SUMMARY

MOMENTUM PRINCIPLE

DR. MUNZER EBAID

MECH. DEPT.

MOMENTUM PRINCIPLE

$$\frac{d(Mass)_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho dQ + \int_{CS} \rho V \bullet dA$$

Multiply the above equation by velocity (V), we have

$$\frac{d(Mom)_{sys}}{dt} = \frac{d}{dt}\int_{cv} v\rho dQ + \int_{cs} v\rho V \bullet dA$$

$$\sum F = \frac{d}{dt}\int_{CV} (\rho Q)V + \sum_{CS} (\dot{m}V)_{out} - \sum_{CS} (\dot{m}V)_{in}$$

The momentum principle for a control surface

MOMENTUM PRINCIPLE

3. Momentum Accumulation

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The momentum principle for a control surface is given by,

$$\sum F = \frac{d}{dt}\int_{cv} v\rho dQ + \sum_{cs}(\dot{m}v)_{out} - \sum_{cs}(\dot{m}v)_{in}$$

$$b = V$$

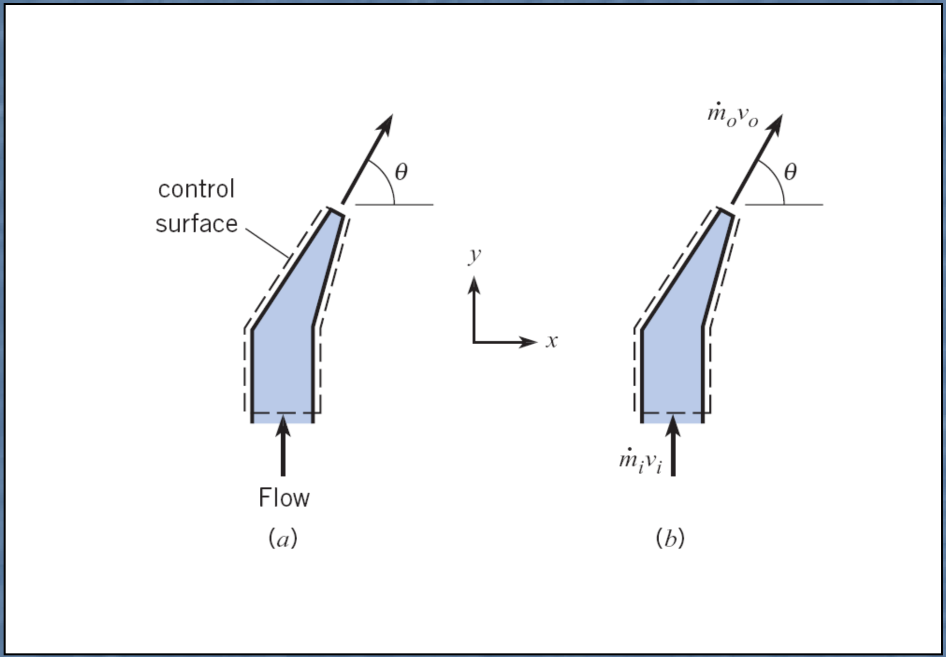
The momentum accumulation $= \frac{d}{dt}\int_{cv} v\rho dQ$

The momentum accumulation for a steady flow **= zero**

The momentum accumulation for a stationary structure **= zero**

MOMENTUM PRINCIPLE

Momentum Diagramme



Momentum flow:

$$\sum_{CS}(\dot{m}v)_{out} - \sum_{CS}(\dot{m}v)_{in} = [\dot{m}v_{out}\cos\theta]i + [\dot{m}v_{out}\sin\theta - \dot{m}v_{in}]j$$

MOMENTUM PRINCIPLE

The Momentum Equation for Cartesian Coordinates

X-direction:

$$\sum F_X = \frac{d}{dt}\int_{cv} v_X \rho dQ + \sum_{CS} (\dot{m}v)_{outX} - \sum_{CS} (\dot{m}v)_{inX}$$

Y-direction:

$$\sum F_Y = \frac{d}{dt}\int_{cv} v_Y \rho dQ + \sum_{CS} (\dot{m}v)_{outY} - \sum_{CS} (\dot{m}v)_{inY}$$

Z-direction:

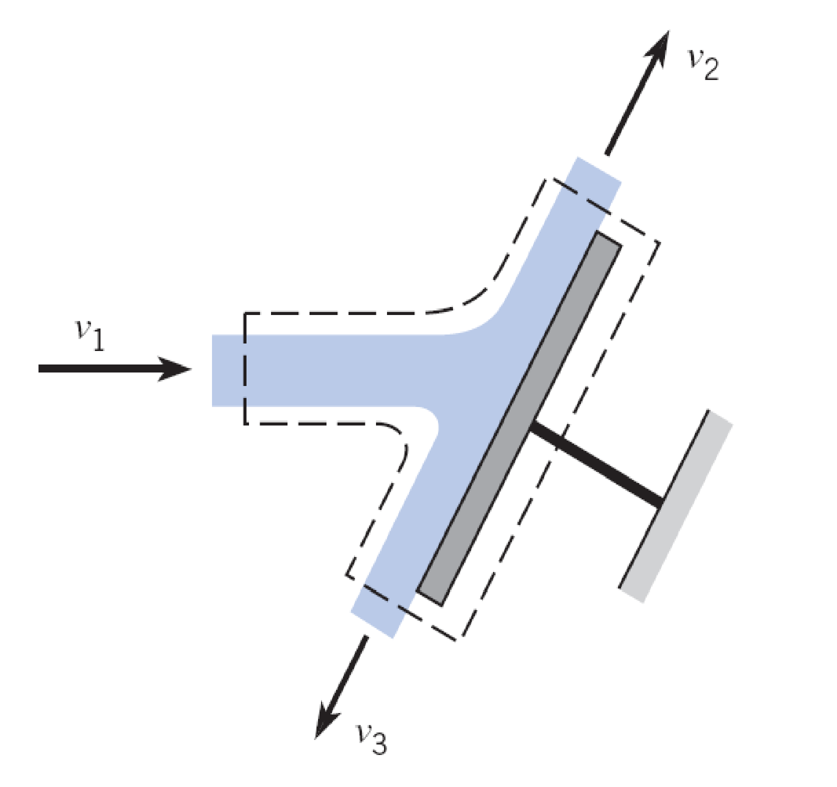
$$\sum F_Z = \frac{d}{dt}\int_{cv} v_Z \rho dQ + \sum_{CS} (\dot{m}v)_{outZ} - \sum_{CS} (\dot{m}v)_{inZ}$$

Vanes

A vane is a structural component, typically thin, that is used to turn a fluid jet or be turned by a fluid jet. **(example: a blade in a turbine)**

For a vane or a blade, the following assumptions are considered:

1. Pressure forces are atmospheric.
2. Neglect changes in elevations.
3. Neglect viscous forces



Using Bernoulli's equation

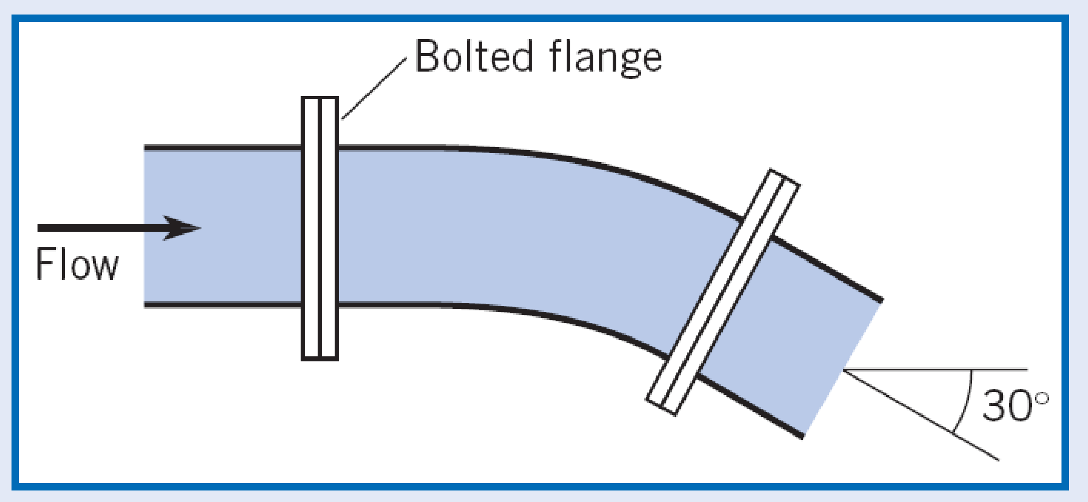
$$\text{Bernoulli's Eqn.},\ p_1 + \gamma z_1 + \frac{\rho v_1^{\,2}}{2} = p_2 + \gamma z_2 + \frac{\rho v_2^{\,2}}{2}$$

$$V_1 = V_2 = V_3$$

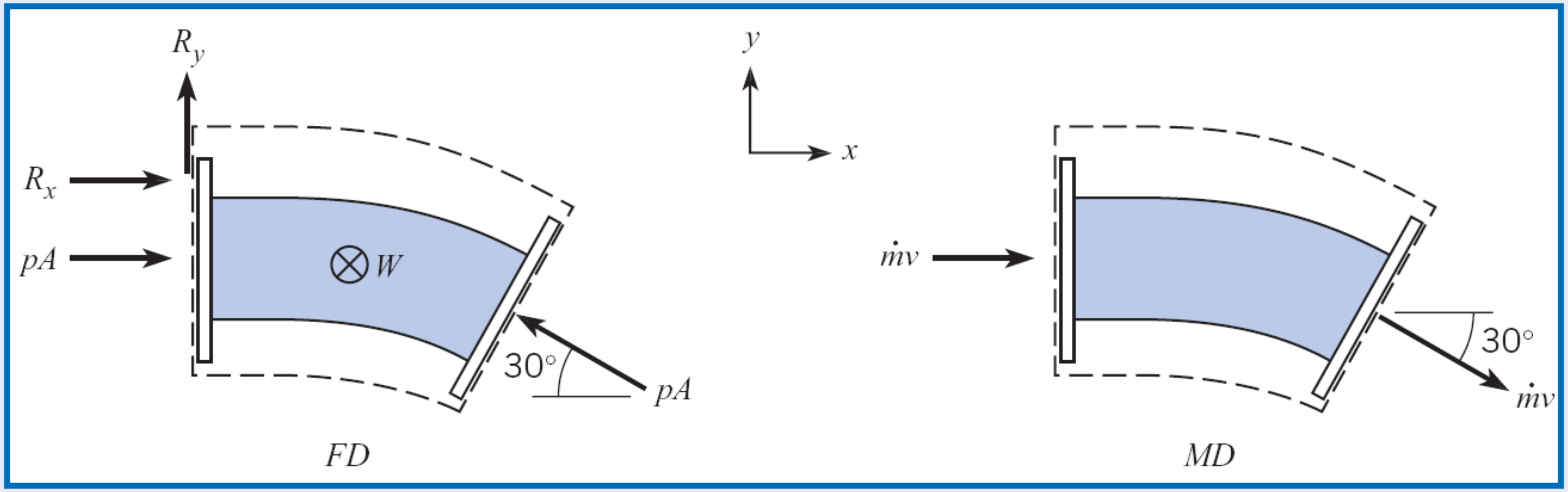
PIPES

Example 6.6

As shown in the figure, a 1-m-diameter pipe bend is carrying crude oil ($S = 0.94$) with a steady flow rate of 2 m^3/s. The bend has an angle of 30° and lies in a horizontal plane. The volume of oil in the bend is 1.2 m^3, and the empty weight of the bend is 4 kN. Assume the pressure along the centerline of the bend is constant with a value of 75 kPa gage. Find the net force required to hold the bend in place.



Find the force required to hold the bend in place?



This problem involves forces in the (x, y, z)

<u>The momentum accumulation</u> = $\frac{d}{dt}\int_{cv} v_z \rho dQ = 0$ (Since Flow is steady)

From the force diagram, $\sum F_x = R_x + pA - pA\cos 30$

$$\sum F_y = R_y + pA\sin 30$$

$$\sum F_z = R_z - W$$

From the momentum diagram, $V_{in} = V_{out}$ *From continuity* $(\dot{m} = \rho AV)$

$$\sum F_x = \sum_{CS}(\dot{m}v)_{out} - \sum_{CS}(\dot{m}v)_{in} = (\dot{m}v\cos 30) - (\dot{m}v)$$

$$R_x + pA - pA\cos 30 = \dot{m}v\cos 30 - \dot{m}v$$

$$\sum F_Y = \sum_{CS}(\dot{m}v)_{outY} - \sum_{CS}(\dot{m}v)_{inY} = -(\dot{m}v\sin 60) - 0$$

$$R_y + pA\sin 30 = -\dot{m}v\sin 30$$

$$\sum F_Z = \sum_{CS}(\dot{m}v)_{outZ} - \sum_{CS}(\dot{m}v)_{inZ} = 0$$

$$R_z - W = 0$$

Note: $W = \gamma Q + W_{bend}$

Resultant Force $\sum F = \sum F_x + \sum F_y + \sum F_z$

The net force $R = R_x + R_y + R_z$

The pressure force is

$$pA = (75\ \text{kN/m}^2)(\pi \times 0.5^2\ \text{m}^2) = 58.9\ \text{kN}$$

The fluid speed is

$$v = Q/A = \frac{(2\ \text{m}^3/\text{s})}{(\pi \times 0.5^2\ \text{m}^2)} = 2.55\ \text{m/s}$$

The momentum flow rate is

$$\dot{m}v = \rho Q v = (0.94 \times 1000\ \text{kg/m}^3)(2\ \text{m}^3/\text{s})(2.55\ \text{m/s}) = 4.80\ \text{kN}$$

The value of R_x is

$$\begin{aligned} R_x &= -(pA + \dot{m}v)(1 - \cos 30°) \\ &= -(58.9 + 4.80)(\text{kN})(1 - \cos 30°) = -8.53\ \text{kN} \end{aligned}$$

The value of R_y is

$$\begin{aligned} R_y &= -(pA + \dot{m}v)\sin 30° \\ &= -(58.9 + 4.80)(\text{kN})(\sin 30°) = -31.8\ \text{kN} \end{aligned}$$

The bend weight includes the oil plus the empty pipe:

$$\begin{aligned} W &= \gamma \forall + 4\ \text{kN} \\ &= (0.94 \times 9.81\ \text{kN/m}^3)(1.2\ \text{m}^3) + 4\ \text{kN} = 15.1\ \text{kN} \end{aligned}$$

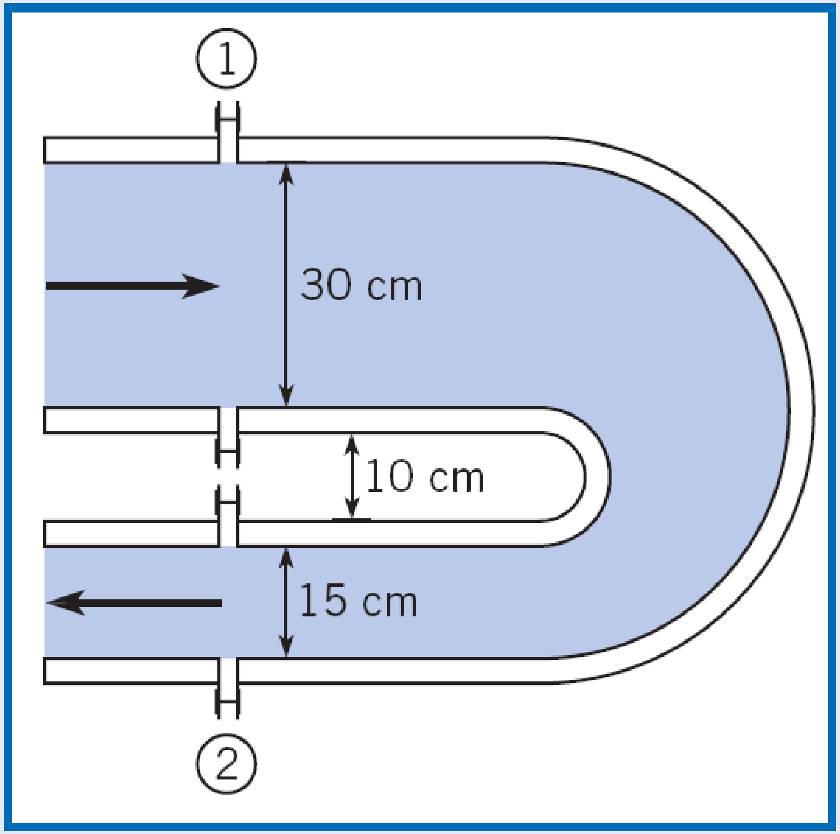
So $R_z = 15.1$ kN. The net force acting on the bend to hold it stationary is

$$\mathbf{R} = (-8.53\ \text{kN})\mathbf{i} + (-31.8\ \text{kN})\mathbf{j} + (15.1\ \text{kN})\mathbf{k} \qquad \triangleleft$$

Example 6.7

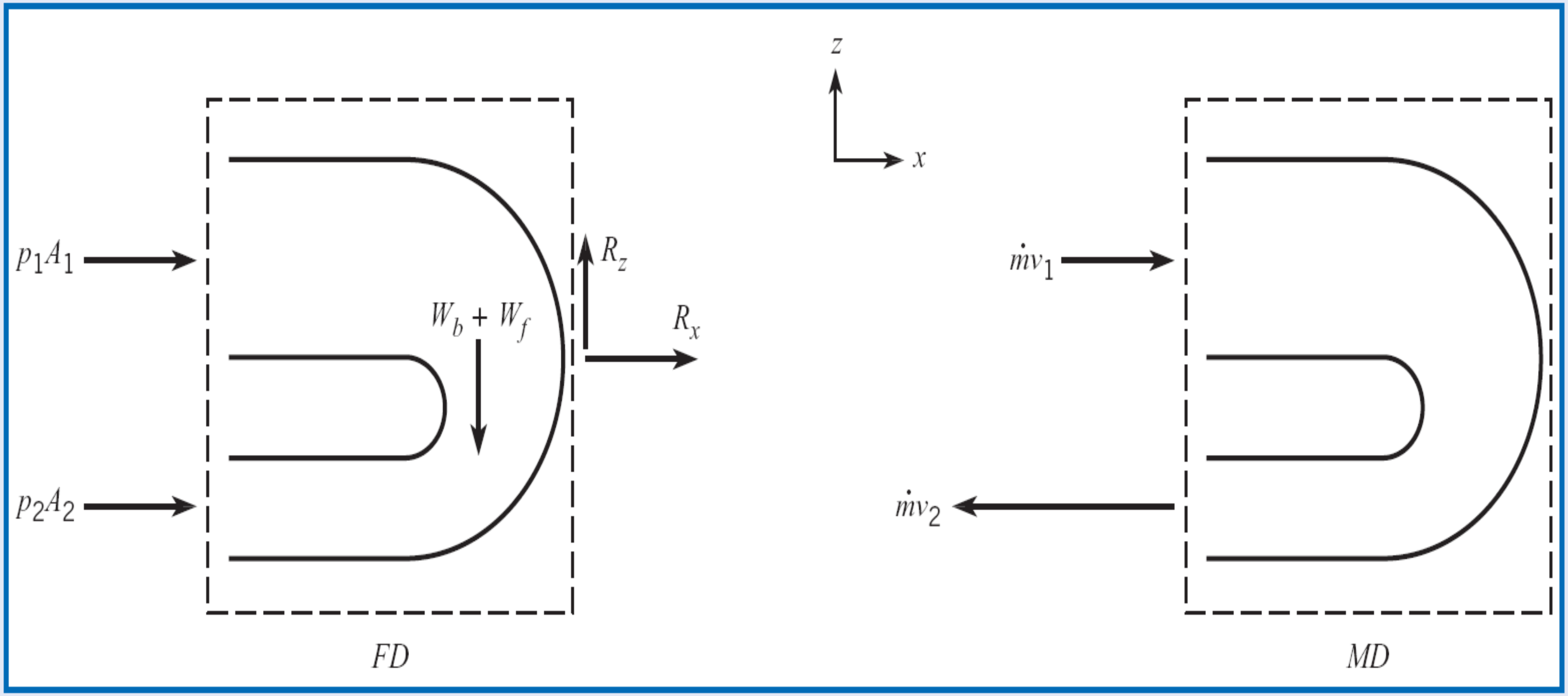
Water flows through a 180° reducing bend, as shown. The discharge is 0.25 m^3/s, and the pressure at the center of the inlet section is 150 kPa gage. If the bend volume is 0.10 m^3, and it is assumed that the Bernoulli equation is valid, what force is required to hold the bend in place? The metal in the bend weighs 500 N.

Find the force required to hold the bend in place?



Given:

$\dot{Q}_2 = 0.25\,kg/m^3, \quad p_1 = 150\,kPa\ gauge, \quad Q_{bend} = 0.10\,m^3/s, \quad W_{bend} = 500\,N$



This problem involves forces in the (x, z) directions

The momentum accumulation = $\frac{d}{dt}\int_{cv} v_z \rho dQ = 0$ (Flow is steady)

From the force diagram, $\sum F_x = R_x + p_1 A_1 + p_2 A_2$

$$\sum F_Z = R_z - W_b - W_f$$

From the momentum diagram, $\sum F_x = \sum_{CS} (\dot{m}v)_{out} - \sum_{CS} (\dot{m}v)_{in}) = -(\dot{m}v_2) - (\dot{m}v_1) = -\dot{m}(v_2 + v_1)$

$$\sum F_Z = \sum_{CS} (\dot{m}v)_{outZ} - \sum_{CS} (\dot{m}v)_{inZ} = o$$

$$R_x + p_1 A_1 + p_2 A_2 = -\dot{m}(v_2 + v_1)$$

$$R_z - W_b - W_f = 0$$

$$W_f = \gamma Q$$

Using Bernoulli's equation between section 1 & 2, we have,

$$p_1 + \gamma z_1 + \frac{1}{2}\rho {v_1}^2 = p_2 + \gamma z_2 + \frac{1}{2}\rho {v_2}^2$$

$$R_x + p_1 A_1 + p_2 A_2 = \dot{m}(v_2 - v_1$$

$$R_z - W_b - W_f = 0$$

$$W_f = \gamma Q$$

From Continuity equation between section 1 & 2, we have,

$$Q = A_1 v_1 = A_2 v_2 = \frac{\pi {d_1}^2}{4} {v_1}^2 = \frac{\pi {d_2}^2}{4} v_2$$

(R_x, R_Z) **can be found**

Speeds are given by

$$v_1 = \frac{Q}{A_1} = \frac{0.25 \text{ m}^3/\text{s}}{\pi/4 \times 0.3^2 \text{ m}^2} = 3.54 \text{ m/s}$$

$$v_2 = \frac{Q}{A_2} = \frac{0.25 \text{ m}^3/\text{s}}{\pi/4 \times 0.15^2 \text{ m}^2} = 14.15 \text{ m/s}$$

Mass flow rate is given by

$$\dot{m} = \rho Q = (1000 \text{ kg/m}^3)(0.25 \text{ m}^3)$$
$$= 250 \text{ kg/s}$$

The net outward momentum flow rate is

$$\dot{m}(v_2 + v_1) = (250 \text{ kg/s})(14.15 + 3.54)(\text{m/s}) = 4420 \text{ N}$$

Pressure at section 2 is given by the Bernoulli equation:

$$p_2 = p_1 + \frac{\rho(v_1^2 - v_2^2)}{2} + \gamma(z_1 - z_2)$$

$$= 150 \text{ kPa} + \frac{(1000)(3.54^2 - 14.15^2)\text{Pa}}{2} + (9810)(0.325)\text{Pa}$$

$$= 59.3 \text{ kPa}$$

R_x is given by $\quad R_x = -(p_1 A_1 + p_2 A_2) - \dot{m}(v_2 + v_1)$

The net pressure force is

$$p_1 A_1 + p_2 A_2 = (150 \text{ kPa})(\pi \times 0.3^2/4 \text{ m}^2) + (59.3 \text{ kPa})(\pi \times 0.15^2/4 \text{ m}^2)$$

$$= 11.6 \text{ kN}$$

The x component of the support force is

$$R_x = -(p_1 A_1 + p_2 A_2) - \dot{m}(v_2 + v_1)$$

$$= -(11.6 \text{ kN}) - (4.42 \text{ kN})$$

$$= -16.0 \text{ kN}$$

and the z component is

$$R_z = W_b + W_f$$

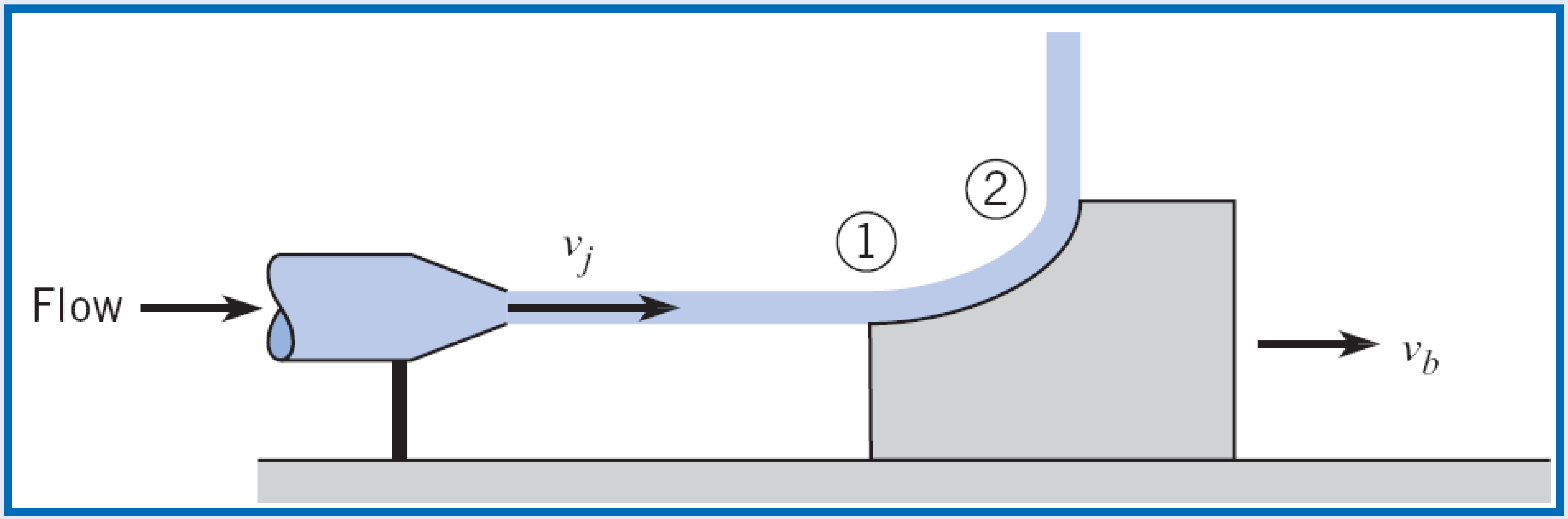
$$= 500 \text{ N} + (9810 \text{ N/m}^3)(0.1 \text{ m}^3)$$

$$= 1.48 \text{ kN}$$

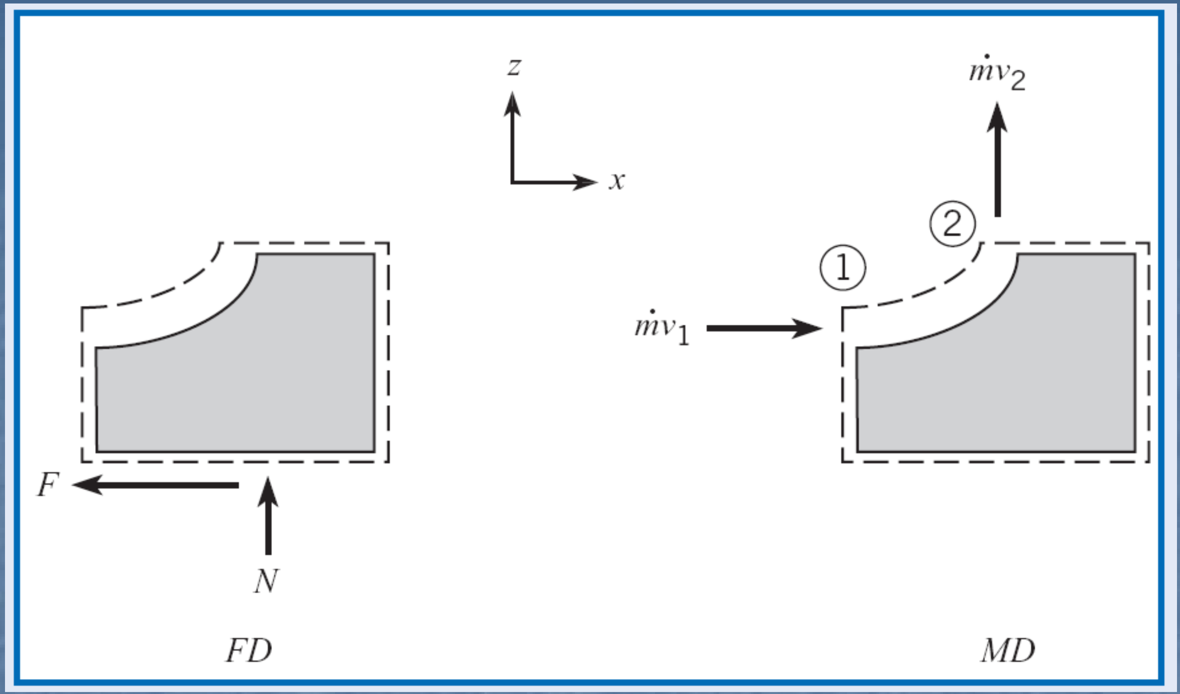
Non-uniform Velocity Distribution

Example (6.9)

A stationary nozzle produces a jet with a speed v_j and an area A_j. The jet strikes a moving block and is deflected 90° relative to the block. The block is sliding with a constant speed v_b on a rough surface. Find the frictional force F acting on the block.



Find: The frictional force (F) acting on the block?



This problem involves forces in the (x, z) **direction**

The momentum accumulation = $\frac{d}{dt}\int_{cv} v_z \rho dQ = 0$ (Flow is steady)

From the force diagram,

$$\sum F_x = -F$$

$$\sum F_z = N - W_{block}$$

From the momentum diagram,

$$\sum F_x = \sum_{CS} (\dot{m}v)_{out} - \sum_{CS} (\dot{m}v)_{in}) = 0 - (\dot{m}v_1) = -\dot{m}v_1$$

$$\sum F_x = -F = -\dot{m}v_1$$

$$\sum F_Z = \sum_{CS} (\dot{m}v)_{outZ} - \sum_{CS} (\dot{m}v)_{inZ} = \dot{m}v_2 - 0 = \dot{m}V_2$$

$$\sum F_Z = N - W_{block} = \dot{m}v_2$$

The mass flow rate is calculated using the velocity relative to the control surface $(v_j - v_b)$, and so

$$\dot{m} = \rho A_j (v_j - v_b)$$

Notice that $\dot{m} \longrightarrow 0$ as $v_b \longrightarrow v_j$, which should be the case because $\dot{m}$ is the rate at which mass is crossing the control surface.

The speed v_1 is relative to the moving reference frame, and so

$$v_1 = v_j - v_b$$

Combining terms results in

$$F = \dot{m} v_1 = \rho A_j (v_j - v_b)^2$$

$$\frac{d}{dt}(m_r v_r) = (\dot{m} v_e + p_e A_e) - W - D$$

$$\dot{m} = \rho A_e v_e$$

$$\frac{d}{dt}(m_r v_r) = (\rho A_e v_e{}^2 + p_e A_e) - W - D$$

$$m_r \frac{d}{dt} v_r = (\rho A_e v_e{}^2 + p_e A_e) - W - D$$

The acceleration of the rocket $= \dfrac{dv_r}{dt} = \dfrac{\left(\rho A_e v_e{}^2 + p_e A_e\right) - D - W}{m_r}$

The term $(\rho A_e v_e{}^2 + p_e A_e)$ is known **as the thrust of the rocket motor (T)**

Momentum - of - Momentum Equation

For rotational motion, the angular momentum of a system is given by,

$$\sum M = \frac{d(H)_{sys}}{dt}$$

$M = The\ moment$

$H_{sys} = angular\ momentum\ of\ the\ system$

Using the Reynolds transport theorem which is

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{cv} b\rho dQ + \int_{cs} b\rho V \bullet dA$$

$$B_{sys} = (H)_{sys} = (mvr)_{sys}$$

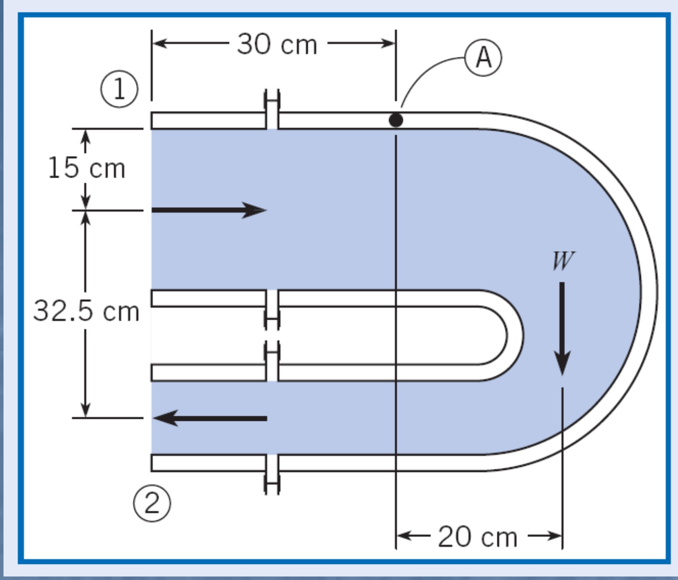
$$b = vr$$

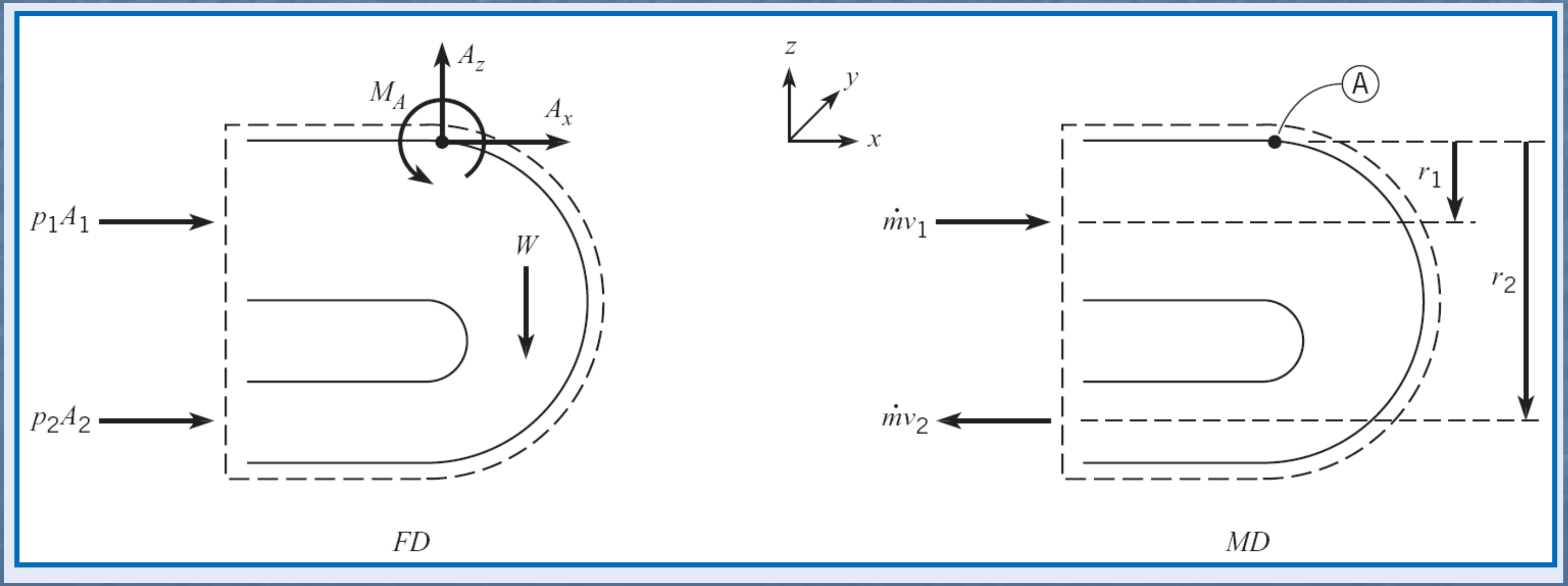
$$\frac{d(H)_{sys}}{dt} = \frac{d}{dt}\int_{cv} (v \times r)\rho dQ + \int_{cs} (v \times r)\rho V \bullet dA = \sum M$$

$$\sum M = \frac{d}{dt}\int_{CV} (r \times v)\rho dQ + \sum_{CS} r \times (\dot{m}v)_{out} - \sum_{CS} r \times (\dot{m}v)_{in}$$

Example (6.12)

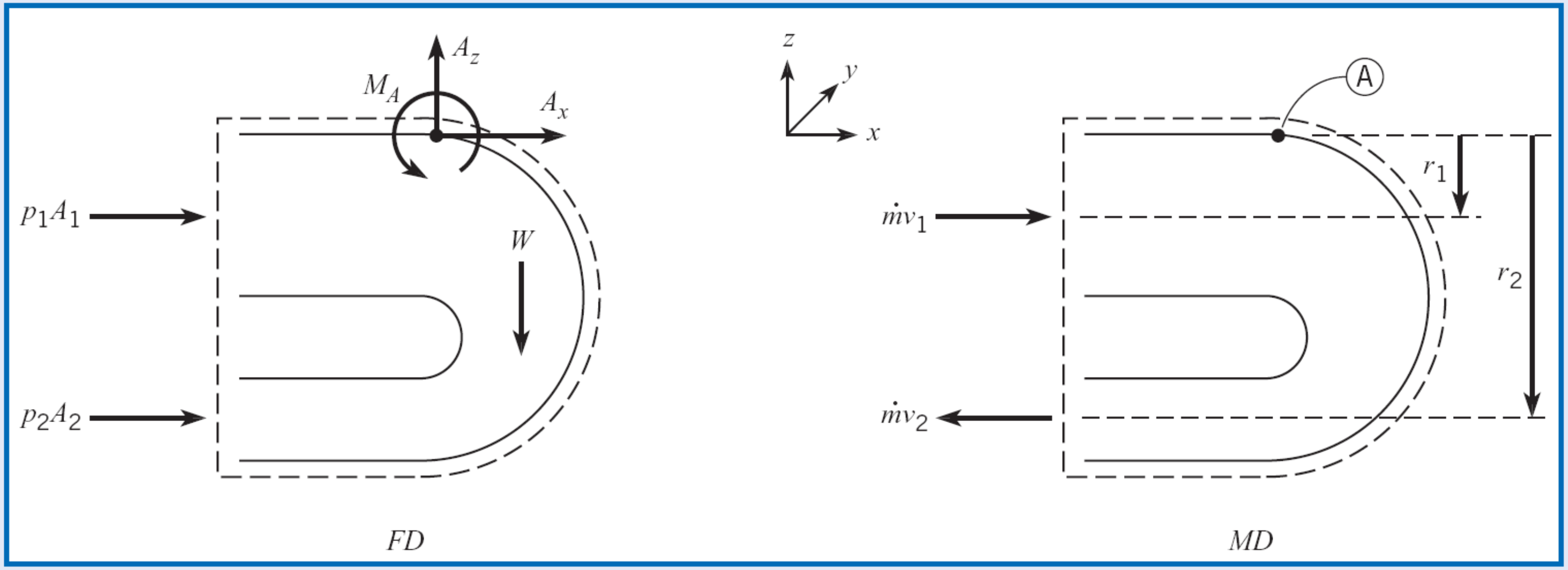
Find: The moment the support system must resist?





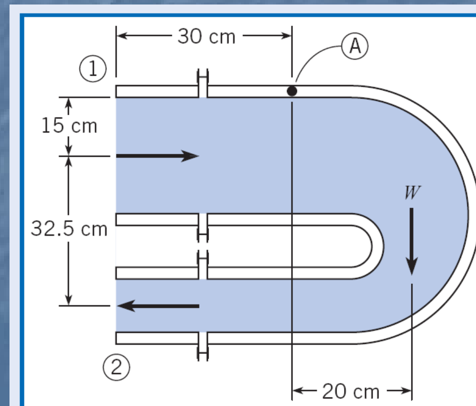
ANALYSIS:

The momentum accumulation = $\frac{d}{dt}\int_{cv} v_Z \rho dQ = 0$



From the force diagram,

$$\sum M = -(M_A + 0.15p_1A_1 + 0.475p_2A_2 - 0.2W)$$



From the momentum diagram,

$$\sum M = \sum_{CS} r_{out} \times (\dot{m}v)_{out} - \sum_{CS} r_{in} \times (\dot{m}v)_{in}) = r_2 \times (\dot{m}v_2) + r_1 \times (\dot{m}v_1)$$

Equating the above Eqns. we have

$$\sum M = -(M_A + 0.15 p_1 A_1 + 0.475 p_2 A_2 - 0.2W) = \dot{m}(r_2 v_2 + r_1 v_1)$$

$$M_A = -3.6\, kNm$$

THE END