Thermodynamics: An Engineering Approach, 6th Edition
Yunus A. Cengel, Michael A. Boles
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Chapter 5

MASS AND ENERGY ANALYSIS OF CONTROL VOLUMES

SUMMARY

CONSERVATION OF MASS

Conservation of mass: Mass, like energy, is a conserved property, and it cannot be created or destroyed during a process.

Closed systems: The mass of the system remain constant during a process.

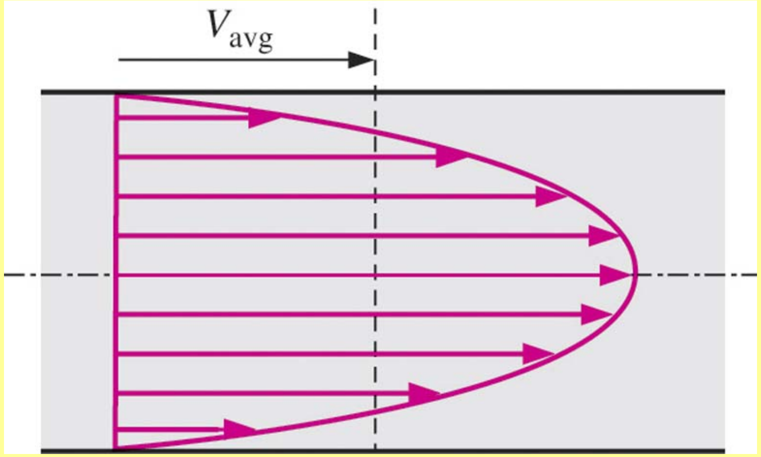
Control volumes: Mass can cross the boundaries, and so we must keep track of the amount of mass entering and leaving the control volume.

Mass and Volume Flow Rates

$$\delta \dot{m} = \rho V_n \, dA_c \qquad \dot{m} = \int_{A_c} \delta \dot{m} = \int_{A_c} \rho V_n \, dA_c \qquad \dot{m} = \rho V_{avg} A_c \quad \text{(kg/s)}$$

$$\dot{m} = \rho \dot{V} = \frac{\dot{V}}{v}$$

Definition of average velocity



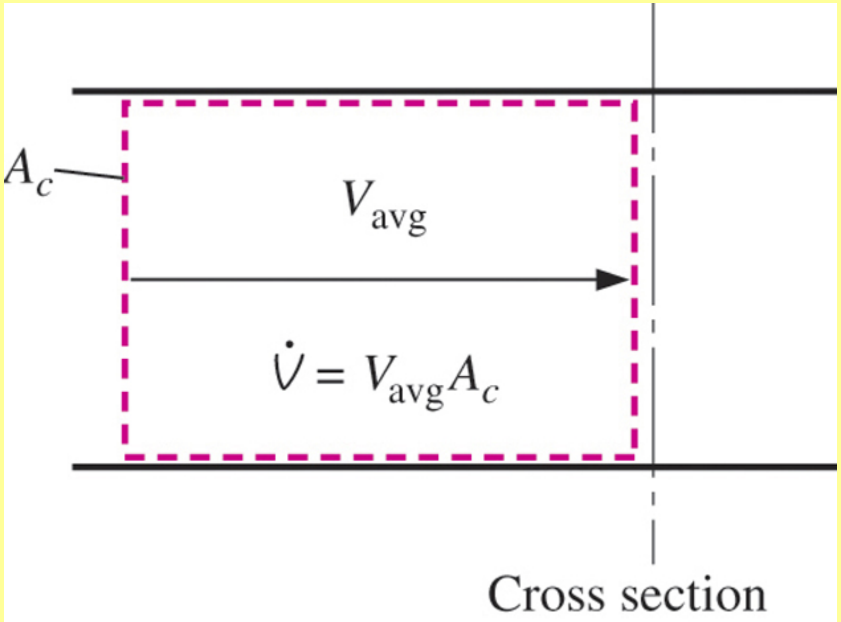
The average velocity V_{avg} is defined as the average speed through a cross section.

Mass and Volume Flow Rates

Volume flow rate

$$\dot{V} = \int_{A_c} V_n \, dA_c = V_{avg} A_c = V A_c \qquad (m^3/s)$$

The volume flow rate is the volume of fluid flowing through a cross section per unit time.



Conservation of Mass Principle

The conservation of mass principle for a control volume: The net mass transfer to or from a control volume during a time interval Δt is equal to the net change (increase or decrease) in the total mass within the control volume during Δt.

$$\begin{pmatrix}\text{Total mass entering} \\ \text{the CV during } \Delta t\end{pmatrix} - \begin{pmatrix}\text{Total mass leaving} \\ \text{the CV during } \Delta t\end{pmatrix} = \begin{pmatrix}\text{Net change in mass} \\ \text{within the CV during } \Delta t\end{pmatrix}$$

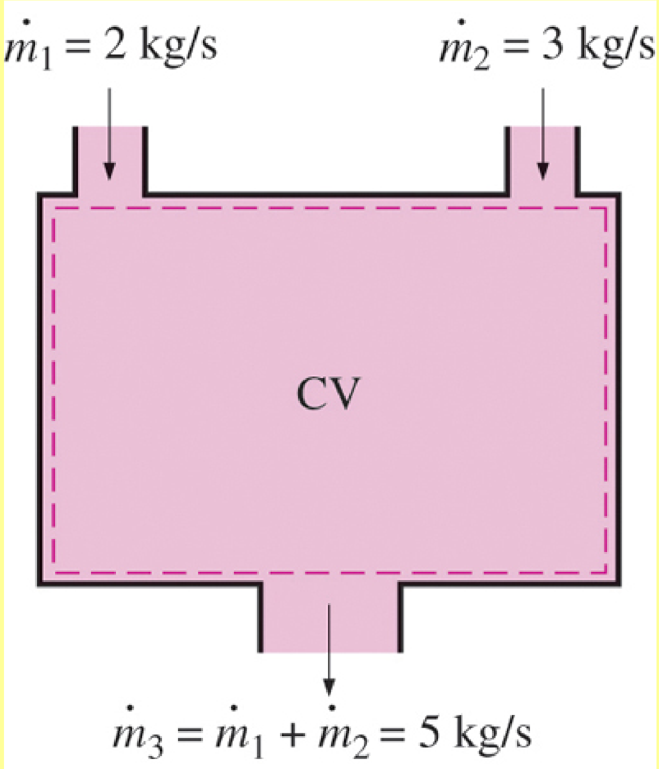
$$m_{\text{in}} - m_{\text{out}} = \Delta m_{\text{CV}} \qquad (\text{kg})$$

$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = dm_{\text{CV}}/dt \qquad (\text{kg/s})$$

Mass Balance for Steady-Flow Processes

During a steady-flow process, the total amount of mass contained within a control volume does not change with time (m_{CV} = constant).

Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it.



Conservation of mass principle for a two-inlet–one-outlet steady-flow system.

For steady-flow processes, we are interested in the amount of mass flowing per unit time, that is, *the mass flow rate.*

$$\sum_{in} \dot{m} = \sum_{out} \dot{m} \qquad \text{(kg/s)}$$

Multiple inlets and exits

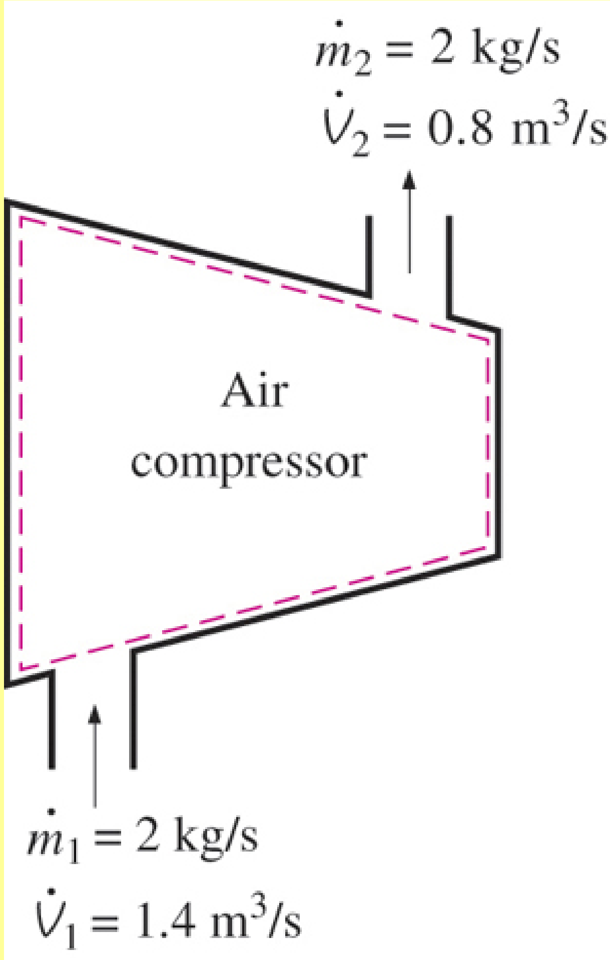
$$\dot{m}_1 = \dot{m}_2 \quad \rightarrow \quad \rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

Single stream

Many engineering devices such as nozzles, diffusers, turbines, compressors, and pumps involve a single stream (only one inlet and one outlet).

Special Case: Incompressible Flow

The conservation of mass relations can be simplified even further when the fluid is incompressible, which is usually the case for liquids.



$$\sum_{in} \dot{V} = \sum_{out} \dot{V} \qquad (m^3/s)$$

Steady, incompressible

$$\dot{V}_1 = \dot{V}_2 \rightarrow V_1 A_1 = V_2 A_2$$

Steady, incompressible flow (single stream)

There is no such thing as a "conservation of volume" principle.

However, for steady flow of liquids, the volume flow rates, as well as the mass flow rates, remain constant since liquids are essentially incompressible substances.

During a steady-flow process, volume flow rates are not necessarily conserved although mass flow rates are.

FLOW WORK AND THE ENERGY OF A FLOWING FLUID

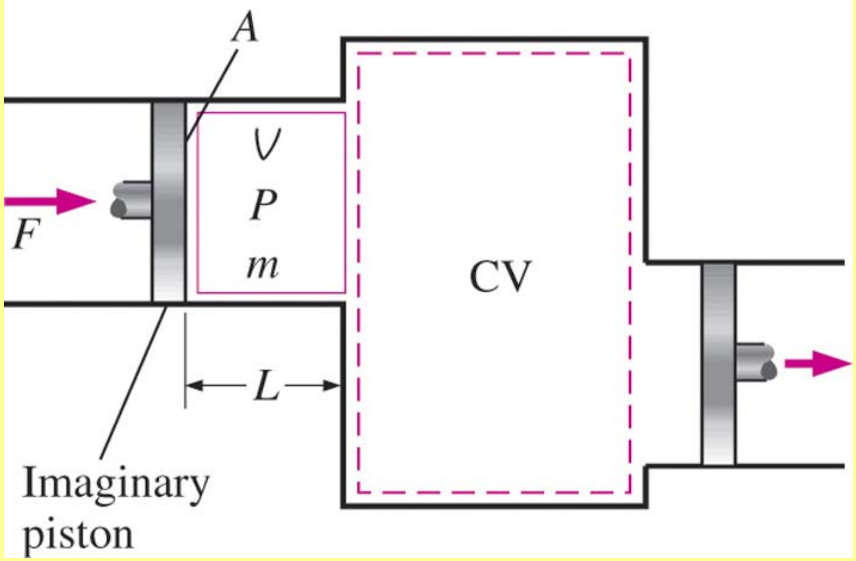
Flow work, or flow energy:

The work (or energy) required to push the mass into or out of the control volume. This work is necessary for maintaining a continuous flow through a control volume.

$$F = PA$$

$$W_{\text{flow}} = FL = PAL = PV \qquad (\text{kJ})$$

$$w_{\text{flow}} = Pv \qquad (\text{kJ/kg})$$



Schematic for flow work

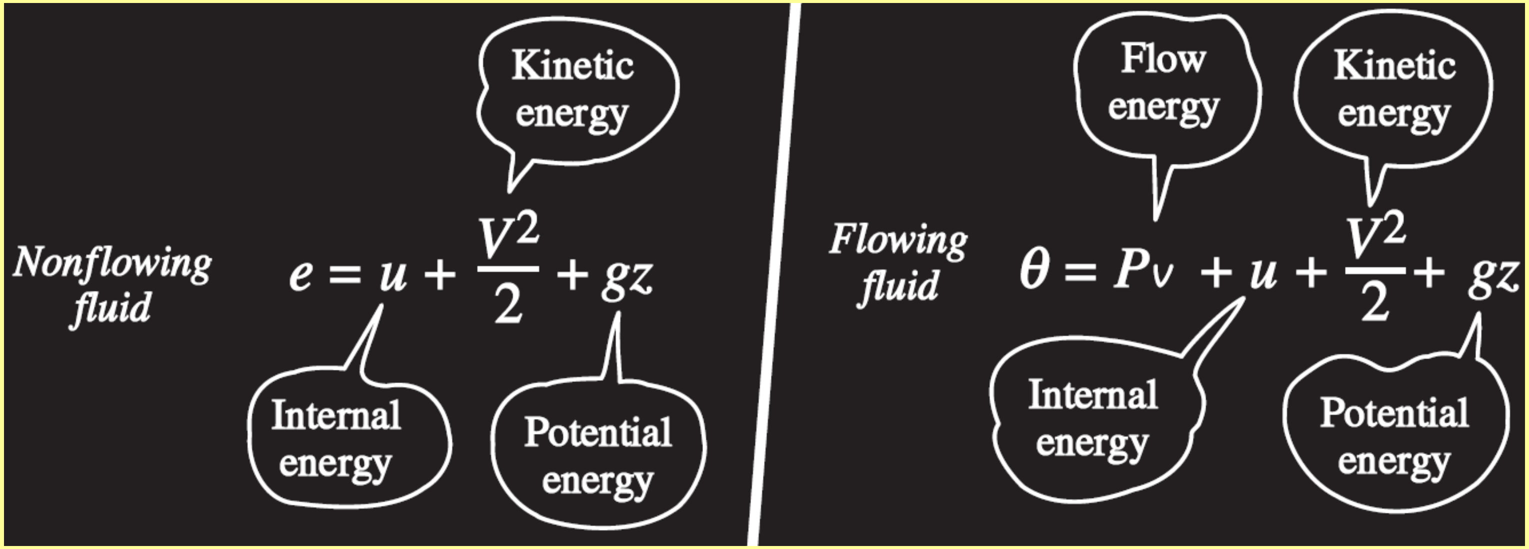
Total Energy of a Flowing Fluid

$$e = u + \text{ke} + \text{pe} = u + \frac{V^2}{2} + gz \qquad (\text{kJ/kg})$$

$$\theta = Pv + e = Pv + (u + \text{ke} + \text{pe}) \qquad h = u + Pv$$

$$\theta = h + \text{ke} + \text{pe} = h + \frac{V^2}{2} + gz \qquad (\text{kJ/kg})$$

The flow energy is automatically taken care of by enthalpy. In fact, this is the main reason for defining the property enthalpy.

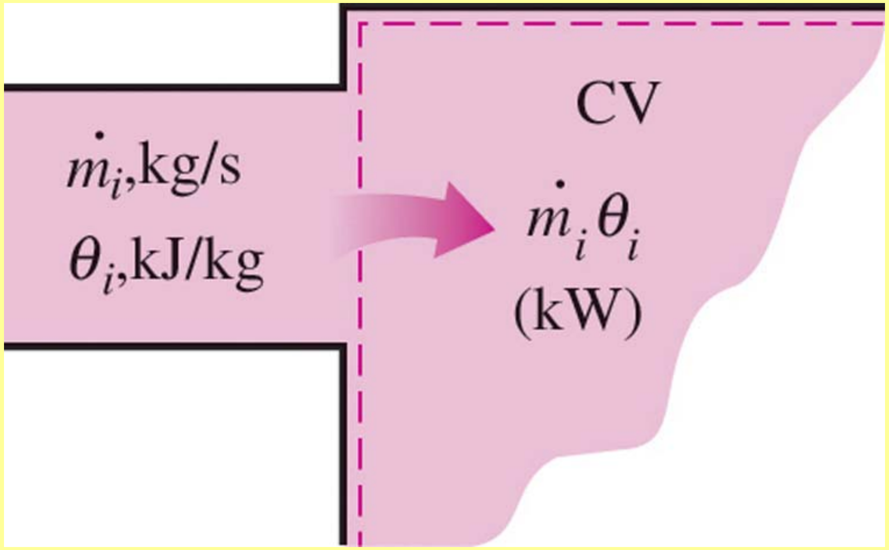


The total energy consists of three parts for a non-flowing fluid and four parts for a flowing fluid.

Energy Transport by Mass

Amount of energy transport: $$E_{\text{mass}} = m\theta = m\left(h + \frac{V^2}{2} + gz\right) \quad \text{(kJ)}$$

Rate of energy transport: $$\dot{E}_{\text{mass}} = \dot{m}\theta = \dot{m}\left(h + \frac{V^2}{2} + gz\right) \quad \text{(kW)}$$



When the kinetic and potential energies of a fluid stream are negligible

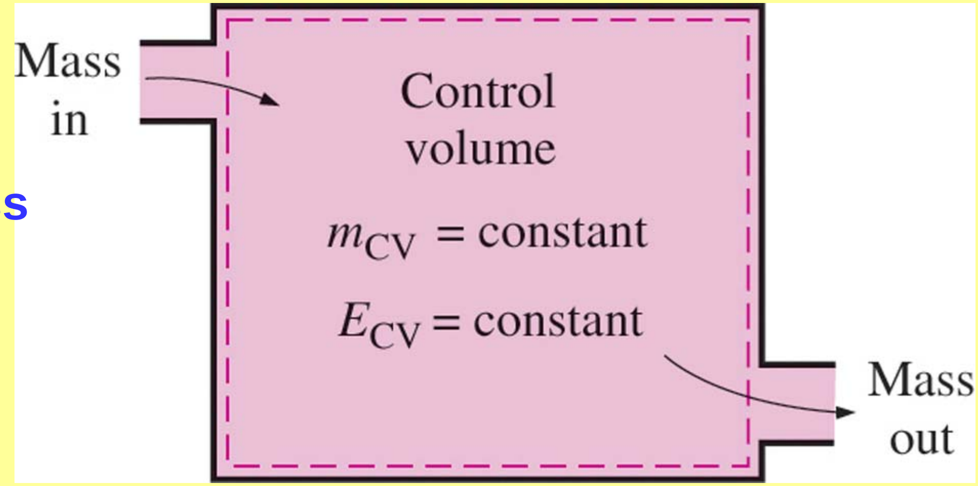
$$E_{\text{mass}} = mh \qquad \dot{E}_{\text{mass}} = \dot{m}h$$

When the properties of the mass at each inlet or exit change with time as well as over the cross section

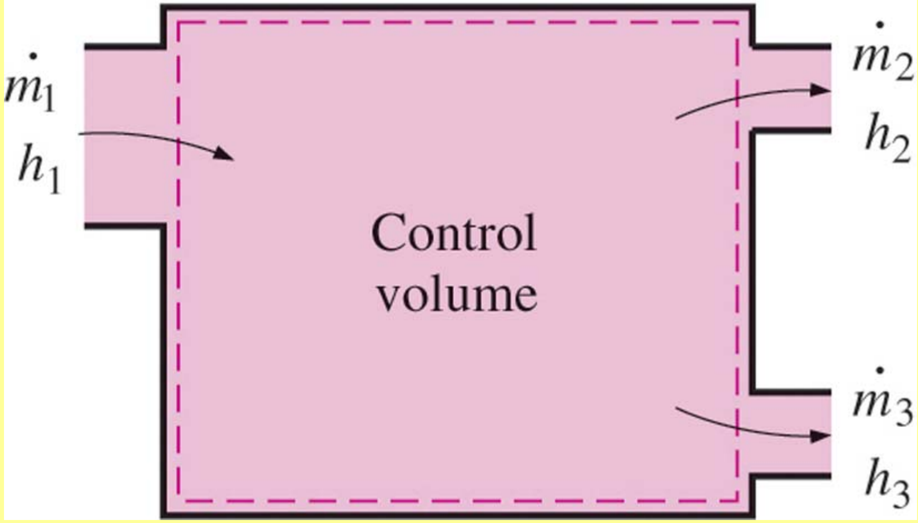
$$E_{\text{in,mass}} = \int_{m_i} \theta_i \, \delta m_i = \int_{m_i} \left(h_i + \frac{V_i^2}{2} + gz_i\right) \delta m_i$$

The product $\dot{m}_i\theta_i$ is the energy transported into control volume by mass per unit time.

ENERGY ANALYSIS OF STEADY-FLOW SYSTEMS



Under steady-flow conditions, the mass and energy contents of a control volume remain constant.



Under steady-flow conditions, the fluid properties at an inlet or exit remain constant (do not change with time).

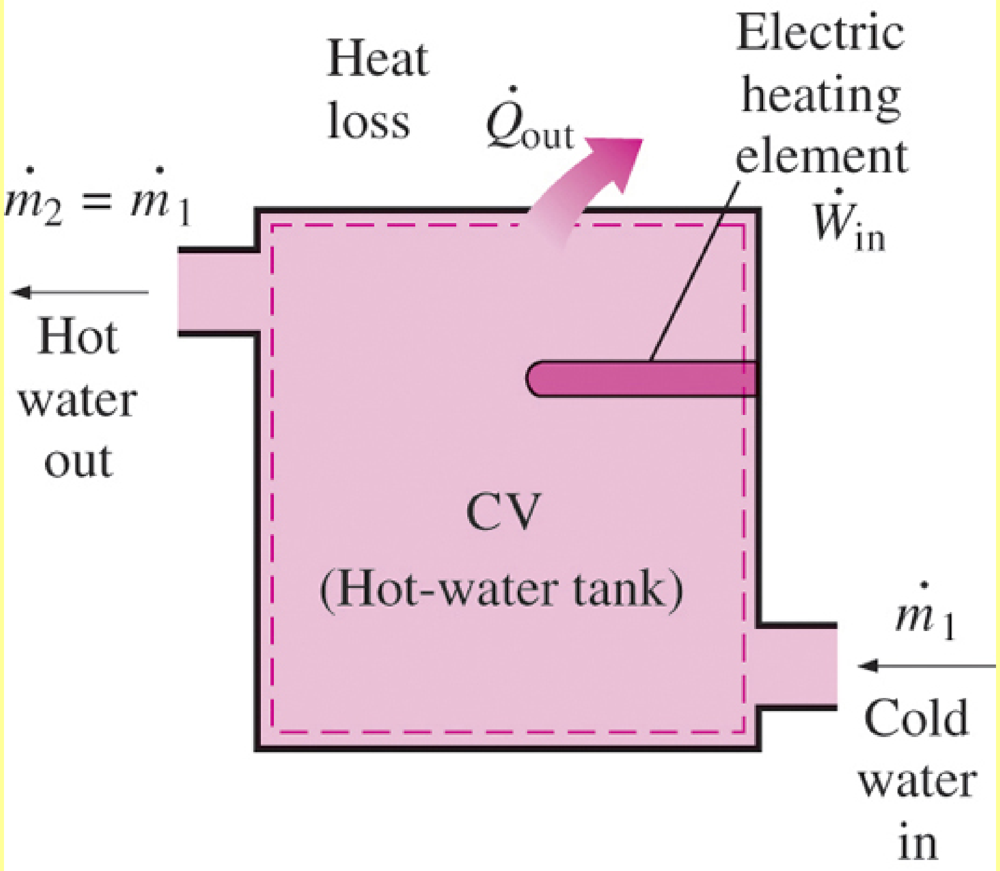
Mass for a steady-flow process

$$\sum_{\text{in}} \dot{m} = \sum_{\text{out}} \dot{m} \qquad (\text{kg/s})$$

$$\dot{m}_1 = \dot{m}_2$$

Mass balance

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$



A water heater in steady operation

Energy balances for a steady-flow process

Energy balance

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{dE_{system}/dt}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc., energies}}} \overset{\nearrow 0 \text{ (steady)}}{} = 0$$

$$\underbrace{\dot{E}_{in}}_{\substack{\text{Rate of net energy transfer in} \\ \text{by heat, work, and mass}}} = \underbrace{\dot{E}_{out}}_{\substack{\text{Rate of net energy transfer out} \\ \text{by heat, work, and mass}}} \quad \text{(kW)}$$

$$\dot{Q}_{in} + \dot{W}_{in} + \sum_{in} \underbrace{\dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each inlet}} = \dot{Q}_{out} + \dot{W}_{out} + \sum_{out} \underbrace{\dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each exit}}$$

Energy balance relations with sign conventions (i.e., heat input and work output are positive)

$$\dot{Q} - \dot{W} = \sum_{\text{out}} \underbrace{\dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each exit}} - \sum_{\text{in}} \underbrace{\dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each inlet}}$$

$$\dot{Q} - \dot{W} = \dot{m}\left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right]$$

$$q - w = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

$$q - w = h_2 - h_1$$

when kinetic and potential energy changes are negligible

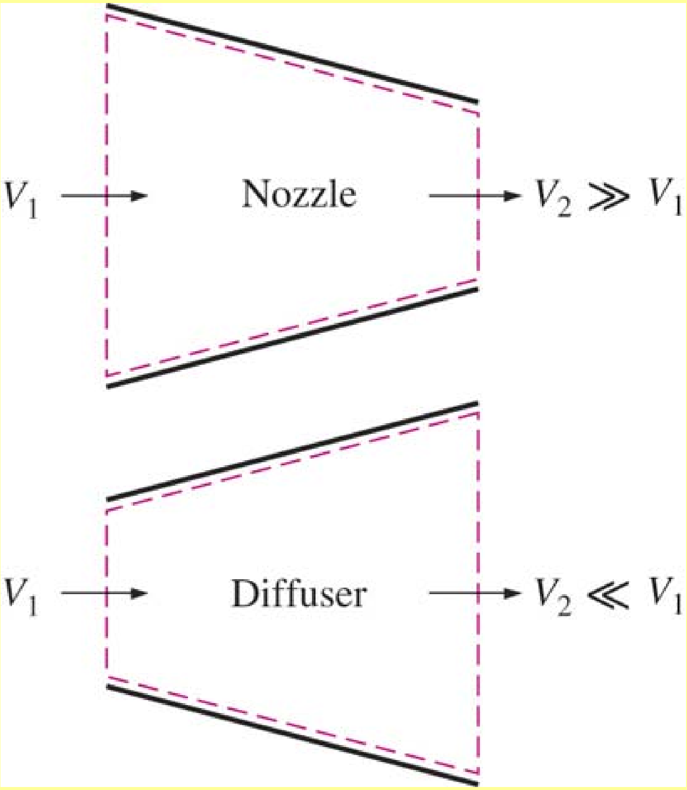
$$q = \dot{Q}/\dot{m} \quad w = \dot{W}/\dot{m}$$

Nozzles and Diffusers

A **nozzle** is a device that *increases the velocity of a fluid* at the expense of pressure.

A **diffuser** is a device that *increases the pressure of a fluid* by slowing it down.

The cross-sectional area of a nozzle decreases in the flow direction for subsonic flows and increases for supersonic flows. The reverse is true for diffusers.



Energy balance for a nozzle or diffuser:

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right)$$

$$(\text{since } \dot{Q} \cong 0,\ \dot{W} = 0, \text{ and } \Delta\text{pe} \cong 0)$$

Turbines and Compressors

Compressors, as well as **pumps** and **fans**, are devices used to increase the pressure of a fluid. Work is supplied to these devices from an external source through a rotating shaft.

A ***compressor*** is capable of compressing the gas to very high pressures.

A ***fan*** increases the pressure of a gas slightly and is mainly used to mobilize a gas.

Pumps work very much like compressors except that they handle liquids instead of gases.

Turbine drives the electric generator in steam, gas, or hydroelectric power plants.

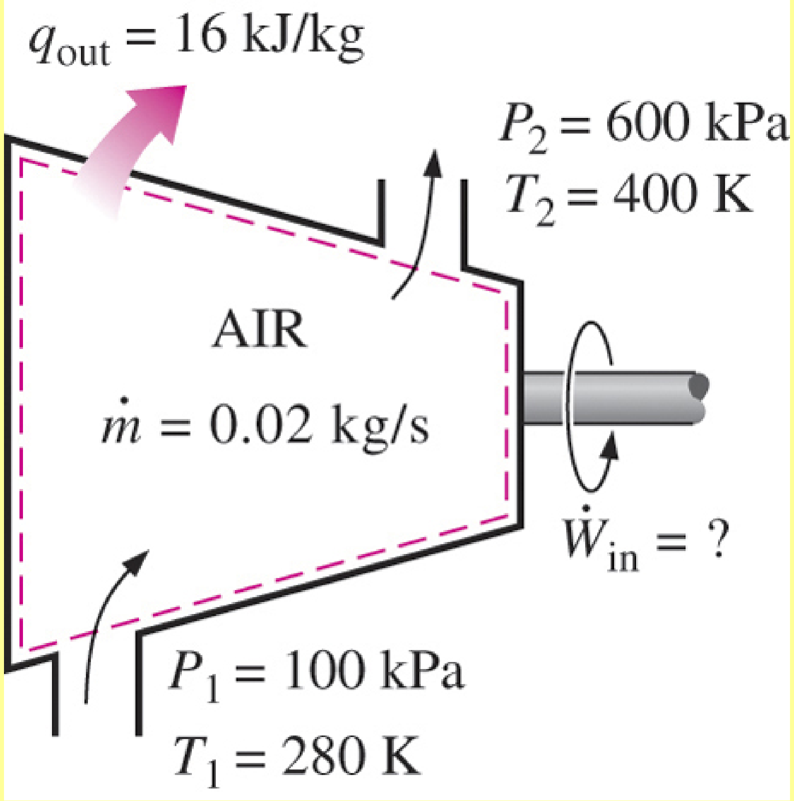
Compressors

Energy balance for the compressor in this Figure

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{in} + \dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2$$

$$(\text{since } \Delta ke = \Delta pe \cong 0)$$



Throttling valves

Throttling valves are *any kind of flow-restricting devices* that cause a significant pressure drop in the fluid.

What is the difference between a turbine and a throttling valve?

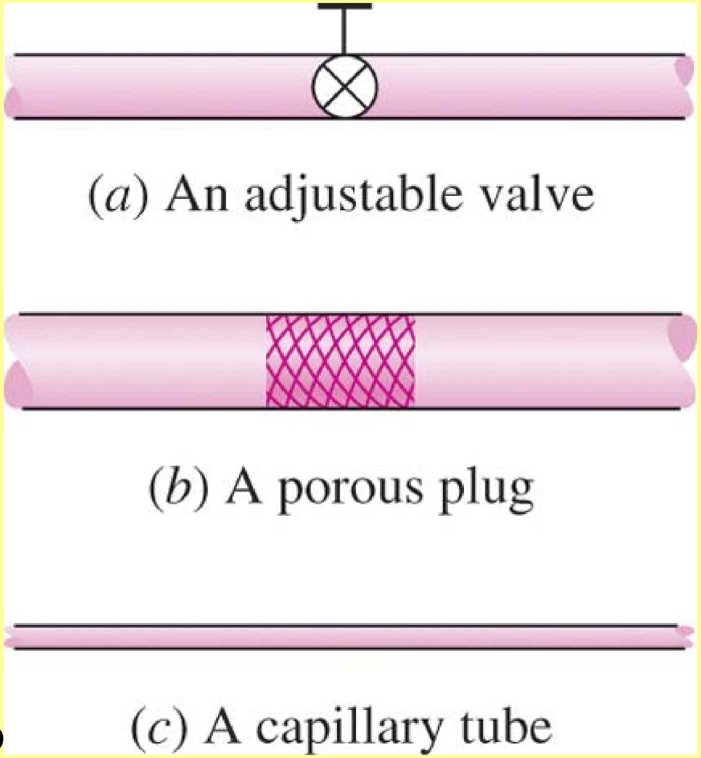
The pressure drop in the fluid is often accompanied by a *large drop in temperature*, and for that reason throttling devices are commonly used in refrigeration and air-conditioning applications.

Energy balance

$$h_2 \cong h_1$$

$$u_1 + P_1 v_1 = u_2 + P_2 v_2$$

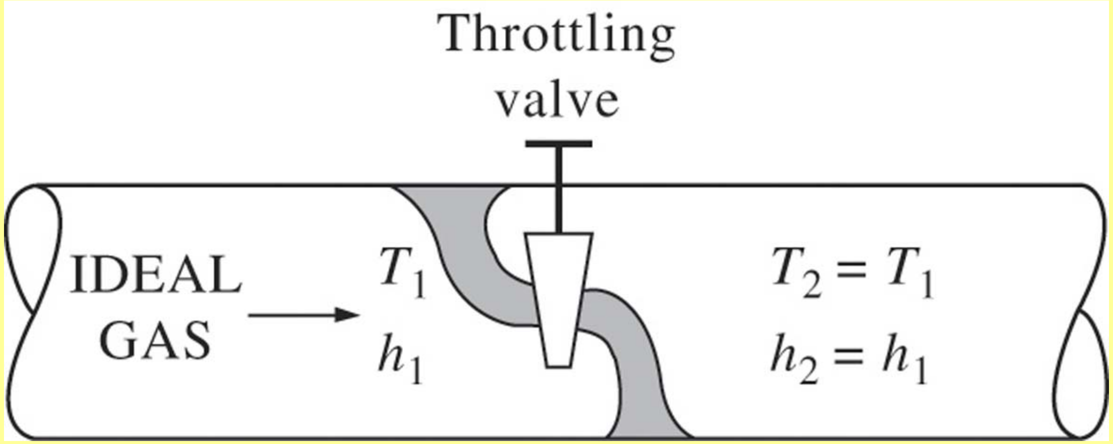
Internal energy + Flow energy = Constant


(*a*) An adjustable valve

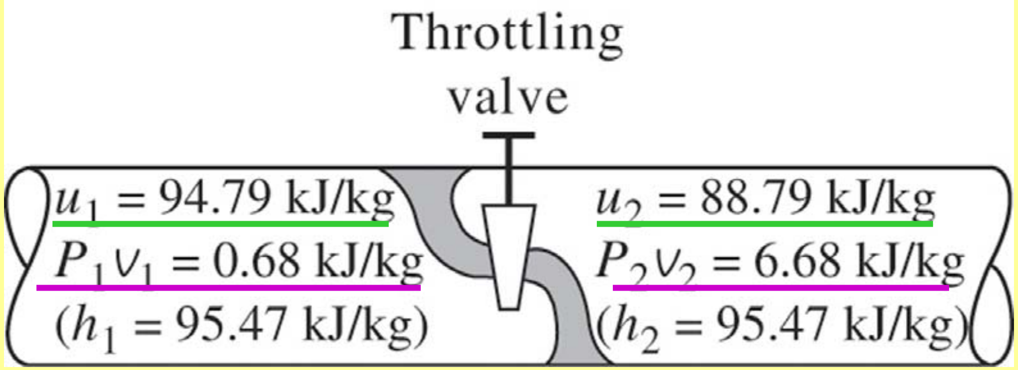
(*b*) A porous plug

(*c*) A capillary tube

Throttling valves



The temperature of an ideal gas does not change during a throttling (*h* = constant) process since *h* = *h*(*T*).



During a throttling process, the enthalpy of a fluid remains constant. But internal and flow energies may be converted to each other.

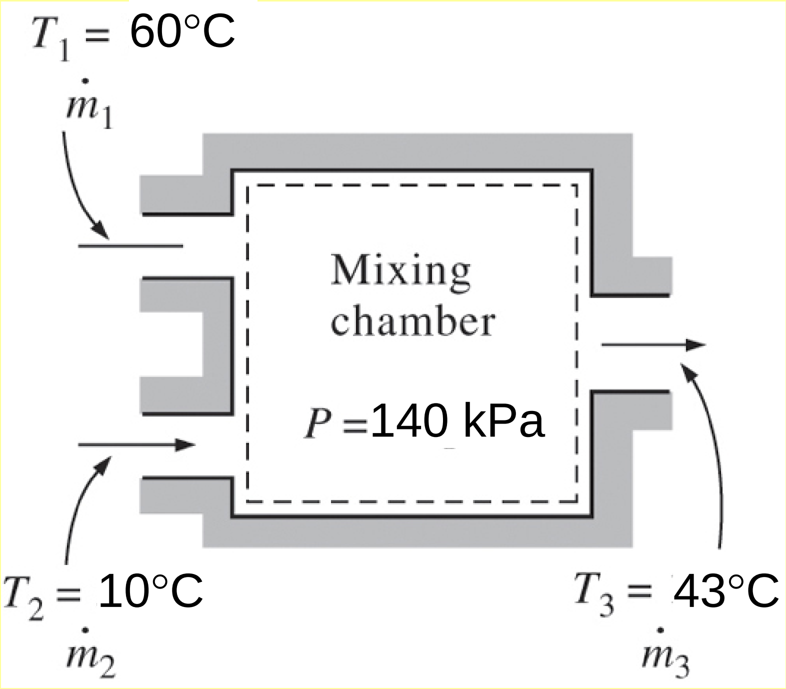
Mixing chambers

Energy balance for the **Adiabatic** mixing chamber in the Figure is:

$$\dot{E}_{in} = \dot{E}_{out}$$

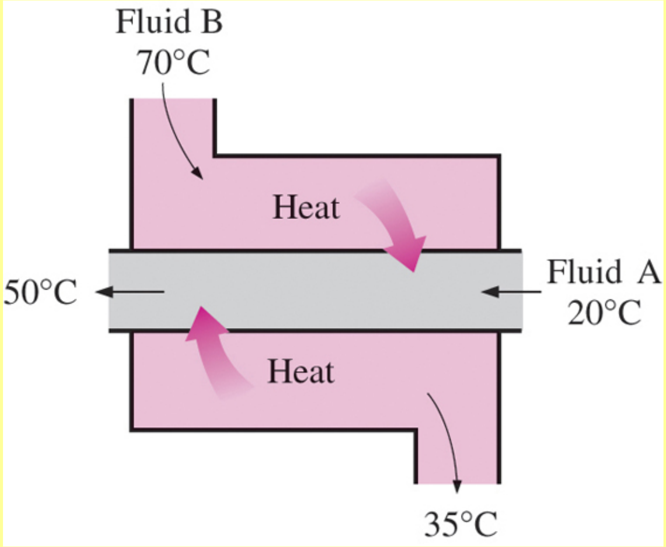
$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

$$(\text{since } \dot{Q} \cong 0,\ \dot{W} = 0,\ \text{ke} \cong \text{pe} \cong 0)$$

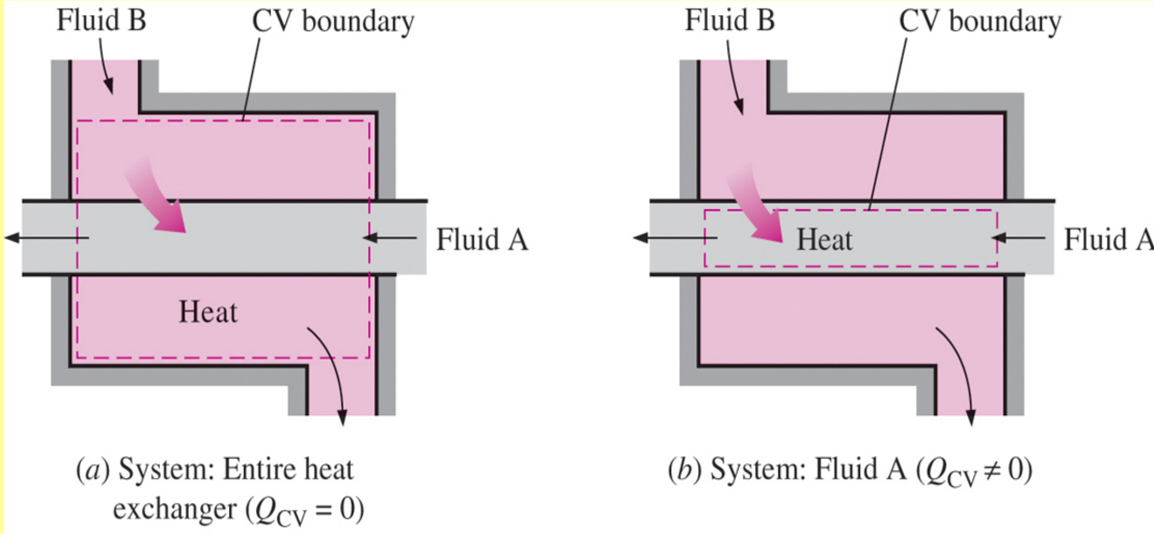


Heat exchangers

Heat exchangers are devices where two moving fluid streams **exchange heat** without mixing. Heat exchangers are widely used in various industries, and they come in various designs.



A heat exchanger can be as simple as two concentric pipes.



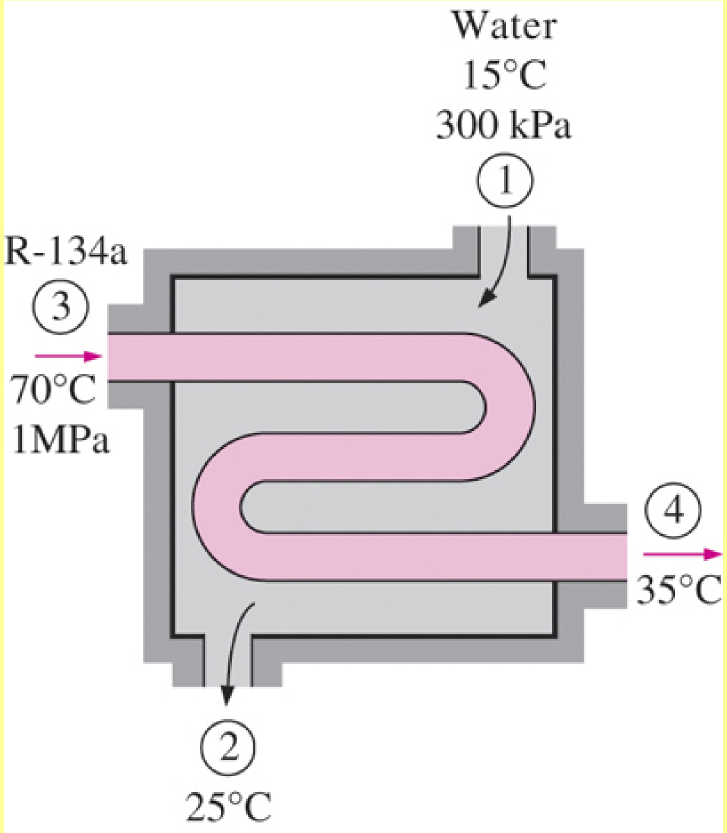
(*a*) System: Entire heat exchanger ($Q_{CV} = 0$)

(*b*) System: Fluid A ($Q_{CV} \neq 0$)

The heat transfer associated with a heat exchanger may be zero or nonzero depending on how the control volume is selected.

Heat exchangers

Mass and energy balances for the Adiabatic heat exchanger in the Figure is:



$$\dot{m}_1 = \dot{m}_2 = \dot{m}_w$$

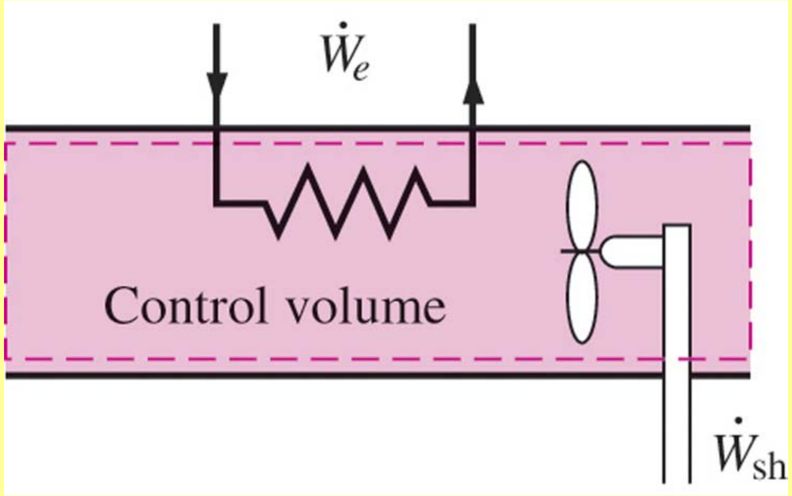
$$\dot{m}_3 = \dot{m}_4 = \dot{m}_R$$

$$\dot{E}_{in} = \dot{E}_{out}$$

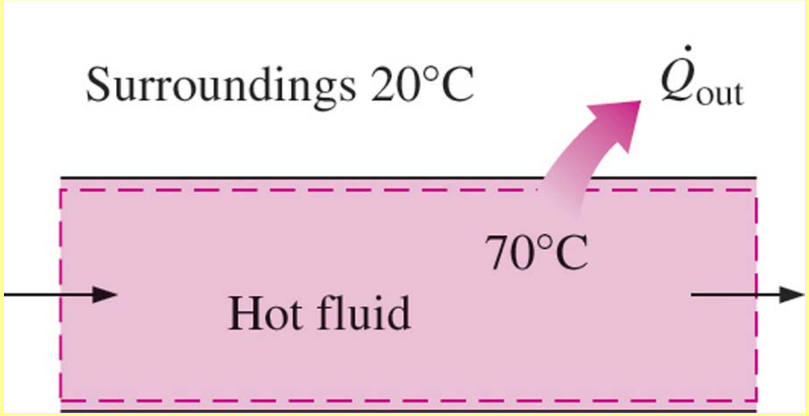
$$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_2 h_2 + \dot{m}_4 h_4$$

Pipe and Duct Flow

The transport of liquids or gases in pipes and ducts is of great importance in many engineering applications. Flow through a pipe or a duct usually satisfies the steady-flow conditions.



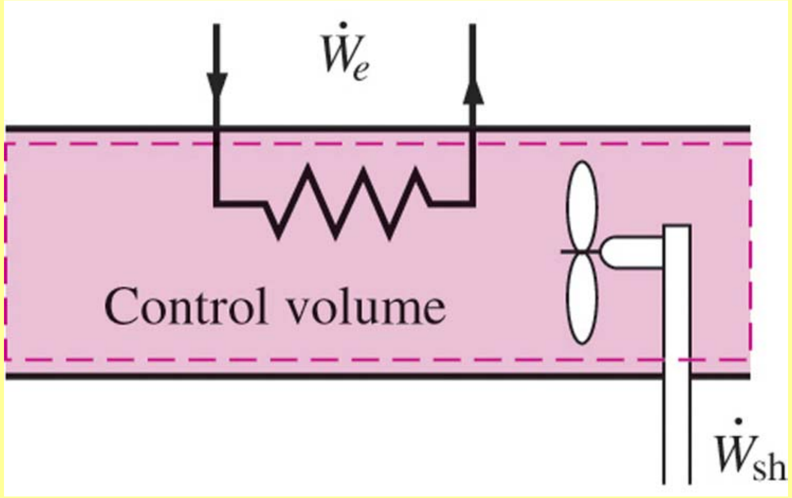
Pipe or duct flow may involve more than one form of work at the same time



Heat losses from a hot fluid flowing through an un-insulated pipe or duct to the cooler environment may be very significant.

Pipe and Duct Flow

Pipe or duct flow may involve more than one form of work at the same time

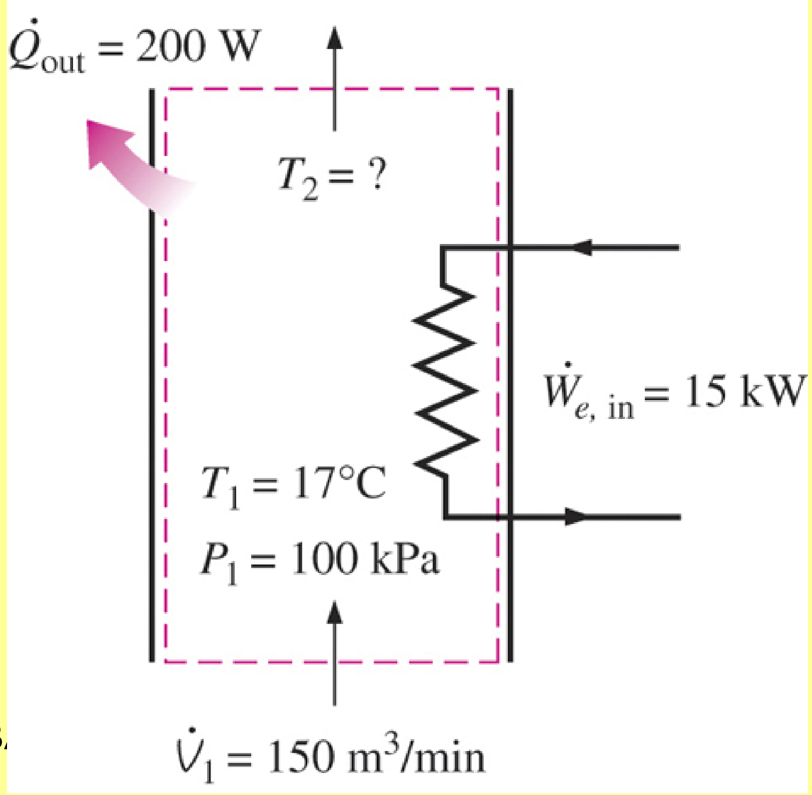


Energy balance for the pipe flow shown in the Figure is

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{e,in} + \dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2$$

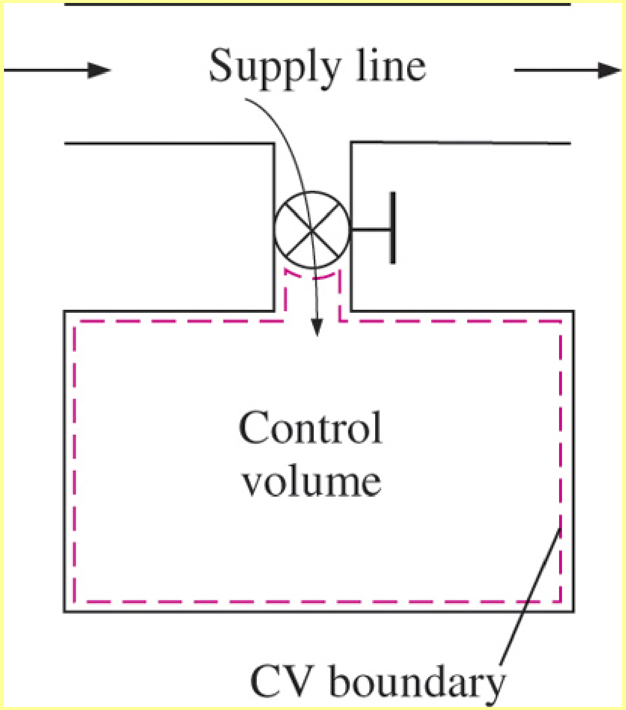
$$\dot{W}_{e,in} - \dot{Q}_{out} = \dot{m}c_p(T_2 - T_1)$$



ENERGY ANALYSIS OF UNSTEADY-FLOW PROCESSES

Many processes of interest, however, involve *changes* within the control volume with time. Such processes are called ***unsteady-flow*, or *transient-flow*, processes.**

Charging of a rigid tank from a supply line is an unsteady-flow process since it involves changes within the control volume.

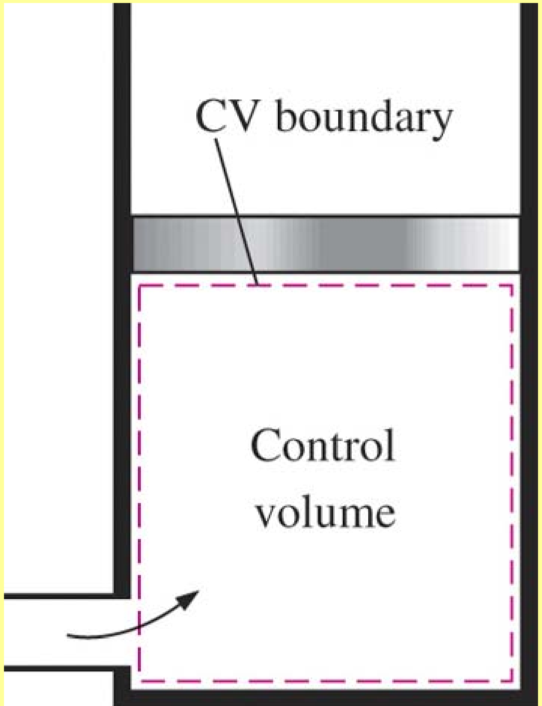


ENERGY ANALYSIS OF UNSTEADY-FLOW PROCESSES

Most unsteady-flow processes can be represented reasonably well by the ***uniform-flow process***.

Uniform-flow process: The fluid flow at any inlet or exit is uniform and steady, and thus the fluid properties do not change with time or position over the cross section of an inlet or exit. If they do, they are averaged and treated as constants for the entire process.

The ***shape*** and ***size*** of a control volume may change during an unsteady-flow process.



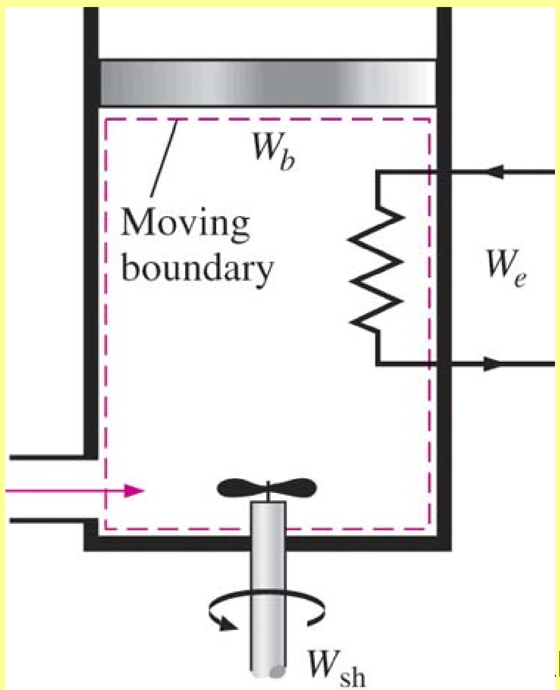
Mass balance

$$m_{\text{in}} - m_{\text{out}} = \Delta m_{\text{system}}$$

$$\Delta m_{\text{system}} = m_{\text{final}} - m_{\text{initial}}$$

$$m_i - m_e = (m_2 - m_1)_{\text{CV}}$$

i = inlet, e = exit, 1 = initial state, and 2 = final state



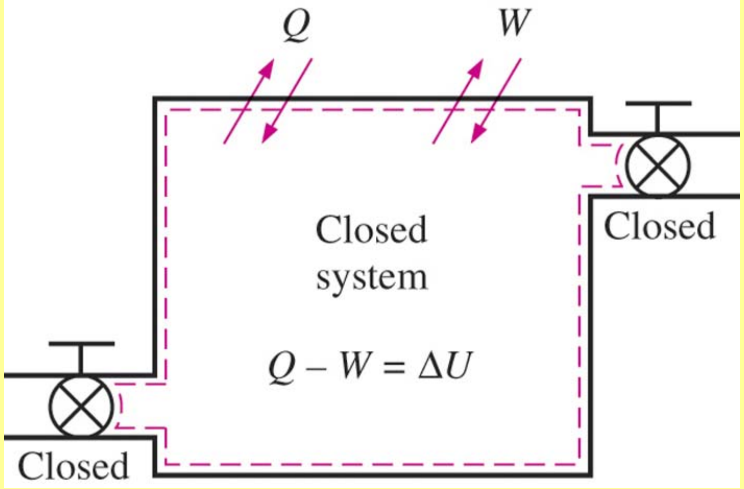
Energy balance

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

$$\left(Q_{in} + W_{in} + \sum_{in} m\theta\right) - \left(Q_{out} + W_{out} + \sum_{out} m\theta\right) = (m_2 e_2 - m_1 e_1)_{system}$$

$$\theta = h + \text{ke} + \text{pe} \qquad e = u + \text{ke} + \text{pe}$$

The energy equation of a uniform-flow system reduces to that of a closed system when all the inlets and exits are closed.



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