CHAPTER (7)

MOMENTUM PRINCIPLE

HOMEWORK (6)

7.17, 7.23, 7.32, 7.51

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1. Steady Flow Energy Equation

$$\dot{Q} - \dot{W}_S = \sum_{CS} \dot{m}_{out} \left(\frac{V^2}{2} + gz + h \right)_{out} - \sum_{CS} \dot{m}_{in} \left(\frac{V^2}{2} + gz + h \right)_{in}$$

2. Energy Equation for Steady Flow of an Incompressible Fluid in a pipe with a Pump and a Turbine

$$\left(h_P + \frac{p_1}{\rho g} + z_1 + \alpha_1 \frac{\overline{V}_1^{\,2}}{2g} \right)_{Mech.part} = \left(h_T + \frac{p_2}{\rho g} + gz_2 + \alpha_2 \frac{\overline{V}_2^{\,2}}{2g} \right)_{Mech.part} + h_{Loss}$$

The coefficients (α_1, α_2) *are called kinetic energy correction factor and can be evaluated as follows :*

$$\alpha = \frac{1}{A} \int_A \left(\frac{V}{\overline{V}} \right)^3 dA$$

4. The head loss due to the expansion as a function of flow velocities in the two pipes?

$$i.e.\quad h_{loss} = \left(\frac{V_1^{\,2} - V_2^{\,2}}{2g} \right)$$

Where (V_1) *is upstream velocity and* (V_2) *is downstream velocity*

$$Pump\ Head = h_p = \frac{\dot{W}_p}{\dot{m}g}$$

$$\dot{W}_p = \dot{m}gh_p = g\,Qh_p$$

$$\dot{W}_T = \dot{m}gh_T = g\,Qh_T$$

$$Pump\ Efficiency = h_P = \frac{(\dot{W}_P)}{(\dot{W}_P)_{actual}}$$

$$Turbine\ Efficiency = h_T = \frac{(\dot{W}_T)_{actual}}{(\dot{W}_T)}$$

Problem 7.17

Assumptions: $\alpha_2 = 1$

APPROACH

To find pressure at point A, apply the energy equation between point A and the pipe exit. Then, then apply energy equation between top of tank and the exit.

ANALYSIS

Energy equation (point A to pipe exit).

$$\frac{p_A}{\gamma} + z_A + \alpha_A \frac{V_A^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \alpha_2 \frac{V_2^2}{2g} + h_t + h_L$$

Term by term analysis: $V_A = V_2$ (continuity); $p_2 = 0$-gage; $(z_A - z_B) = y$; $h_p = 0$, $h_t = 0$, $h_L = 0$. Thus

$$\begin{aligned} p_A &= -\gamma y \\ &= -62.4 \times 4 \\ &\boxed{p_A = \text{-250 lb/ft}^2} \end{aligned}$$

Energy equation (top of tank and pipe exit)

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 + h_p &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + h_t + h_L \\ z_1 &= V_2^2/2g + z_2 \\ V_2 &= \sqrt{2g(z_1 - z_2)} \\ &= \sqrt{2 \times 32.2 \times 14} \\ &\boxed{V_2 = 30.0 \text{ ft/s}} \end{aligned}$$

Problem 7.23

Assumptions: $\gamma = 9810 \text{ N/m}$

APPROACH

Apply the energy equation.

ANALYSIS

Energy equation

$$\begin{aligned} p_{reser.}/\gamma + V_r^2/2g + z_r &= p_{outlet}/\gamma + V_0^2/2g + z_0 \\ 0 + 0 + 5 &= 0 + V_0^2/2g \\ V_0 &= 9.90 \text{ m/s} \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= V_0 A_0 \\ &= 9.90 \times (\pi/4) \times 0.20^2 \end{aligned}$$

$$\boxed{Q = 0.311 \text{ m}^3/\text{s}}$$

Energy equation from reservoir surface to point B:

$$0 + 0 + 5 = p_B/\gamma + V_B^2/2g + 3.5$$

where

$$\begin{aligned} V_B &= Q/V_B = 0.311/[(\pi/4) \times 0.4^2] = 2.48 \text{ m/s} \\ V_B^2/2g &= 0.312 \text{ m} \end{aligned}$$

$$p_B/\gamma - 5 - 3.5 = 0.312$$

$$\boxed{p_B = 11.7 \text{ kPa}}$$

Problem 7.32

ANALYSIS

Let V_n = velocity of jet from nozzle:

Flow rate equation

$$V_n = \frac{Q}{A_n} = \frac{0.10}{((\pi/4)\times 0.10^2)} = 12.73 \text{ m/s}$$

$$\frac{V_n^2}{2g} = 8.26 \text{ m}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.10}{((\pi/4)\times 0.3^2)} = 1.41 \text{ m/s}$$

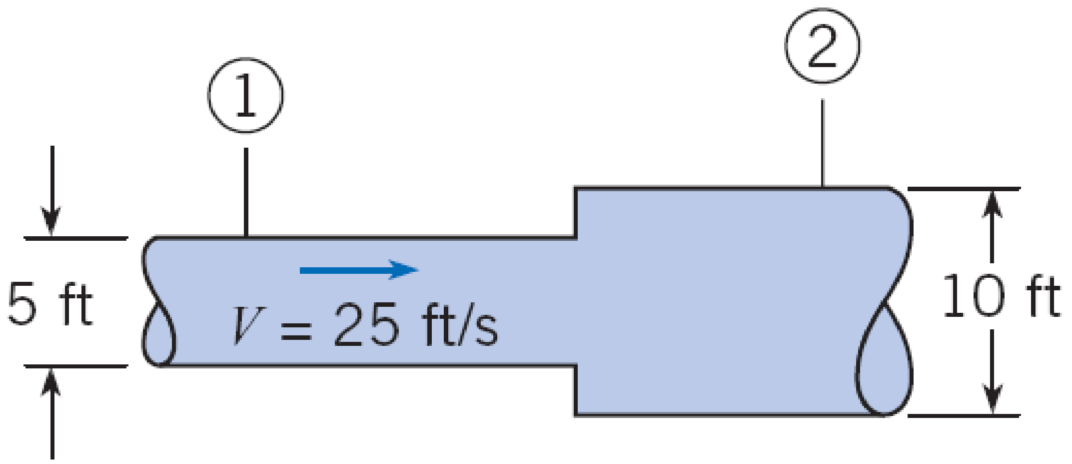
$$\frac{V_2^2}{2g} = .102 \text{ m}$$

Energy equation

$$\frac{p_2}{\gamma} + 0.102 + 2 = 0 + 8.26 + 7$$

$$\boxed{\frac{p_2}{\gamma} = 13.16 \text{ m}}$$

Problem 7.51



Situation: Flow through a pipe is described in the problem statement.

Find:
(a) Horsepower lost.
(b) Pressure at section 2.
(c) Force needed to hold expansion.

Find the head loss by applying the sudden expansion head loss equation, first solving for V_2 by applying the continuity principle. Then apply the power equation, the energy equation, and finally the momentum principle.

ANALYSIS

Continuity equation

$$\begin{aligned} V_2 &= V_1(A_1/A_2) \\ &= 25(1/4) \\ &= 6.25 \text{ ft/s} \end{aligned}$$

Sudden expansion head loss equation

$$\begin{aligned} h_L &= (V_1 - V_2)^2/(2g) \\ h_L &= (25 - 6.25)^2/64.4 \\ &= 5.46 \text{ ft} \end{aligned}$$

a) Power equation

$$\begin{aligned} P(\text{hp}) &= Q\gamma h/550 \\ Q &= VA = 25(\pi/4)(5^2) = 490.9 \text{ ft}^3/\text{s} \\ P &= (490.9)(62.4)(5.46)/550 \\ & \boxed{P = 304 \text{ hp}} \end{aligned}$$

b) Energy equation

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\ (5 \times 144)/62.4 + 25^2/64.4 &= p_2/\gamma + 6.25^2/64.4 + 5.46 \\ p_2/\gamma &= 15.18 \text{ ft} \\ p_2 &= 15.18 \times 62.4 \\ &= 947 \text{ psfg} \\ & \boxed{p_2 = 6.58 \text{ psig}} \end{aligned}$$

Problem 7.51

c) Momentum equation

$$\begin{aligned}
\sum F_x &= \dot{m}_o V_{x,o} - \dot{m}_i V_{x,i} \\
\dot{m} &= 1.94 \times (\pi/4) \times 5^2 \times 25 \\
&= 952.3 \text{ kg/s} \\
p_1 A_1 - p_2 A_2 + F_x &= \dot{m}(V_2 - V_1) \\
(5)(14)\pi/4)(5^2) - (6.57)(144)(\pi/4)(10^2) + F_x &= 952.3 \times (6.25 - 25) \\
& \boxed{F_x = 42{,}426 \text{ lbf}}
\end{aligned}$$

<u>THE END</u>