

SUMMARY

ENERGY PRINCIPLE

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ENERGY PRINCIPLE

First Law of Thermodynamics

For a given system

$$\Delta E = Q - W$$

$$\frac{dE}{dt} = \dot{Q} - \dot{W}$$

Where:

Q = Heat transferred to the system (positive sign)

W = Work transferred out of the system (negative sign)

E = Energy of the system (Kinetic Energy, Potential Energy, Chemical Energy, Electrical Energy)

$$E = U + K.E + P.E$$

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Derivation of Energy Equation

Using the Reynolds transport theorem which is

$$\frac{dB_{sys}}{dt} = \frac{d}{dt} \int_{cv} b \rho dQ + \int_{cs} b \rho V \cdot dA$$

$$B_{sys} = (E)_{sys} = (mu + me_K + me_P)_{sys}$$

$$b = e = u + e_K + e_P$$

$$\frac{d(E)_{sys}}{dt} = \frac{d}{dt} \int_{cv} e \rho dQ + \int_{cs} e \rho V \cdot dA$$

$$\dot{Q} - \dot{W} = \frac{d}{dt} \int_{cv} e \rho dQ + \int_{cs} e \rho V \cdot dA$$

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$$\dot{Q} - \dot{W} = \frac{d}{dt} \int_{cv} (u + e_K + e_P) \rho dQ + \int_{cs} (u + e_K + e_P) \rho V \cdot dA$$

$$e_K = \frac{(1/2)mv^2}{m} = \frac{v^2}{2}$$

$$e_P = \frac{mgz}{m} = gz$$

$$\dot{Q} - \dot{W} = \frac{d}{dt} \int_{cv} (u + \frac{v^2}{2} + gz) \rho dQ + \int_{cs} (u + \frac{v^2}{2} + gz) \rho V \cdot dA$$

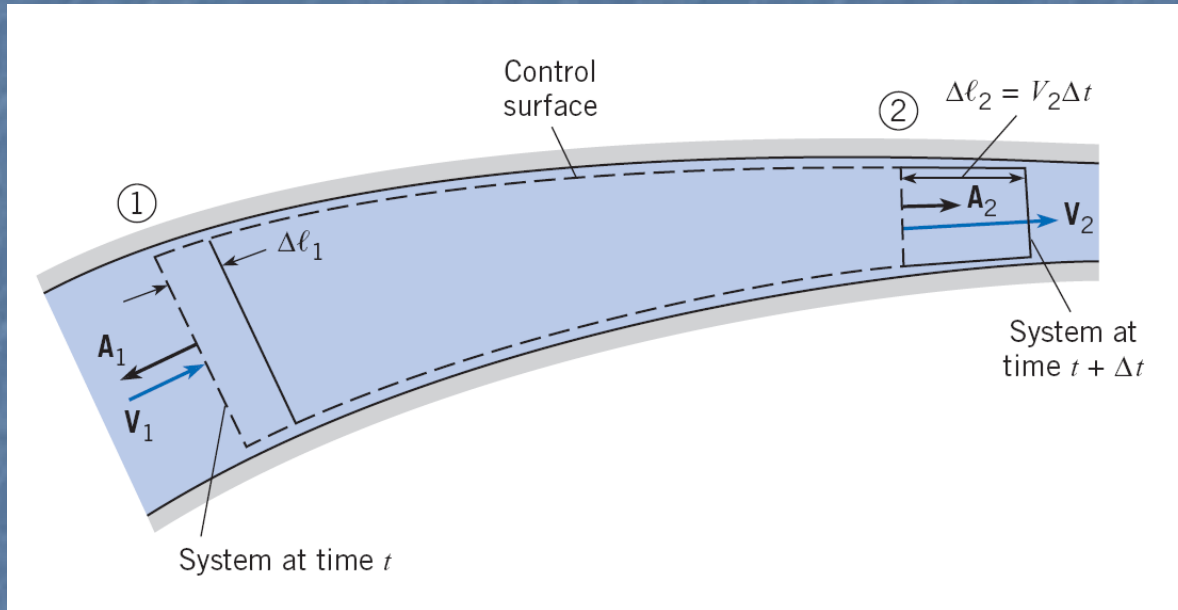
The Work $\dot{W} = \dot{W}_f + \dot{W}_s$

$\dot{W}_s =$ Shaft Work

$\dot{W}_f =$ Flow Work

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Flow Work



The flow of work done by the system at station (2) is given by

$$W_{f2} = F \times \Delta l_2 = (p_2 A_2) \times v_2 \Delta t$$

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$$\dot{W}_{f2} = (p_2 A_2) \times v_2 = p_2 v \cdot A$$

The flow of work by the system at station (1) is given by

$$W_{f1} = F \times \Delta l_1 = (p_1 A_1) \times V_1 \Delta t$$

$$\dot{W}_{f1} = (p_1 A_1) \times V_1 = p_1 V \cdot A$$

In general form, the work done by the system for a constant velocity

$$\dot{W}_f = \sum_{CS} pV \cdot A$$

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In general form, the work done by the system for a variable velocity

$$\dot{W}_f = \int_{CS} p V \cdot dA = \int_{CS} \frac{p}{\rho} (\rho V \cdot dA)$$

Shaft Work (\dot{W}_s)

Consider Eqn.

$$\dot{Q} - \dot{W} = \frac{d}{dt} \int_{cv} \left(u + \frac{v^2}{2} + gz \right) \rho dQ + \int_{CS} \left(u + \frac{v^2}{2} + gz \right) \rho V \cdot dA$$

$$\dot{Q} - \dot{W}_s - \dot{W}_f = \frac{d}{dt} \int_{cv} \left(u + \frac{v^2}{2} + gz \right) \rho dQ + \int_{CS} \left(u + \frac{v^2}{2} + gz \right) \rho V \cdot dA$$

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$$\dot{W}_f = \int_{CS} p \mathbf{V} \cdot d\mathbf{A} = \int_{CS} \frac{p}{\rho} (\rho \mathbf{V} \cdot d\mathbf{A})$$

$$\dot{Q} - \dot{W}_s - \int_{CS} \frac{p}{\rho} (\rho \mathbf{V} \cdot d\mathbf{A}) = \frac{d}{dt}_{CV} \int (u + \frac{v^2}{2} + gz) \rho dQ + \int_{CS} (u + \frac{v^2}{2} + gz) \rho \mathbf{V} \cdot d\mathbf{A}$$

$$\dot{Q} - \dot{W}_s = \frac{d}{dt}_{CV} \int (u + \frac{v^2}{2} + gz) \rho dQ + \int_{CS} (u + \frac{v^2}{2} + gz) \rho \mathbf{V} \cdot d\mathbf{A} + \int_{CS} \frac{p}{\rho} (\rho \mathbf{V} \cdot d\mathbf{A})$$

$$\dot{Q} - \dot{W}_s = \frac{d}{dt}_{CV} \int (u + \frac{v^2}{2} + gz) \rho dQ + \int_{CS} (u + \frac{p}{\rho} + \frac{v^2}{2} + gz) \rho \mathbf{V} \cdot d\mathbf{A}$$

$$\text{The term enthalpy} = h = \left(u + \frac{p}{\rho} \right)$$

$$\dot{Q} - \dot{W}_s = \frac{d}{dt}_{CV} \int (u + \frac{v^2}{2} + gz) \rho dQ + \int_{CS} (h + \frac{v^2}{2} + gz) \rho \mathbf{V} \cdot d\mathbf{A}$$

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Steady Flow Energy Equation

For steady flow, the flow accumulation equal zero and if the velocity distribution is constant, then

the Eqn. $\dot{Q} - \dot{W}_s = \frac{d}{dt} \int_{CV} (u + \frac{v^2}{2} + gz) \rho dQ + \int_{CS} (h + \frac{v^2}{2} + gz) \rho V \cdot dA$ becomes

$$\dot{Q} - \dot{W}_s = \sum_{CS} \dot{m}_{out} \left(\frac{V^2}{2} + gz + h \right)_{out} - \sum_{CS} \dot{m}_{in} \left(\frac{V^2}{2} + gz + h \right)_{in}$$

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The coefficients (α_1, α_2) are called kinetic energy correction factor and can be evaluated as follows :

$$\alpha \frac{\rho \bar{V}^3 A}{2} = \int_A \frac{\rho V^3 dA}{2}$$

$$\alpha = \frac{1}{A} \int_A \left(\frac{V}{\bar{V}} \right)^3 dA$$

$\alpha = 1$ Velocity is uniform

$\alpha > 1$ Velocity is non – uniform

$\alpha = 2$ Laminar Flow

$\alpha = 1.05$ Turbulent Flow

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Pump – Turbine Combination

When a compressor and a turbine are connected together, the resultant shaft work \dot{W}_S is equal to

$$\dot{W}_S = \dot{W}_T - \dot{W}_p$$

Substituting for the above expression in Eqn. below

$$\frac{1}{\dot{m}}(\dot{Q} - \dot{W}_s) + \left(\frac{p_1}{\rho} + gz_1 + u_1 + \alpha_1 \frac{\bar{V}_1^2}{2} \right) = \left(\frac{p_2}{\rho} + gz_2 + u_2 + \alpha_2 \frac{\bar{V}_2^2}{2} \right)$$

and divide by (g), we have

$$\frac{\dot{Q}}{\dot{m}g} - \left(\frac{\dot{W}_T - \dot{W}_P}{g} \right) + \left(\frac{p_1}{\rho g} + z_1 + \frac{u_1}{g} + \alpha_1 \frac{\bar{V}_1^2}{2g} \right) = \left(\frac{p_2}{\rho g} + gz_2 + \frac{u_2}{g} + \alpha_2 \frac{\bar{V}_2^2}{2g} \right)$$

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Rearranging, we have

$$\left(\frac{\dot{W}_p}{\dot{m}g}\right) + \left(\frac{p_1}{\rho g} + z_1 + \alpha_1 \frac{\bar{V}_1^2}{2g}\right) = \left(\frac{\dot{W}_T}{\dot{m}g}\right) + \left(\frac{p_2}{\rho g} + gz_2 + \alpha_2 \frac{\bar{V}_2^2}{2g}\right) + \left(\frac{u_2 - u_1}{g}\right) - \left(\frac{\dot{Q}}{\dot{m}g}\right)$$

$$\left(h_P + \frac{p_1}{\rho g} + z_1 + \alpha_1 \frac{\bar{V}_1^2}{2g}\right)_{\text{Mech. part}} = \left(h_T + \frac{p_2}{\rho g} + gz_2 + \alpha_2 \frac{\bar{V}_2^2}{2g}\right)_{\text{Mech. part}} + \left(\frac{u_2 - u_1}{g} - \frac{\dot{Q}}{\dot{m}g}\right)_{\text{thermal part}}$$

$$h_P = \frac{\dot{W}_C}{\dot{m}g} = \text{Pump Head}$$

$$h_T = \frac{\dot{W}_T}{\dot{m}g} = \text{Turbine Head}$$

$$\text{The Head Loss} = h_{\text{Loss}} = \left(\frac{1}{g}\right) \left(u_2 - u_1 - \frac{\dot{Q}}{\dot{m}}\right)$$

$$\left(h_P + \frac{p_1}{\rho g} + z_1 + \alpha_1 \frac{\bar{V}_1^2}{2g}\right)_{\text{Mech. part}} = \left(h_T + \frac{p_2}{\rho g} + gz_2 + \alpha_2 \frac{\bar{V}_2^2}{2g}\right)_{\text{Mech. part}} + h_{\text{Loss}}$$

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$$\text{Pump Head} = h_C = \frac{\dot{W}_C}{\dot{m}g}$$

$$\dot{W}_C = \dot{m}gh_C = \gamma Qh_C$$

$$\dot{W}_T = \dot{m}gh_T = \gamma Qh_T$$

$$\text{Pump Efficiency} = \eta_P = \frac{(\dot{W}_P)}{(\dot{W}_P)_{\text{actual}}}$$

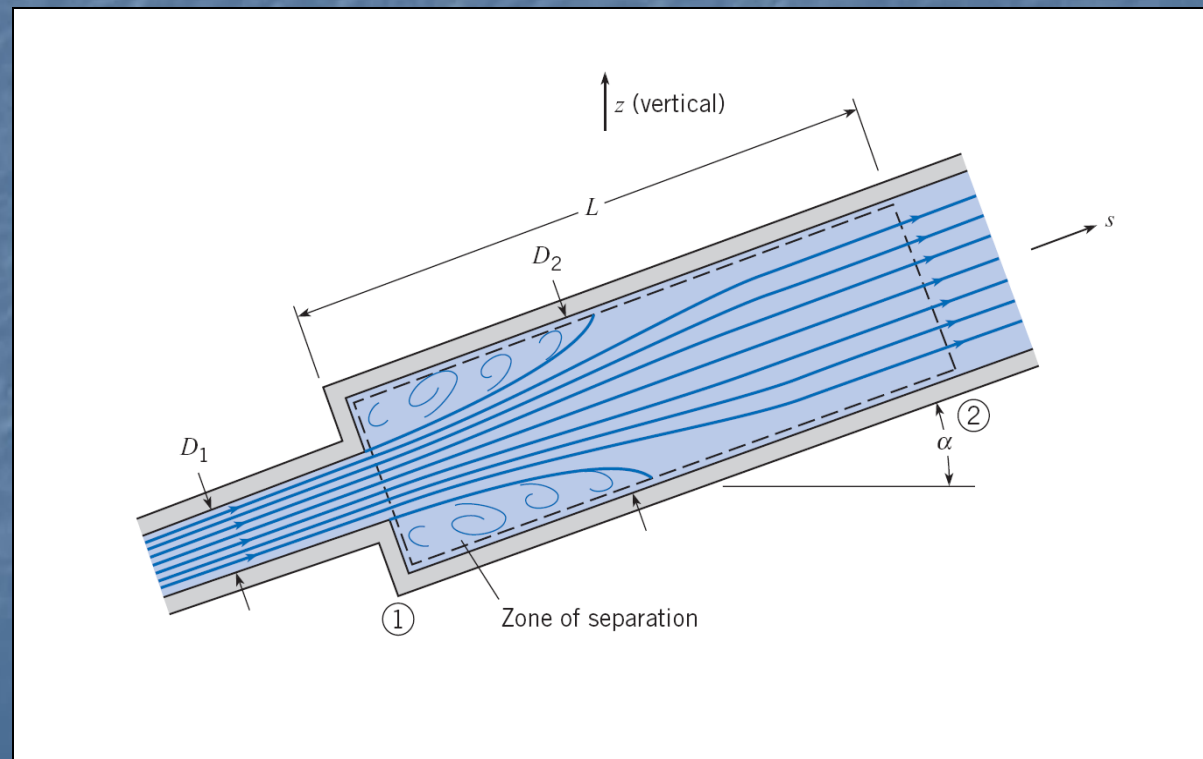
$$\text{Turbine Efficiency} = \eta_T = \frac{(\dot{W}_T)_{\text{actual}}}{(\dot{W}_T)}$$

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Application of the Energy Equation, Momentum and Continuity Principles in Combination

Abrupt Expansion

$$i.e. \quad h_{loss} = \left(\frac{V_1^2 - V_2^2}{2g} \right)$$



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THE END