SURFACE RESISTANCE

Chapter (9)

SUMMARY

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SURFACE RESISTANCE

Chapter (9)

1. Definition

A force resistance exerted by a fluid medium (Liquid or Gas) to the motion of bodies such as aircrafts, ships or cars is called a **DRAG FORCE** **OR** **SURFACE RESISTANCE.**

2. Types of a Drag

(2.1) Skin Friction Drag: **Formed due to shear forces.**

(Shear forces are forces act Parallel to motion).

(2.2) Form Drag: **Formed due to pressure forces.**

(Pressure forces are forces act Normal to motion).

Three Cases to consider in a uniform laminar flow over surfaces

CASE (1)

Flow Produced by a Moving Plate (Couette Flow)

CASE (2)

Liquid Flow Down an Inclined Plane

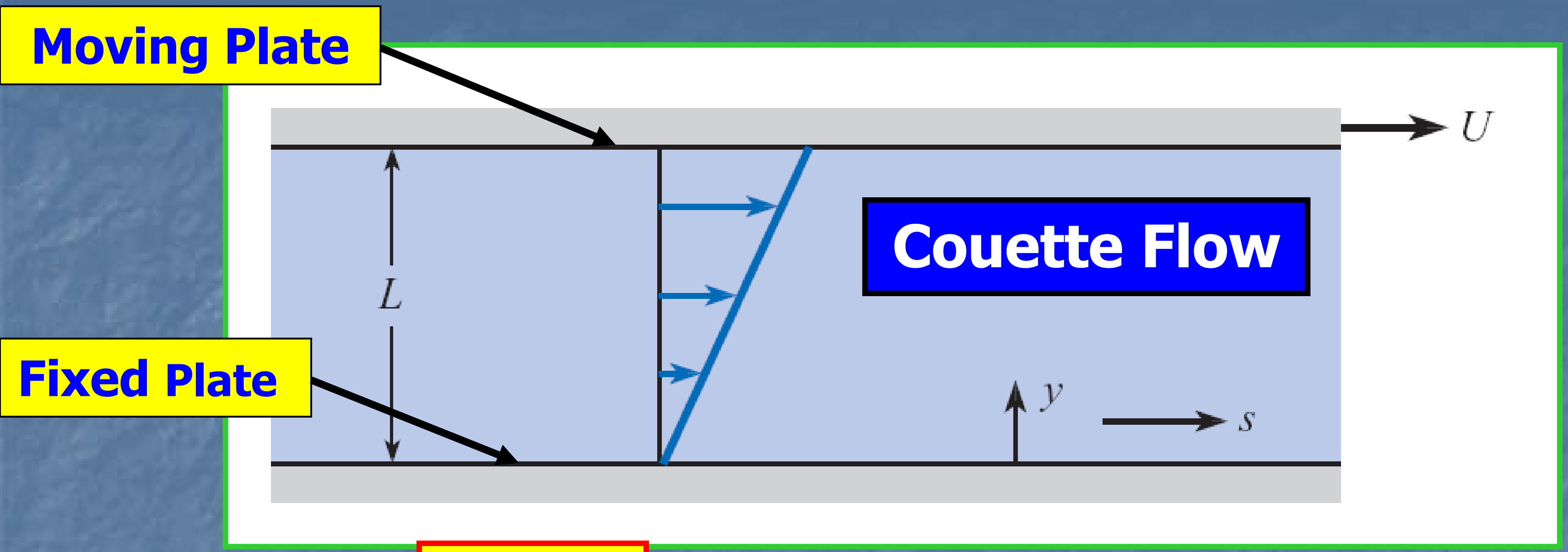
CASE (3)

Flow Produced by a Moving Plate (Couette Flow)

Always start with this equation →

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{d}{ds}(p + \gamma z)$$

Flow Produced by a Moving Plate (Couette Flow)



$$u = \frac{y}{L} U$$

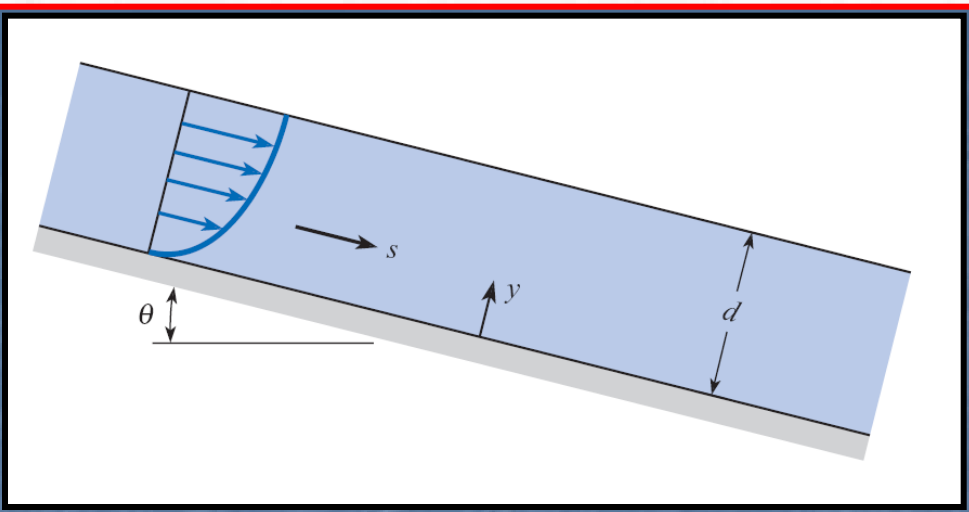
Linear Distribution

which shows that the velocity profile is linear between the two plates. The shear stress is constant and equal to

Shear stress is constant across the plates

$$\tau = \mu \frac{du}{dy} = \mu \frac{U}{L}$$

Liquid Flow Down an Inclined Plane



- Ve

$$u = \frac{gS_0}{2\nu} y(2d - y)$$

$$u_{max} = \frac{gS_0 d^2}{2\nu}$$

$$u_{max} = \frac{\gamma d^2}{2\mu} \sin\theta$$

Volume flow per unit width

$$q = \frac{1}{3}\frac{\gamma}{\mu} S_0 d^3 = \frac{1}{3}\frac{\gamma}{\rho\nu} S_0 d^3 = \frac{1}{3}\frac{g}{\nu} S_0 d^3$$

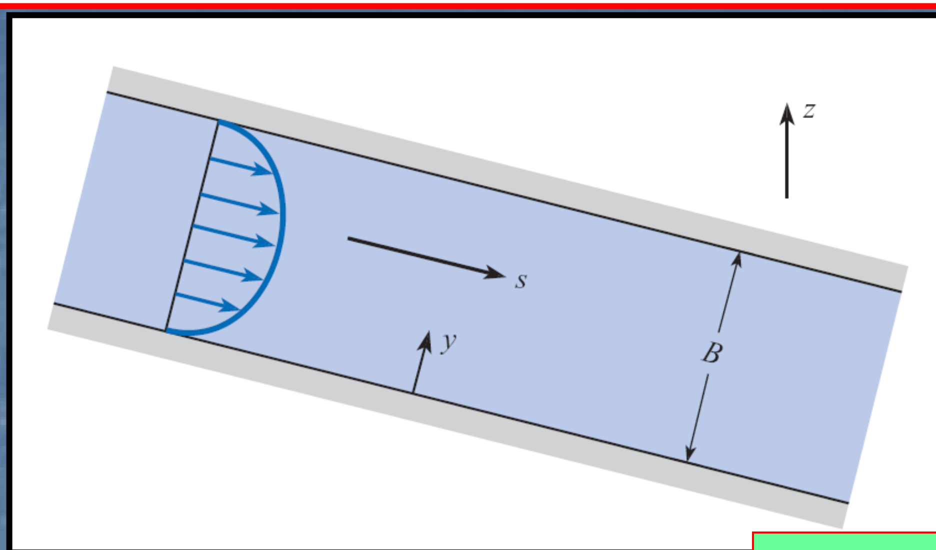
Average Velocity

$$V = \frac{q}{d} = \frac{1}{3}\frac{\gamma}{\mu} d^2 \sin\theta$$

$$V = \frac{gd^2}{3\nu} \sin\theta$$

$$V = \frac{gS_0 d^2}{3\nu}$$

Flow Between Stationary Parallel Plates Hele-Shaw flow



$$u = -\frac{1}{2\mu}\frac{d}{ds}(p+\gamma z)(By-y^2) = -\frac{\gamma}{2\mu}(By-y^2)\frac{dh}{ds}$$

$$u_{\max} = -\left(\frac{B^2\gamma}{8\mu}\right)\frac{dh}{ds}$$

Average Velocity

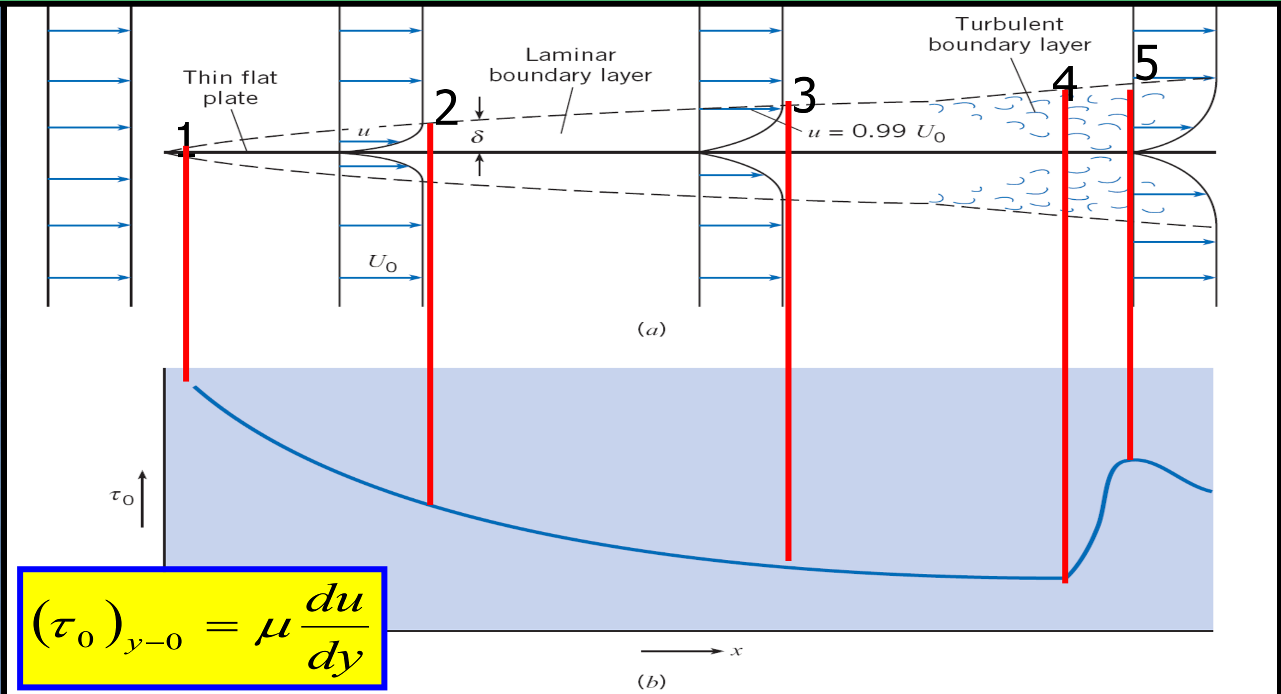
$$V = \frac{q}{B} = -\left(\frac{B^2}{12\mu}\right)\frac{d}{ds}(p+\gamma z) = \frac{2}{3}u_{\max}$$

Volume flow per unit width

$$q = \int_0^B u\,dy = -\left(\frac{B^3}{12\mu}\right)\frac{d}{ds}(p+\gamma z) = -\left(\frac{B^3\gamma}{12\mu}\right)\frac{dh}{ds}$$

Qualitative Description of a Laminar Boundary Layer

Is defined as the distance from the boundary to the point in the fluid **where the velocity is 99% of the free steam velocity.**



$$(\tau_0)_{y-0} = \mu \frac{du}{dy}$$

The boundary layer thickness is very thin, occurs because of the viscosity of the fluid.

The velocity gradient at the surface is responsible for the **viscous shear and surface resistance.**

The actual thickness of the boundary layer may be (2 -3 %) of the plate length.

Boundary Layers whether Laminar or Turbulent are cases of Rotational flow

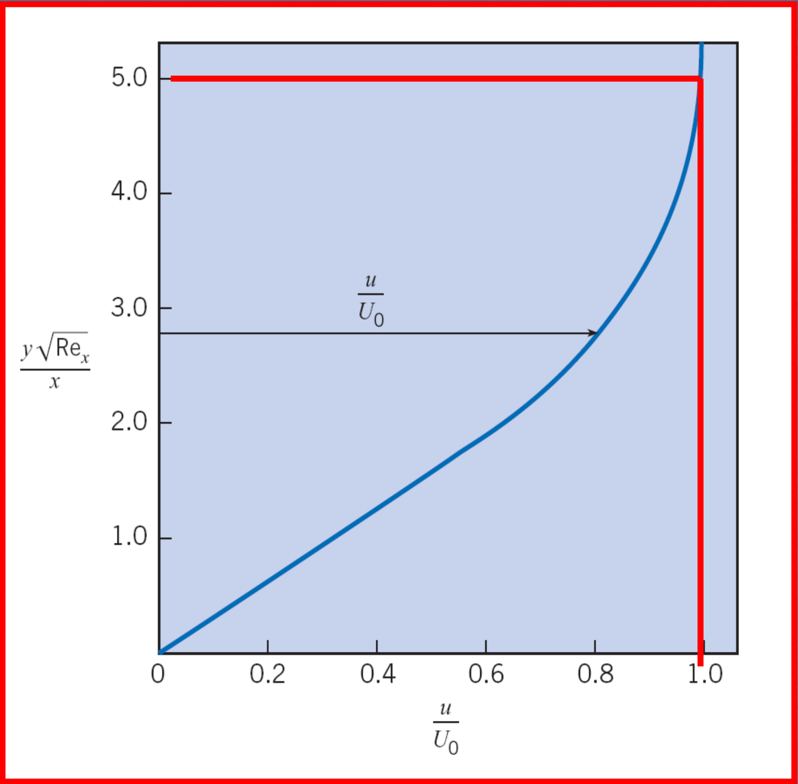
Laminar Boundary Layer Thickness (δ)

Blasius Solution

$$\frac{\delta (Re_x)^{1/2}}{x} = 5 \quad OR$$

$$\delta = \frac{5x}{(Re_x)^{1/2}}$$

$$(R_e)_X = \frac{U_0 X}{\nu}$$



Blasius Curve Assume $\frac{dp}{dx} = 0$

Velocity gradient at the boundary

$$\left.\frac{du}{dy}\right|_{y=0} = 0.332\frac{U_0}{x}Re_x^{1/2}$$

$$\left.\frac{du}{dy}\right|_{y=0} = 0.332\frac{U_0}{x}\left(\frac{U_0 x}{\nu}\right)^{1/2}$$

$$\left.\frac{du}{dy}\right|_{y=0} = 0.332\frac{U_0^{3/2}}{x^{1/2}\nu^{1/2}}$$

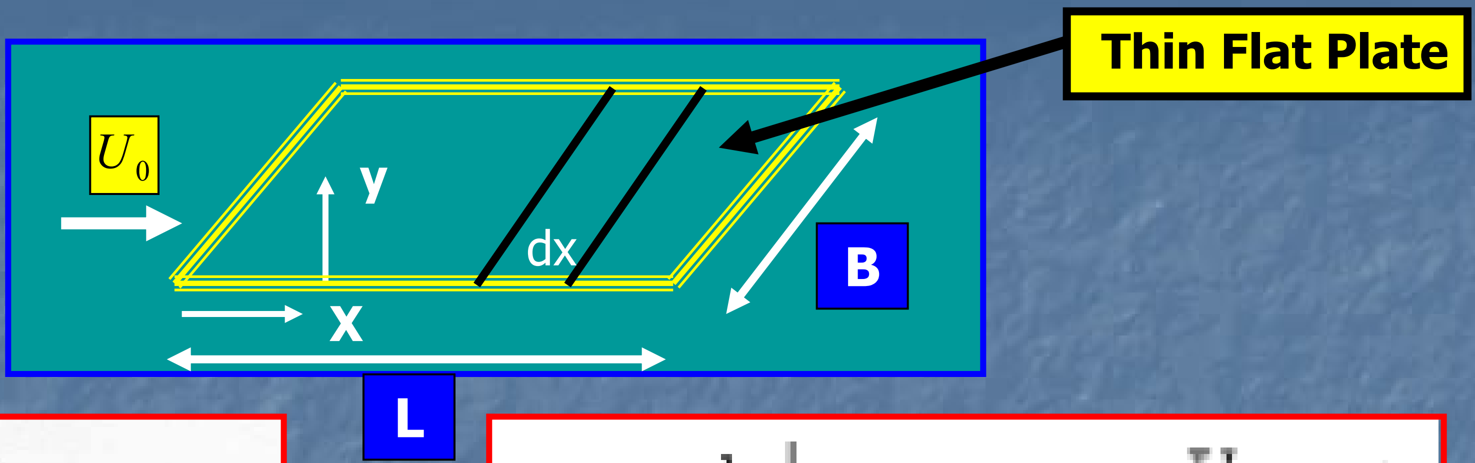
$$\left(\frac{du}{dy}\right)_{y=0} \downarrow AS \ X \uparrow$$

$$(\tau_0)_{y=0} \downarrow AS \ X \uparrow$$

The shear stress at the boundary is obtained from

$$\tau_0 = \mu \left.\frac{du}{dy}\right|_{y=0} = 0.332\mu\frac{U_0}{x}Re_x^{1/2}$$

Surface Resistance (Shear Force) at the Boundary Layer



$$F_s = \int_0^L \tau_0 B\, dx$$

$$\tau_0 = \mu \frac{du}{dy}\bigg|_{y=0} = 0.332\mu \frac{U_0}{x} \mathrm{Re}_x^{1/2}$$

Shearing Force = $$F_s = \int_0^L 0.332 B\mu \frac{U_0 U_0^{1/2} x^{1/2}}{x \nu^{1/2}}\, dx$$

$$= 0.664 B\mu U_0 \frac{U_0^{1/2} L^{1/2}}{\nu^{1/2}} \tag{9.19}$$

$$= 0.664 B\mu U_0 \mathrm{Re}_L^{1/2}$$

In Eq. (9.19) Re_L is the Reynolds number based on the approach velocity and the length of the plate.

Shear Stress Coefficients at a Laminar Boundary Layer

The *local sheer stress coefficient*, c_f, is defined as

$$c_f = \frac{\tau_0}{\rho U_0^2/2}$$

The shear stress at the boundary is obtained from

$$\tau_0 = \mu \frac{du}{dy}\bigg|_{y=0} = 0.332\mu \frac{U_0}{x} \mathrm{Re}_x^{1/2}$$

Combining the above two equations , we obtain:

$$c_f = \frac{0.664}{\mathrm{Re}_x^{1/2}}$$

$$C_f = \frac{1.33}{\sqrt{(\mathrm{Re})_L}}$$

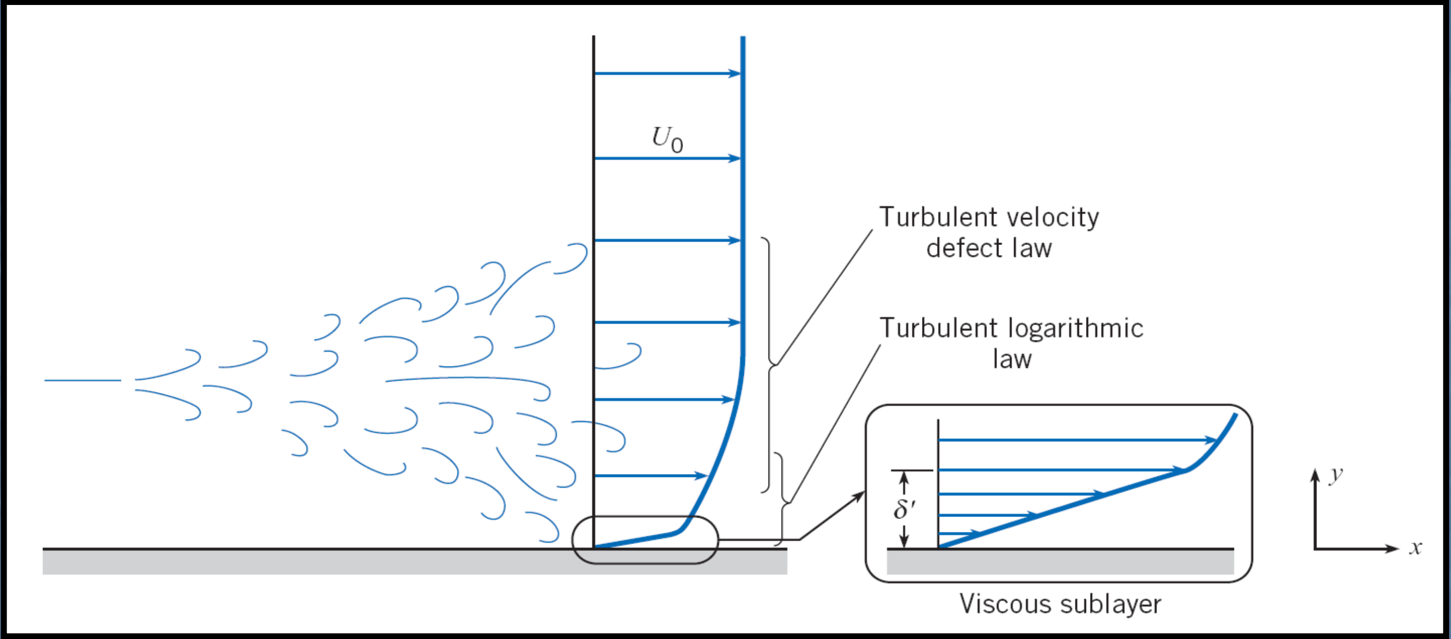
Resistance force on one side of plate is given by:

$$F_S = (C_f)(1/2\,\rho U_0^2)(BL)$$

Total Resistance force of plate is given by:

$$F_S = 2\times(C_f)(1/2\,\rho U_0^2)(BL)$$

Turbulent Boundary Layer



Sketch of zones in turbulent boundary layer.

<u>Turbulent boundary Layers Consists of Three Layers:</u>

1. **Viscous Sub-Layer**
2. **Logarithmic Layer**
3. **Velocity Defect Layer**

Viscous Sub-Layer Zone

Shear Velocity

OR

Friction Velocity

$$u_* = \sqrt{\frac{\tau_0}{\rho}}$$

$$At\ Viscous\ Sublayer\ Limit,\ y = \delta'$$

$$\delta' = \frac{5\nu}{u_*} = \frac{5\nu}{\sqrt{\tau_0/\rho}} \qquad \tau_0 = \mu \frac{du}{dy} \qquad (9.28)$$

$$\delta' \uparrow\ AS\ \tau_0 \downarrow$$

$$\delta' \uparrow\ AS\ \frac{du}{dy} \downarrow$$

Nominal Thickness of the Viscous Sub-Layer (δ'_N)

$$\frac{y u_*}{\nu} = 11.84 \rightarrow \delta'_N = 11.84 \frac{\nu}{u_*}$$

Logarithmic Zone Layer

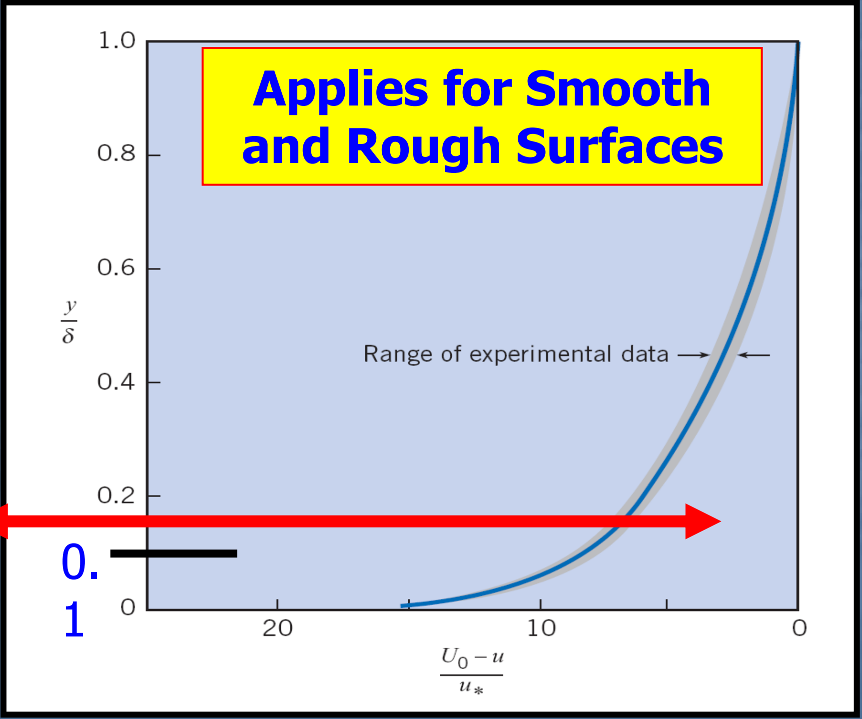
$$\frac{u}{u_*} = 5.75 \log \frac{y u_*}{\nu} + 5.56$$

This distribution is valid for values of $y u_*/\nu$ ranging from approximately 30 to 500.

Velocity Defect Zone

$$\frac{U_0 - u}{u_*} = f\left(\frac{y}{\delta}\right)$$

0.15



Velocity Defect Zone

For $\frac{y}{\delta} > 0.15$

and $\frac{yu_*}{\nu} \geq 500$

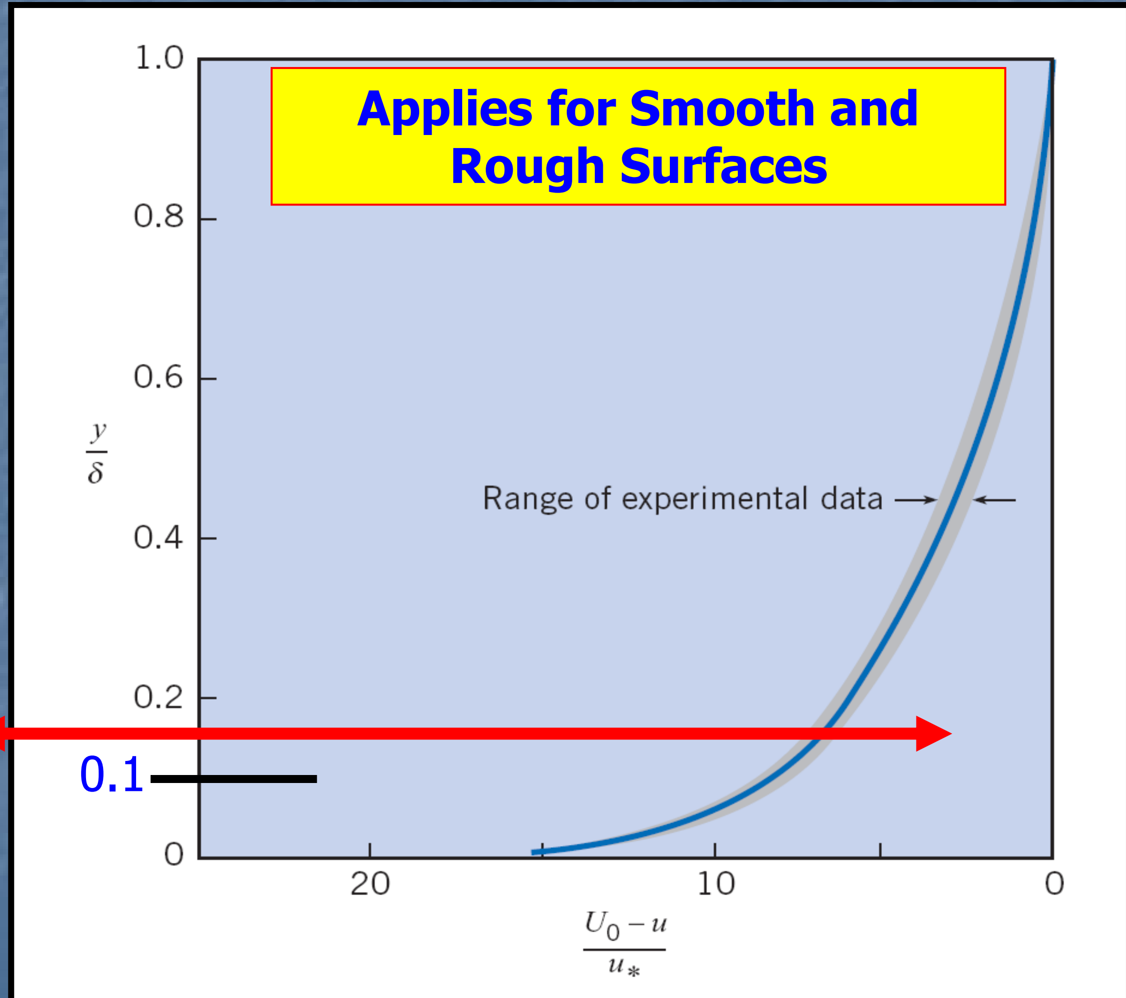
Law of the wall is not applicable

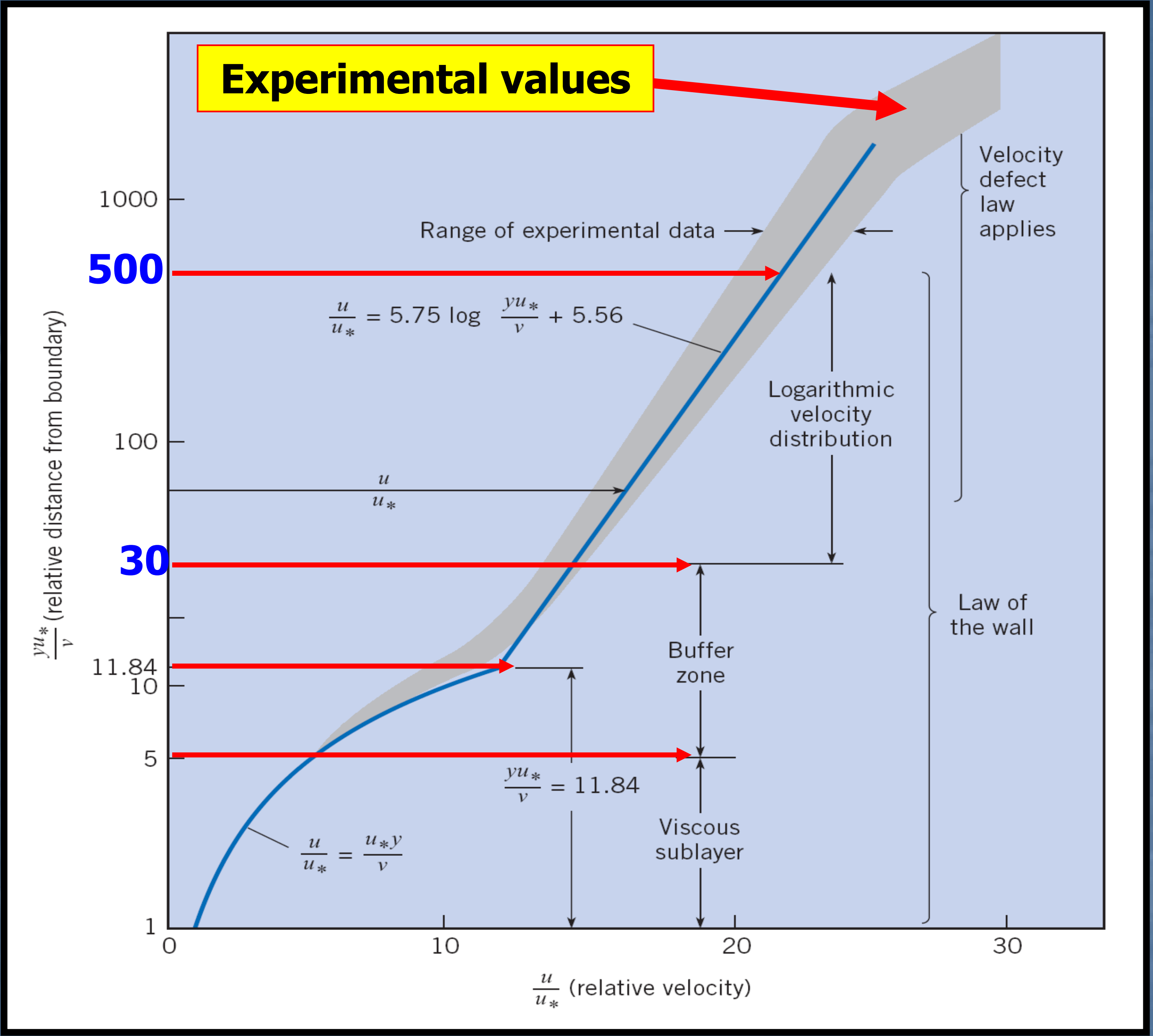
Lead to

Velocity defect law for boundary layers

$$\frac{U_0 - u}{u_*} = f\left(\frac{y}{\delta}\right)$$

0.15





Velocity distribution in a turbulent boundary layer

ZONES FOR TURBULENT BOUNDARY LAYER ON FLAT PLATE

Zone	Velocity Distribution	Range
Viscous Sublayer	$\frac{u}{u_*} = \frac{yu_*}{\nu}$	$0 < \frac{yu_*}{\nu} < 11.84$
Logarithmic Velocity Distribution	$\frac{u}{u_*} = 2.44 \ln \frac{yu_*}{\nu} + 5.56$	$11.84 \leq \frac{yu_*}{\nu} < 500$
Velocity Defect Law	$\frac{u_0 - u}{u_*} = f\left(\frac{y}{\delta}\right)$	$500 \leq \frac{yu_*}{\nu}, \frac{y}{\delta} > 0.15$

Power Law Formula for Velocity Distribution for Turbulent Flow

Analyses have shown that for a wide range of Reynolds numbers ($10^5 < Re < 10^7$), the velocity profile in the turbulent boundary layer is reasonably approximated by the *power-law* equation

$$\frac{u}{U_0} = \left(\frac{y}{\delta}\right)^{1/7} \quad (9.37)$$

Boundary Layer Thickness for Turbulent Flow

$$\delta = \frac{0.16x}{Re_x^{1/7}}$$

Shear Stress Coefficients & shearing Resistance of the Turbulent Boundary Layer on a Flat Plate

1. Local Shear Stress Coefficient

$$c_f = \frac{\tau_0}{\rho U_0^2/2} = \frac{0.027}{\mathrm{Re}_x^{1/7}}$$

Not suitable for high Reynolds numbers

2. Average Shear Stress Coefficient

$$C_F = \frac{0.032}{(R_e)_L^{1/7}}$$

3. Shearing Resistance Over the Area of the Boundary

$$F_S = \tau_0 \times A \; where \; (A = BL)$$

$$\tau_0 = \rho \frac{U_0^2}{2}\left(\frac{0.027}{\mathrm{Re}_x^{1/7}}\right)$$

By integration over the area (BL)

$$F_s = \frac{0.032BL}{\mathrm{Re}_L^{1/7}} \rho \frac{U_0^2}{2}$$

Shear Stress Coefficients of the Turbulent Boundary Layer on a Flat Plate at Higher Reynolds Numbers

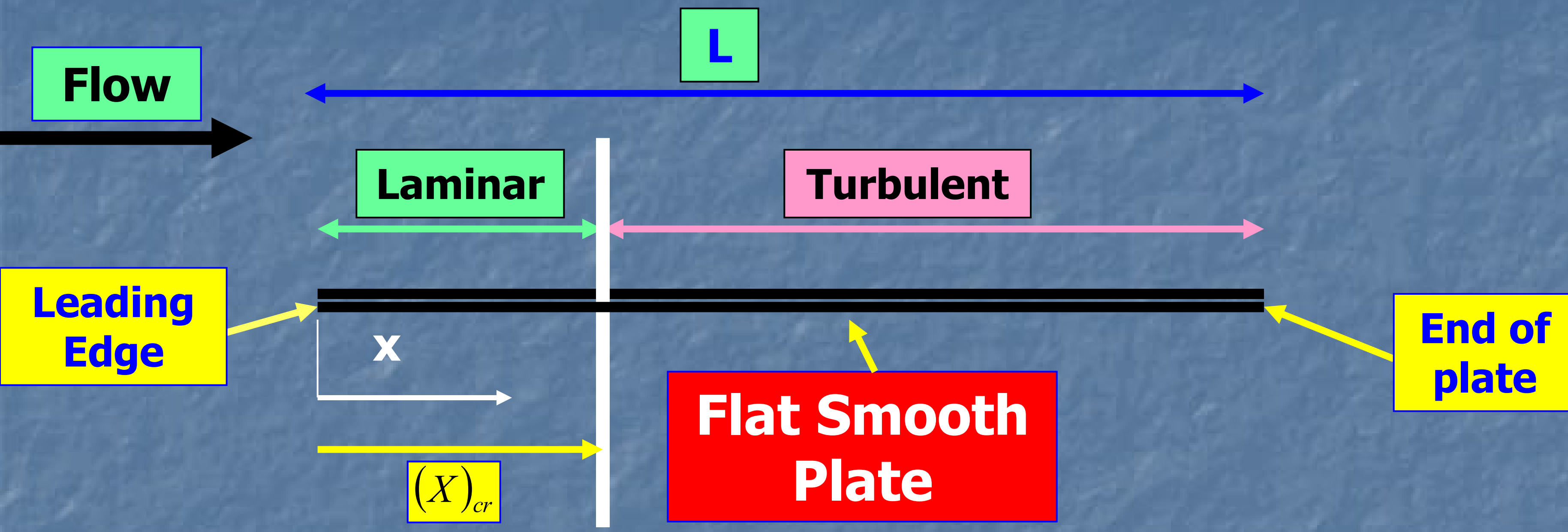
For Reynolds Numbers $R_e \leq 10^{10}$, the below correlations are valid

$$c_f = \frac{0.455}{\ln^2(0.06\mathrm{Re}_x)} \tag{9.52a}$$

The corresponding average shear stress coefficient is

$$C_f = \frac{0.523}{\ln^2(0.06\mathrm{Re}_L)} \tag{9.52b}$$

Laminar and Turbulent Boundary Layers Coexist Together on a Flat Smooth Plate



Critical Transition Point at $R_e = 500{,}000$

<u>**Calculating the resistance of the boundary layer**</u> **$(F_S)_L$:**

1. Compute the resistance as though the entire layer were turbulent. $(F_{tur})_L$
2. Compute the resistance of the laminar part, $(F_{lam})_{Ltr}$ and the resistance of the turbulent part $(F_{tur})_{Ltr}$ up to the transition zone.
3. $\mathbf{(F_S)_L} = (F_{tur})_L - (F_{tur})_{Ltr} + (F_{Lam})_{Ltr}$

Shear Stress Coefficients & Boundary Layer Thickness of Tripped Boundary Layer on a Flat Plate

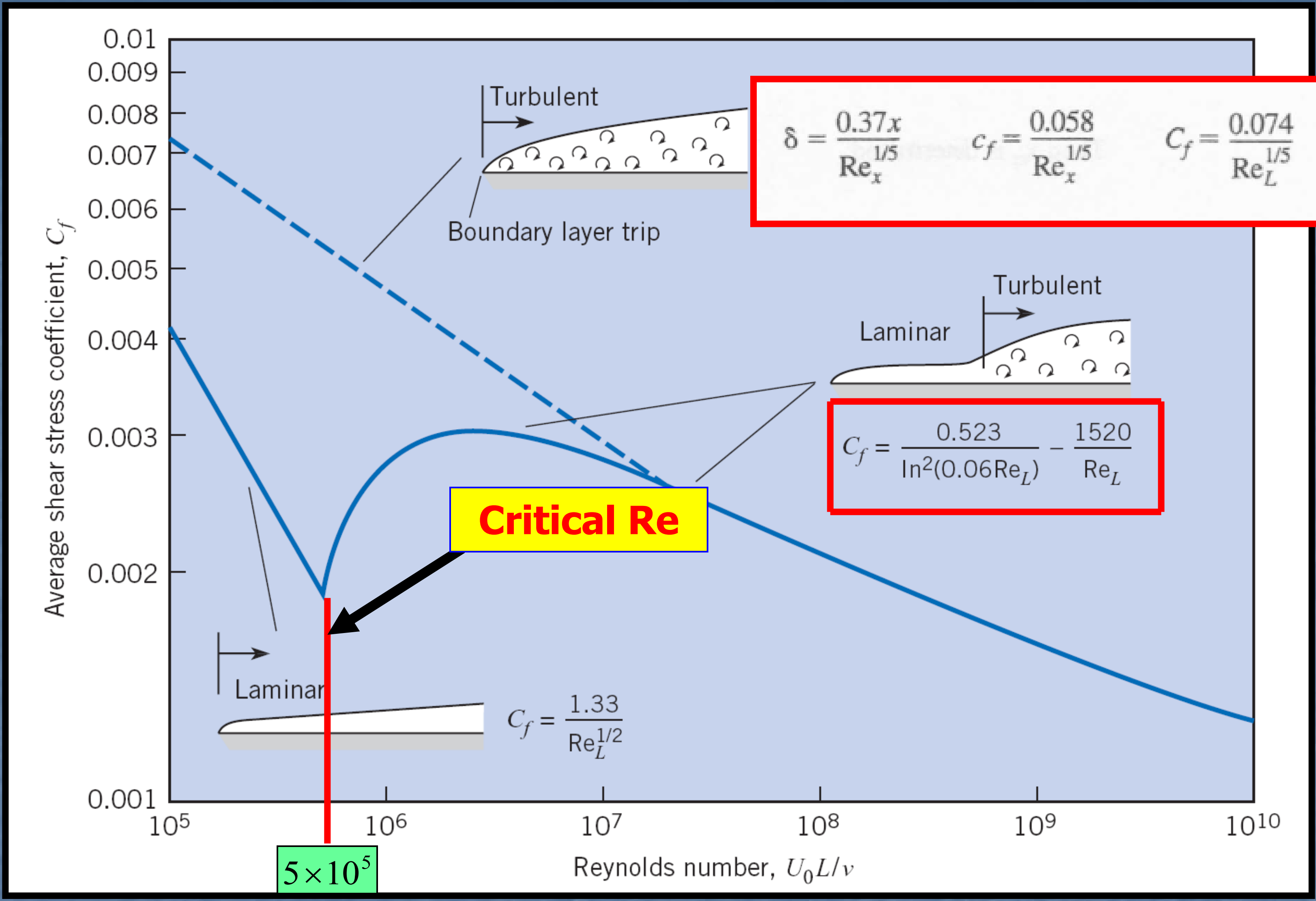
If the boundary layer is "tripped" by some roughness or leading-edge disturbance, the boundary layer is turbulent from the beginning. This is shown by the dashed line in Fig. 9.14. For this condition the boundary layer thickness, local shear stress coefficient, and average shear stress coefficient are fit by the empirical expressions

$$\delta = \frac{0.37x}{\mathrm{Re}_x^{1/5}} \qquad c_f = \frac{0.058}{\mathrm{Re}_x^{1/5}} \qquad C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}}$$

The above Eqns. are Valid only for $(R_e) \leq 10^7$

For Reynolds numbers beyond $(R_e) > 10^7$

$$C_f = \frac{0523}{In^2(0.06\,\mathrm{Re}_L)}$$



Average Shear Stress Coefficient Variation with Reynolds Number

TABLE 9.2 SUMMARY OF EQUATIONS FOR BOUNDARY LAYER ON A FLAT PLATE

	Laminar flow Re_x, $\mathrm{Re}_L < 5 \times 10^5$	**Turbulent flow** $\mathrm{Re}_x, \mathrm{Re}_L \geq 5 \times 10^5$
Boundary layer thickness, δ	$\delta = \frac{5x}{\mathrm{Re}_x^{1/2}}$	$\delta = \frac{0.16x}{\mathrm{Re}_x^{1/7}}$
Local shear stress coefficient, c_f	$c_f = \frac{0.664}{\mathrm{Re}_x^{1/2}}$	$c_f = \frac{0.455}{\ln^2(0.06\mathrm{Re}_x)}$
Average shear stress coefficient, C_f	$C_f = \frac{1.33}{\mathrm{Re}_L^{1/2}}$	$C_f = \frac{0.523}{\ln^2(0.06\mathrm{Re}_L)} - \frac{1520}{\mathrm{Re}_L}$

END OF SUMMARY CHAPTER (9)