

CHAPTER (12)

COMPRESSIBLE FLOW

HOMEWORK (4)

12.13, 12.28, 12.40, 12.59

Problem (12.13)



Situation: An aircraft is flying at Mach 1.8 through air at 10000 m, 30.5 kPa, and -44°C .

- Find:
- (a) Speed of aircraft.
 - (b) Total temperature.
 - (c) Total pressure.
 - (d) Speed for $M = 1$.

ANALYSIS

Speed of sound (at 10,000 m)

$$\begin{aligned} c &= \sqrt{kRT} \\ c &= \sqrt{(1.40)(287)(229)} \\ c &= 303.3 \text{ m/s} \end{aligned}$$

Mach number

$$\begin{aligned} V &= (1.8)(303.3)(3,600/1,000) \\ &= 1,965 \text{ km/hr} \end{aligned}$$

Problem (12.13)

Mach number

$$\begin{aligned}V &= (1.8)(303.3)(3,600/1,000) \\&= \boxed{1,965 \text{ km/hr}}\end{aligned}$$

Total temperature

$$T_t = T \left(1 + \frac{k-1}{2} M^2 \right)$$
$$\begin{aligned}T_t &= 229(1 + ((1.4 - 1)/2) \times 1.8^2) \\&= 377K = \boxed{104 \text{ }^\circ\text{C}}\end{aligned}$$

Total pressure

$$P_t = P \left(1 + \frac{k-1}{2} M^2 \right)^{k/(k-1)}$$
$$\begin{aligned}p_t &= (30.5)(1 + 0.2 \times 1.8^2)^{(1.4/(1.4-1))} \\&= \boxed{175 \text{ kPa}}\end{aligned}$$

Mach number

$$\begin{aligned}M &= 1; V = 1 \times c = c \\V &= (303.3)(3,600/1,000) \\&= \boxed{1092 \text{ km/hr}}\end{aligned}$$

Problem (12.28)



Situation: A normal shock wave is described in the problem statement.

Find: (a) Mach number.

(b) Pressure downstream of wave.

(c) Temperature downstream of wave.

(d) Entropy increase.

ANALYSIS

Speed of sound

$$\begin{aligned} c_1 &= \sqrt{kRT} \\ &= \sqrt{(1.4)(297)(223)} \\ &= 304.5 \text{ m/s} \end{aligned}$$

Mach number

$$\begin{aligned} M_1 &= V/c \\ &= 500/304.8 \\ &= 1.64 \end{aligned}$$

Problem (12.28)

Normal shock wave (Mach number)

$$\begin{aligned} M_2^2 &= [(k - 1)M_1^2 + 2]/[2kM_1^2 - (k - 1)] \\ &= [(0.4)(1.64)^2 + 2]/[(2)(1.4)(1.64)^2 - 0.4] \\ &\boxed{M_2 = 0.657} \end{aligned}$$

Normal shock wave

You can use table A-1

Pressure ratio

$$\begin{aligned} p_2 &= p_1(1 + k_1M_1^2)/[(1 + k_1M_2^2)] \\ &= (70)(1 + 1.4 \times 1.64^2)/(1 + 1.4 \times 0.657^2) \\ &\boxed{p_2 = 208 \text{ kPa}} \end{aligned}$$

Temperature ratio

$$\begin{aligned} T_2 &= T_1(1 + ((k - 1)/2)M_1^2)/(1 + ((k - 1)/2)M_2^2) \\ &= 223[1 + 0.2 \times 1.64^2]/[1 + 0.2 \times 0.657^2] \\ &\boxed{T_2 = 316 \text{ K} = 43^\circ\text{C}} \end{aligned}$$

Entropy

$$\begin{aligned} \Delta s &= R\ln[(p_1/p_2)(T_2/T_1)^{k/(k-1)}] \\ &= R[\ln(p_1/p_2) + (k/(k-1))\ln(T_2/T_1)] \\ &= 297[\ln(70/208) + 3.5\ln(315/223)] \\ &\boxed{\Delta s = 35.6 \text{ J/kg K}} \end{aligned}$$

Problem (12.40)

Properties: From Table A.2 $k = 1.66$, $R_{\text{He}} = 2077 \text{ J/kg}\cdot\text{K}$.

$$A_e = 10 \text{ cm}^2$$

$$T_t = 28^\circ\text{C}$$

$$P_b = 100 \text{ kPa}$$

SOLUTION

(a) $p_t = 130 \text{ kPa}$

If sonic at exit,

$$\left(\frac{P_*}{P_t}\right)_{\text{He}} = 0.487$$

$$\begin{aligned} p_* &= [2/(k+1)]^{k/(k-1)} p_t \\ &= 0.487 \times 130 \text{ kPa} \\ &= 63.3 \text{ kPa} \end{aligned}$$

Back pressure is higher so flow must exit subsonically.

Find Mach number

$$\left(\frac{P_b}{P_t}\right)_{\text{He}} = \frac{100}{130} = 0.769$$

$$\begin{aligned} M_e^2 &= (2/(k-1))[(p_t/p_b)^{(k-1)/k} - 1] \\ &= 3.03[(130 \text{ kPa}/100 \text{ kPa})^{0.4} - 1] = 0.335 \end{aligned}$$

$$M_e = 0.579$$

Problem (12.40)

Exit temperature

$$\begin{aligned}\therefore T_e &= T_t / (1 + ((k - 1)/2)M^2) \\ &= 301 \text{ K} / (1 + (0.33)(0.335)) \\ &= 271 \text{ K}\end{aligned}$$

Density at exit from ideal gas law

$$\begin{aligned}\rho_e &= p / RT_e \\ &= 100 \times 10^3 \text{ Pa} / [(2,077 \text{ J/kg-K})(271 \text{ K})] \\ &= 0.178 \text{ kg/m}^3\end{aligned}$$

Velocity at exit

$$\begin{aligned}V_e &= M_e \sqrt{kRT_e} \\ &= 0.579 \times \sqrt{1.66 \times 2077 \text{ J/kg-K} \times 271 \text{ K}} \\ &= 560 \text{ m/s}\end{aligned}$$

Flow rate equation

$$\begin{aligned}\dot{m} &= \rho_e A_e V_e \\ &= 0.178 \text{ kg/m}^3 \times 10 \times 10^{-4} \text{ m}^2 \times 560 \text{ m/s} \\ &\boxed{\dot{m} = 0.100 \text{ kg/s}}\end{aligned}$$

Problem (12.40)

b) Exit pressure is 350 kPa.

$$p_t = 350 \text{ kPa}$$

$$\therefore p_* = (0.487)(350) = 170 \text{ kPa}$$

\therefore Flow exits sonically

Flow rate equation for sonic flow at exit

$$\begin{aligned}\dot{m} &= 0.727 p_t A_* / \sqrt{RT_t} \\ &= (0.727)(350 \times 10^3 \text{ Pa})(10 \times 10^{-4} \text{ m}^2) / \sqrt{2,077 \text{ J/kg-K} \times 301 \text{ K}}\end{aligned}$$

$$\boxed{\dot{m} = 0.322 \text{ kg/s}}$$

Problem (12.59)

12.59: PROBLEM DEFINITION

Situation: A shock wave in air exists in a Laval nozzle where cross-sectional area is 120 cm². The inlet Mach number, area and static pressure are 0.3, 200 cm² and 400 kPa. Exit area is 140 cm².

Find: Back pressure for shock position.

SOLUTION From Table A.1

$$M = 0.3$$

$$A/A_* = 2.0351$$

$$p/p_t = 0.9395$$

Therefore

$$A_* = 200 \text{ cm}^2 / 2.0351 = 98.3 \text{ cm}^2$$

$$p_t = 400 \text{ kPa} / 0.9395 = 426 \text{ kPa}$$

The area ratio at the shock location

$$A_s/A_* = 120/98.3 = 1.2208$$

Problem (12.59)

By interpolation from Table A.1:

$$M_{s1} = 1.562; p_1/p_t = 0.2490 \rightarrow p_1 = 0.249(426) = 106 \text{ kPa}$$

$$M_{s2} = 0.680; p_{s2}/p_1 = 2.679 \rightarrow p_{s2} = 2.679(106) = 284 \text{ kPa}$$

$$A_s/A_{*2} = 1.1097 \rightarrow A_{*2} = 120/1.1097 = 108 \text{ cm}^2$$

$$p_{s2}/p_{t2} = 0.7338; p_{t2} = 284/0.7338 = 387 \text{ kPa}$$

$$A_2/A_{*2} = 140/108 = 1.296 \rightarrow M_2 = 0.525$$

$$p_2/p_{t2} = 0.8288$$

$$p_2 = 0.8288(387 \text{ kPa})$$

$$\boxed{p_2 = 321 \text{ kPa}}$$

END OF HOMEWORK (4)