Chapter (10)

Flow in conduits

SUMMARY

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Laminar Flow and Turbulent Flow

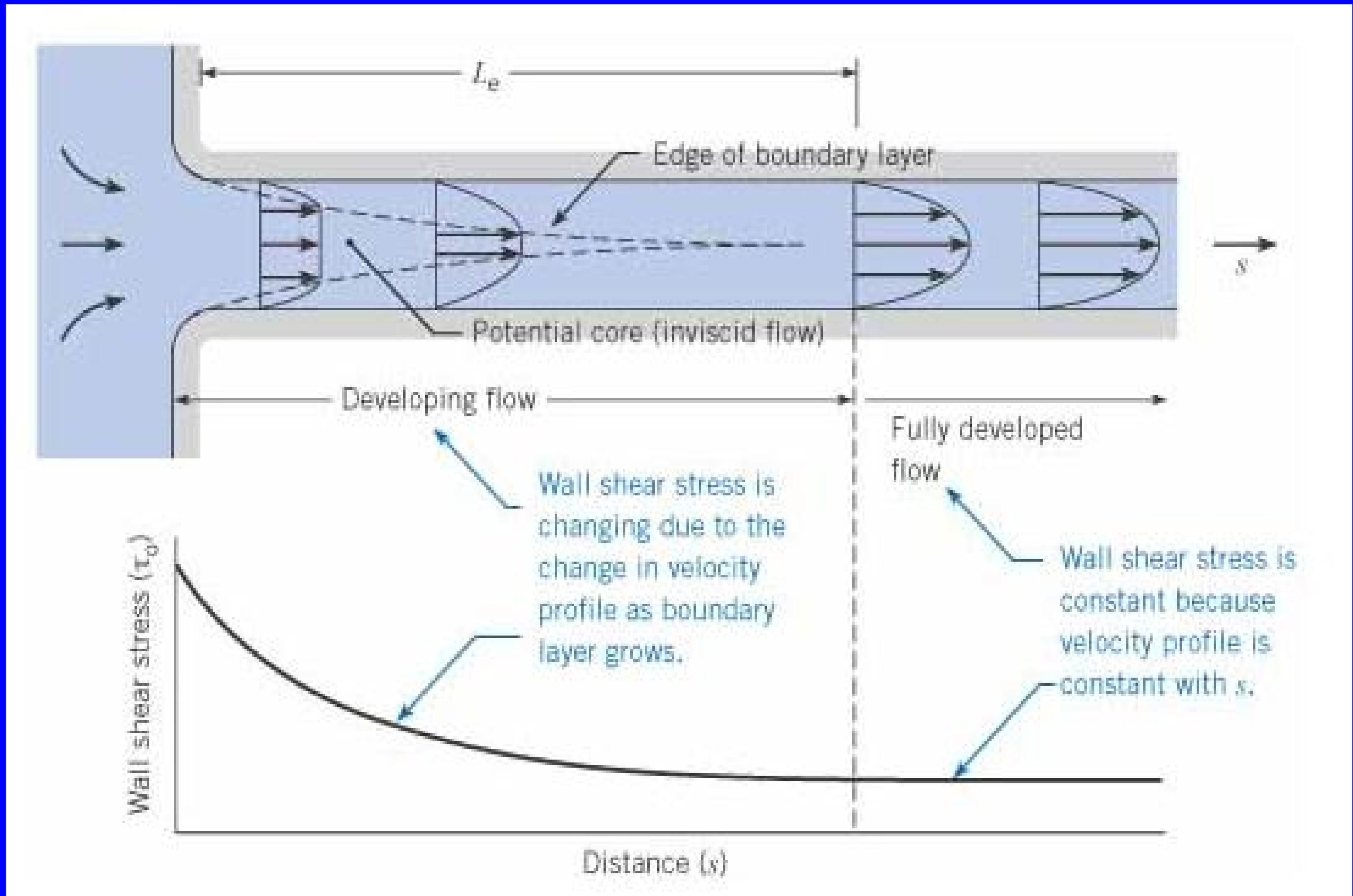
Identification of Flow is based on Reynolds Number

- $Re \leq 2000$ → Laminar
- $Re \geq 3000$ → Turbulent
- $2000 < Re < 3000$ → Unpredictable

There are several equations for calculating Reynolds number in a pipe.

$$Re = \frac{VD}{\nu} = \frac{\rho VD}{\mu} = \frac{4Q}{\pi D \nu} = \frac{4\dot{m}}{\pi D \mu}$$

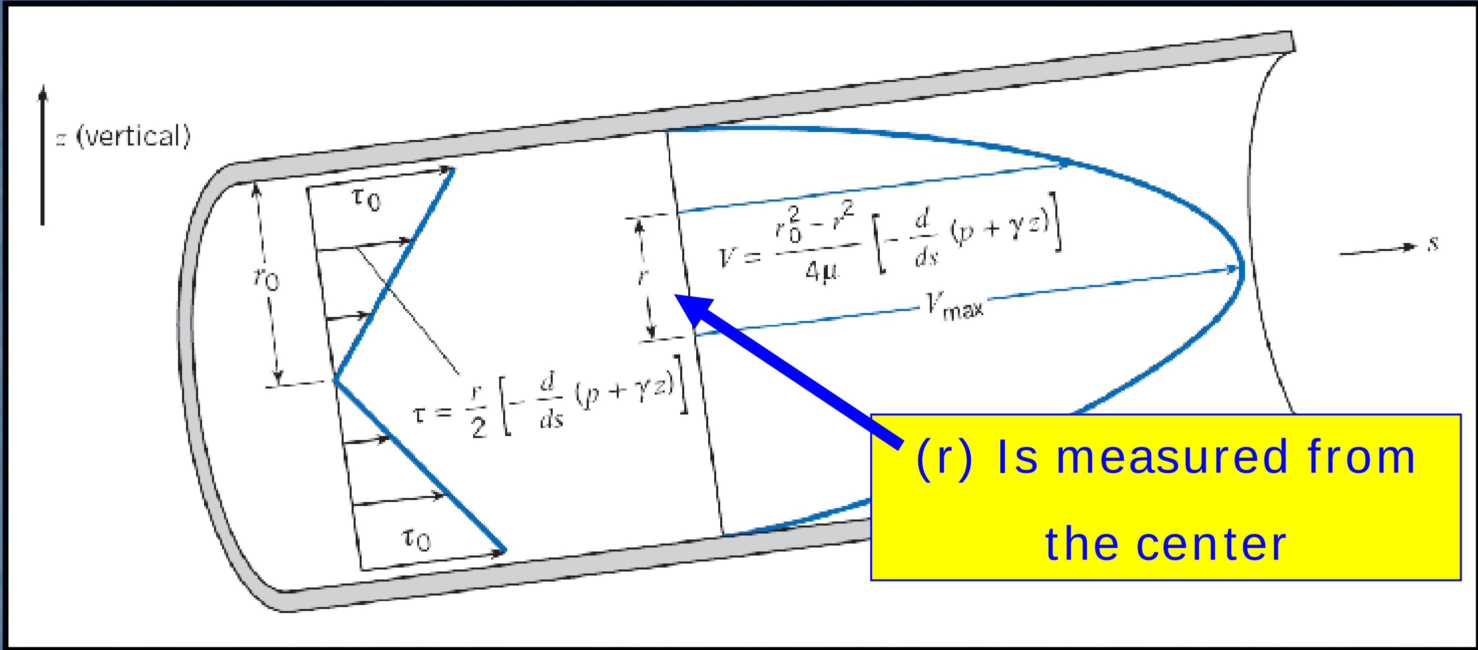
Developing Flow and Fully Developed Flow



Developing flow is the region whereby the velocity profile is changing, hence wall shear stress changes as well. Flow non-uniform.

Developed flow is the region whereby the velocity profile is constant, hence wall shear stress remain constant. Flow uniform.

Shear Stress Distribution in a Pipe Flow



1. Shear distribution is <u>Linear</u>.

2. Velocity distribution is <u>Parabolic</u>.

Laminar Flow in a round Pipe is Known as <u>Hagen-Poiseuille Flow</u>

Velocity Profile In A Laminar Flow

$$V = \frac{r_0^2 - r^2}{4\mu}\left[-\frac{d}{ds}(p + \gamma z)\right] = -\left(\frac{r_0^2 - r^2}{4\mu}\right)\left(\frac{\gamma \Delta h}{\Delta L}\right)$$

Eqn. (A)

The maximum velocity occurs at $r = r_0$

$$V_{max} = -\left(\frac{r_0^2}{4\mu}\right)\left(\frac{\gamma \Delta h}{\Delta L}\right)$$

Eqn. (B)

Combining Eqn. (A) and Eqn. (B), we have:

$$V(r) = -\left(\frac{r_0^2 - r^2}{4\mu}\right)\left(\frac{\gamma \Delta h}{\Delta L}\right) = V_{max}\left(1 - \left(\frac{r}{r_0}\right)^2\right)$$

Volume Flow

$$Q = \frac{\pi r_0^4}{8\mu}\left[-\frac{d}{ds}(p + \gamma z)\right]$$

Head Loss In A Laminar Flow

Head Loss →

$$h_f = \frac{32\mu LV}{\gamma D^2} \qquad (10.17)$$

Here the bar over the V has been omitted to conform to the standard practice of denoting the mean velocity in one-dimensional flow analyses by V without the bar.

Equation above shows that Head Loss in Laminar Flow varies Linearly with Velocity

Head Loss in Laminar Flow is influenced by:

1. Viscosity.
2. Pipe length.
3. Specific weight.
4. Pipe diameter.

Velocity Distribution in a *Smooth Pipe* for Turbulent Flow

The time-average velocity distribution is often described using an equation called the power-law formula.

$$\frac{u(r)}{u_{max}} = \left(\frac{r_0 - r}{r_0}\right)^m$$

2. Turbulent Boundary Layer, Logarithmic Velocity Equations

$$\frac{u}{u_*} = 2.44\, In \frac{u_*(r - r_0)}{n} + 5.56$$

$$100 < \frac{u_*(r - r_0)}{n} < 500$$

Velocity Distribution in a *Rough Pipe* for Turbulent Flow

$$\frac{u}{u_*} = 5.75 \log \frac{y}{k} + B \qquad (10.25)$$

Y = Distance from rough wall.

K = Height of the roughness.

B = Function of type, concentration and size variation of the roughness.

Nikusadse carried out a number of tests on flow in pipes that were roughened with ***uniform sized sand grain***. The results were plotted graphically as shown in the next slide

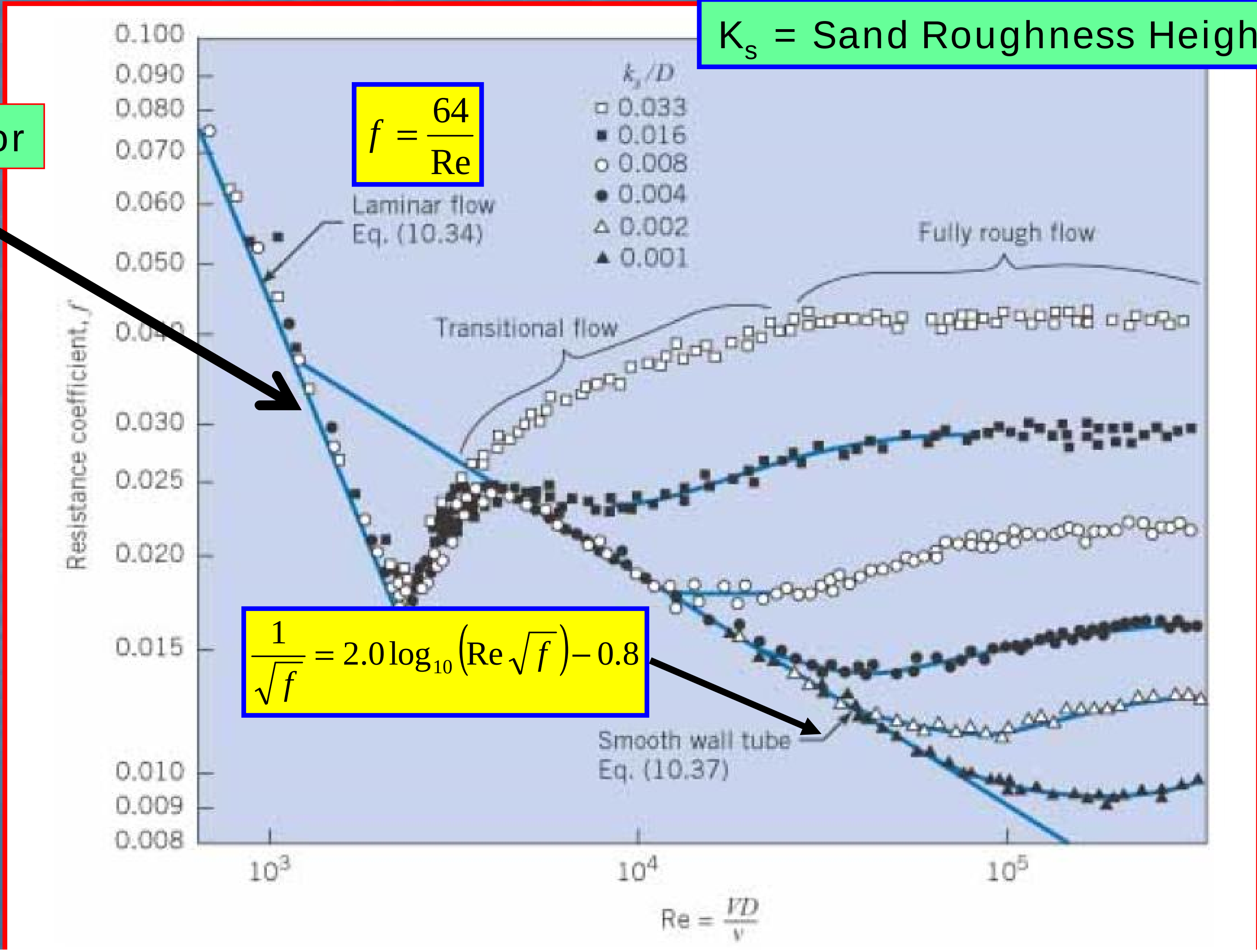
Nikusadse found out from these tests on flow in pipes that (B=8.5)

Friction Factor, (f) for Rough Pipes

Nikuradse' Curve

K_s = Sand Roughness Height

No effect for $\frac{K_S}{D}$



The results from Nikusadse's graph shows the followings:

Table 10.3 EFFECTS OF WALL ROUGHNESS

Type of Flow	Parameter Ranges		Influence of Parameters on f
Laminar Flow	Re < 2000	NA	f depends on Reynolds number f is independent of wall roughness (k_s/D)
Turbulent Flow, Smooth Tube	Re > 3000	$\left(\frac{k_s}{D}\right)\text{Re} < 10$	f depends on Reynolds number f is independent of wall roughness (k_s/D)
Transitional Turbulent Flow	Re > 3000	$10 < \left(\frac{k_s}{D}\right)\text{Re} < 1000$	f depends on Reynolds number f depends on wall roughness (k_s/D)
Fully Rough Turbulent Flow	Re > 3000	$\left(\frac{k_s}{D}\right)\text{Re} > 1000$	f is independent of Reynolds number f depends on wall roughness (k_s/D)

Head Loss in Pipes

Combined (total) Head Loss

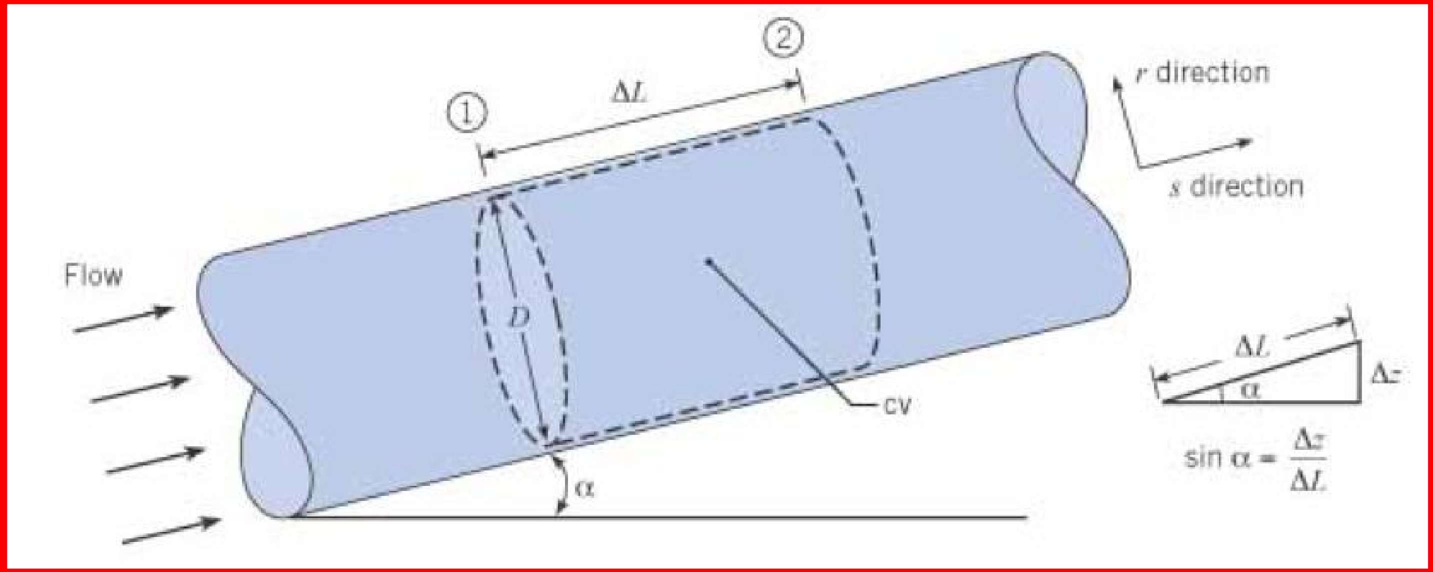
Combined (total) Head Loss

Pipe Head Loss (major) + Component Head Loss minor)

Component head loss: is associated with flow through devices such as devices, bends, and tees and is called *Minor Head Loss*.

Pipe head loss: is associated with fully developed flow in conduits and is *called major head loss*. This loss is predicted with the *Darcy-Weisbach equation*.

Darcy-Weisbach Equation



Assumptions: Fully developed, steady flow in a round pipe.

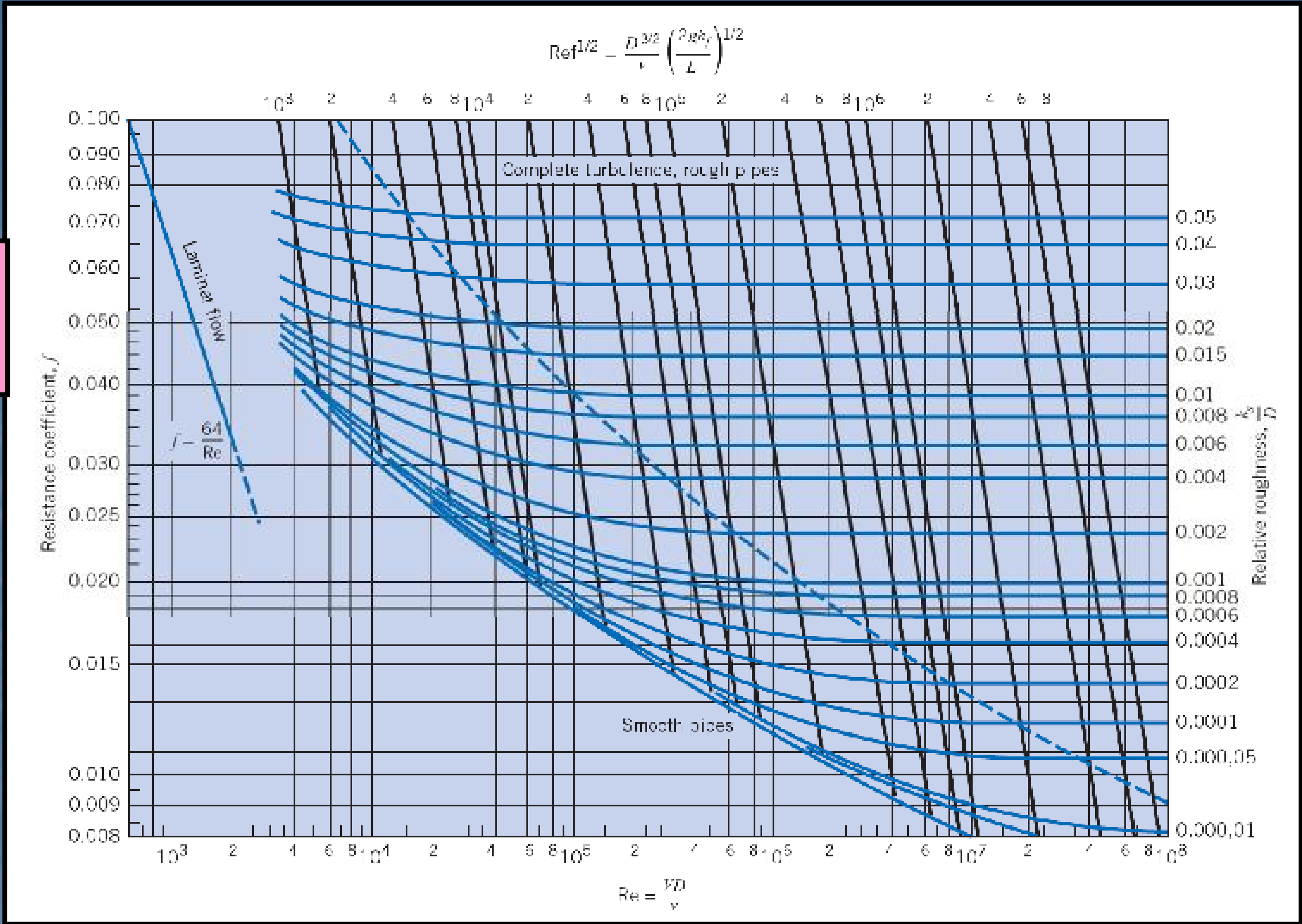
$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

1. The flow should be Fully developed and steady.
2. Applies for Laminar & Turbulent flow.
3. Applies for either round pipes or non round pipes.

Friction factor

$$f = \frac{(4 \cdot \tau_0)}{(\rho V^2 / 2)} \approx \frac{\text{shear stress acting at the wall}}{\text{kinetic pressure}}$$

Moody's Diagram



When (V) *and* $\left(\frac{K_S}{D}\right)$ *are known, Hence* (R_e), $\left(\frac{K_S}{D}\right)$ *plot in Moody's diagram is used.*

When (V) *is not known and only* (h_f) $\left(\frac{K_S}{D}\right)$ *are known, Hence* $\left(R_e\sqrt{f}\right)$, $\left(\frac{K_S}{D}\right)$ *plot in Moody's diagram is used.*

From the previous slide , the problems for uniform flow in a pipe can be summarized as shown in the table below:

What to Calculate	Given Values	Calculated Values in order	Values read from Moody's diagram	Final Value required
Case (A) Head Loss	$K_S, D, L, \dot{Q} \text{ or } \dot{m}$	$V, R_e, \frac{K_S}{D}$	f	h_f
Case (B) Flow Rate	K_S, D, L, h_f	$\frac{K_S}{D}, \frac{D^{3/2}}{n}\left(\frac{2gh_f}{L}\right)^{1/2}$	f, R_e	$V, \dot{m}, \dot{Q}$
Case (C) Pipe Size	$K_{S}, h_f, \dot{Q}, L$	Assume a value for (D) then calculate $\left(\frac{K_S}{D}\right), V, R_e$	Iterative procedure is used to compute f, R_e	D

$$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

Explicit Equations for Head Loss h_f, Discharge (Q) and Diameter (D) as an alternative to Moody's Diagrams

Case (A)

Given $K_S, D, L, Q\ or\ \dot{m}$ $\quad h_f = ?$

Swamee & Jain Formula

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \quad \text{(10.26)}$$

$$\rightarrow \quad h_f = f\,\frac{L}{D}\,\frac{V^2}{2g}$$

Case (B)

Given K_S, D, L, h_f $\quad Q = ?$

$$Q = -2.22D^{5/2}\sqrt{gh_f/L}\,\log\left(\frac{k_s}{3.7D} + \frac{1.78\nu}{D^{3/2}\sqrt{gh_f/L}}\right) \quad (10.27)$$

Case (C)

Given K_{S}, h_f, Q, L $\quad D = ?$

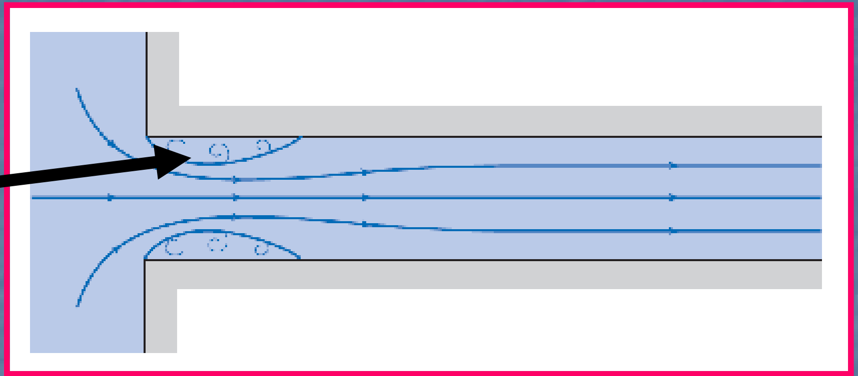
Streeter & Wile

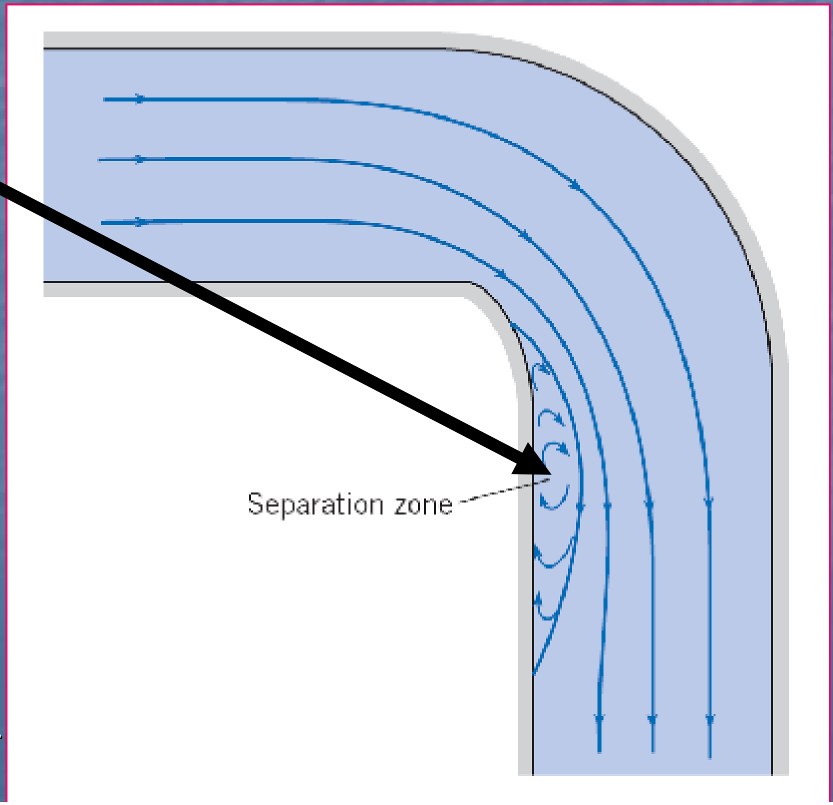
$$D = 0.66\left[k_s^{1.25}\left(\frac{LQ^2}{gh_f}\right)^{4.75} + \nu Q^{9.4}\left(\frac{L}{gh_f}\right)^{5.2}\right]^{0.04} \quad (10.28)$$

Minor Losses from Flow in Pipe Inlets and Fittings

Minor Loss Equation

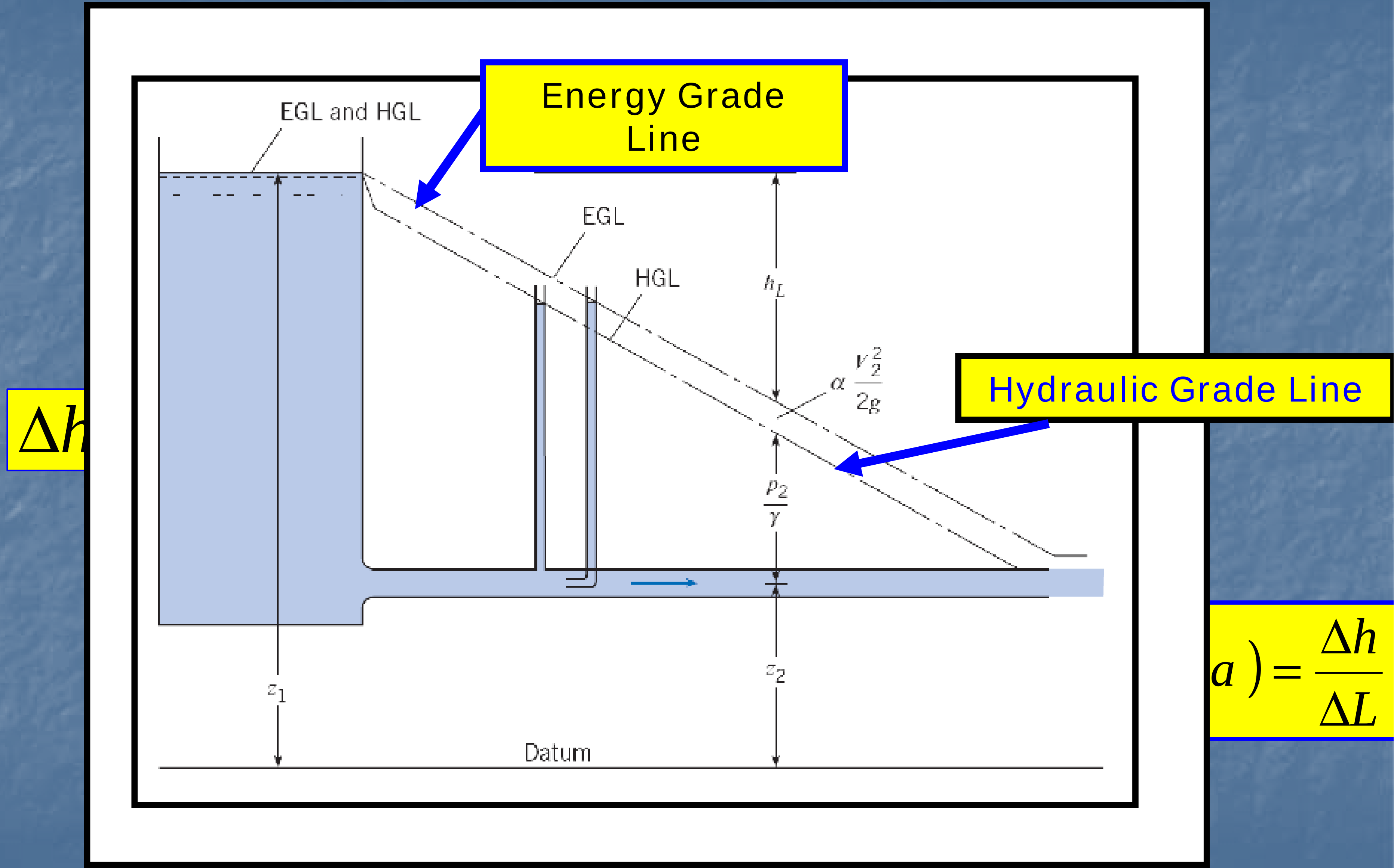
$$h_L = K \frac{V^2}{2g}$$





Separation Zone where streamlines converges and velocity increases, hence pressure decreases , hence Losses Occurs

GRADE LINES


EGL and HGL
Energy Grade Line
EGL
HGL
h_L
$\alpha \frac{V_2^2}{2g}$
Hydraulic Grade Line
$\frac{p_2}{\gamma}$
z_1
z_2
Datum
Δh
$a) = \frac{\Delta h}{\Delta L}$

ENERGY PRINCIPLE

1. The height of the $\left[\frac{p}{g}\right]$ line is called the Hydraulic Grade Line (HGL)

2. The height of the $\left[\frac{p}{g}+a\frac{V^2}{2g}\right]$ line is called the Energy Grade Line (HGL)

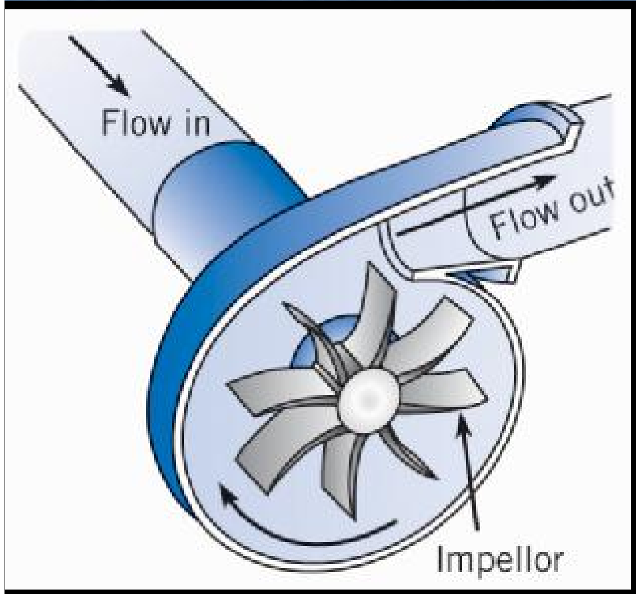
3. EGL is above HGL by a distance $\left[a\frac{V^2}{2g}\right]$

4. For a lake or a reservoir, EGL and HGL will Coincide because the velocity is zero.

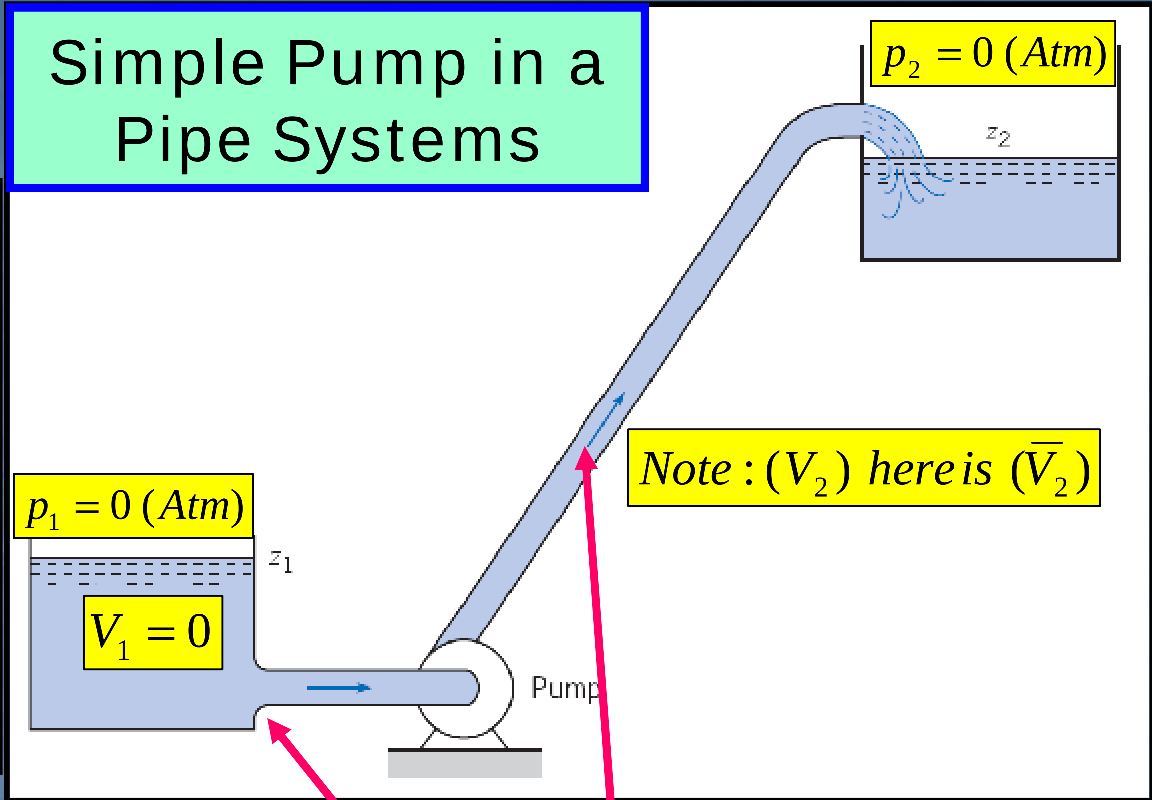
5. Head loss for a flow in a pipe as shown in the Figure always means that the EGL will slope downwards in the direction of flow. (i.e in the direction of decreasing pressure head)

6. For a steady flow in a pipe where the diameter, roughness and shape is the same, the slope $\left[\frac{\Delta h}{\Delta L}\right]$ is constant

Pipe Systems



Simple Pump in a Pipe Systems



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum K_L \frac{V^2}{2g} + \sum \frac{fL}{D}\frac{V^2}{2g}$$

0 0

For a system with one size of pipe, this simplifies to

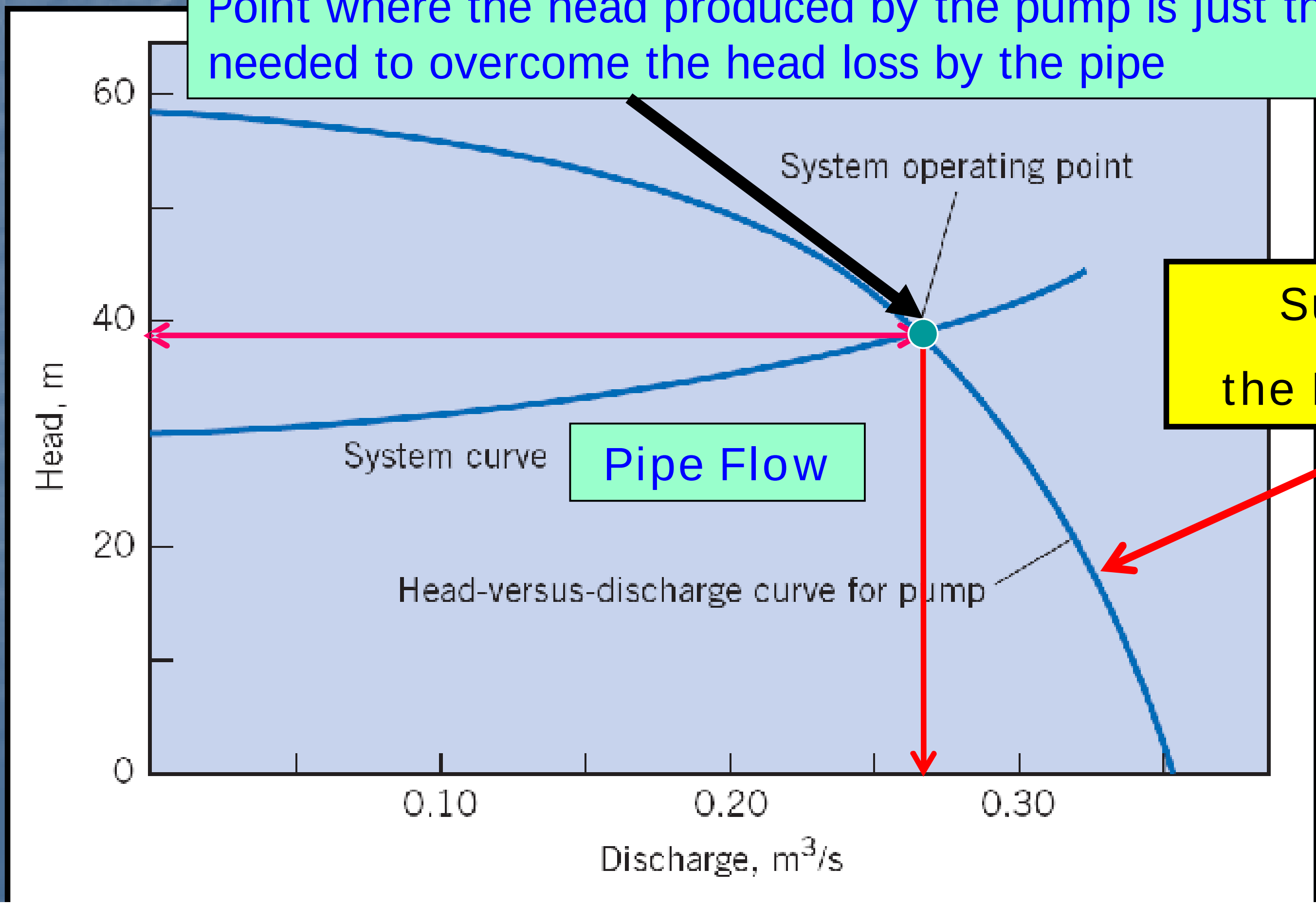
$$h_p = (z_2 - z_1) + \frac{V^2}{2g}\left(1 + \sum K_L + \frac{fL}{D}\right) \qquad (10.30)$$

$$h_p = (z_2 - z_1) + \frac{V^2}{2g}\left(1 + +\sum K_L + \frac{fL}{D}\right) \rightarrow h_p = (z_2 - z_1) + \frac{\dot{Q}^2/A^2}{2g}\left(1 + +\sum K_L + \frac{fL}{D}\right)$$

$h_p = f(\dot{Q})$ Provided other parameter are given

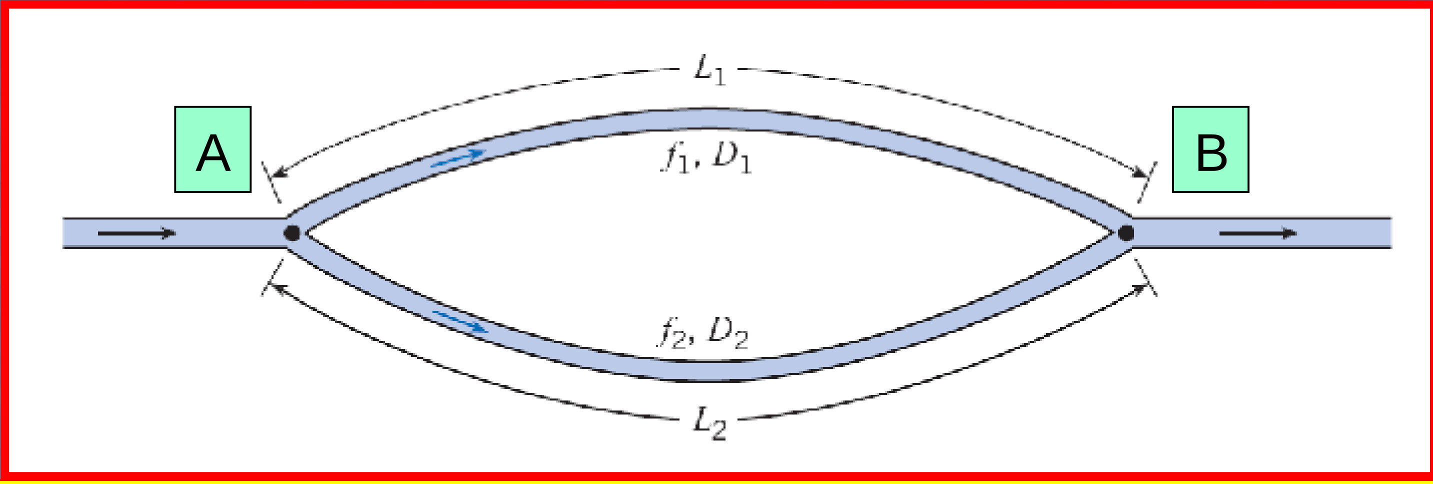
The Eqn. Above shows that for any Given Discharge, a certain Head must be supplied to maintain that flow.

Point where the head produced by the pump is just the amount needed to overcome the head loss by the pipe



Supplied by the Manufacturer

Pipe in Parallel



$$Head\ Loss = h_L = \left(\frac{p_1}{g} + z_1\right) - \left(\frac{p_2}{g} + z_2\right) \qquad Eqn.(10.16),\ page(373)$$

$$h_{L_1} = h_{L_2}$$

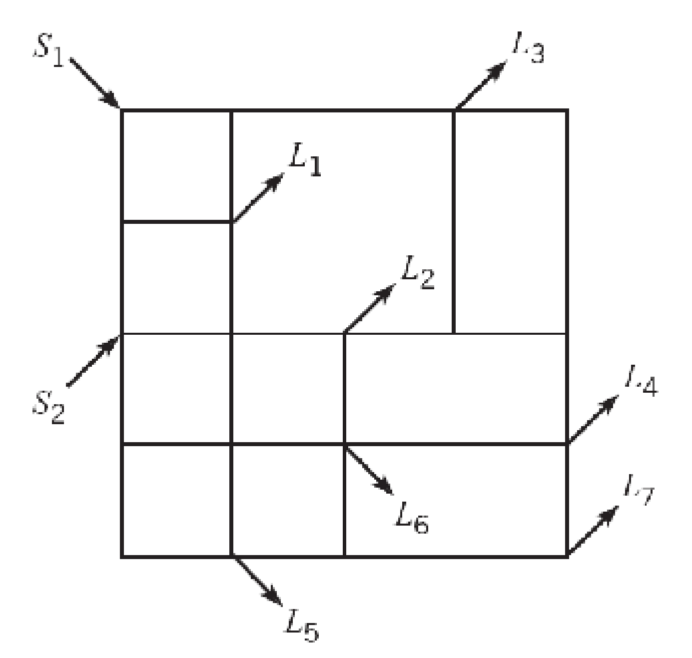
$$f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g}$$

Then $\left(\frac{V_1}{V_2}\right)^2 = \frac{f_2 L_2 D_1}{f_1 L_1 D_2}$ or $\frac{V_1}{V_2} = \left(\frac{f_2 L_2 D_1}{f_1 L_1 D_2}\right)^{1/2}$

Pipe Networks

S = Sources

L = Loads



Water Distribution System

The requirements of the pipe networks design are:

1. Layout of the pipes.
1. Pipe sizes.
2. Future loads.

The objective of the Pipe Network Design is to arrive at a network of pipes that will deliver the *Design Flow* at the *Design Pressure* for Minimum Cost

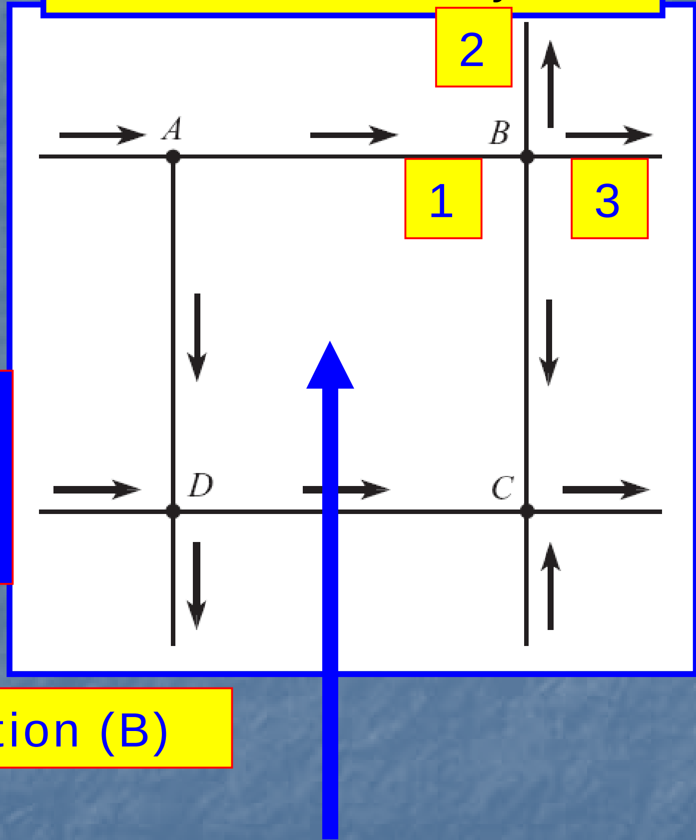
Cost will include (materials and construction, maintenance and operating cost)

Pipe Networks

<u>For the design of the Pipe Networks, two conditions must be satisfied:</u>

1. Continuity must be satisfied.
2. Head loss between any two junctions must be the same regardless of the path.

Water Distribution System



The algebraic sum of the head losses around a given loop must be equal to zero

Condition (1) $\dot{Q}_1 = \dot{Q}_2 + \dot{Q}_3$ Junction (B)

Condition (2) $h_{AB} + h_{BC} = h_{AD} + h_{DC}$

END OF SUMMARY