Karnaugh Map Methods

objectives:

1. Karnaugh Map definition
2. Karnaugh map construction.
   - Two-variables K-maps.
   - Three-variables K-maps.
   - four variables K-maps
3. Summary.

① Karnaugh map definition:

- The Karnaugh map (K-map) is a graphical tool used to simplify (minimize) the logic functions, so that it can be implemented with minimum number of gates (minimum number of product terms and minimum number of literals).
- The K-map is used to convert a truth table to its corresponding logic circuit.

② Karanaugh map construction (2-variables)

- A two-variable function has four possible minterms, We can re-arrange these minterms into a Karnaugh map:

X	Y	minterms
0	0	$\bar{X}\bar{Y}$
0	1	$\bar{X}Y$
1	0	$X\bar{Y}$
1	1	XY

$\Longrightarrow$

X \ Y	0 ($\bar{Y}$)	1 (Y)
0 ($\bar{X}$)	$\bar{X}\bar{Y}$	$\bar{X}Y$
1 (X)	$X\bar{Y}$	XY

$\Rightarrow$

X \ Y	0	1
0	m_0	m_1
1	m_2	m_3

example:

$F = xy + x\bar{y} \Rightarrow$

Karnaugh map will be:

X \ Y	0	1
0	0	0
1	1	1

$\Rightarrow F = xy + x\bar{y}$

Note: We can easily from the K-map see which minterms contain common literal

- Minterms on the left and right sides contain $\bar{Y}$ and Y respectively.
- Minterms in the top and bottom rows contain $\bar{X}$ and X respectively.

	$\bar{Y}$	Y
$\bar{X}$	$\bar{X}\bar{Y}$	$\bar{X}Y$
X	$X\bar{Y}$	XY

Note: each case in the truth table corresponds to a square in the K-map

- **Karnaugh map simplification:**

1- The K-map squares labeled so that horizontally adjacent square differ only in <u>one variable</u>

example 1: Imagine a two-variable sum of minterms, both of these minterms appear in the top row of a Karnaugh map, which means that they both contain the literal $\bar{X}$

	$\bar{Y}$	Y
$\bar{X}$	$\bar{X}\bar{Y}$	$\bar{X}Y$
X	$X\bar{Y}$	XY

$$F = \bar{X}\bar{Y} + \bar{X}Y = \bar{X}(\bar{Y}+Y) = X$$

example 2:

$F = \bar{X}Y + XY$ - minimize it using K-map.

X \ Y	0	1
0	$\bar{x}\bar{y}$ 0	$\bar{x}y$ 1
1	$x\bar{y}$ 0	xy 1

- both minterms appear in the right-side where Y is Uncomplemented
- Thus, we can reduce $\bar{X}Y + XY$ just to $\underline{Y}$

$$F = \bar{X}Y + XY = Y(X+\bar{X}) = Y$$

example 3:

$Z = \bar{X}\bar{Y} + \bar{X}Y + XY$

X \ Y	$\bar{Y}$	Y
$\bar{X}$	1	1
X	0	1

- We have $\bar{X}\bar{Y} + \bar{X}Y$ in the top row, corresponding to $\underline{\bar{X}}$
- There's also $\bar{X}Y + XY$ in the right side corresponding to Y. ⇒ The result ⇒ $Z = \bar{X} + Y$.

using algebraic simplification:

$$Z = \bar{X}\bar{Y} + \bar{X}Y + XY = \bar{X}(\bar{Y}+Y) + XY$$

$$= \bar{X} + XY = (X+\bar{X})(\bar{X}+Y) = \boxed{\bar{X}+Y}$$

. Three-variables K-map

1- representation truth table using K-map.

$F(x,y,z) \Rightarrow 2^3 = 8$ minterms.

$x \backslash yz$	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
$\bar{x}$ 0	① $\bar{x}\bar{y}\bar{z}$	② $\bar{x}\bar{y}z$	③ $\bar{x}yz$	④ $\bar{x}y\bar{z}$
x 1	⑤ $x\bar{y}\bar{z}$	⑥ $x\bar{y}z$	⑦ xyz	⑧ $xy\bar{z}$
	$\bar{z}$	z	z	$\bar{z}$

$\Rightarrow$

$x \backslash yz$	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6
	$\bar{z}$	z	z	$\bar{z}$

. Grouping (ordering, Looping)

- the groups can be 2, 4, or 8 adjacent squares,
  - 2 squares $\rightarrow$ 1 variable canceled.
  - 4 squares $\rightarrow$ 2 variables —"—.
  - 8 squares $\rightarrow$ 3 variables —"—.

examples:

① and ② squares : the common literals :$\Rightarrow$ $\bar{x}\bar{y}$

② and ③ squares : the common literals :$\Rightarrow$ $\bar{x}z$

① and ④ squares : the common literals :$\Rightarrow$ $\bar{x}\bar{z}$

①, ②, ⑤ and ⑥ squares : the common literal : $\Rightarrow$ $\bar{y}$

③, ④, ⑦ and ⑧ squares : the common literal : $\Rightarrow$ y

①, ②, ③ and ④ squares : —"— —"— $\Rightarrow$ $\bar{x}$

⑤, ⑥, ⑦ and ⑧ squares : —"— —"— $\Rightarrow$ x

①, ⑤, ④ and ⑧ squares : —"— —"— $\Rightarrow$ $\bar{z}$

(wrapping case is also adjacent), to proof this case algebraically, we can write:

$$F = \bar{x}\bar{y}\bar{z} + x\bar{y}\bar{z} + \bar{x}y\bar{z} + xy\bar{z}$$

$$= \bar{z}(\bar{x}\bar{y} + x\bar{y} + \bar{x}y + xy) = \bar{z}(\bar{y}(\bar{x}+x) + y(\bar{x}+x))$$

$$= \bar{z}(\bar{y} + y) = \bar{z}$$

- "Adjacency" includes wrapping around the left and right sides.

$x \backslash yz$				
$\bar{x}$	m_0	m_1	m_3	m_2
x	m_4	m_5	m_7	m_6

$\rightarrow$ wrapping adjacency (group).

Example 1:

Simplify the following Logical function using K-map.

$F(x,y,z) = xy + \bar{y}z + xz$

Solution:

Step 1: the expression must be in a sum of minterms form, so we should convert it: (Two ways to do that)

1. using logical rules (algebraically)

$F = xy + \bar{y}z + xz$

$= xy(z+\bar{z}) + \bar{y}z(x+\bar{x}) + xz(y+\bar{y}) =$

$= xyz + xy\bar{z} + x\bar{y}z + \bar{x}\bar{y}z + \cancel{xyz} + \cancel{x\bar{y}z}$

$= xyz + xy\bar{z} + x\bar{y}z + \bar{x}\bar{y}z = m_1 + m_5 + m_6 + m_7$

2. make the truth table and read the minterms.

$F = xy + \bar{y}z + xz$

x	y	z	F(x,y,z)	
0	0	0	0	
0	0	1	1	
0	1	0	0	$\bar{x}y$
0	1	1	0	
1	0	0	0	
1	0	1	1	xz
1	1	0	1	xy
1	1	1	1	xz

$F(x,y,z) = \bar{x}\bar{y}z + x\bar{y}z + xy\bar{z} + xyz$

$= m_1 + m_5 + m_6 + m_7$

- Step 2: fill one's (for the minterms) in Karnaugh map: Zero's for other squares.

x \ yz	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
$\bar{x}$ 0	0	① 1	0	0
x 1	0	② 1	③ 1	④ 1

$\bar{y}z$ xy

- Step 3: Grouping (Looping):

2 Groups. ① and ②
③ and ④

- Step 4: Simplify:

$\boxed{F = xy + \bar{y}z}$

To proof the result:

$F = \bar{x}\bar{y}z + x\bar{y}z + xyz + xy\bar{z}$

$= \bar{y}z(x+\bar{x}) + xy(z+\bar{z})$

$= \boxed{\bar{y}z + xy}$

Grouping the minterms:

- Grouping together all the 1s in the K-map.
  - make rectangles of 2^n $(1, 2, 4, \ldots)$
  - all the 1s in the map should be included in at least one rectangle.
  - Do not include any of the 0s.
  - each group corresponds to one product term.
  - Make each rectangle as large as possible.
  - We can overlap the rectangles if that makes them larger.

Example 2:

Simplify the boolean function

$Z = AB + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} = AB\bar{C} + ABC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$

Answer:

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	1	0	0	0
A	1	0	1	1

$\Rightarrow F = \bar{B}\bar{C} + AB$

Note: Ignore!!

Example 3: Simplify the boolean function

$Z = \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	1	0	0
A	1	1	1	1

1 couple

1 quad

$Z = A + \bar{B}C$

quad: $A\bar{B}C + A\bar{B}\bar{C} + AB\bar{C} + ABC = A(\bar{B}C + \bar{B}\bar{C} + B\bar{C} + BC)$
$= A(\bar{B}(C+\bar{C}) + B(\bar{C}+C)) = A(\bar{B}+B) = \boxed{A}$

couple: $\bar{A}\bar{B}C + A\bar{B}C = \bar{B}C(A+A) = \boxed{\bar{B}C}$