SUMMARY

Fluid Statics

DR. MUNZER EBAID

MECH. DEPT.

(CHAPTER 3)

Fluid Statics

1. PRESSURE

For a static Fluid, hydrostatic pressure by definition

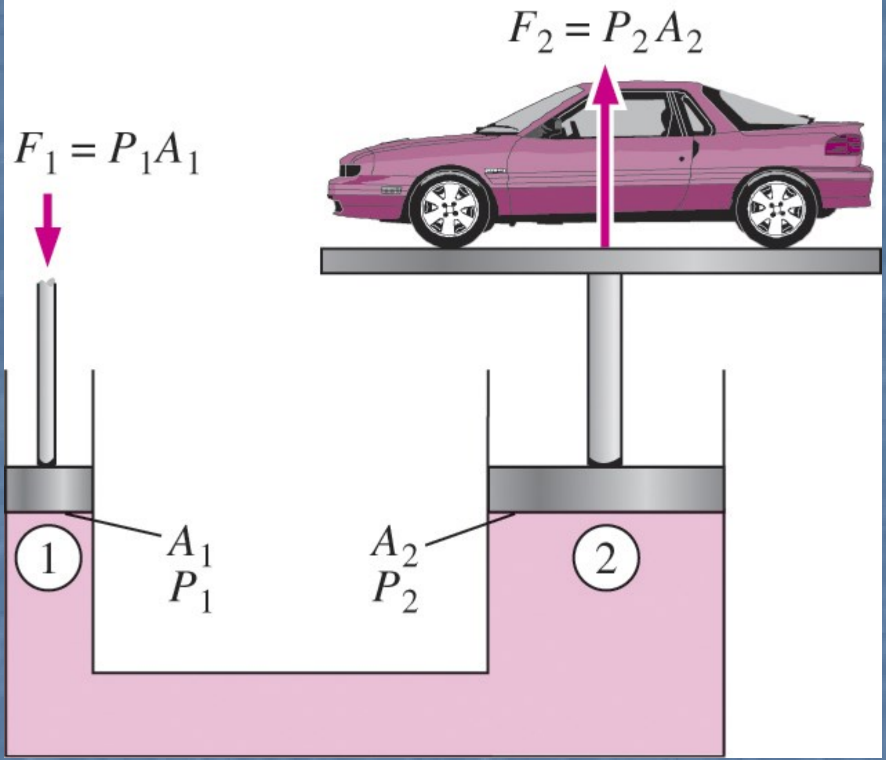
$$P = \frac{F}{A}$$

To prove this, consider a wedge-shaped element of fluid in equilibrium as shown in Fig. 1

Fluid Statics

Pascal's law: The pressure applied to a confined fluid increases the pressure throughout by the same amount.

$$P_1 = P_2 \quad \rightarrow \quad \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \rightarrow \quad \frac{F_2}{F_1} = \frac{A_2}{A_1}$$



Lifting of a large weight by a small force by the application of Pascal's law.

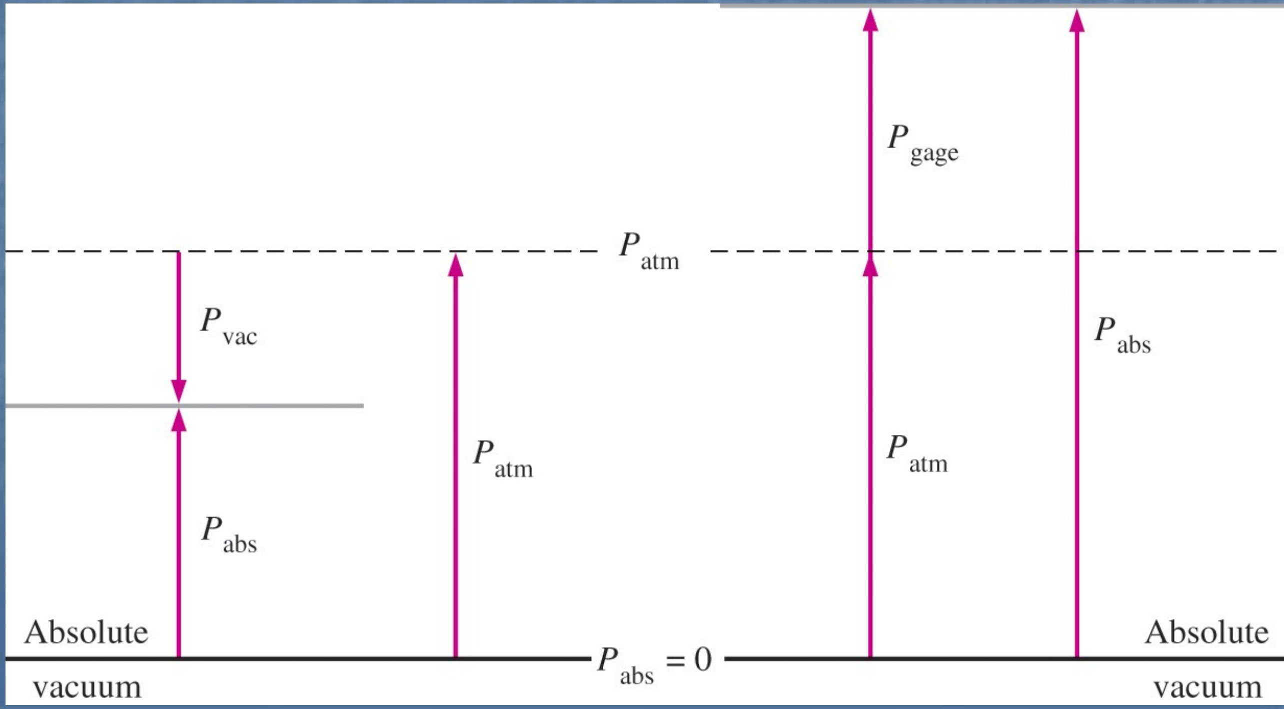
Fluid Statics

ABSOLUTE, GAUGE & VACUUM PRESSURE

- **Absolute pressure:** The actual pressure at a given position. It is measured relative to absolute vacuum (i.e., absolute zero pressure).
- **Gauge pressure:** The difference between the absolute pressure and the local atmospheric pressure. Most pressure-measuring devices are calibrated to read zero in the atmosphere, and so they indicate gage pressure.
- **Vacuum pressures:** Pressures below atmospheric pressure.

$$p_{(gauge)} + p_{(atm)} = p_{(absolute)}$$

$$P_{vac} = P_{atm} - P_{abs}$$



Fluid Statics

$$\frac{dp}{dz} = -\gamma = -\rho g$$

Equation above proves that pressure changes with height or elevation inversely.

By considering fluid density constant with the fluid under consideration, then by integrating Eqn. above, we obtain,

$$p + \rho g z = C$$

Where:

$$p + \gamma z = C$$

is called piezometric Pressure Eqn. (a)

$$\frac{P}{\gamma} + Z = C$$

is called piezometric Head Eqn. (2)

Fluid Statics

The pressure difference between two points with a given fluid can be given as

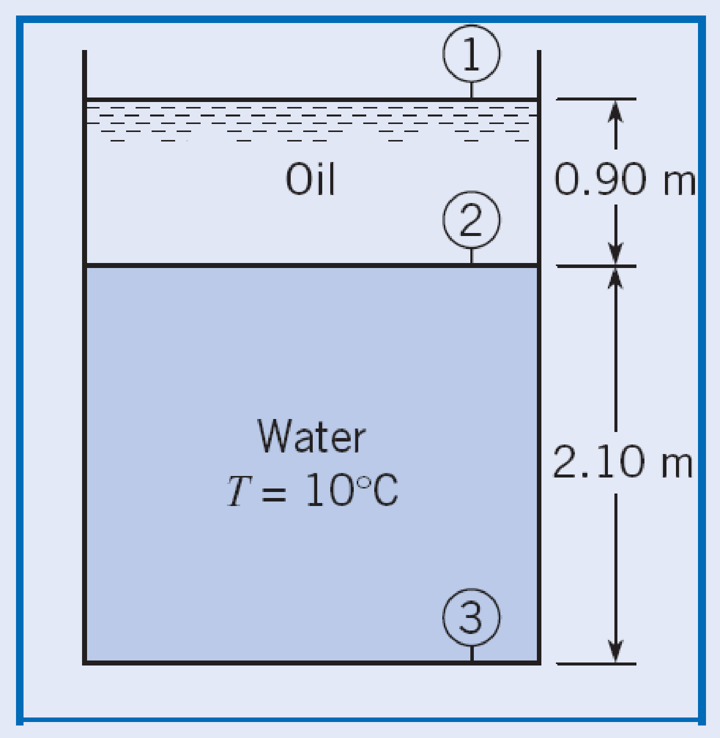
$$p_2 - p_1 = \gamma(z_1 - z_2) \quad OR \quad \Delta p = -\gamma\Delta z$$

For illustration consider Figure below

$$S_{oil} = 0.8$$

Calculate P(gauge) at bottom

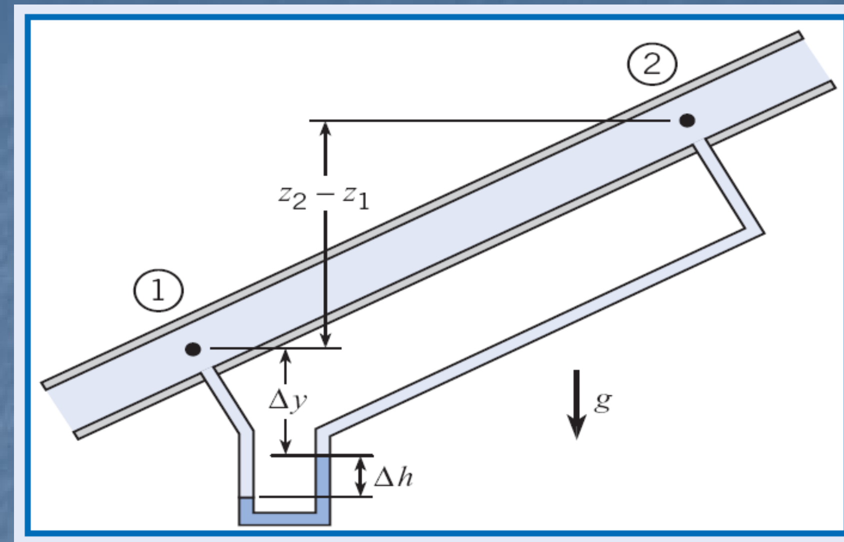
Of tank?



Fluid Statics

Required

1. Piezometric Pressure (1-2).
2. Piezometric Head(1-2).



Pie

$$p_{z_2} - p_{z_1} = (p_2 + \gamma_w z_2) - (p_1 + \gamma_w z_1)$$

$$h_2 - h_1 = \frac{p_{z_2} - p_{z_1}}{\gamma_w}$$

Piezometric Head (1-2)

$$p_2 = p_1 + \gamma_w(\Delta y + \Delta h) - (\gamma_m \Delta h) - \gamma_w(\Delta y + z_2 - z_1)$$

$$p_2 - p_1 = (\gamma_w \Delta h) - (\gamma_m \Delta h) - \gamma_w(z_2 + z_1)$$

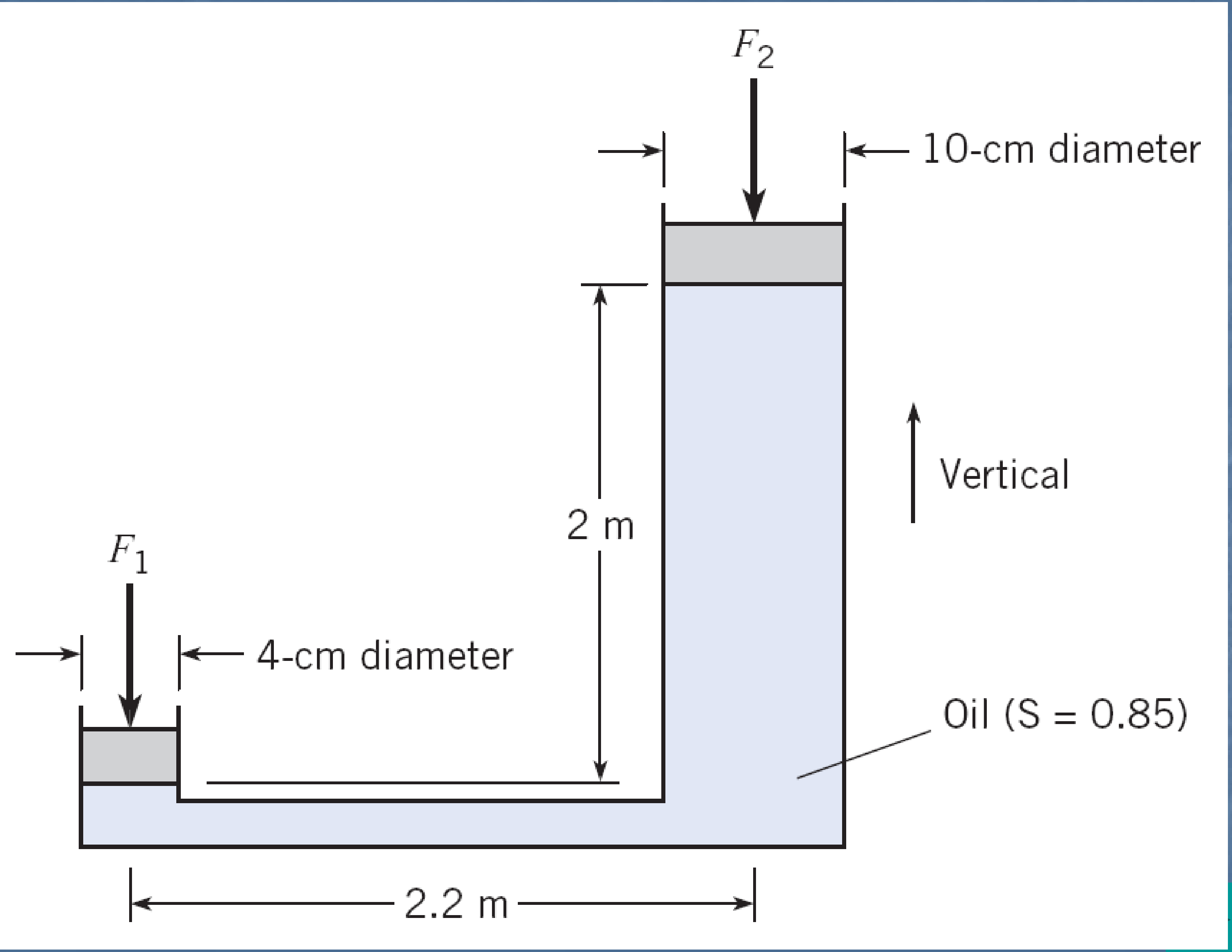
$$(p_2 + \gamma_w z_2) - (p_1 + \gamma_w z_1) = \Delta h(\gamma_w - \gamma_m)$$

Piezometric Pressure (1-2) =

$$h_2 - h_1 = \frac{\Delta h\,(\gamma_w - \gamma_m)}{\gamma_w}$$

Piezometric Head(1-2)=

Problem (3.9)



PROBLEM 3.9

Situation: A force is applied to a piston–additional details are provided in the problem statement.

Find: Force resisted by piston.

APPROACH

Apply the hydrostatic equation and equilibrium.

ANALYSIS

Equilibrium (piston 1)

$$\begin{aligned} F_1 &= p_1 A_1 \\ p_1 &= \frac{F_1}{A_1} \\ &= \frac{4 \times 200\,\mathrm{N}}{\pi \cdot 0.04^2\,\mathrm{m}^2} \\ &= 1.592 \times 10^5\,\mathrm{Pa} \end{aligned}$$

Hydrostatic equation

Fluid Statics

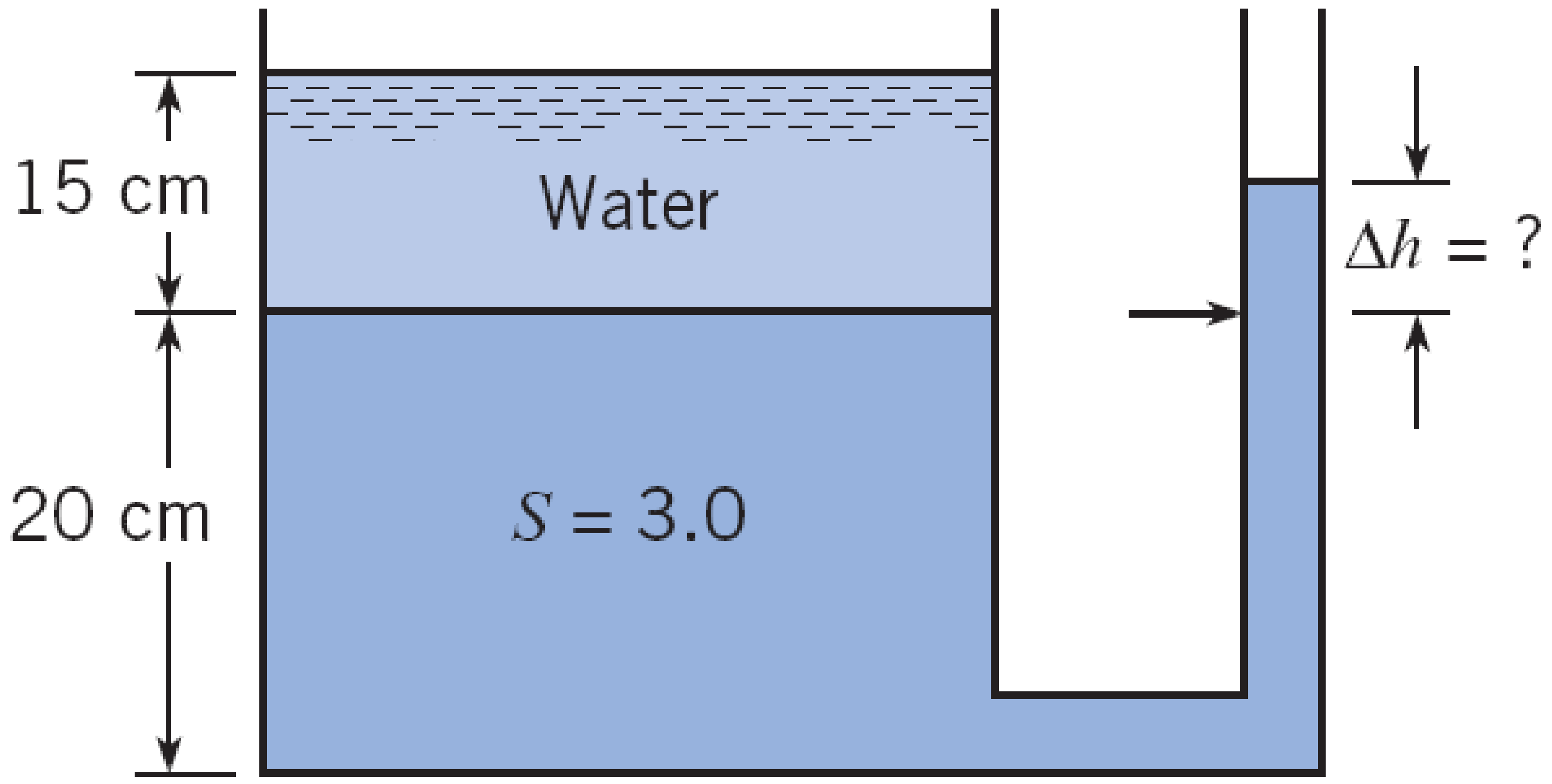
$$
\begin{aligned}
p_2 + \gamma z_2 &= p_1 + \gamma z_1 \\
p_2 &= p_1 + (S\gamma_{\text{water}})(z_1 - z_2) \\
&= 1.592 \times 10^5 \text{ Pa} + \left(0.85 \times 9810 \text{ N/m}^3\right)(-2 \text{ m}) \\
&= 1.425 \times 10^5 \text{ Pa}
\end{aligned}
$$

Equilibrium (piston 2)

$$
\begin{aligned}
F_2 &= p_2 A_2 \\
&= \left(1.425 \times 10^5 \text{ N/m}^2\right)\left(\frac{\pi (0.1 \text{ m})^2}{4}\right) \\
&= 1119 \text{ N}
\end{aligned}
$$

$$\boxed{F_2 = 1120 \text{ N}}$$

Problem (3.15)



PROBLEM 3.15

Situation: A tank fitted with a manometer is described in the problem statement.

Find: Deflection of the manometer.(Δh)

APPROACH

Apply the hydrostatic principle to the water and then to the manometer fluid.

ANALYSIS

Hydrostatic equation (location 1 is on the free surface of the water; location 2 is the interface)

$$\begin{aligned} \frac{p_1}{\gamma_{\text{water}}} + z_1 &= \frac{p_2}{\gamma_{\text{water}}} + z_2 \\ \frac{0\,\text{Pa}}{9810\,\text{N/m}^3} + 0.15\,\text{m} &= \frac{p_2}{9810\,\text{N/m}^3} + 0\,\text{m} \\ p_2 &= (0.15\,\text{m})\left(9810\,\text{N/m}^3\right) \\ &= 1471.5\,\text{Pa} \end{aligned}$$

Hydrostatic equation (manometer fluid; let location 3 be on the free surface)

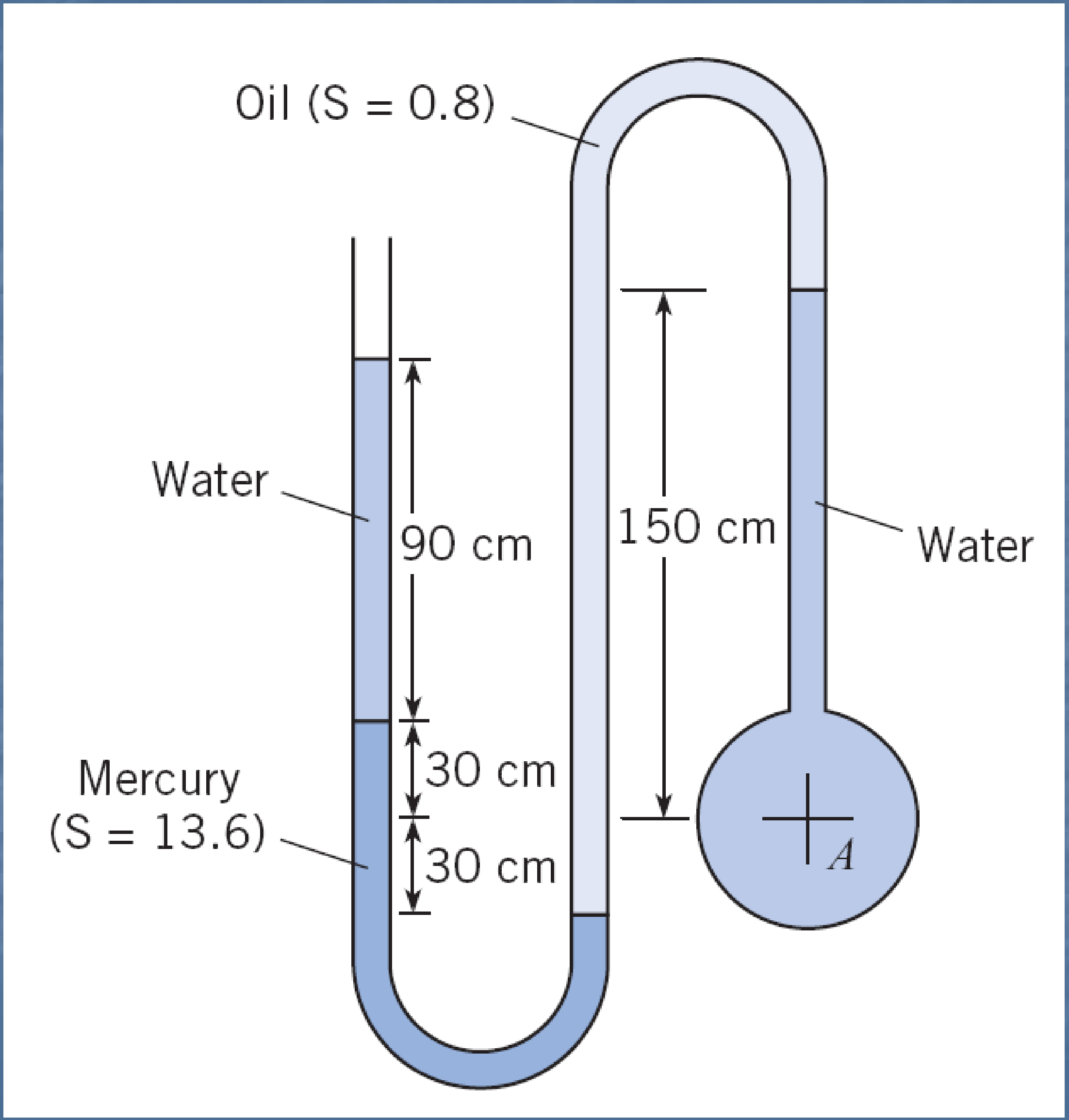
$$\begin{aligned} \frac{p_2}{\gamma_{\text{man. fluid}}} + z_2 &= \frac{p_3}{\gamma_{\text{man. fluid}}} + z_3 \\ \frac{1471.5\,\text{Pa}}{3\,(9810\,\text{N/m}^3)} + 0\,\text{m} &= \frac{0\,\text{Pa}}{\gamma_{\text{man. fluid}}} + \Delta h \end{aligned}$$

Solve for Δh

$$\begin{aligned} \Delta h &= \frac{1471.5\,\text{Pa}}{3\,(9810\,\text{N/m}^3)} \\ &= 0.0500\,\text{m} \end{aligned}$$

$$\boxed{\Delta h = 5.00\text{ cm}}$$

Problem (3.39)



Fluid Statics

PROBLEM 3.39

Situation: A pipe system is described in the problem statement.

Find: Pressure at center of pipe A.

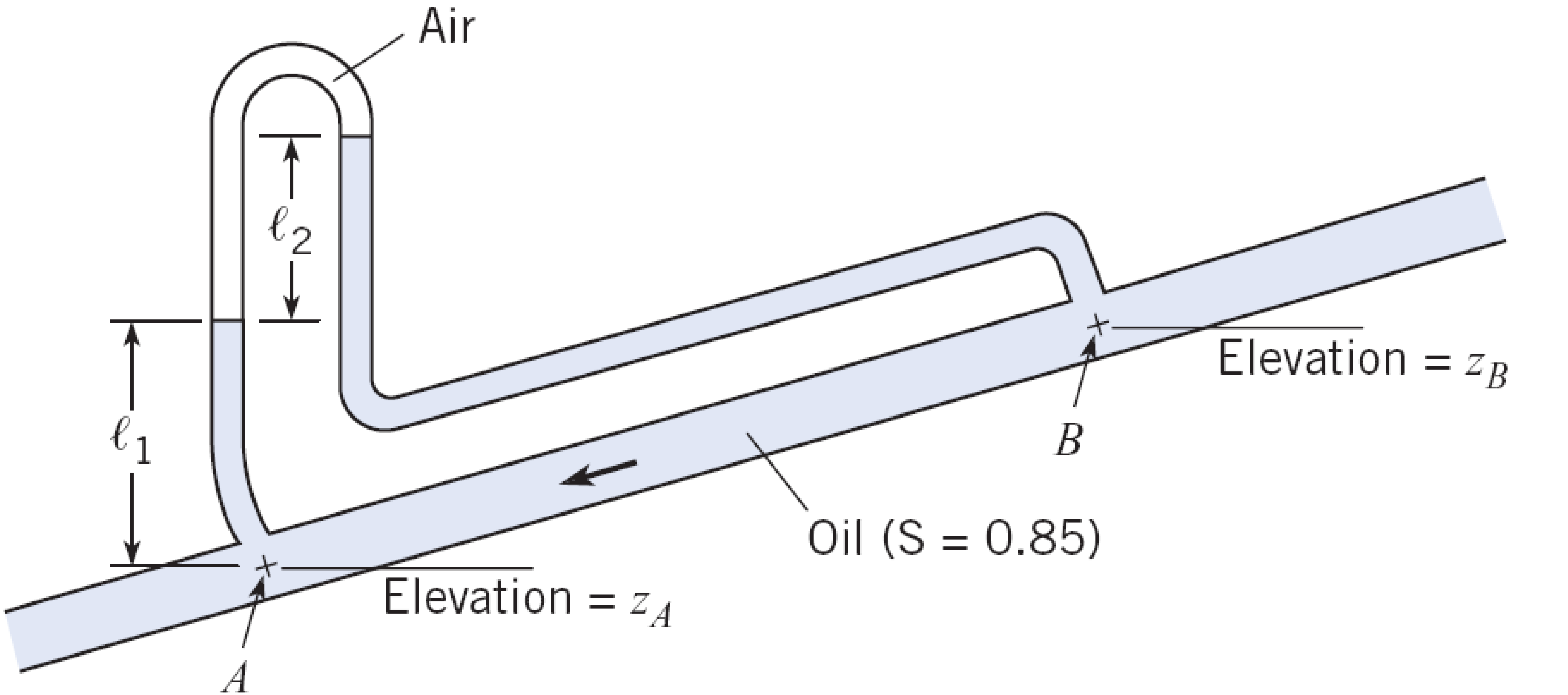
ANALYSIS

Manometer equation

$$p_A = (0.9 + 0.6 \times 13.6 - 1.8 \times 0.8 + 1.5)9,810 = 89,467 \text{ Pa}$$

$$\boxed{p_A = 89.47 \text{ kPa}}$$

Problem (3.40)



PROBLEM 3.40

Situation: A pipe system is described in the problem statement.

Find: (a) Difference in pressure between points A and B.
(b) Difference in piezometric head between points A and B.

APPROACH

Apply the manometer equation.

ANALYSIS

Manometer equation

$$\begin{aligned} p_A - (1\,\text{m})\left(0.85 \times 9810\,\text{N/m}^3\right) + (0.5\,\text{m})\left(0.85 \times 9810\,\text{N/m}^3\right) &= p_B \\ p_A - p_B &= 4169\,\text{Pa} \end{aligned}$$

$$\boxed{p_A - p_B = 4.169\ \text{kPa}}$$

Piezometric head

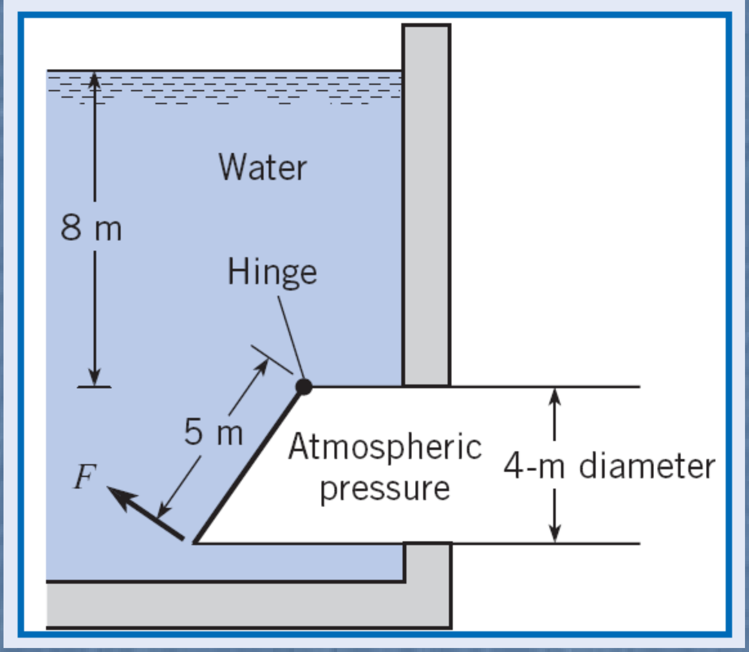
$$\begin{aligned} h_A - h_B &= \left(\frac{p_A}{\gamma} + z_A\right) - \left(\frac{p_B}{\gamma} + z_B\right) \\ &= \frac{p_A - p_B}{\gamma} + (z_A - z_B) \\ &= \frac{4169\,\text{N/m}^2}{0.85 \times 9810\,\text{N/m}^3} - 1\,\text{m} \\ &= -0.5\ \text{m} \end{aligned}$$

$$\boxed{h_A - h_B = -0.50\ \text{m}}$$

Example(3.12)

Calculate F to open the gate

Hydrostatic Force $F = \bar{p}A = \gamma$



Solution First evaluate the magnitude of the hydrostatic force:

$$F = \overline{p} A$$

The area in question is an ellipse with major and minor axes of 5 m and 4 m. The area i given by the formula $A = \pi ab$ (from Fig. A.1 in the Appendix). Then

$$F = 10 \text{ m} \times 9810 \text{ N/m}^3 \times \pi \times 2 \text{ m} \times 2.5 \text{ m} = 1.541 \text{ MN}$$

Now calculate the slant distance between the centroid of the elliptical area and the center of pressure:

$$y_{cp} - \overline{y} = \frac{\overline{I}}{\overline{y} A} = \frac{\frac{1}{4}\pi a^3 b}{\overline{y}\,\pi ab} = \frac{\frac{1}{4}a^2}{\overline{y}}$$

Here $\overline{y} = 12.5$ m (slant distance from the water surface to the centroid). Thus

$$y_{cp} - \overline{y} = \frac{1}{4} \times \frac{6.25 \text{ m}^2}{12.5 \text{ m}} = 0.125 \text{ m}$$

Now take moments about the hinge at the top of the gate to obtain F:

$$\sum M_{\text{hinge}} = 0$$

$$1.541 \times 10^6 \text{ N} \times 2.625 \text{ m} - F \times 5 \text{ m} = 0$$

$$F = 809 \text{ kN}$$

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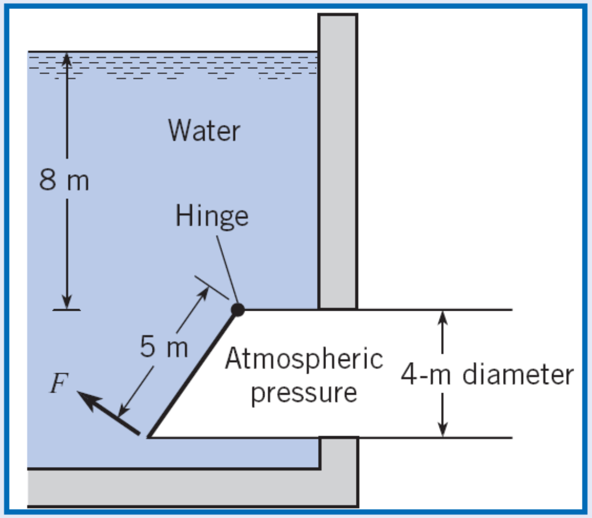
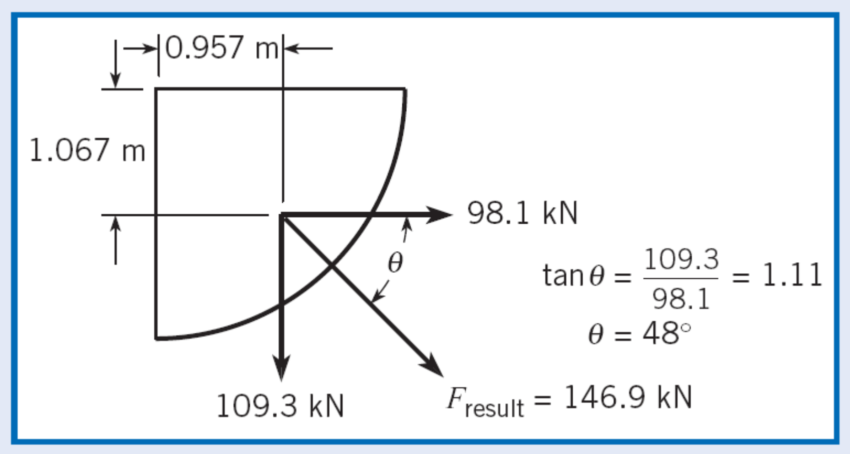


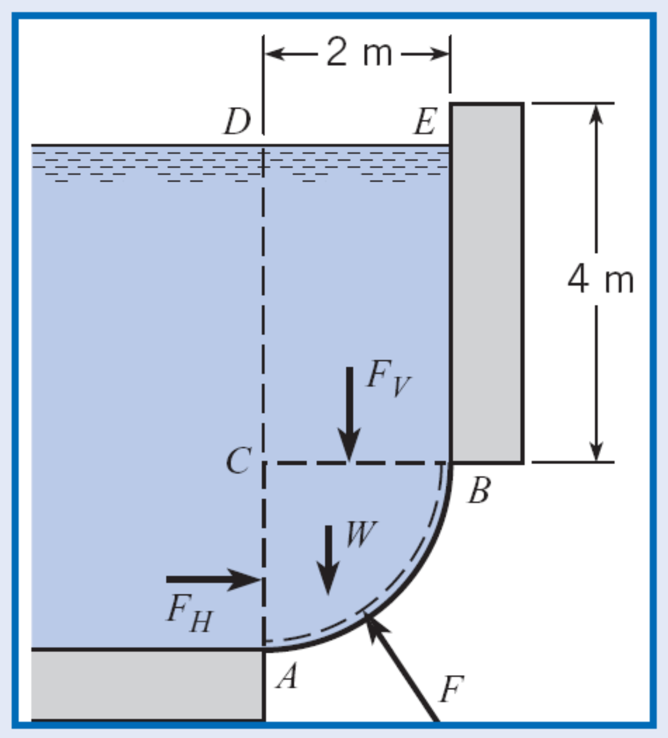
Fluid Statics

Calculate

(F_r)

$(y_{cp}), (x_{cp})$





$$F_x = F_H = (\bar{p}\,A)_H = (\gamma\,\bar{y}\,A)_H = 9810 \times 5 \times (1 \times 2) = 98.1\ KN$$

$$F_V = (\bar{p}\,A)_V = (\gamma\,\bar{y}\,A)_V = 9810 \times 4 \times (1 \times 2) = 78.5\ KN$$

$$W = mg = \rho V_{ABC}\,g = \gamma V_{ABC} = \gamma \times (\frac{1}{4} \times \pi r^2) \times h = 9810 \times \frac{\pi \times 2^2}{4} \times 1 = 30.8\ KN$$

$$F_y = F_V + W = 78.5 + 30.8 = 109.3$$

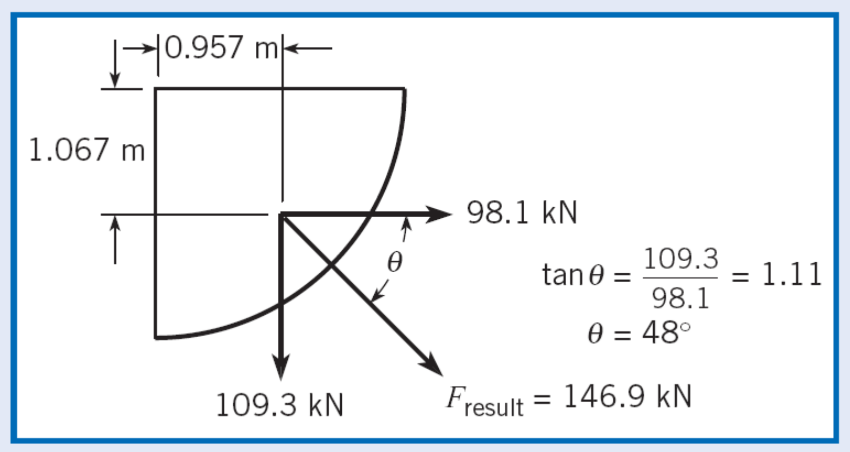
Fluid Statics

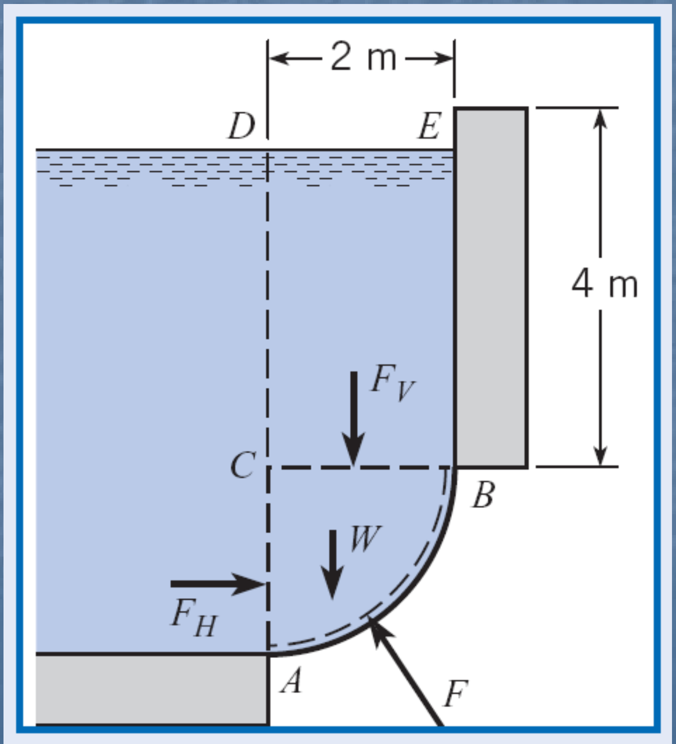
Line of action (y_{cp}) **for the** (F_x) $= y_{cp} = \bar{y}_c + \dfrac{\bar{I}}{\bar{y}_c A} = (4+1) + \dfrac{(1\times 2^3/12)}{(1+4)(2\times 1)} = 5.067m$

Line of action (x_{cp}) **for the** (F_v) **can be found by taking moments about point (C) as follows:**

$$x_{cp}F_y = (F_v \times 1) + W \times \bar{x}_{cp}$$

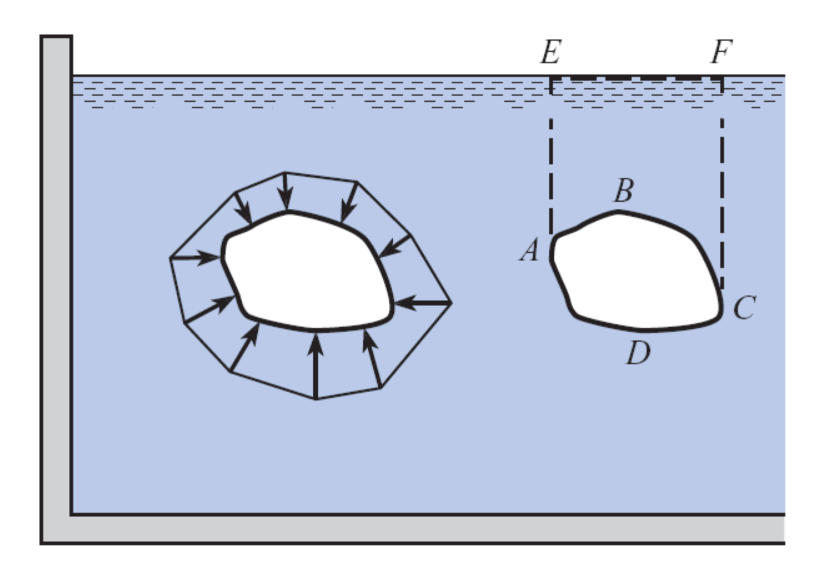
$$x_{cp} = \frac{(78.5\times 1) + (30.8)\times 0.849}{109.3} = 0.957m$$





Fluid Statics

PRINCIPLE OF BUOYANCY



Force acting upwards due to pressure,

$$F\uparrow = \gamma(V)_{EADCF} = \gamma(V_{ABCD} + V_{EABCF})$$

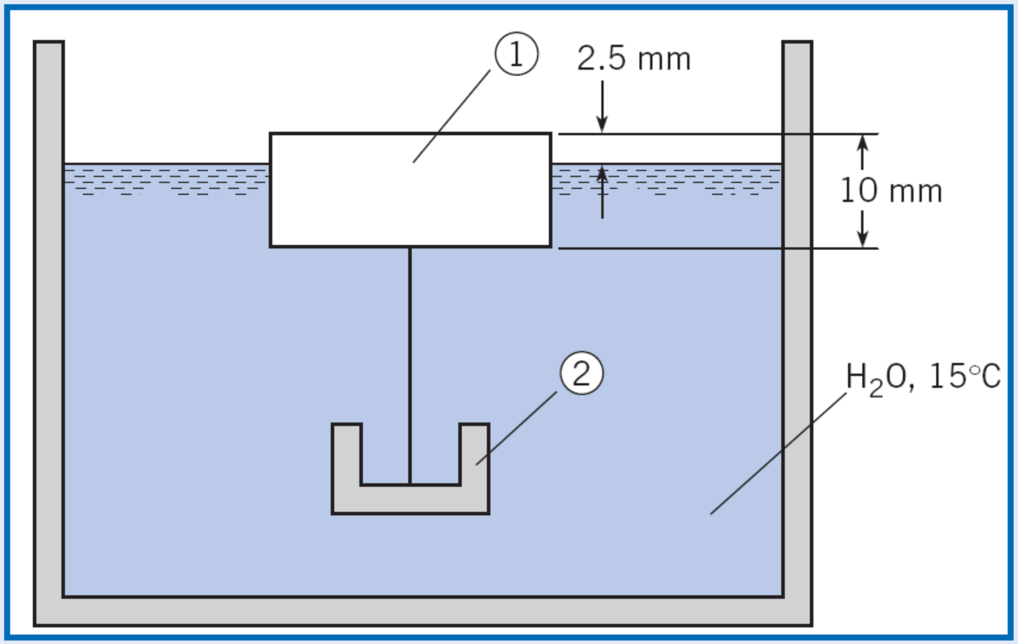
Force acting downwards due to pressure,

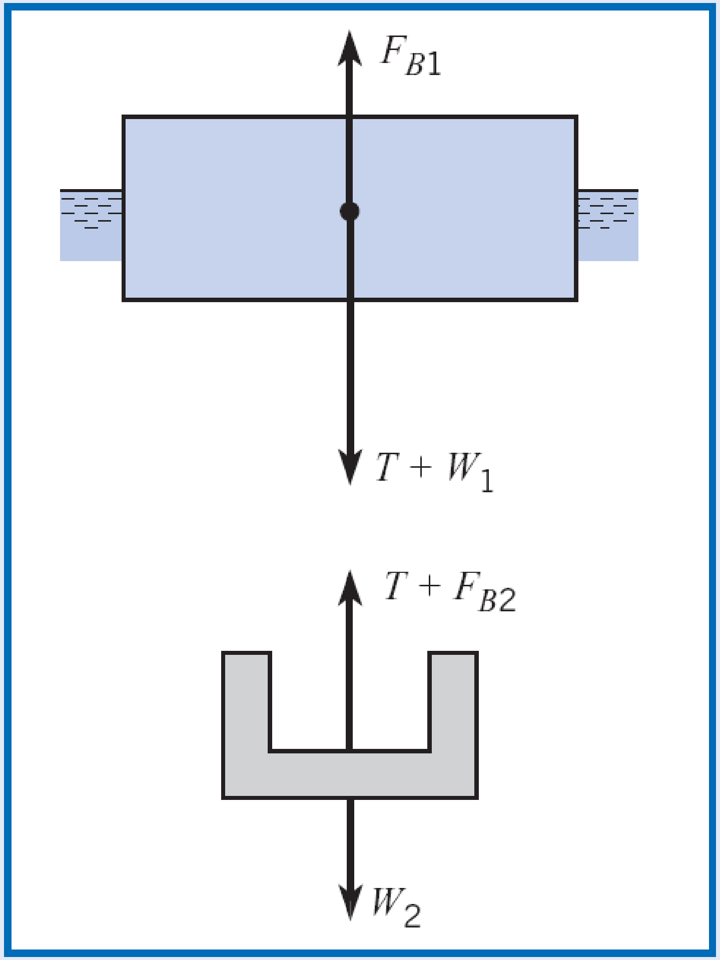
$$F\downarrow = \gamma V_{EABCF}$$

The net force= buoyant force = $F_B \uparrow = \gamma V_{ABCD}$

Equation above means that the buoyant force (net force) equals the weight of liquid that would be needed to occupy the volume of the body.

Example (3.15)





Sum forces on the block:

$$T = F_{B1} - W_1$$

The buoyant force on the floating block is $F_{B1} = \gamma \mathcal{V}_{D1}$, where $\mathcal{V}_{D1}$ is the submerged volume:

$$F_{B1} = \gamma \mathcal{V}_{D1} = (9800 \text{ N/m}^3)(50 \times 50 \times 7.5 \text{ mm}^3)(10^{-9} \text{ m}^3/\text{mm}^3)$$
$$= 0.184 \text{ N}$$

The weight of the block is

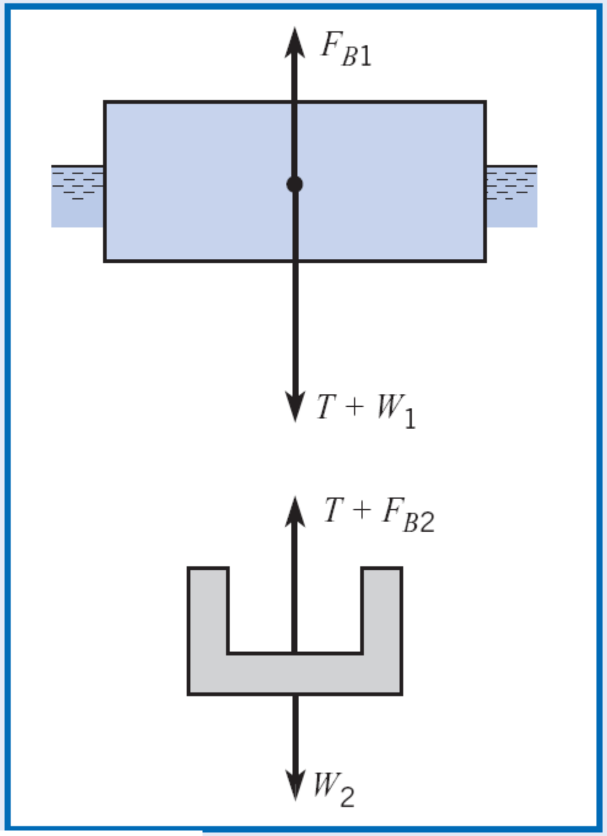
$$W_1 = \gamma S_1 \mathcal{V}_1 = (9800 \text{ N/m}^3)(0.3)(50 \times 50 \times 10 \text{ mm}^3)(10^{-9} \text{ m}^3/\text{mm}^3)$$
$$= 0.0735 \text{ N}$$

Hence the tension on the cord is

$$T = (0.184 - 0.0735) = 0.110 \text{ N}$$

Apply force equilibrium to the metal part:

$$W_2 = T + F_{B2}$$



Because the metal part is submerged, use the volume of the part to calculate the buoyant force:

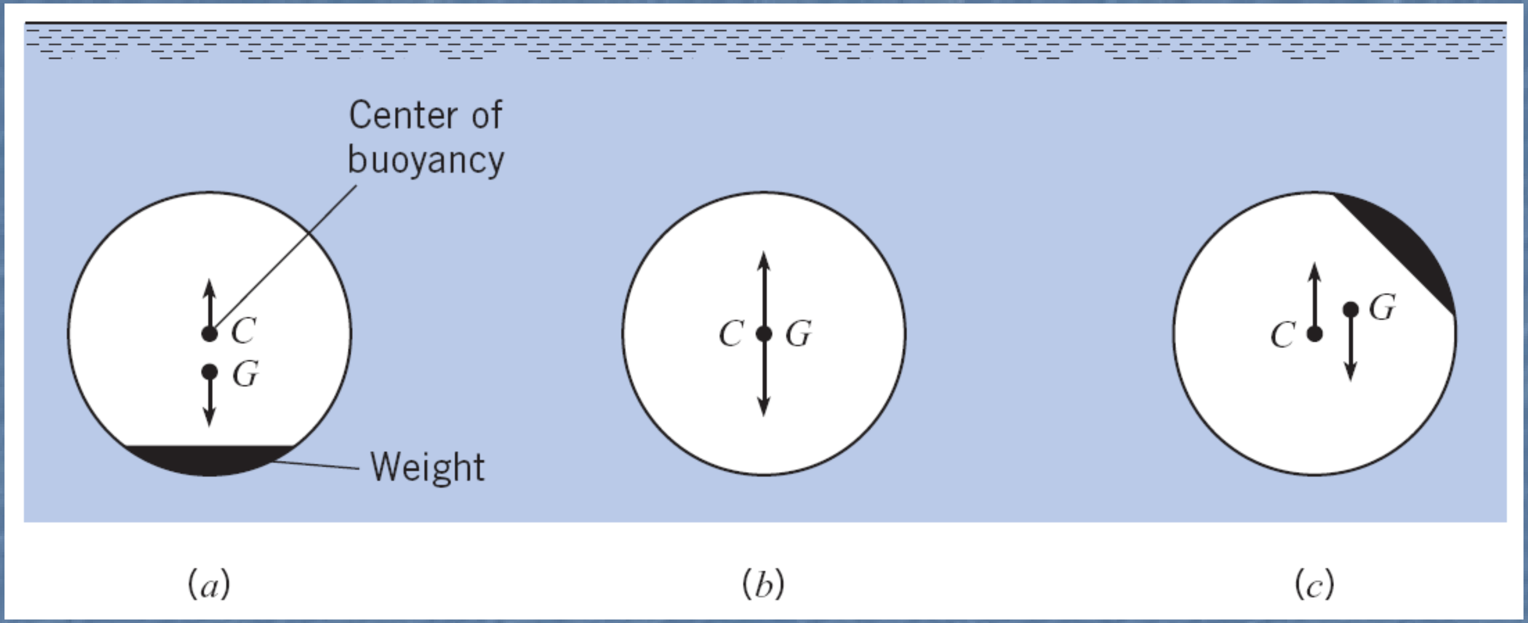
$$F_{B2} = \gamma \mathcal{V}_2 = (9800 \text{ N/m}^3)(6600 \text{ mm}^3)(10^{-9}) = 0.0647 \text{ N}$$

Hence, the weight is given by $W_2 = (0.110 + 0.0647) = 0.175$ N, and the mass is found from

$$m_2 = W_2/g = 17.8 \text{ g} \quad \triangleleft$$

Fluid Statics

STABILITY OF IMMERSED BODIES

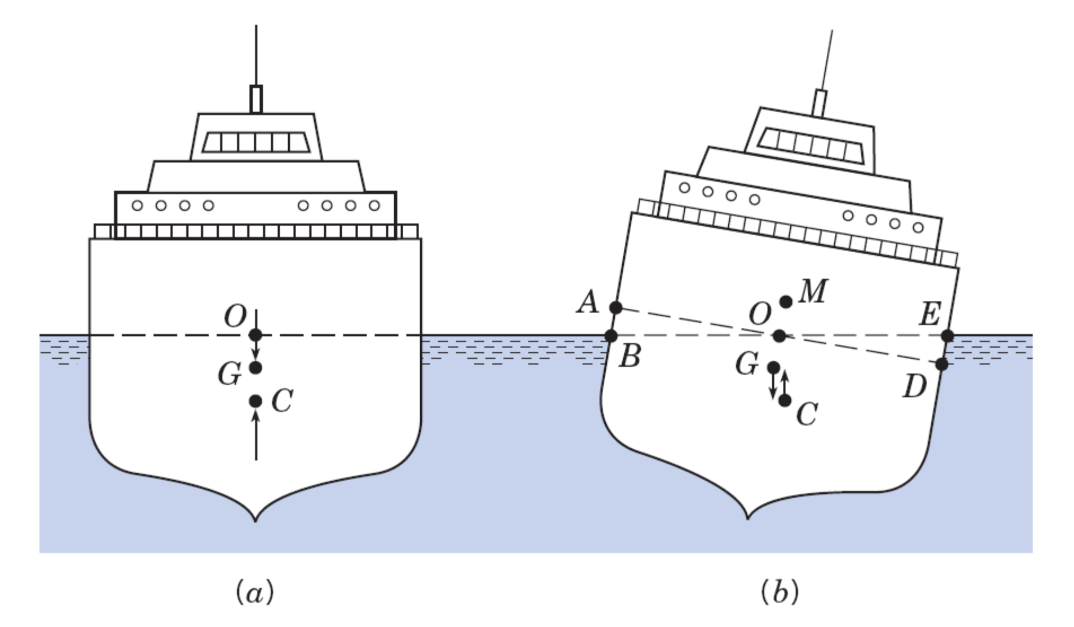


C = Center of buoyancy
G = Center of gravity

Case (a): Body is stable as (C above G)
Case (b): Body is neutral as (C & G are coincident)
Case (c): Body is unstable as (C below G)

Fluid Statics

STABILITY OF FLOATING BODIES



The point of intersection of the lines of action of the buoyant force before and after heel is called metacenter (M) and the distance (GM) is called the metacentric height

If GM is positive (i.e. M above G), the ship is Stable.

If GM is negative (i.e. M below G), the ship is Unstable.