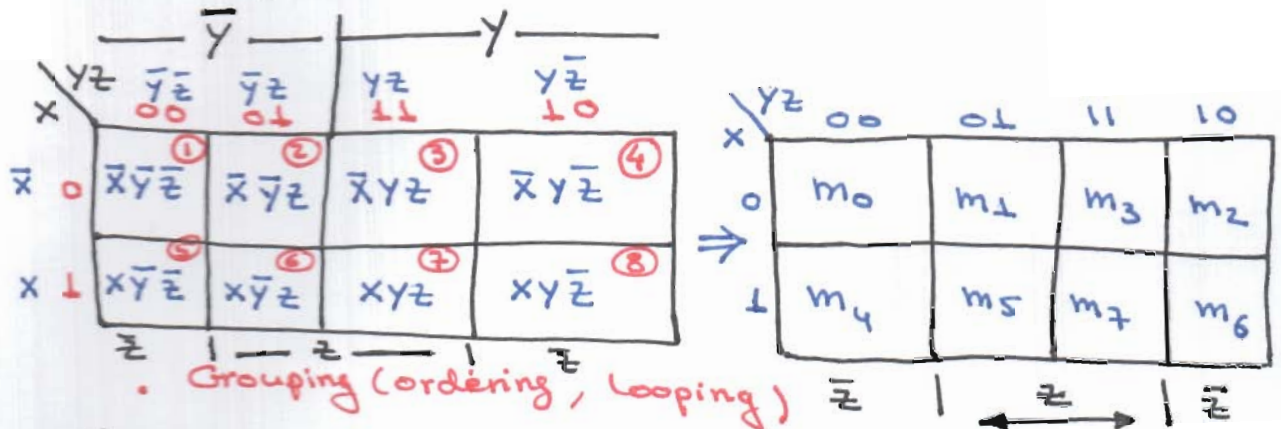
using algebraic simplification:

$$\begin{aligned} z &= \bar{x}\bar{y} + \bar{x}y + xy = \bar{x}(\bar{y} + y) + xy \\ &= \bar{x} + xy = (x + \bar{x})(\bar{x} + y) = \boxed{\bar{x} + y} \end{aligned}$$

Three-variables K-map

1- representation truth table using K-map.

$F(x,y,z) \Rightarrow 2^3 = 8$ minterms.



- the groups can be 2, 4, or 8 adjacent squares,
- 2 squares \rightarrow 1 variable canceled.
- 4 squares \rightarrow 2 variables - " - .
- 8 squares \rightarrow 3 variables - " - .

examples:

① and ② squares : the common literals $\Rightarrow \bar{x}\bar{y}$

② and ③ squares : the common literals $\Rightarrow \bar{x}z$

① and ④ squares : the common literals $\Rightarrow \bar{x}\bar{z}$

①, ②, ⑤ and ⑥ squares : the common literal $\Rightarrow \bar{y}$

③, ④, ⑦ and ⑧ squares : the common literal $\Rightarrow y$

①, ②, ③ and ④ squares : - " - - " - $\Rightarrow \bar{x}$

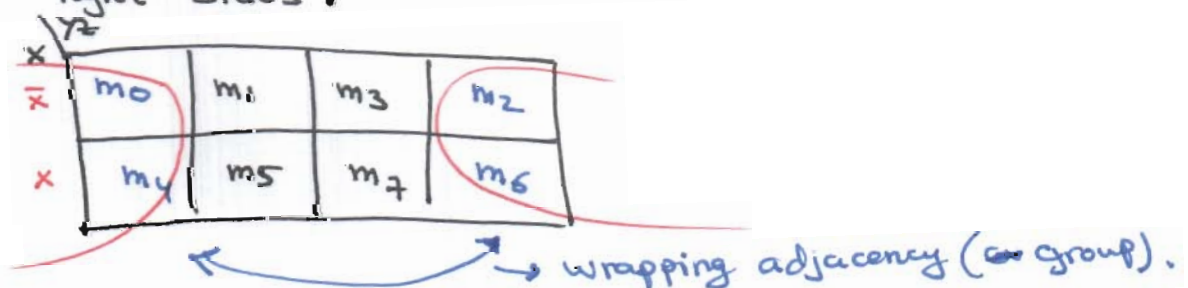
⑤, ⑥, ⑦ and ⑧ squares : - " - - " - $\Rightarrow x$

①, ⑤, ④ and ⑧ squares : - " - - " - $\Rightarrow \bar{z}$

(wrapping case is also adjacent), to proof this case algebraically, we can write:

$$\begin{aligned}
 F &= \bar{x}\bar{y}\bar{z} + x\bar{y}\bar{z} + \bar{x}y\bar{z} + xy\bar{z} \\
 &= \bar{z}(\bar{x}\bar{y} + x\bar{y} + \bar{x}y + xy) = \bar{z}(\bar{y}(\bar{x}+x) + y(\bar{x}+x)) \\
 &= \bar{z}(\bar{y} + y) = \bar{z}
 \end{aligned}$$

"Adjacency" includes wrapping around the left and right sides.



Example 1:

Simplify the following Logical function using K-map.

$$F(x,y,z) = xy + \bar{y}z + xz$$

Solution:

Step 1: the expression must be in a sum of minterms form, so we should convert it: (Two ways to do that)

1. using logical rules (algebraically)

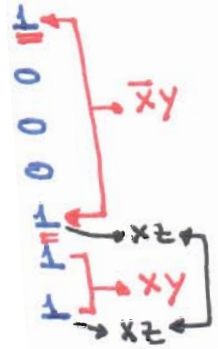
$$\begin{aligned}
 F &= xy + \bar{y}z + xz \\
 &= xy(z + \bar{z}) + \bar{y}z(x + \bar{x}) + xz(y + \bar{y}) = \\
 &= xyz + xy\bar{z} + x\bar{y}z + \bar{x}\bar{y}z + \underline{xy\bar{z}} + \underline{x\bar{y}z} \\
 &= xyz + xy\bar{z} + x\bar{y}z + \bar{x}\bar{y}z = m_1 + m_5 + m_6 + m_7
 \end{aligned}$$

2. make the truth table and read the minterms.

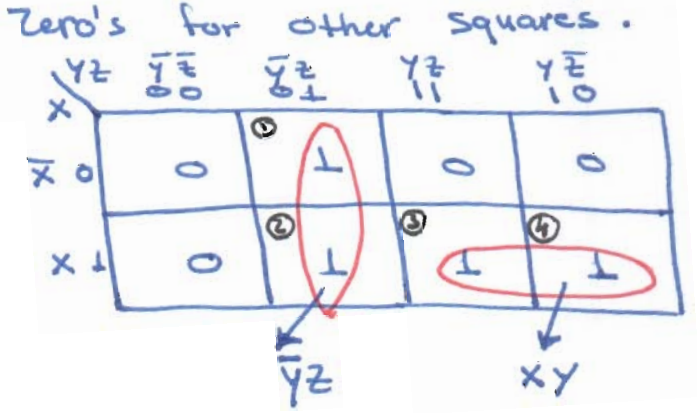
$F = xy + \bar{y}z + xz$

x	y	z	F(x,y,z)
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$F(x,y,z) = \bar{x}\bar{y}z + x\bar{y}z + xy\bar{z} + xyz$
 $= m_1 + m_5 + m_6 + m_7$



Step 2: fill one's (for the minterms) in Karnaugh map:



- Step 3: Grouping (Looping):
2 Groups: ① and ②, ③ and ④

Step 4: Simplify:

$$F = xy + \bar{y}z$$

To proof the result:

$$\begin{aligned}
 F &= \bar{x}\bar{y}z + x\bar{y}z + xy\bar{z} + xyz \\
 &= \bar{y}z(x + \bar{x}) + xy(z + \bar{z}) \\
 &= \bar{y}z + xy
 \end{aligned}$$

Grouping the minterms:

- Grouping together all the 1s in the K-map.
 - make rectangles of 2^n (1, 2, 4, ...)
 - all the 1s in the map should be included in at least one rectangle.
 - Do not include any of the 0s.
 - each group corresponds to one product term.
 - Make each rectangle as large as possible.
 - We can overlap the rectangles, if that makes them larger.

Example 2:

simplify the boolean function
 $Z = AB + \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} = \underline{\bar{A}B\bar{C} + AB\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C}}$

Answer:

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	1	0	0	0
A	1	0	1	1

$\Rightarrow F = \bar{B}\bar{C} + AB$

Note: Ignore!!

Example 3: simplify the boolean function

$Z = \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	0	1	0	0
A	1	1	1	1

1 couple
 1 quad
 $Z = A + \bar{B}C$

quad: $\bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} = A(\bar{B}C + \bar{B}\bar{C} + B\bar{C} + BC)$
 $= A(\bar{B}(C + \bar{C}) + B(\bar{C} + C)) = A(\bar{B} + B) = A$

couple: $\bar{A}\bar{B}C + A\bar{B}C = \bar{B}C(A + \bar{A}) = \bar{B}C$