SUMMARY

CONTROL VOLUME APPROACH

DR. MUNZER EBAID

MECH. DEPT.

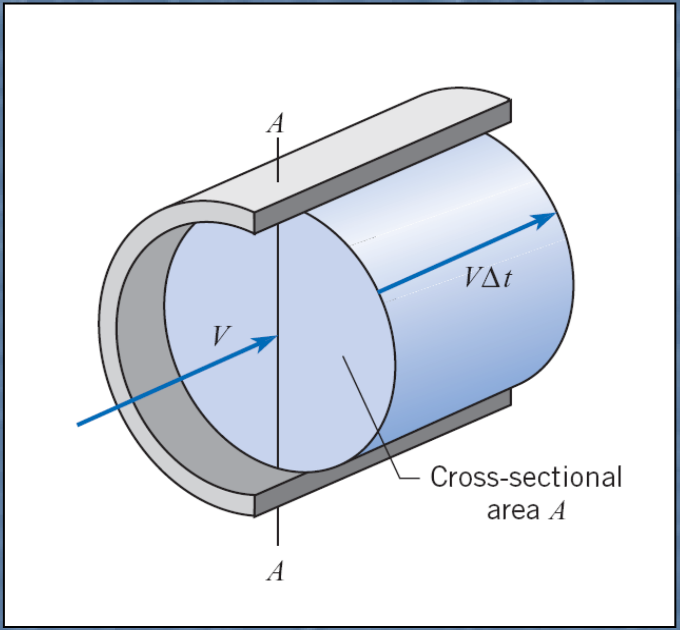
CONTROL VOLUME APPROACH

MASS FLOW RATE

Volume Flow Rate $\dot{Q} = AV$

Mass Flow Rate $\dot{m} = \rho \dot{Q} = \rho AV$

Note: Velocity is constant across the section (A-A)

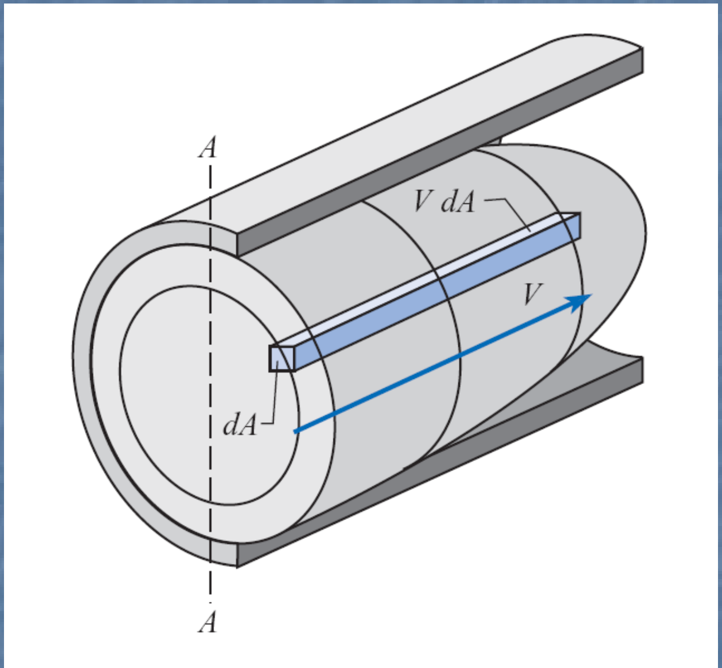


CONTROL VOLUME APPROACH

a. Considering the velocity across the section A-A is variable
b. Considering the density is constant across the section A-A
c. Velocity is always normal to cross sectional area

Volume Flow Rate $\dot{Q} = \int_A V dA$

Mass Flow Rate $\dot{m} = \int_A \rho V dA = \rho \int_A V dA = \rho \dot{Q}$

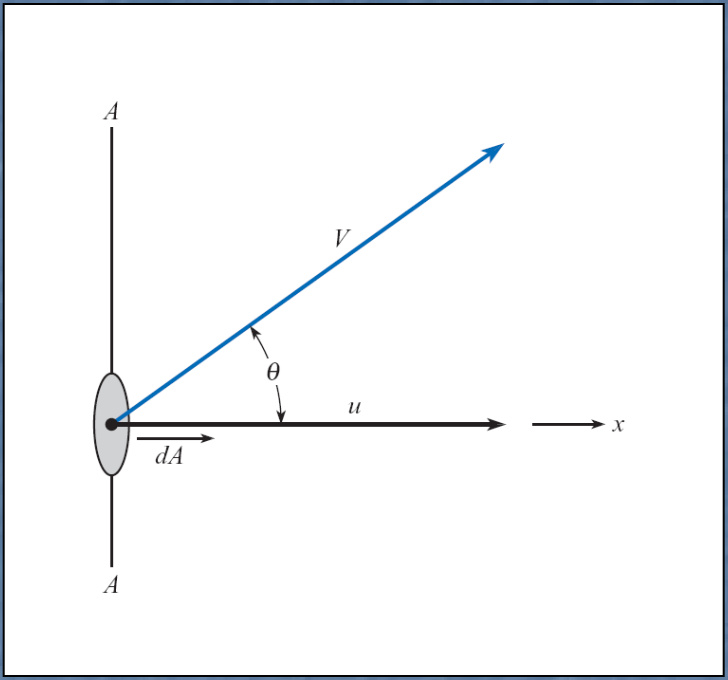


CONTROL VOLUME APPROACH

In the above Figure, the flow rate is given as:

$$\dot{m} = \int_A \rho V \cos\theta \, dA$$

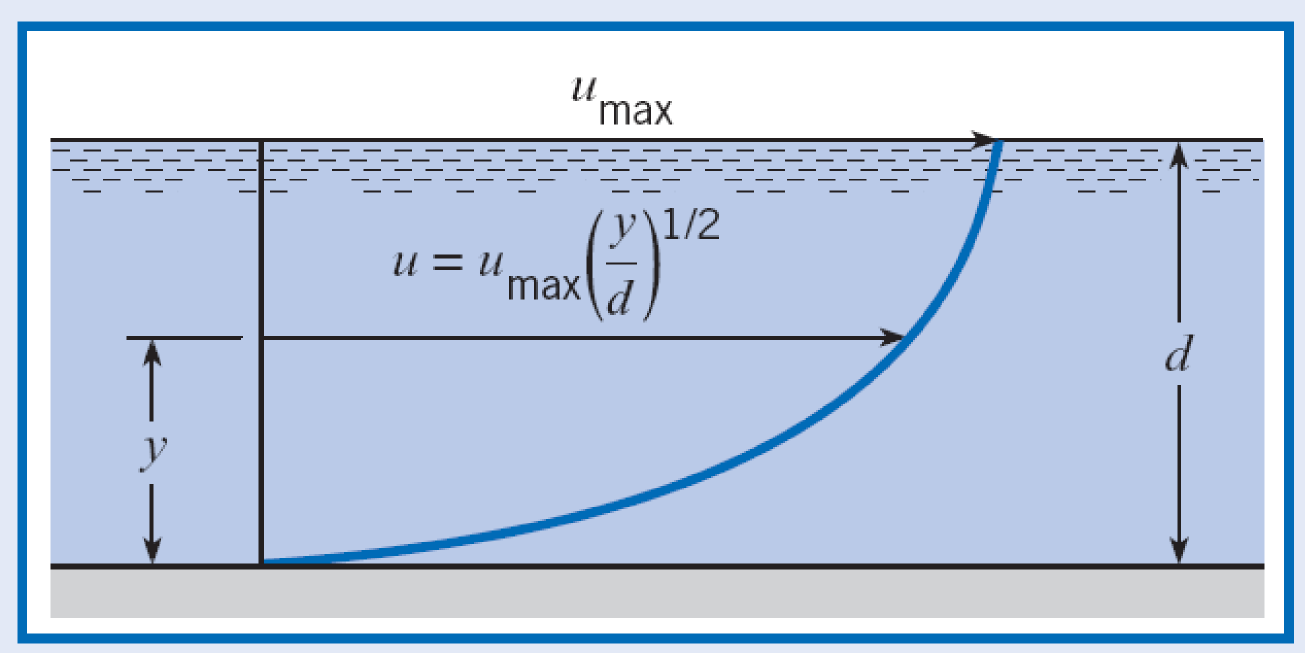
Where the differential area dA **is normal to velocity flow**



MEAN VELOCITY $\overline{V} = \dfrac{\dot{Q}}{A}$

Example (5.3)

The water velocity in the channel shown in the accompanying figure has a distribution across the vertical section equal to $u/u_{max} = (y/d)^{1/2}$. What is the discharge in the channel if the channel is 2 m deep (d = 2 m) and 5 m wide and the maximum velocity is 3 m/s?



Example (5.3)

Solution The discharge is given by

$$Q = \int_0^d u\,dA$$

The channel is 5 m wide, so the differential area is 5 *dy*. Thus

$$Q = \int_0^2 u_{\max}(y/d)^{1/2}\,5\,dy$$

$$= \frac{5u_{\max}}{d^{1/2}} \int_0^2 y^{1/2}\,dy$$

$$= \frac{5u_{\max}}{d^{1/2}} \frac{2}{3} y^{3/2} \Big|_0^2$$

$$= \frac{5 \times 3}{2^{1/2}} \times \frac{2}{3} \times 2^{3/2} = 20\ \text{m}^3/\text{s}$$

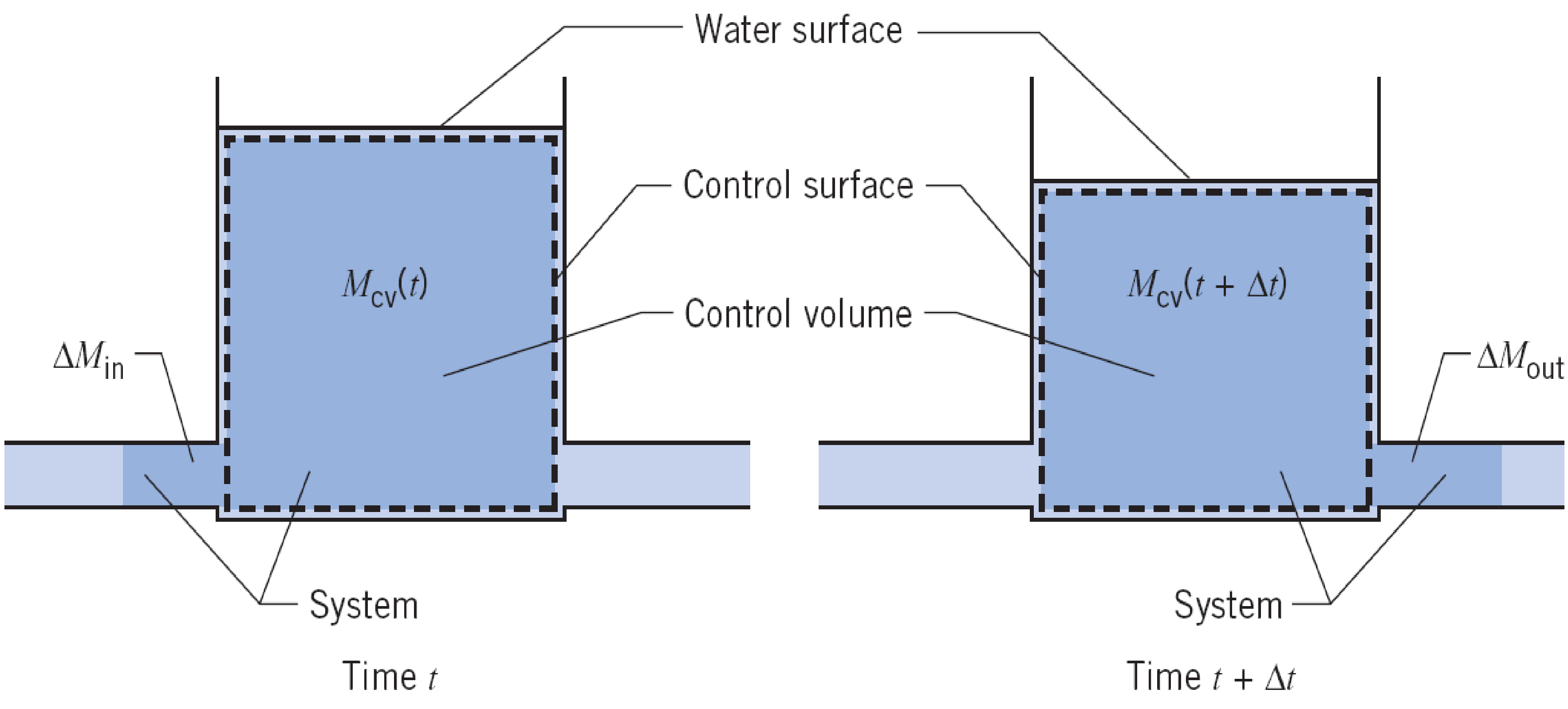
◁

CONTROL VOLUME APPROACH

a. **System:** The mass of a system is constant

b. **Control Volume** (*CV*) : Is defined as a volume in space

c. **Control Surface** (*CS*) : is the surface enclosing the control volume.



CONTROL VOLUME APPROACH

$$M_{sys}(t) = M_{cv}(t) + \Delta M_{in}$$

$$M_{sys}(t+\Delta t) = M_{cv}(t+\Delta t) + \Delta M_{out}$$

By definition, the mass of the system is constant

i.e. $$M_{cv}(t+\Delta t) + \Delta M_{out} = M_{cv}(t) + \Delta M_{in}$$

$$\Delta M_{cv} = \Delta M_{in} - \Delta M_{out}$$

In the limit, $$\frac{dM_{cv}}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

Example 5.4 (P149)

Area of a tank = $10\,m^2$

$$\dot{m}_{in} = 7\,kg/s$$
$$\dot{m}_{out} = 5\,kg/s$$

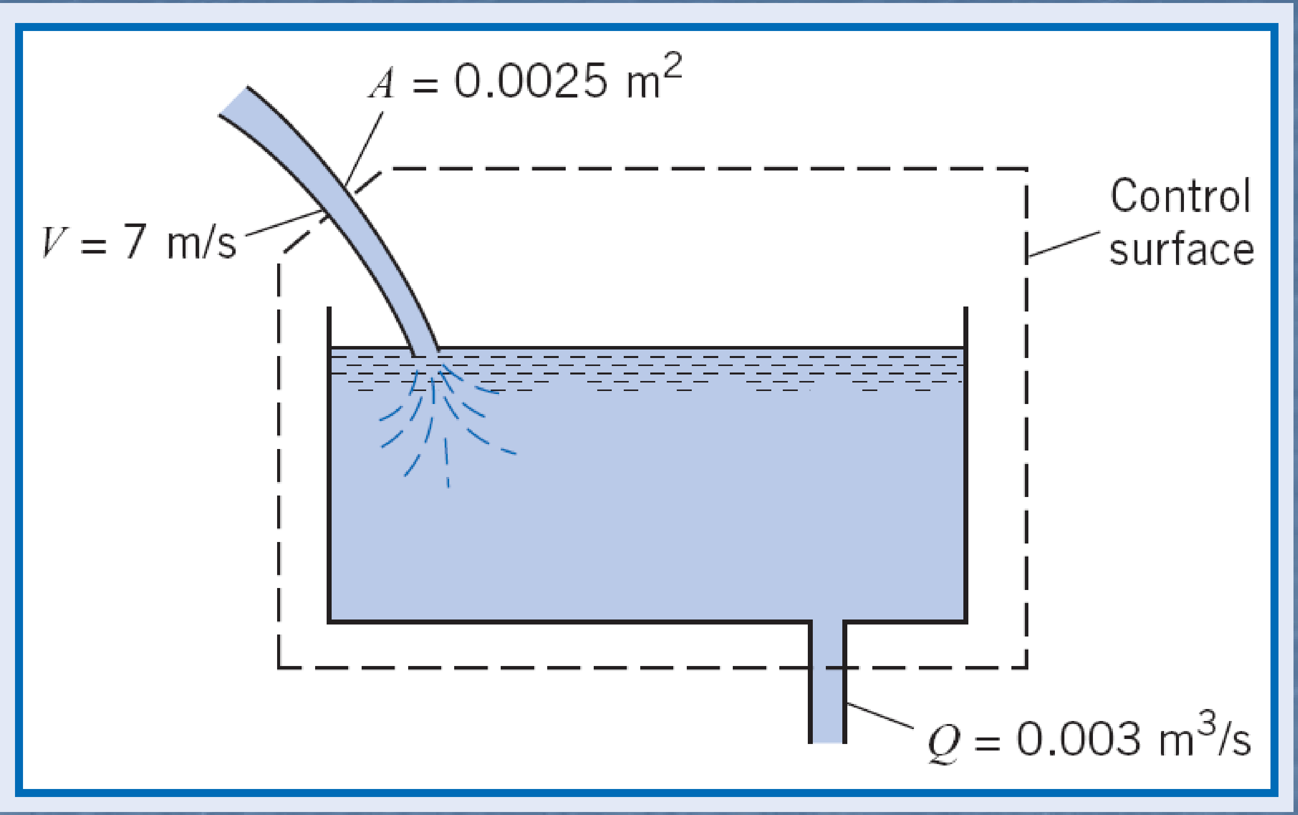
Find the rate at which the water level in the tank changing?

$$\frac{dM_{cv}}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$\frac{d}{dt}(\rho A h) = \dot{m}_{in} - \dot{m}_{out}$$

$$\frac{dh}{dt} = \frac{\dot{m}_{in} - \dot{m}_{out}}{\rho A} = \frac{7-5}{1000 \times 10} = 0.0002\,m/s = 0.72\,m/h$$

Example 5.5 (p. 155)



Find the rate of water accumulating in or evacuating from the tank?

$$\frac{d}{dt}M_{cv} = \dot{m}_{in} - \dot{m}_{out}$$

Solution By drawing a control surface to enclose the entire tank, we observe that there are two streams crossing the control surface, one entering and one leaving. Applying the continuity equation, we have

$$\frac{dM_{cv}}{dt} = \dot{m}_i - \dot{m}_o$$

The mass flow rate out is

$$\dot{m}_o = \rho Q = 1000\ \text{kg/m}^3 \times 0.003\ \text{m}^3/\text{s}$$
$$= 3\ \text{kg/s}$$

The mass flow rate in is

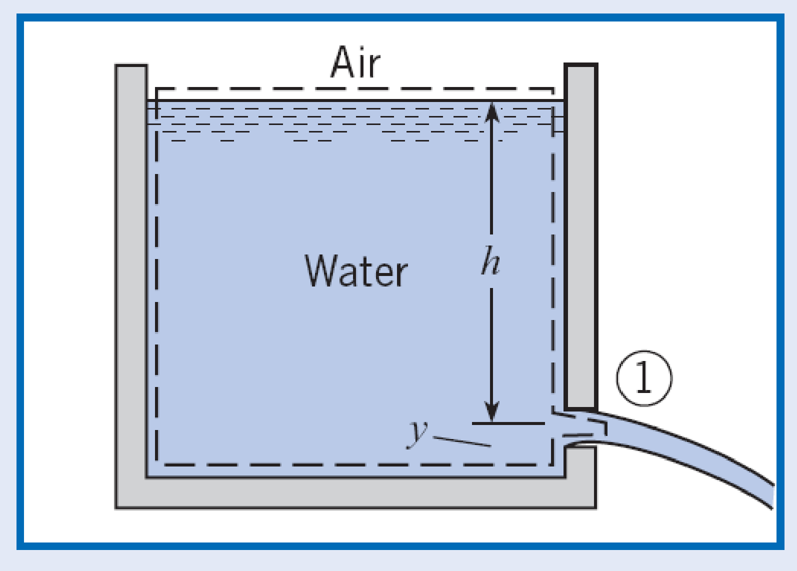
$$\dot{m}_i = \rho VA$$
$$= 1000\ \text{kg/m}^3 \times 7\ \text{m/s} \times 0.0025\ \text{m}^2$$
$$= 17.5\ \text{kg/s}$$

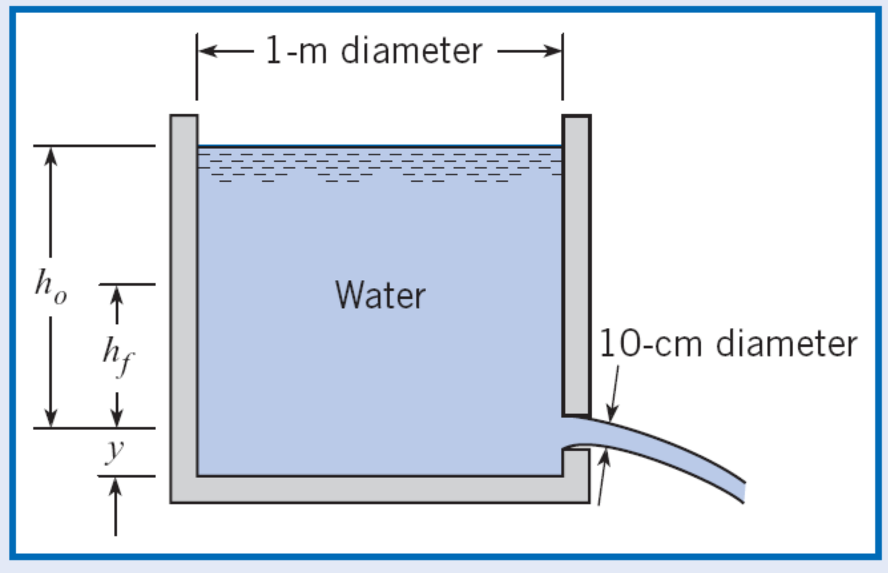
From the continuity equation,

$$\frac{dM_{cv}}{dt} = 17.5 - 3$$
$$= 14.5\ \text{kg/s} \qquad \triangleleft$$

so the mass is accumulating in the tank at the rate of 14.5 kg/s.

Example 5.7a (p. 157)





How long will take (t=?) for the water to drop from h0= 2m to hf=0.5 ?

$$\frac{dM_{CV}}{dt} = \sum_{cs} \dot{m}_{in} - \sum_{cs} \dot{m}_{out}$$

$$\sum_{cs} \dot{m}_{in} = 0$$

$$\frac{dM_{CV}}{dt} = -\sum_{cs} \dot{m}_{out}$$

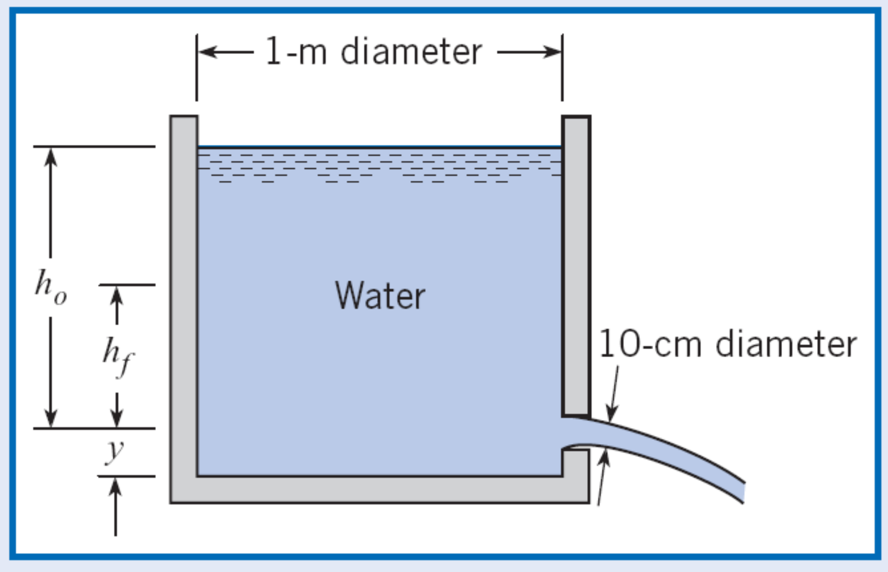
$$-(\rho AV)_{out} = \frac{d}{dt}(\rho A_T)(h+y)$$

$$\frac{dy}{dt} = 0$$

$$-(AV)_{out} = \frac{dh}{dt}(A_T)$$

$$-(A\sqrt{2gh})_{out} = \frac{dh}{dt}(A_T)$$

$$dt = -\frac{A_T}{\sqrt{2g}A_{out}} h^{-1/2} dh$$



Noting now that $A_T/\sqrt{2g}A_1$ is constant, we integrate the differential equation and get

$$t = \frac{-2A_T}{\sqrt{2g}A_1}h^{1/2} + C$$

The constant of integration is evaluated by arbitrarily letting $t = 0$ when $h = h_0$. Then

$$C = +\frac{2A_T}{\sqrt{2g}A_1}h_0^{1/2}$$

So we have

$$t = \frac{2A_T}{\sqrt{2g}A_1}(h_0^{1/2} - h^{1/2})$$

Thus, for this particular example, the elapsed time for the water level to drop from $h_0 = 2$ m to $h_f = 0.50$ m will be

$$t = \frac{2A_T}{\sqrt{2g}A_1}(2^{1/2} - 0.5^{1/2})$$

But

$$A_T = \frac{\pi}{4}D^2 = \frac{\pi}{4} \times 1^2 = \frac{\pi}{4}\ \text{m}^2$$

$$A_1 = \frac{\pi}{4}(0.10)^2 = 0.01\left(\frac{\pi}{4}\right)\ \text{m}^2$$

Hence

$$t = \frac{2\pi/4}{\sqrt{2g}(\pi/4 \times 0.01)}(1.414 - 0.707) = 31.9\ \text{s}$$ ◁

Continuity Equation for flow in a Pipe

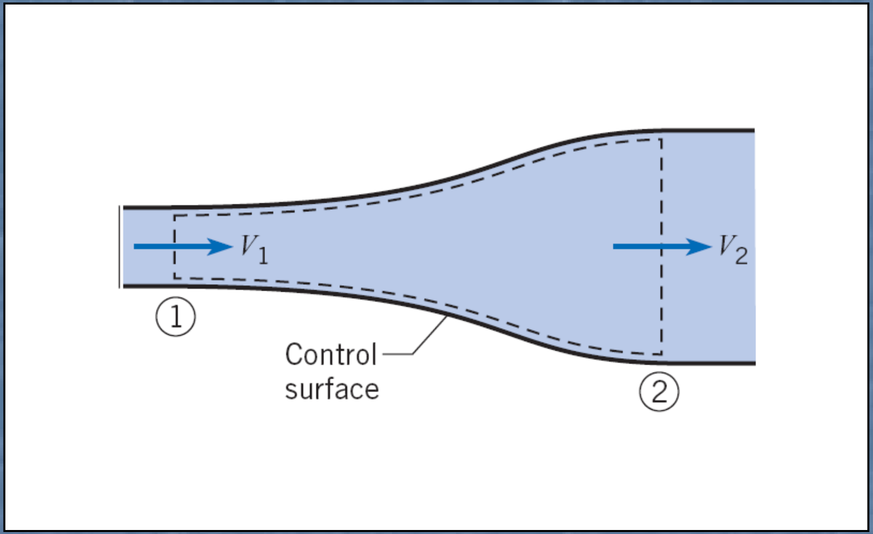
If the flow is steady, then $\frac{dM_{CV}}{dt} = 0$

i.e. $\dot{m}_1 = \dot{m}_2$

$$(\rho AV)_1 = (\rho AV)_2$$

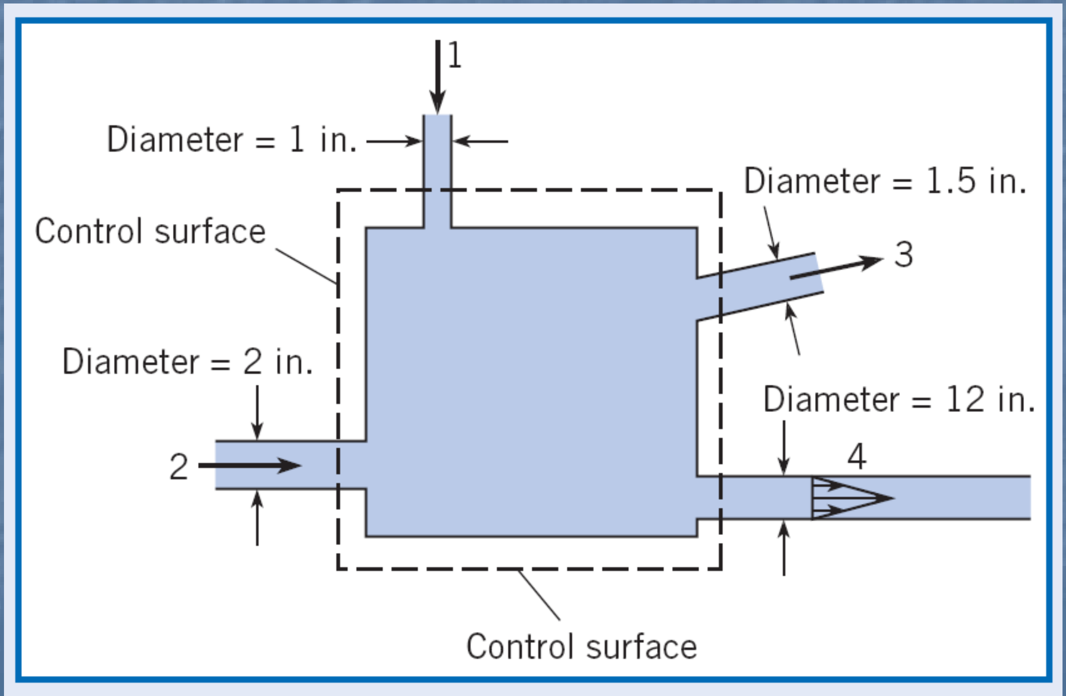
$$(AV)_1 = (AV)_2$$

$$\dot{Q}_1 = \dot{Q}_2$$



Example 5.10a (p. 161)

As shown in the accompanying figure, water flows steadily into a tank through pipes 1 and 2 and discharges at a steady rate out of the tank through pipes 3 and 4. The mean velocity of inflow and outflow in pipes 1, 2, and 3 is 50 ft/s, and the hypothetical outflow velocity in pipe 4 varies linearly from zero at the wall to a maximum at the center of the pipe. What are the mass rate of flow and the discharge from pipe 4, and what is the maximum velocity in pipe 4?



Find the mass and volume flow rate at station (4), also V(max) at (4)?

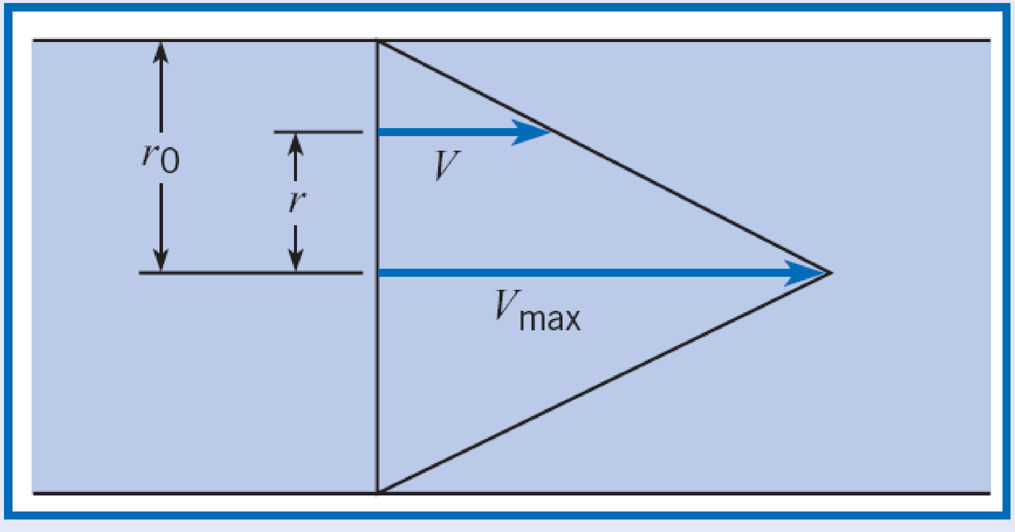
$$\sum_{cs} Q_o = \sum_{cs} Q_i$$

$$Q_3 + Q_4 = Q_1 + Q_2$$

Solving for Q_4, we have

$$\begin{aligned} Q_4 &= Q_1 + Q_2 - Q_3 \\ &= V_1A_1 + V_2A_2 - V_3A_3 \\ &= 50 \times \frac{\pi}{4}(D_1^2 - D_2^2 + D_3^2) \\ &= 50 \times \frac{\pi}{4} \times \frac{1}{144}(1 + 4 - 2.25) \end{aligned}$$

$$Q_4 = 0.750 \text{ ft}^3/\text{s}$$



Therefore, by proportions we have

$$\frac{V}{r_0 - r} = \frac{V_{max}}{r_0} \qquad \text{or} \qquad V = V_{max}\left(1 - \frac{r}{r_0}\right)$$

Thus

$$Q = \int_A V\,dA = \int_0^{r_0} V_{max}\left(1 - \frac{r}{r_0}\right) 2\pi r\,dr$$

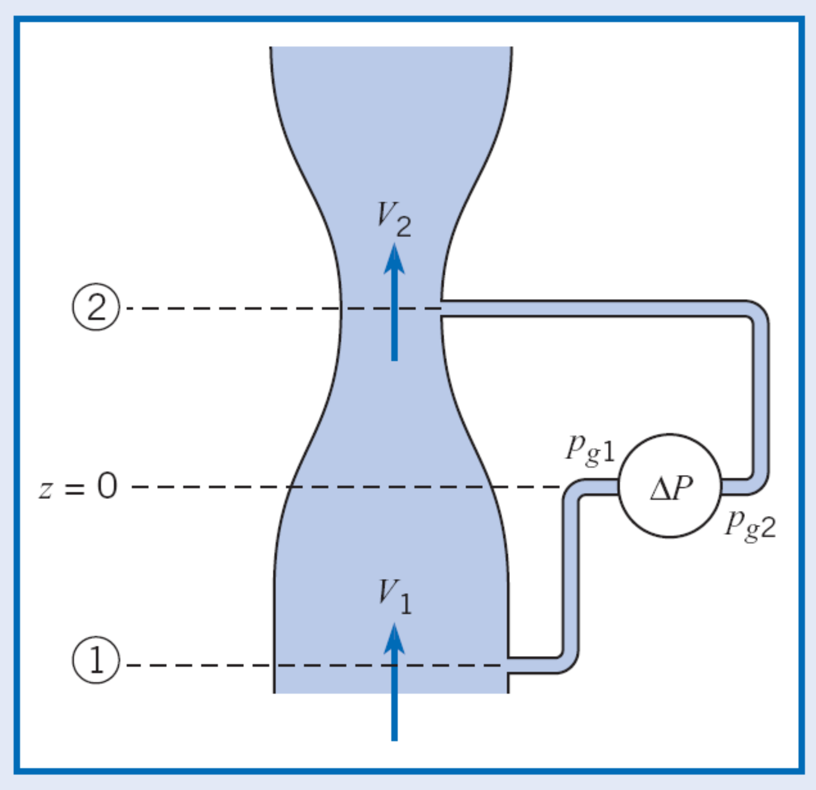
$$Q = 2\pi V_{max} \int_0^{r_0} \left(1 - \frac{r}{r_0}\right) r\,dr = 2\pi V_{max} r_0^2 \left(\tfrac{1}{2} - \tfrac{1}{3}\right) = \tfrac{1}{3}\pi r_0^2 V_{max}$$

Thus

$$V_{max} = \frac{Q}{\frac{1}{3}\pi r_0^2} = \frac{0.75\ \text{ft}^3/\text{s}}{\frac{1}{3}\pi \times \left(\frac{1}{2}\right)^2\ \text{ft}^2} = 2.86\ \text{ft/s} \qquad \triangleleft$$

Example 5.12 (p. 163)

Water with a density of 1000 kg/m^3 flows through a vertical venturimeter as shown. A pressure gage is connected across two taps in the pipe (1) and the throat (2). The area ratio A_{throat}/A_{pipe} is 0.5. The velocity in the pipe is 10 m/s. Find the pressure difference recorded by the pressure gage. Assume the flow has a uniform velocity distribution and that viscous effects are not important.



Find the pressure difference recorded by the pressure gauge?

Solution The Bernoulli equation is used to relate the pressures at stations (1) and (2):

$$p_1 + \gamma z_1 + \rho\frac{V_1^2}{2} = p_2 + \gamma z_2 + \rho\frac{V_2^2}{2}$$

The steady-flow continuity equation is used to find the velocity ratio between taps 2 and 1:

$$\frac{V_2}{V_1} = \frac{A_1}{A_2}$$

Define the zero elevation at the gage location. The water in the lines from the pressure taps to the gage is static, so the pressure at the upstream connection to the gage, p_{g_1}, is

$$p_{g_1} = p_1 + \gamma z_1$$

and the pressure at the downstream connection, p_{g_2}, is

$$p_{g_2} = p_2 + \gamma z_2$$

CONTROL VOLUME APPROACH

The Bernoulli equation simplifies to

$$p_{g_1} + \rho\frac{V_1^2}{2} = p_{g_2} + \rho\frac{V_2^2}{2}$$

and the pressure across the gage is

$$p_{g_1} - p_{g_2} = \frac{\rho}{2}(V_2^2 - V_1^2)$$

$$\Delta p_g = \rho\frac{V_1^2}{2}\left(\frac{V_2^2}{V_1^2} - 1\right)$$

Using the steady-flow continuity equation,

$$\Delta p_g = \rho\frac{V_1^2}{2}\left(\frac{A_1^2}{A_2^2} - 1\right)$$

$$= \frac{1000\ \text{kg/m}^3 \times 10^2(\text{m}^2/\text{s}^2)}{2}(2^2 - 1)$$

$$= 150\ \text{kPa}$$

◁

CONTROL VOLUME APPROACH

Differential form of continuity equation

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=\frac{\partial \rho}{\partial t}$$

If the flow is steady, the above equation becomes,

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0$$

If the flow is incompressible $\rho = Constant$ **, the above equation becomes,**

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Example 5.13 (p. 169)

The expression $\mathbf{V} = 10x\mathbf{i} - 10y\mathbf{j}$ is said to represent the velocity for a two-dimensional incompressible flow. Check it to see whether it satisfies continuity.

Solution

$$u = 10x \quad \text{so} \quad \frac{\partial u}{\partial x} = 10$$

$$v = -10y \quad \text{so} \quad \frac{\partial v}{\partial y} = -10$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 10 - 10 = 0$$

Continuity is satisfied.

CONTROL VOLUME APPROACH

THE END