CHAPTER (5)

CONTROL VOLUME APPROACH

HOMEWORK (4)

5.8, 5.11, 5.49, 5.50, 5.71

Problem (5.8)

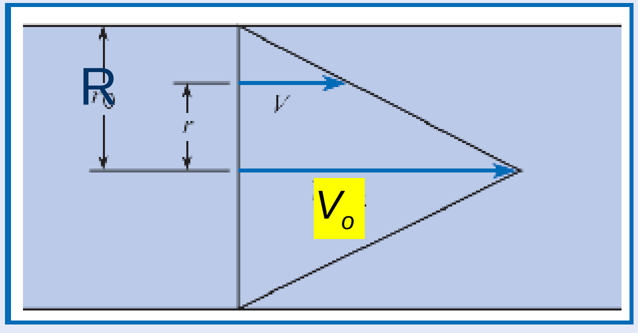
ANALYSIS

Flow rate equation

$\dot{Q} = ?$

$\overline{V} = ?$

$$Q = \int v dA$$



where $dA = 2\pi r dr$. Then

$$\begin{aligned} Q &= \int_0^R V_0(1-(r/R))2\pi r dr \\ &= V_0(2\pi)((r^2/2)-(r^3/(3R)))\,|_0^R \\ &= 2\pi V_0((R^2/2)-(R^2/3)) \\ &= (2/6)\pi V_0 R^2 \end{aligned}$$

$$\frac{V_o}{R} = \frac{V}{R-r}$$

$$V = V_o\left(1-\frac{r}{R}\right)$$

Average Velocity

$$\begin{aligned} \bar{V} &= \frac{Q}{A} \\ \frac{\bar{V}}{V_0} &= \frac{Q}{A}\frac{1}{V_0} \\ &= \frac{(2/6)\pi V_0 R^2}{\pi R^2}\frac{1}{V_0} \end{aligned}$$

$$\boxed{\bar{V}/V_o = 1/3}$$

Problem (5.11)

Situation: Water flows in a 2 m pipe. The velocity profile is linear. The center line velocity is $V_{\max} = 8$ m/s and the velocity at the wall is $V_{\min} = 6$ m/s.

Find: (a) Discharge: Q

(b) Mean velocity: V

APPROACH

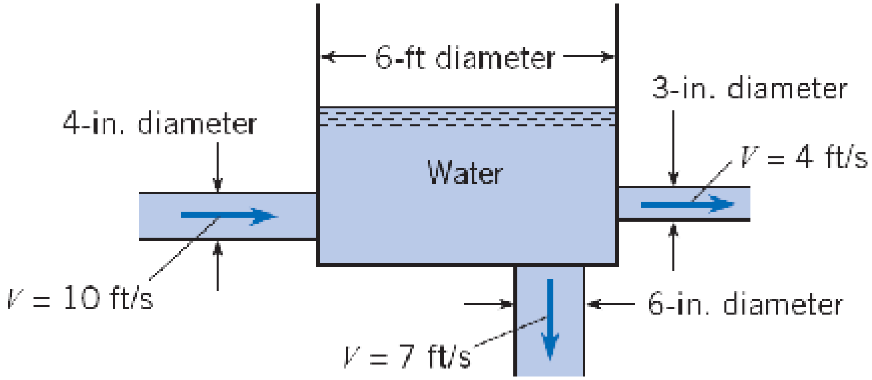
Apply the flow rate equation.

ANALYSIS

Flow rate equation $V = V_{\max} - \frac{2r}{R}$

$$
\begin{aligned}
Q &= 2\pi r_0^2((V_{\max}/2) - (2/3)) \text{ (see problem 5.10 for derivation)} \\
&= 2\pi \times 1((8/2) - (2/3)) \\
& \boxed{Q = 20.94 \text{ m}^3/\text{s}} \\
V &= Q/A = 20.94/(\pi \times 1) \\
& \boxed{V = 6.67 \text{ m/s}}
\end{aligned}
$$

Problem (5.49)



Situation: A tank with one inflow and two outflows indicated by diagram with problem statement.

Find: (a) Is the tank filling or emptying.
(b) Rate at which the tank level is changing: $\frac{dh}{dt}$

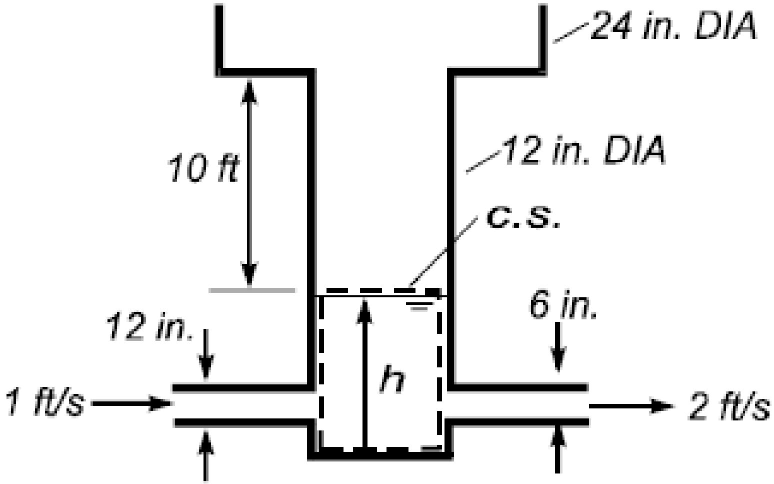
ANALYSIS

$$\begin{aligned}
\text{Inflow} &= 10 \times \pi \times 2^2/144 = 0.8727 \text{ cfs} \\
\text{Outflow} &= (7 \times \pi \times 3^2/144) + (4 \times \pi \times 1.5^2/144) = 1.571 \text{ cfs} \\
\text{Outflow} &> \text{Inflow, Thus, } \boxed{\text{tank is emptying}} \\
\frac{dh}{dt} &= -Q/A \\
&= -(1.571 - 0.8727)/(\pi \times 3^2) \\
&\boxed{\frac{dh}{dt} = -0.0247 \text{ ft/s}}
\end{aligned}$$

Problem 5.50 (p. 176)

PROBLEM 5.50

Situation: The sketch shows a tank filled with water at time $t = 0$ s.



Find: (a) At $t = 22$ s, if the the water surface will be rising or falling.
(b) Rate at which the tank level is changing: $\frac{dh}{dt}$

APPROACH

Apply the continuity principle. Define a control volume in which the control surface (c.s.) is coincident with the water surface and moving with it.

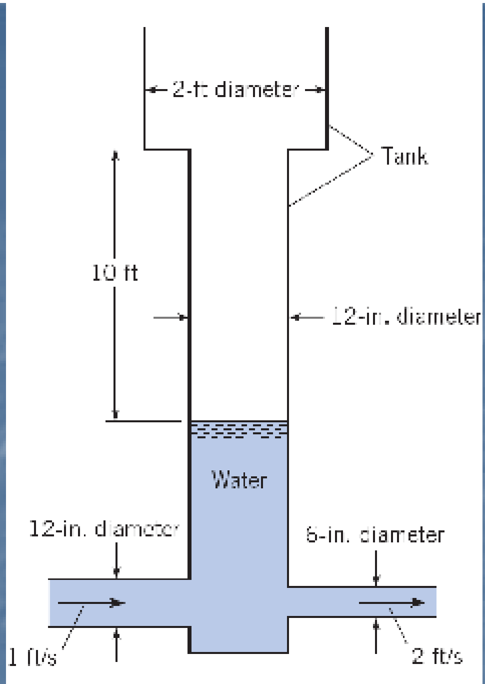
ANALYSIS

Continuity principle

$$
\begin{aligned}
d/dt \int_{cv} \rho d\mathcal{V} &= \dot{m}_i - \dot{m}_o \\
d/dt(\rho A h) &= (\rho A V)_{\text{in}} - (\rho A V)_{\text{out}} \\
d/dt(\rho A h) &= \rho(\pi/4 \times 1^2)(1) + \rho(\pi/4 \times 0.5^2)(2) \\
A dh/dt &= (\pi/4) - (\pi/8) \\
A dh/dt &= (\pi/8)
\end{aligned}
$$

Since $Adh/dt > 0$, the water level must be $\boxed{\text{rising.}}$ While the water column occupies the 12 in. section, the rate of rise is

$$
\begin{aligned}
dh/dt &= (\pi/8)/A \\
&= \pi/(8 \times \pi/4 \times 1^2) \\
&= 1/2 \ \text{ft/s}
\end{aligned}
$$



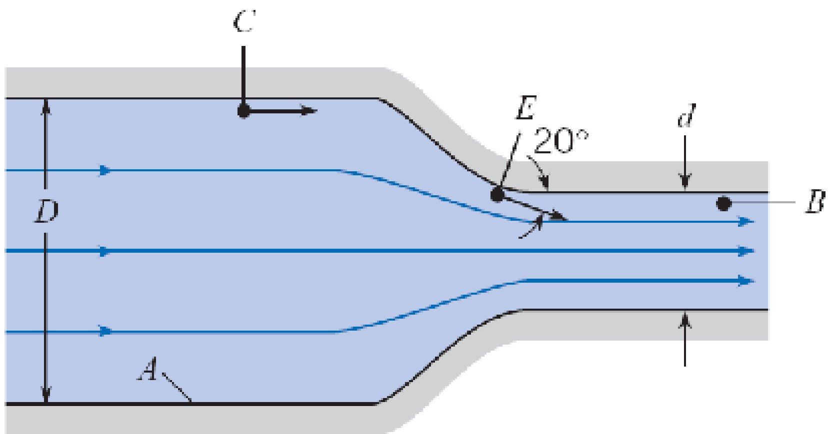
Determine the time it takes the water surface to reach the 2 ft. section:

$$\begin{aligned} 10 &= (dh/dt)t; \\ t &= (10)/(1/2) = 20 \text{ secs.} \end{aligned}$$

Therefore, at the end of 20 sec. the water surface will be in the 2 ft. section. Then the rise velocity will be:

$$\begin{aligned} dh/dt &= \pi/(8A) \\ &= \pi/(8 \times \pi/4 \times 2^2) \\ & \boxed{dh/dt = 1/8 \text{ ft/sec}} \end{aligned}$$

Problem (5.71)



Situation: The flow pattern through a pipe contraction is described in the problem statement. Discharge of water is 70 cfs and pressure at point A is 3500 psf.

Find: Pressure at point B.

APPROACH

Apply the Bernoulli equation and the continuity principle.

ANALYSIS

Continuity principle

$$\begin{aligned} V_A &= Q/A_A = 70/(\pi/4 \times 6^2) = 2.476 \text{ ft/s} \\ V_B &= Q/A_B = 70/(\pi/4 \times 2^2) = 22.28 \text{ ft/s} \end{aligned}$$

Bernoulli equation

$$\begin{aligned} p_A/\gamma + V_A^2/2g + z_A &= p_B/\gamma + V_B^2/2g + z_B \\ p_B/\gamma &= 3500/62.4 - 2.48^2/64.4 - 22.28^2/64.4 - 4 \\ p_B &= 2775 \text{ lbf/ft}^2 \\ & \boxed{p_B = 19.2 \text{ lbf/in}^2} \end{aligned}$$

END OF HOMEWORK (4)