CHAPTER (12)

COMPRESSIBLE FLOW

SUMMARY

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MECH. DEPT.

Application of Compressible Flows

a) High speed flight through air. (Concorde plane)

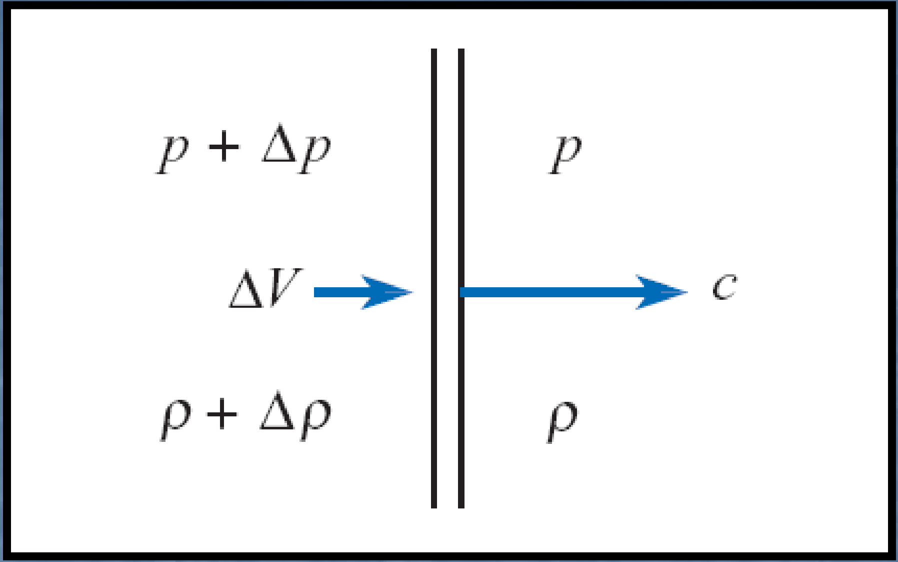
b) Flow of air through a Compressor.

c) Flow of steam through a Turbine.

d) Natural gas piped from producer to consumer.

e) Compressed gas tanks.

Speed of Sound (Sonic Velocity)



Section of a Pressure Wave Propagate at a Velocity (c)

Definition of Speed of Sound

1. **The process of compression of gases layers at a finite velocity is called sonic velocity.**

2. **The process of compression is like a wave traveling through the medium is called sonic velocity.**

<u>Sound Waves by definition can be regarded as:</u>

1) **Reversible.**
2) **Adiabatic.**

<u>Reversible and Adiabatic</u> means *Isentropic Process*, therefore

$$c^2 = \left.\frac{\partial p}{\partial \rho}\right|_s \qquad (12.8)$$

To reiterate, the speed of sound is the speed at which an infinitesimal pressure disturbance travels through a fluid. Waves of finite strength (finite pressure change across the wave) travel faster than sound waves. Sound speed is the *minimum* speed at which a pressure wave can propagate through a fluid.

$$c = \sqrt{kRT}$$

Mach Number

1) $M < 1$ Subsonic Flow: $V < c$

2) $M = 1$ Transonic Flow or Sonic flow: $V = c$

3) $M > 1$ Supersonic Flow: $V > c$

Mach Number

$$M = \frac{V}{c}$$

Mach Number Relationships Summary

(1) Temperature

$$T_t = T\left(1 + \frac{k-1}{2}M^2\right)$$

(2) Pressure

The *total pressure* in a compressible flow is defined as

$$p_t = p\left(1 + \frac{k-1}{2}\mathrm{M}^2\right)^{k/(k-1)} \qquad (12.27)$$

(3) Density

$$\rho_t = \rho\left(1 + \frac{k-1}{2}\mathrm{M}^2\right)^{1/(k-1)}$$

Tables can be used to find the above properties

Adiabatic Flow

Total Pressure at upstream and downstream **are not the same** across a shock wave in an **Adiabatic** flow.

Total Density at upstream and downstream **are not the same** across a shock wave in an **Adiabatic** flow.

Total Temperature at upstream and downstream are the same across a shock wave as the process is **Adiabatic**.

Isentropic Flow

Total Pressure, Total Density and Total Temperature at upstream and downstream **are the same** across a shock wave in an **Isentropic** flow.

Kinetic Pressure

By definition

The kinetic pressure, $q = \rho V^2/2$

$$q = \left(\frac{1}{2}\frac{PV^2}{RT}\right) = \left(\frac{1}{2}kP\frac{V^2}{kRT}\right) = \frac{k}{2}PM^2$$

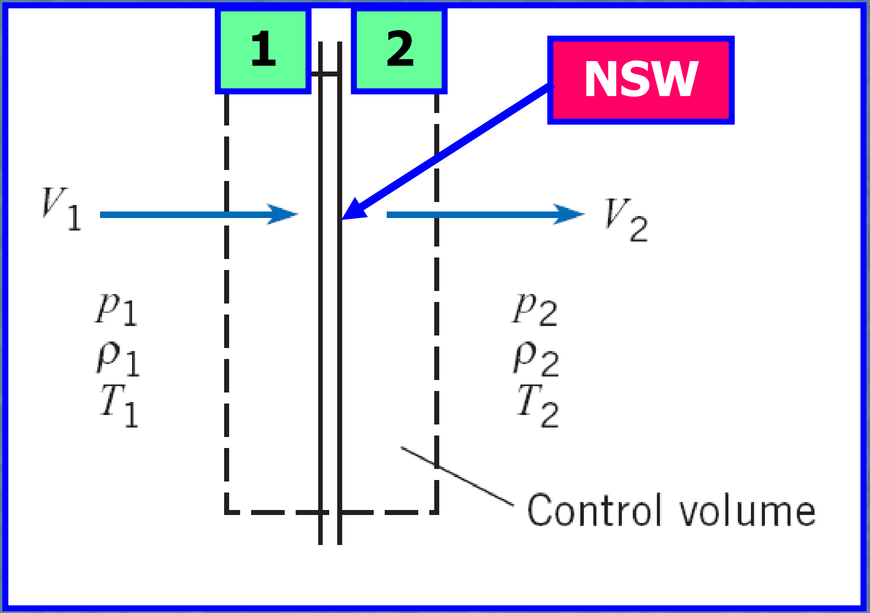
Validation of Bernoulli's Equation for Compressible Flow

Thus applying Bernoulli equation would have led one to say that the flow was **Supersonic**, whereas the flow was actually **Subsonic.**

In the limit of low velocities $p_t/p \rightarrow 1$ the Bernoulli equation is valid for very low (M<<1) Mach numbers

Normal ShockWaves (NSW)

(NSW) are wave fronts normal to the flow across which a supersonic flow **($M>1$)** is decelerated to subsonic flow **($M<1$)** with an increase in static temp., pressure and density.



Control volume enclosing a normal shock wave in Inviscid, Adiabatic, Steady Flow

$$\frac{M_1}{1+kM_1^2}\left(1+\frac{k-1}{2}M_1^2\right)^{1/2}=\frac{M_2}{1+kM_2^2}\left(1+\frac{k-1}{2}M_2^2\right)^{1/2}$$

Then, solving this equation for M_2 as a function of M_1, we obtain two solutions. One solution is trivial, $M_1 = M_2$, which corresponds to no shock wave in the control volume. The other solution gives the Mach number downstream of the shock wave:

$$M_2^2=\frac{(k-1)M_1^2+2}{2kM_1^2-(k-1)} \qquad (12.41)$$

For $M_1 = M_2 = 1$ means we have a **Sound Wave** and in this case, no change in pressure and temperature as the change by definition is **Infinitesimal.**

Values of Mach Numbers can also be found from tables

Total pressure at upstream and downstream

Total Pressure at upstream and downstream **are not the same** across a shock wave in an **Adiabatic** flow.

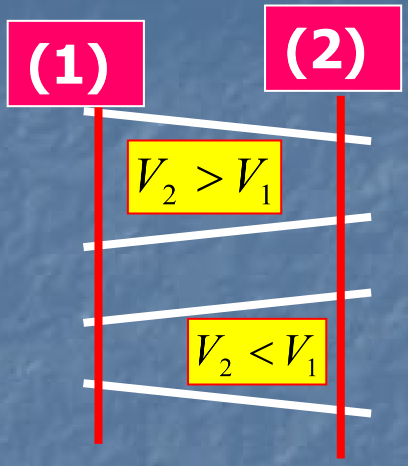
Total Density at upstream and downstream **are not the same** across a shock wave in an **Adiabatic** flow.

Total Temperature at upstream and downstream are the same across a shock wave as the process is **Adiabatic**.

(1) Subsonic Flow (M<1)

$$\frac{1}{V}\frac{dV}{dx} = \frac{(1/A)(dA/dx)}{M^2-1}$$

$M^2 - 1$ *is negative*

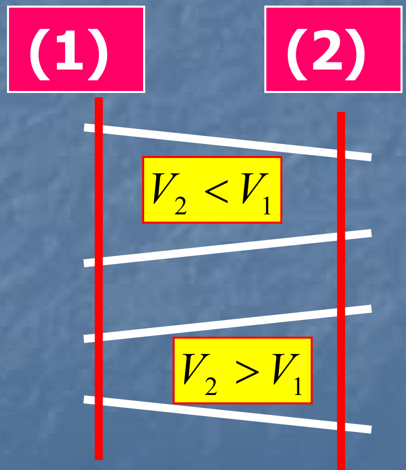


When $\frac{dA}{dx}$ *is decreasing, i.e* $\frac{dA}{dx}$ *is negative, the result* $\frac{dV}{dx}$ *is increasing.*

When $\frac{dA}{dx}$ *is increasing, i.e* $\frac{dA}{dx}$ *is positive, the result* $\frac{dV}{dx}$ *is decreasing.*

(2) Supersonic Flow (M>1)

$M^2 - 1$ *is positive*

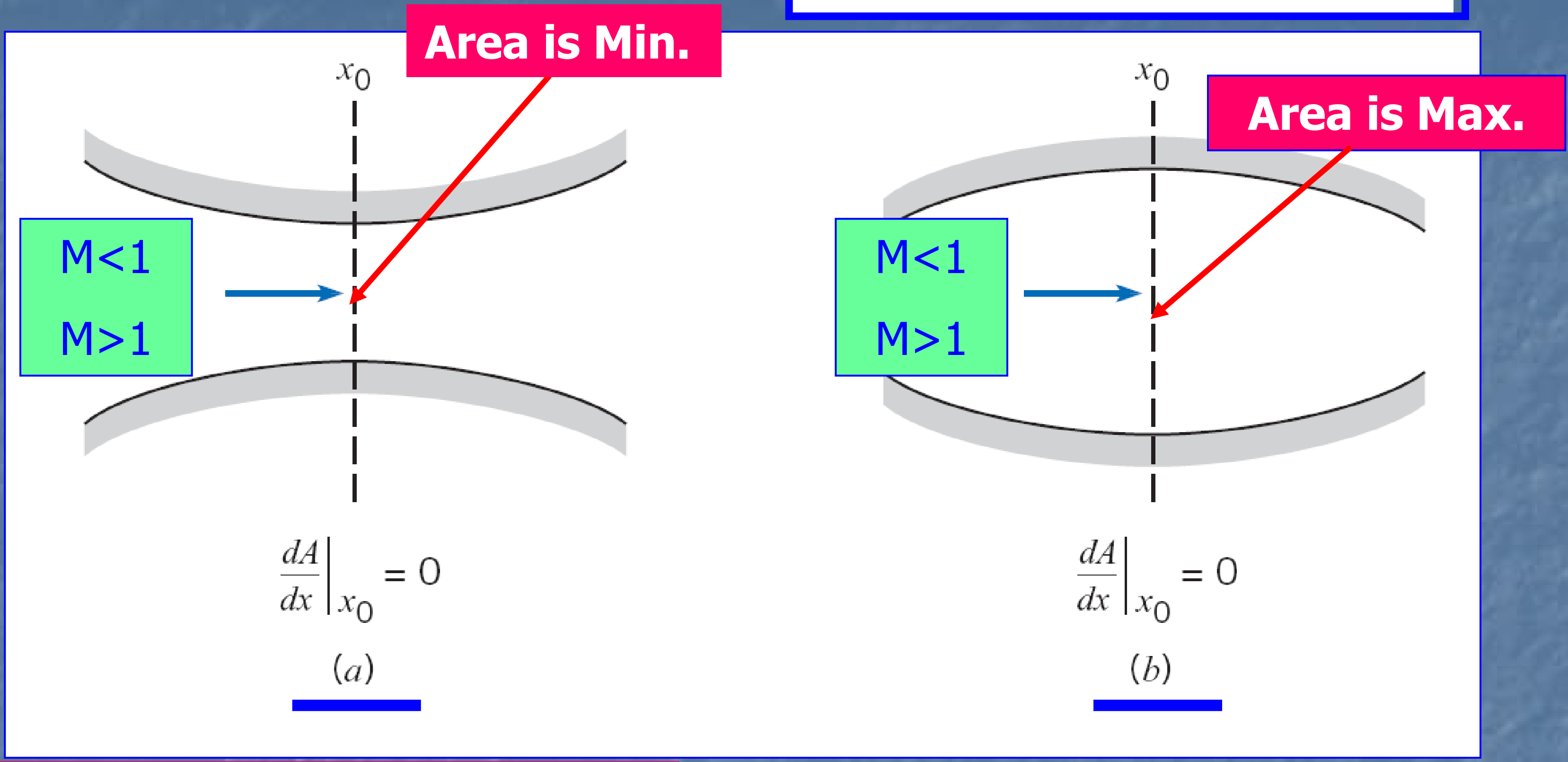


When $\frac{dA}{dx}$ *is decreasing, i.e* $\frac{dA}{dx}$ *is negative, the result* $\frac{dV}{dx}$ *is decreasing.*

When $\frac{dA}{dx}$ *is increasing, i.e* $\frac{dA}{dx}$ *is positive, the result* $\frac{dV}{dx}$ *is increasing.*

(3) Transonic Flow $M \approx 1$

$$\frac{1}{V}\frac{dV}{dx} = \frac{(1/A)(dA/dx)}{M^2 - 1}$$



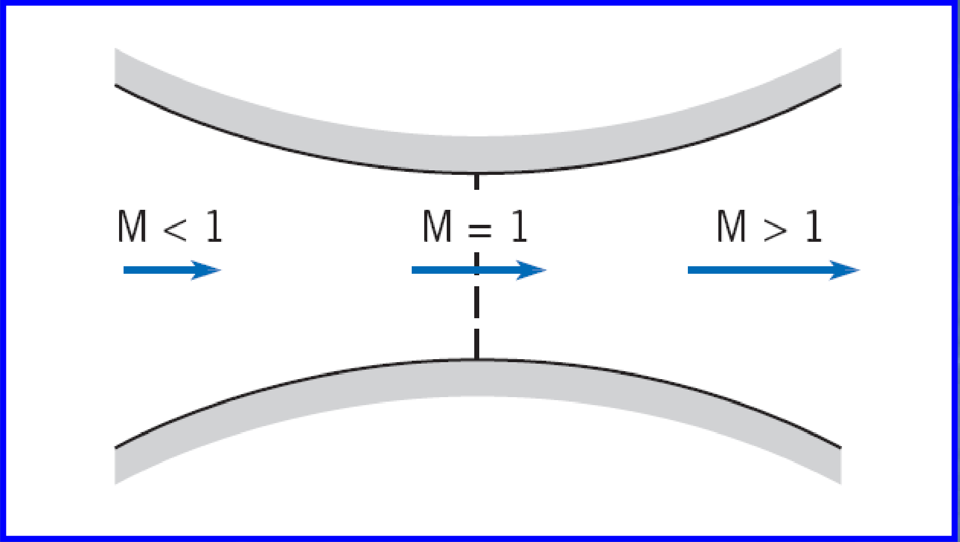
(1) Subsonic Flow (M<1) — Velocity at the throat is maximum

(2) Supersonic Flow (M>1) — Velocity at the throat is minimum

At (a): Sonic conditions can be attained for both M<1 & M>1

At (b): Sonic conditions can not be attained for both M<1 & M>1

Laval Nozzle (Swedish Engineer)



The Laval nozzle is a duct of varying area that produces Supersonic Flow

Flow in Laval nozzle is assumed to be Isentropic, hence Total Temperature, Total Pressure and Total Density are constant throughout the nozzle.

Area Ratio in Laval Nozzle

$$\frac{A}{A_*} = \frac{1}{M}\left\{\frac{1+[(k-1)/2]M^2}{(k+1)/2}\right\}^{(k+1)/2(k-1)}$$

Area Ratio in Laval Nozzle can be found from tables for both subsonic and supersonic flows.

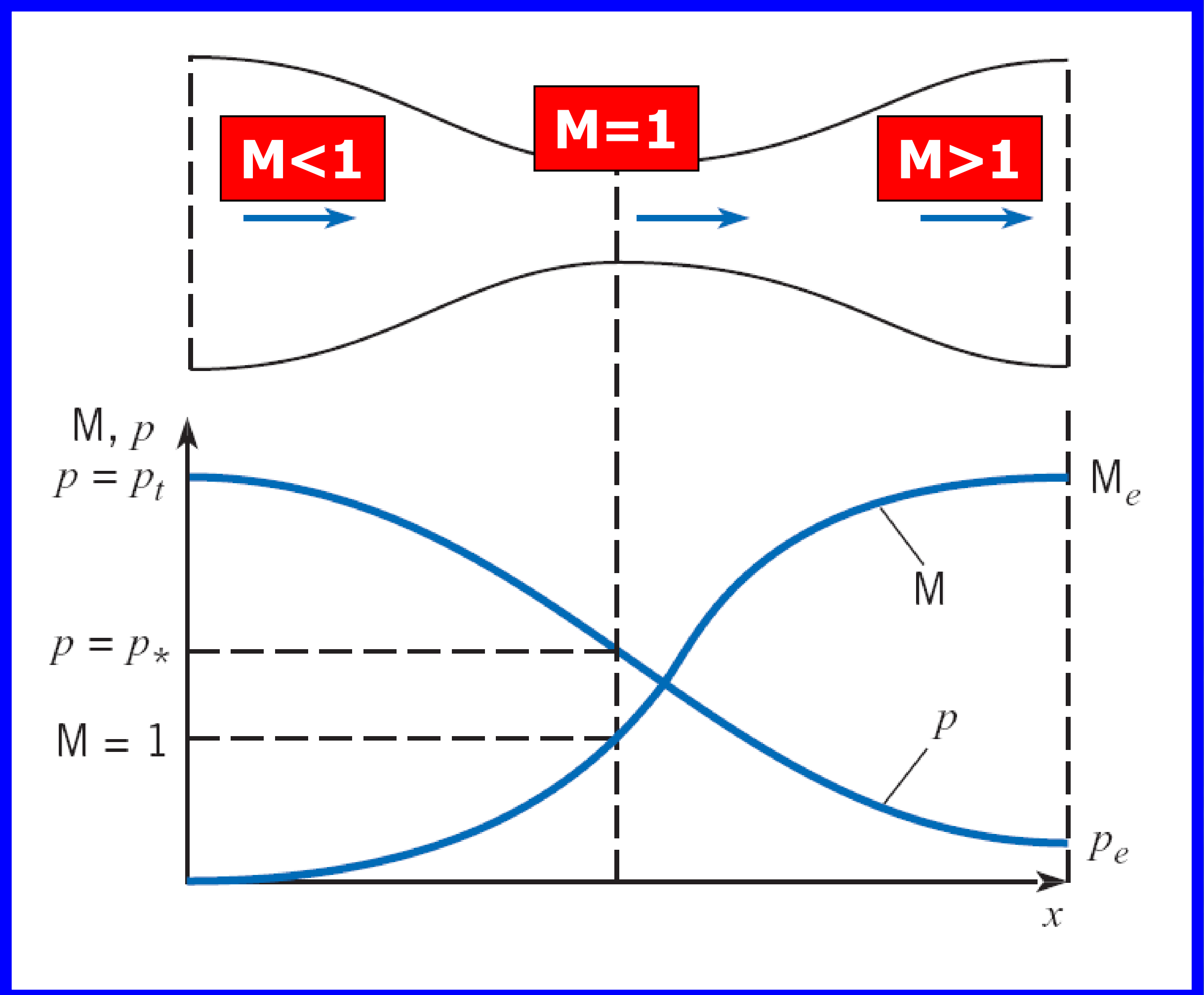
Mass Flow Rate Through Laval Nozzle

$$\dot{m} = 0.685\frac{p_t A_*}{\sqrt{RT_t}} \qquad (12.59)$$

and for gases with $k = 1.67$,

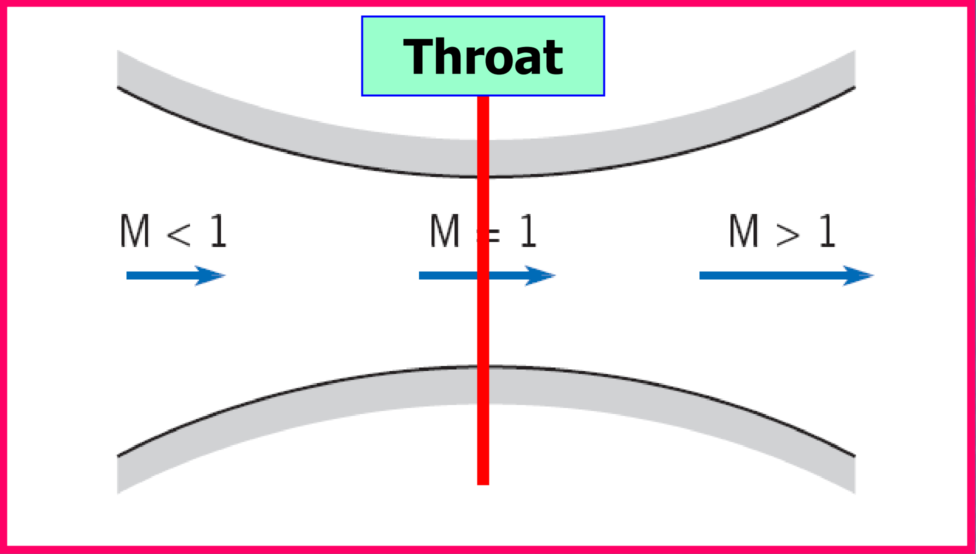
$$\dot{m} = 0.727\frac{p_t A_*}{\sqrt{RT_t}} \qquad (12.60)$$

Classification of Nozzle Flow by Exit Condition



Distribution of Static Pressure and Mach Number in a Laval nozzle

Critical pressure ratio at the throat



The Static Pressure to Total Pressure at the Throat is given as

$$p_t = p\left(1 + \frac{k-1}{2}\mathrm{M}^2\right)^{k/(k-1)}$$

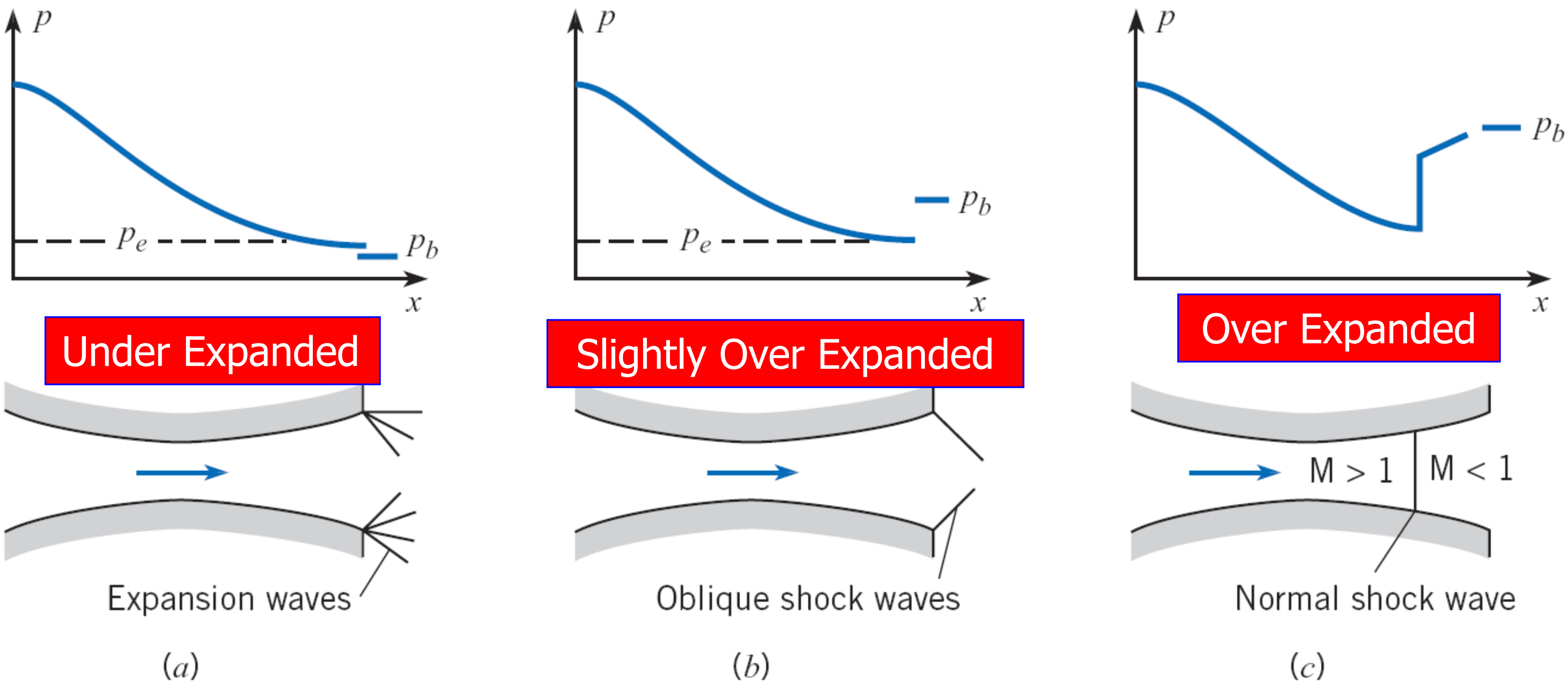
M=1

$$\frac{p_*}{p_t} = \left(\frac{2}{k+1}\right)^{k/(k-1)}$$

which for air with $k = 1.4$ is

Critical Pressure ratio= $\frac{p_*}{p_t} = 0.528$

Classification of Nozzle Flow by Exit Conditions

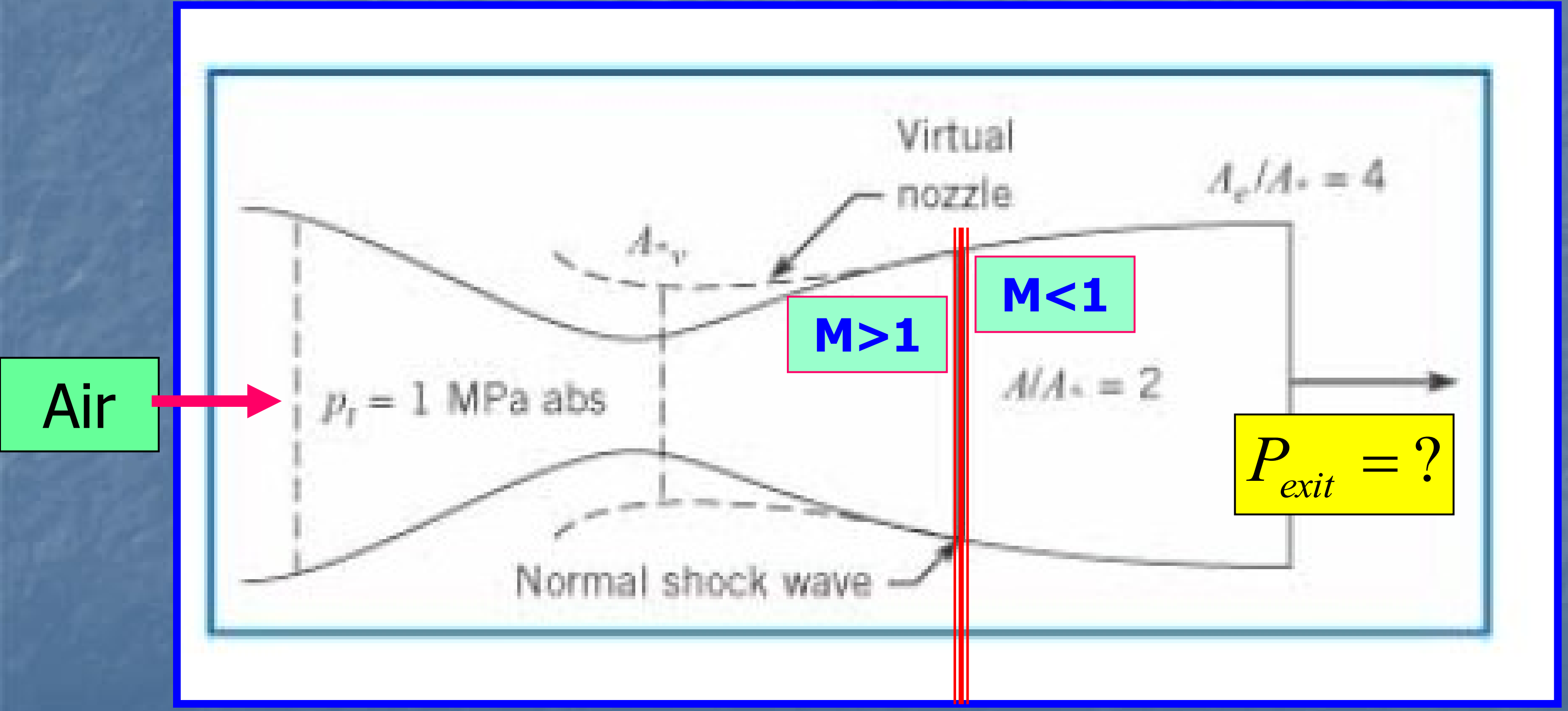


Back Pressure is the pressure outside the Nozzle

Back Pressure = Exit Pressure (Ideally Expanded)

Example (12.11)

The Laval nozzle shown in the figure has an expansion ratio of 4 (exit area/throat area). Air flows through the nozzle, and a normal shock wave occurs where the area ratio is 2. The total pressure upstream of the shock is 1 MPa. Determine the static pressure at the exit.



Truncated Nozzle

A Truncated Nozzle is a **Laval Nozzle** **cut off** at the throat as shown in the figure.

It is important to engineers because it is used as a **Flow metering device** for **Compressible flow**.

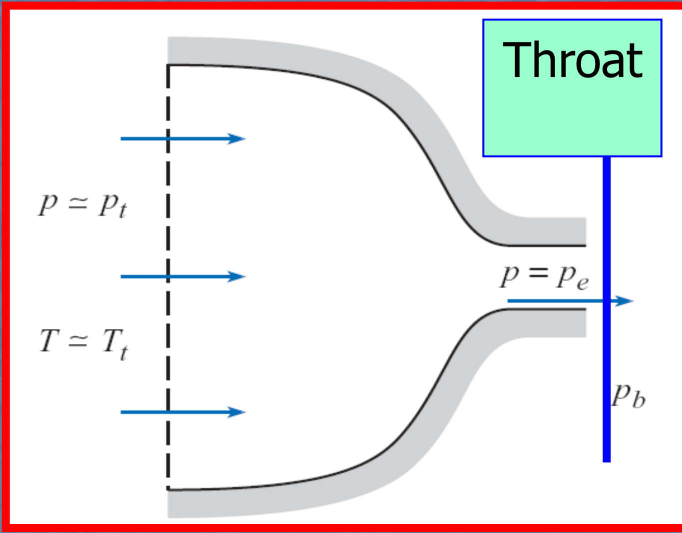
Mass Flow through a Truncated Nozzle

For gases with a ratio of specific heats of 1.4,

$$\dot{m} = 0.685 \frac{p_t A_*}{\sqrt{RT_t}}$$

For gases with $k = 1.67$,

$$\dot{m} = 0.727 \frac{p_t A_*}{\sqrt{RT_t}}$$



1. If $p_b/p_t < p_*/p_t$, the exit pressure is higher than the back pressure, so the exit flow must be sonic. Pressure equilibration is achieved after exit by a series of expansion waves. The mass flow is calculated using Eq. (12.58), where A_* is the area at the truncated station.

2. If $p_b/p_t > p_*/p_t$, the flow exits subsonically. If we were to irrationally assume that the flow exited at the speed of sound, then the exit pressure p_* would be less than p_b. There can be no shock waves in a sonic flow (only sound waves) to raise the exit pressure to the back pressure. Therefore, the flow adjusts itself to the back pressure by exiting subsonically.

Case (1):

$$If\ \frac{p_b}{p_t} < \frac{p_*}{p_t}\ means\ that\ p_b < p_* \rightarrow M_b > M_* = 1 \rightarrow leads\ that\ sonic\ conditions\ at\ exit$$

Case (2):

$$If\ \frac{p_b}{p_t} > \frac{p_*}{p_t}\ means\ that\ p_b > p_* \rightarrow M_b < M_* = 1 \rightarrow leads\ that\ Subsonic\ conditions\ at\ exit$$

END OF SUMMARY