CHAPTER (9)

SURFACE RESISTANCE

SOLVED PROBLEMS

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Problem (9.4)

Application of a flow produced by Moving plate

9.4 A cube weighing 150 N and measuring 35 cm on a side is allowed to slide down an inclined surface on which there is a film of oil having a viscosity of 10^{-2} N · s/m^2. What is the velocity of the block if the oil has a thickness of 0.1 mm?

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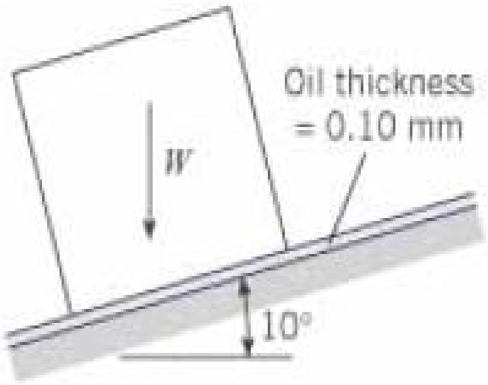


9.4: PROBLEM DEFINITION

Situation:

A 35 cm block on side weighing 150 N slides on an oil film with thickness of 0.1 mm.

Find:

Terminal velocity of block.

Properties:

Viscosity is 10^{-2} N·s/m^2

PLAN

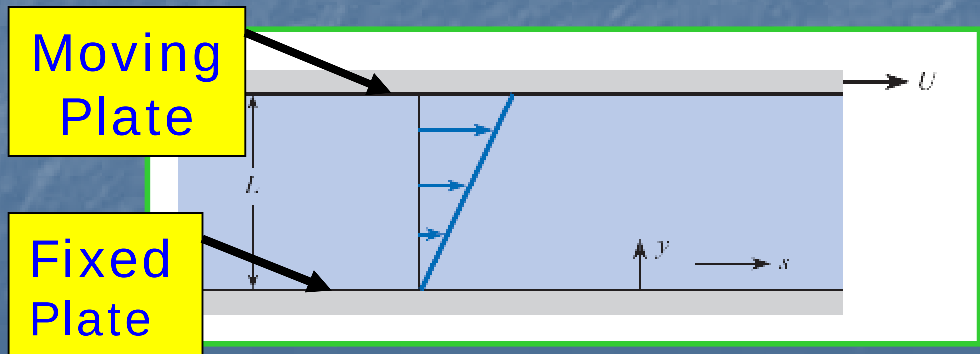
Apply equilibrium. Then relate shear force (viscous drag force) to viscosity and solve the resulting equation.

Related Equations

$$t = m\frac{du}{dy} = m\left(\frac{U}{L}\right) = \frac{F_S}{A}$$

$$L = \Delta y$$

$$u = y\left(\frac{U}{L}\right)$$



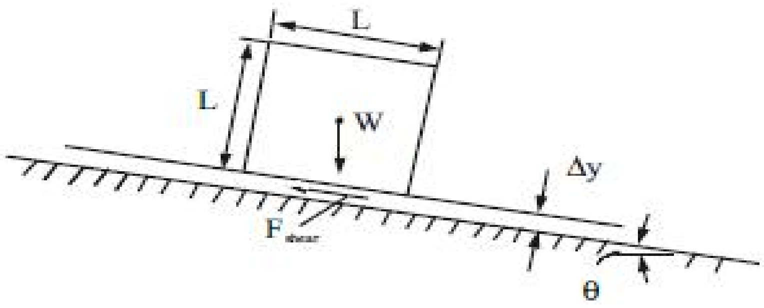
SOLUTION

Force equilibrium

$$F_{\text{shear}} = W \sin\theta$$
$$\tau = \frac{F_{\text{shear}}}{A_s} = \frac{W \sin\theta}{L^2}$$

Shear stress

$$\tau = \mu \frac{dV}{dy} = \mu \times \frac{V}{\Delta y}$$



or

$$V = \frac{\tau \Delta y}{\mu}$$

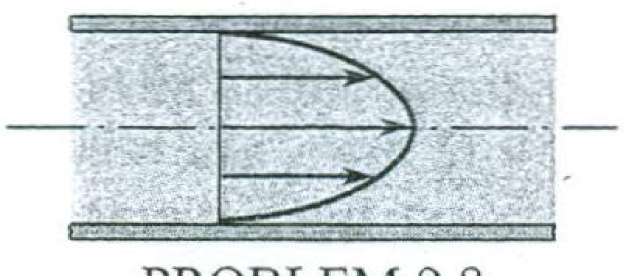
Then

$$V = \frac{W \sin\theta}{L^2} \frac{\Delta y}{\mu}$$
$$V = \left(\frac{150 \sin 10^\circ}{(0.35\ \mathrm{m})^2}\right) \times 1 \times 10^{-4}/10^{-2}$$
$$\boxed{V = 2.13\ \mathrm{m/s}}$$

Problem (9.7)

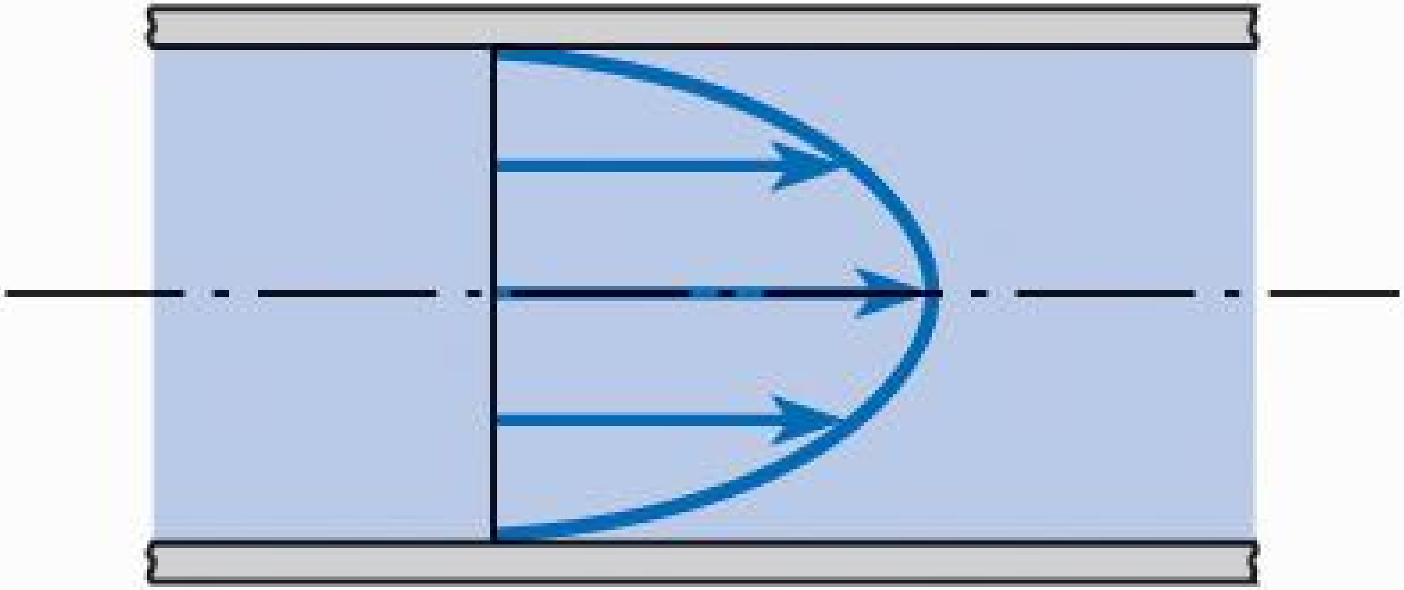
9.8 Under certain conditions (pressure decreasing in the x-direction and a moving plate), the laminar velocity distribution will be as shown here. For such a condition, indicate whether each of the following statements is true or false.

a. The greatest shear stress in the liquid occurs next to the fixed plate.
b. The shear stress midway between the plates is zero.
c. The minimum shear stress in the liquid occurs next to the moving plate.
d. The shear stress is greatest where the velocity is the greatest.
e. The minimum shear stress occurs where the velocity is the greatest.



PROBLEM 9.8

$$-\frac{dp}{dx}$$



9.8: PROBLEM DEFINITION

Situation:
A laminar velocity distribution is shown in figure

Find:
Whether statements (a) through (e) are true or false.

SOLUTION

a). True b). False c). False d). False e). True

Problem (9.20)

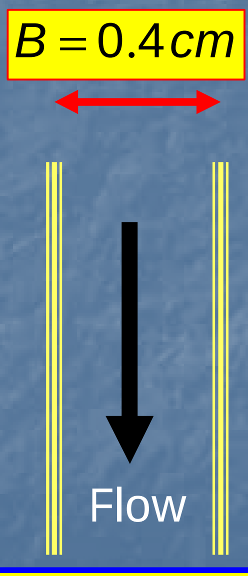
9.20 Glycerine at 20°C flows downward between two vertical parallel plates separated by a distance of 0.4 cm. The ends are open, so there is no pressure gradient. Calculate the discharge per unit width, q, in m^2/s.

Given

$$\frac{dP}{ds} = 0$$

$$u = -\frac{1}{2\mu}\frac{d}{ds}(p+\gamma z)(By-y^2) = -\frac{\gamma}{2\mu}(By-y^2)\frac{dh}{ds}$$

$$q = -\left(\frac{B^3}{12m}\right)\frac{d}{ds}(p+gz) = \left(\frac{gB^3}{12m}\right)\left(\frac{dh}{ds}\right)$$



Plates are of Unit Width

9.20: PROBLEM DEFINITION

Situation:

Glycerin flows downward between two plates spaced 0.4 cm apart. Both ends open — no pressure gradient.

Find:

Discharge per unit width.

$$\frac{q}{width=1}=?$$

Properties:

Table A.4 (Glycerin) $\mu = 1.41$ N·s/m^2, $\nu = 1.12$ m^2/s and $\gamma = 12,300$ N/m^3.

Assumptions:

Flow will be laminar.

SOLUTION

Use Eq. 9.8

$$q=-\left(\frac{B^3}{12m}\right)\frac{d}{ds}(p+gz)=\left(\frac{gB^3}{12m}\right)\left(\frac{dh}{ds}\right)$$

$$q = -\frac{B^3\gamma}{12\mu}\frac{dh}{ds}$$

$$h=\frac{P}{g}+Z$$

$$\frac{dh}{ds} = \frac{d}{ds}\left(\frac{p}{\gamma}+z\right) = \left(\frac{1}{\gamma}\right)\frac{dp}{ds}+\frac{dz}{ds} = -1$$

$$For\ Flow\ Downwards\ \left(\frac{dz}{ds}=-1\right)$$

$$For\ Flow\ Upwards\ \left(\frac{dz}{ds}=+1\right)$$

$$
\begin{aligned}
q &= -\left(\frac{B^3\gamma}{12\mu}\right)(-1) \\
&= -\left[\frac{(0.004\ \text{m})^3 \times 12,300\ \text{N/m}^3}{12 \times 1.41\ \text{N}\cdot\text{s/m}^2}\right](-1)
\end{aligned}
$$

$$
\boxed{q = 4.65 \times 10^{-5}\ \text{m}^2/\text{s}}
$$

Now check to see if the flow is laminar (Reynolds number $< 1,000$)

$$
\begin{aligned}
\text{Re} &= \frac{VB}{v} = \frac{q}{v} \\
&= \frac{4.65 \times 10^{-5}\ \text{m}^2/\text{s}}{1.12 \times 10^{-3}\ \text{m}^2/\text{s}} \\
\text{Re} &= 0.0415 \leftarrow \text{Laminar}
\end{aligned}
$$

Therefore, the original assumption of laminar flow was correct.

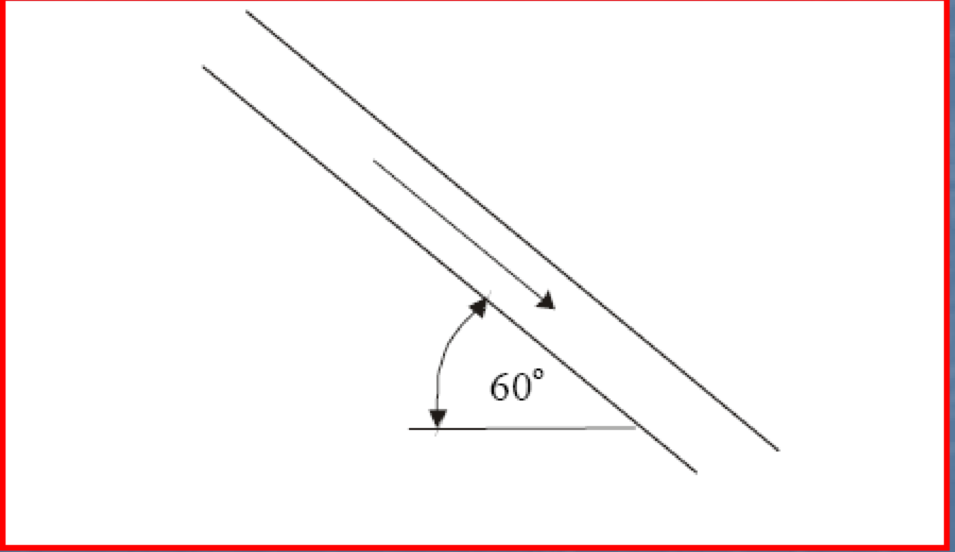
Problem (9.22)

9.22 Two parallel plates are spaced 0.2 cm apart, and motor oil (SAE 30) with a temperature of 40°C flows at a rate of $8 \times 10^{-4}\ m^3/s$ per meter of width between the plates. What is the pressure gradient in the direction of flow if the plates are inclined at 60° with the horizontal and if the flow is downward between the plates?

Given

$B = 2\,mm$

$q = 8\times10^{-4}\ m^3/s$ *per meter of width*



Find

Pressure gradient, $\dfrac{dP}{ds} = ?$

9.22: PROBLEM DEFINITION

Situation:

Flow of SAE 30 at 40°C occurs between two plates spaced 0.2 cm apart and inclined at 60° with a rate of $8 \times 10^{-4} \frac{\text{m}^3/\text{s}}{\text{m}}$. Flow is downward.

Find:

Pressure gradient in the direction of flow.

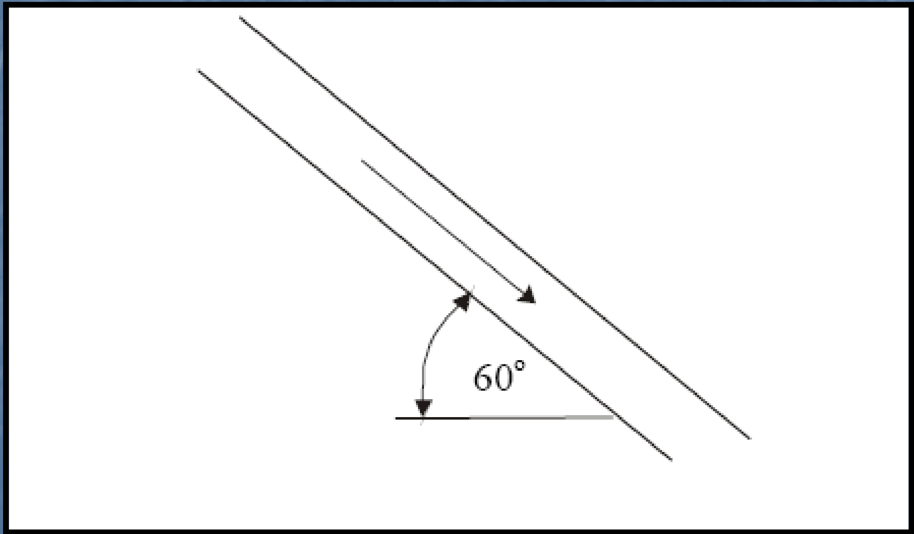
Properties:

From Table A.4 $\mu = 1 \times 10^{-1}$ N·s/m^2; $\gamma = 8,630$ N/m^3.

$$q = -\left(\frac{B^3}{12\mu}\right)\frac{d}{ds}(p+\gamma z) = \left(\frac{\gamma B^3}{12\mu}\right)\left(\frac{dh}{ds}\right)$$

$$Average\ Velocity = \bar{u} = \frac{2}{3}u_{max}$$

$$u_{max} = -\left(\frac{B^2\gamma}{8\mu}\right)\frac{dh}{ds}$$

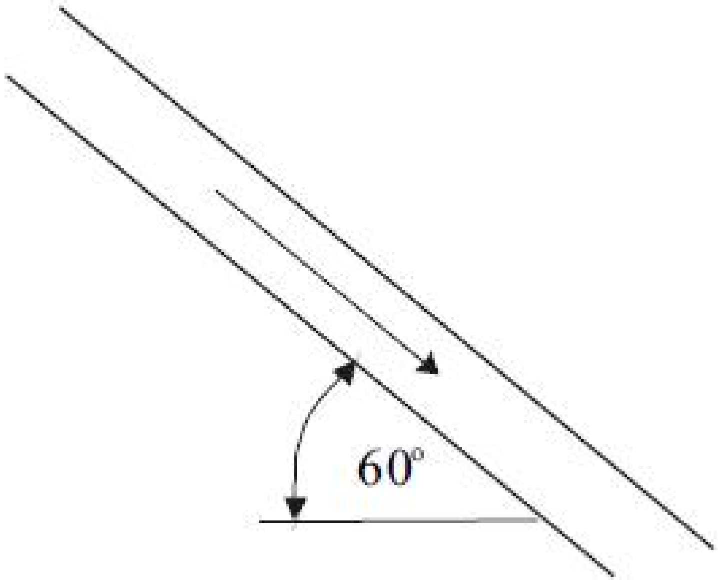


SOLUTION

Flow rate and maximum velocity

$$\bar{V} = \frac{q}{B} = \frac{8 \times 10^{-4}\ \text{m}^2/\text{s}}{0.2\ \text{cm} \times \frac{1\ \text{m}}{100\ \text{cm}}} = 0.4\ \text{m/s}$$

$$u_{\max} = (3/2)\bar{V} = 0.6\ \text{m/s}$$



$$u_{\max} = -\frac{B^2\gamma}{8\mu}\frac{dh}{ds}$$

$$\frac{dh}{ds} = -\left(\frac{8\mu u_{\max}}{\gamma B^2}\right)$$

$$= -\left(\frac{8 \times (1 \times 10^{-1}\ \text{N-s/m}^2) \times 0.6\ \text{m/s}}{8,630\ \text{N/m}^3 \times (0.002\ \text{m})^2}\right)$$

$$= -14.0$$

But

$$\frac{dh}{ds} = \left(\frac{1}{\gamma}\right)\frac{dp}{ds} + \frac{dz}{ds}$$

where $dz/ds = -0.866$. Then

$$-14 = \left(\frac{1}{\gamma}\right)\frac{dp}{ds} - 0.866$$

$$\frac{dp}{ds} = \gamma(-14 + 0.866)$$

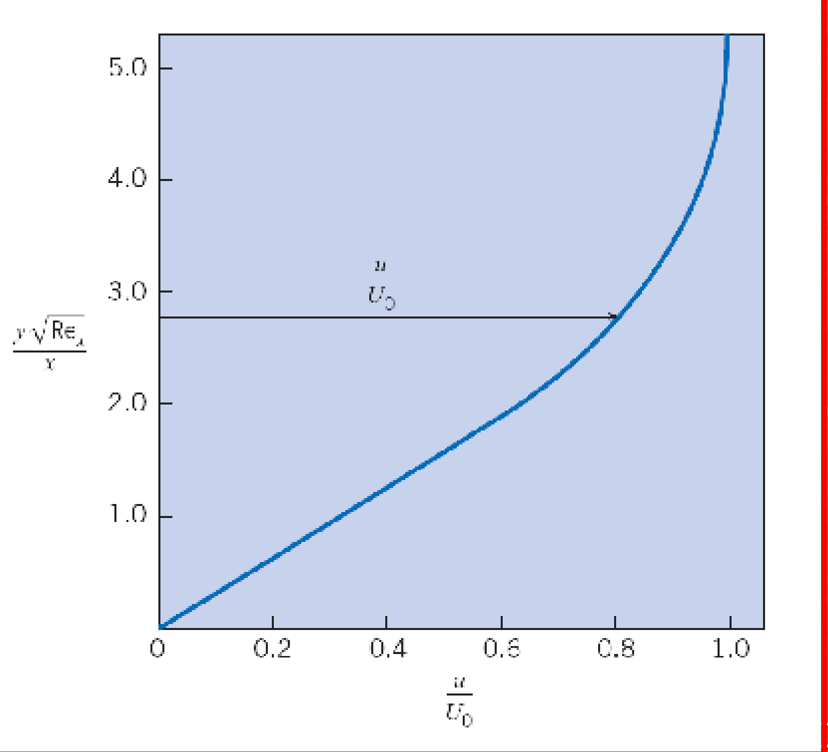
$$= 8,630(-14 + 0.866)\frac{\text{N/m}^2}{\text{m}}$$

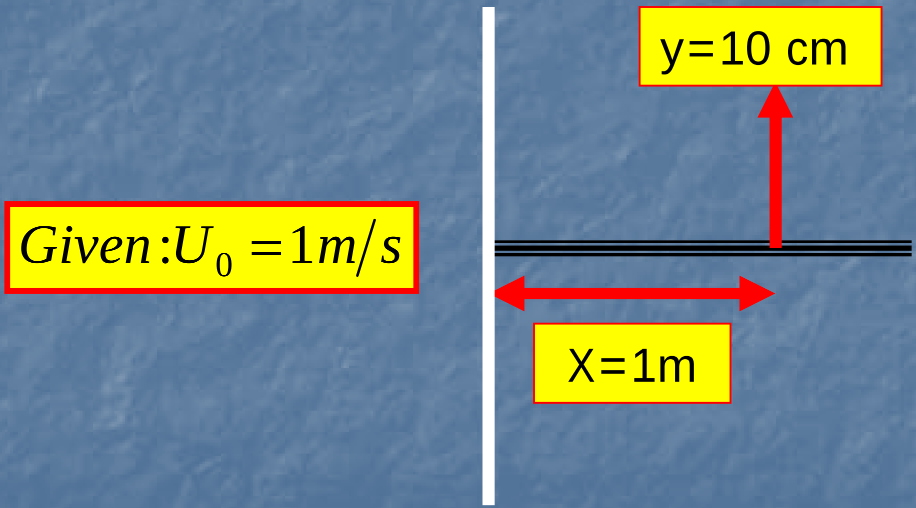
$$= \frac{-113.3\ \text{kPa}}{\text{m}}$$

$$\boxed{\frac{dp}{ds} = -113.3\ \text{kPa/m}}$$

Problem (9.40)

9.40 Oil ($\nu = 10^{-4}$ m^2/s; S = 0.9) flows past a plate in a tangential direction so that a boundary layer develops. If the velocity of approach is 1 m/s, what is the oil velocity 1 m downstream from the leading edge and 10 cm away from the plate?





9.40: PROBLEM DEFINITION

Situation:

Oil flows over a flat plate at 1 m/s.

Find:

Oil velocity 1 m from leading edge and 10 cm from surface.

Properties:

$\nu = 10^{-4}\ \text{m}^2/\text{s}$

PLAN

Calculate Reynolds number and apply figure for velocity distribution in laminar boundary layer.

SOLUTION

Reynolds number

$$\text{Re}_x = \frac{1\ \text{m/s} \times 1\ \text{m}}{10^{-4}\ \text{m}^2/\text{s}} = 10^4$$

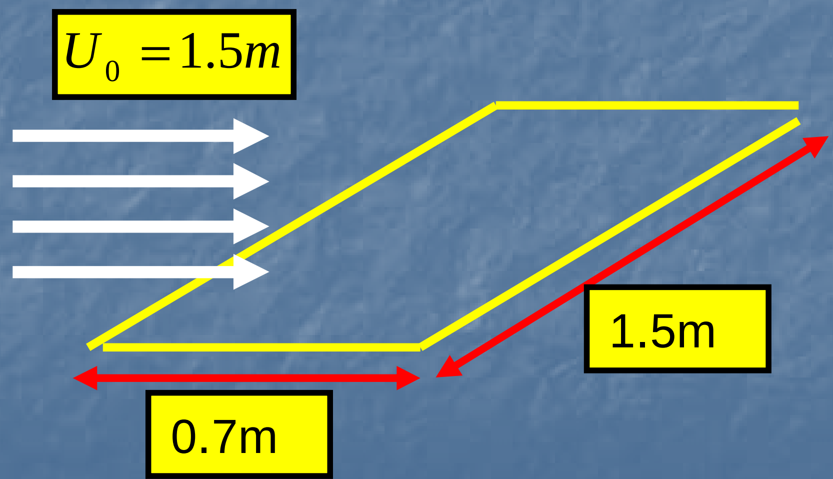
The boundary layer is laminar.

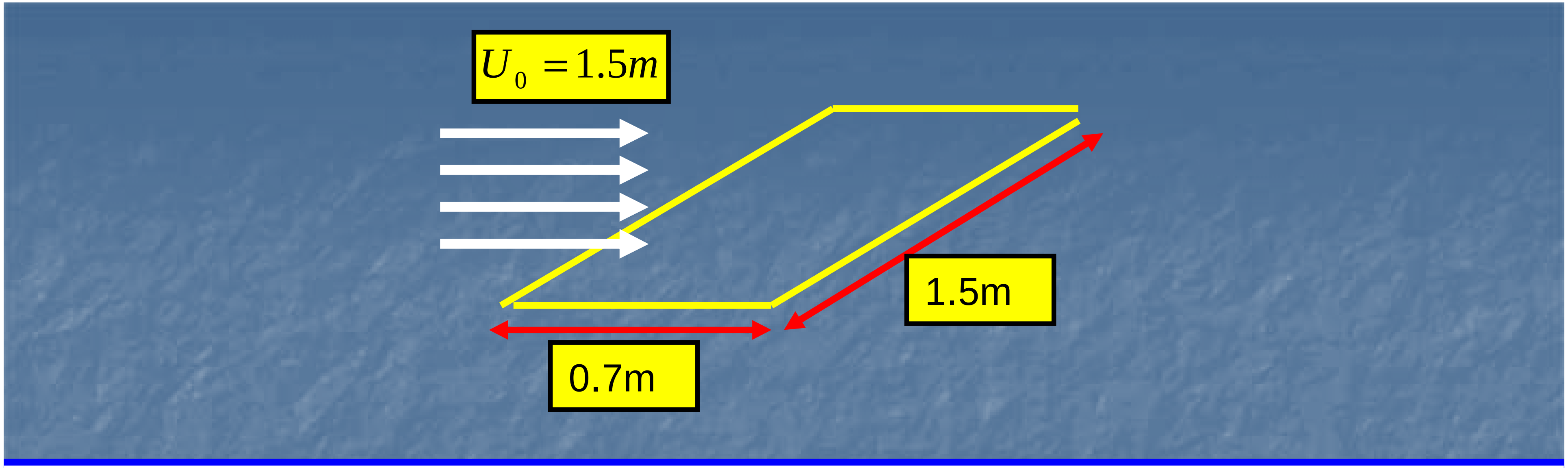
$$\frac{y\text{Re}_x^{0.5}}{x} = \frac{0.10\,\text{m} \times 10^2}{1\,\text{m}} = 10$$

Therefore the point is outside the boundary layer so $\boxed{u = U_0 = 1\ \text{m/s.}}$

Problem (9.41)

9.41 A thin plate 0.7 m long and 1.5 m wide is submerged and held stationary in a stream of water ($T = 10°C$) that has a velocity of 1.5 m/s. What is the thickness of the boundary layer on the plate for $Re_x = 500{,}000$ (assume the boundary layer is still laminar), and at what distance downstream of the leading edge does this Reynolds number occur? What is the shear stress on the plate on this point?





PROBLEM 9.47

Situation: Water flows over a submerged flat plate.
Plate length is $L = 0.7$ m and the width is $W = 1.5$ m.
Free stream velocity is $U_o = 1.5$ m/s.

Find:
(a) Thickness of boundary layer at the location where $Re_x = 500,000$.
(b) Distance from leading edge.where the Reynolds number reaches 500,000.
(c) Local shear stress.at the location where $Re_x = 500,000$.

Properties: Table A.5 (water at 10 °C): $\rho = 1000\,\mathrm{kg/m^3}$, $\mu = 1.31 \times 10^{-3}\,\mathrm{N \cdot s/m^2}$, $\nu = 1.31 \times 10^{-6}\,\mathrm{m^2/s}$.

APPROACH

Calculate Reynolds number. Next calculate boundary layer thickness and local shear stress.

$$Re_x = 500,000$$

$$500,000 = \frac{U_0 x}{\nu}$$

$$x = \frac{500,000\nu}{U_0}$$

$$= \frac{500000 \times (1.31 \times 10^{-6}\, \mathrm{m^2/s})}{1.5\, \mathrm{m/s}}$$

$$= 0.4367\, \mathrm{m}$$

$$\boxed{\text{b.)} \quad x = 0.437\, \mathrm{m}}$$

Boundary layer thickness correlation

$$C_f = \frac{1.33}{(\mathrm{Re})_L^{1/2}}$$

$$t_0 = C_f \left(1/2\, r U_0^{\,2}\right)\left(BX_{cr}\right)$$

$$\delta = \frac{5x}{\mathrm{Re}_x^{1/2}} \quad \text{....Laminar flow}$$

$$= \frac{5 \times 0.4367\, \mathrm{m}}{\sqrt{500000}}$$

$$= 3.09 \times 10^{-3}\ \mathrm{m}$$

$$\boxed{\text{a.)} \quad \delta = 3.09\ \mathrm{mm}}$$

Local shear stress correlation

Laminar Flow

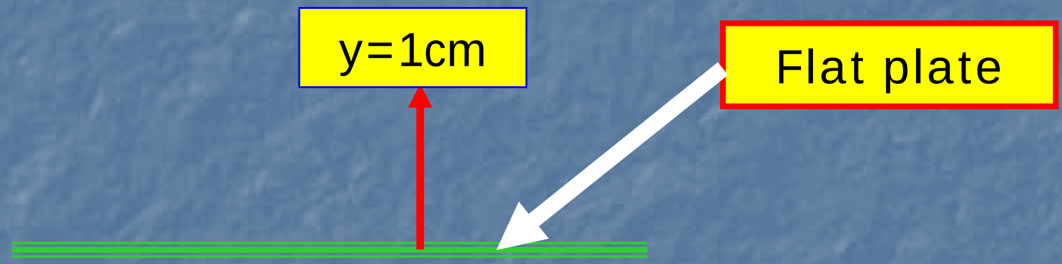
$$\tau_0 = 0.332\mu(U_0/x)\mathrm{Re}_x^{1/2}$$

$$= 0.332 \times 1.31 \times 10^{-3}(1.5/0.4367) \times (500,000)^{1/2}$$

$$\boxed{\text{c.)} \quad \tau_0 = 1.06\ \mathrm{N/m^2}}$$

Problem (9.49)

9.49 A turbulent boundary layer exists in the flow of water at 20°C over a flat plate. The local shear stress measured at the surface of the plate is 0.1 N/m^2. What is the velocity at a point 1 cm from the plate surface?



Situation:

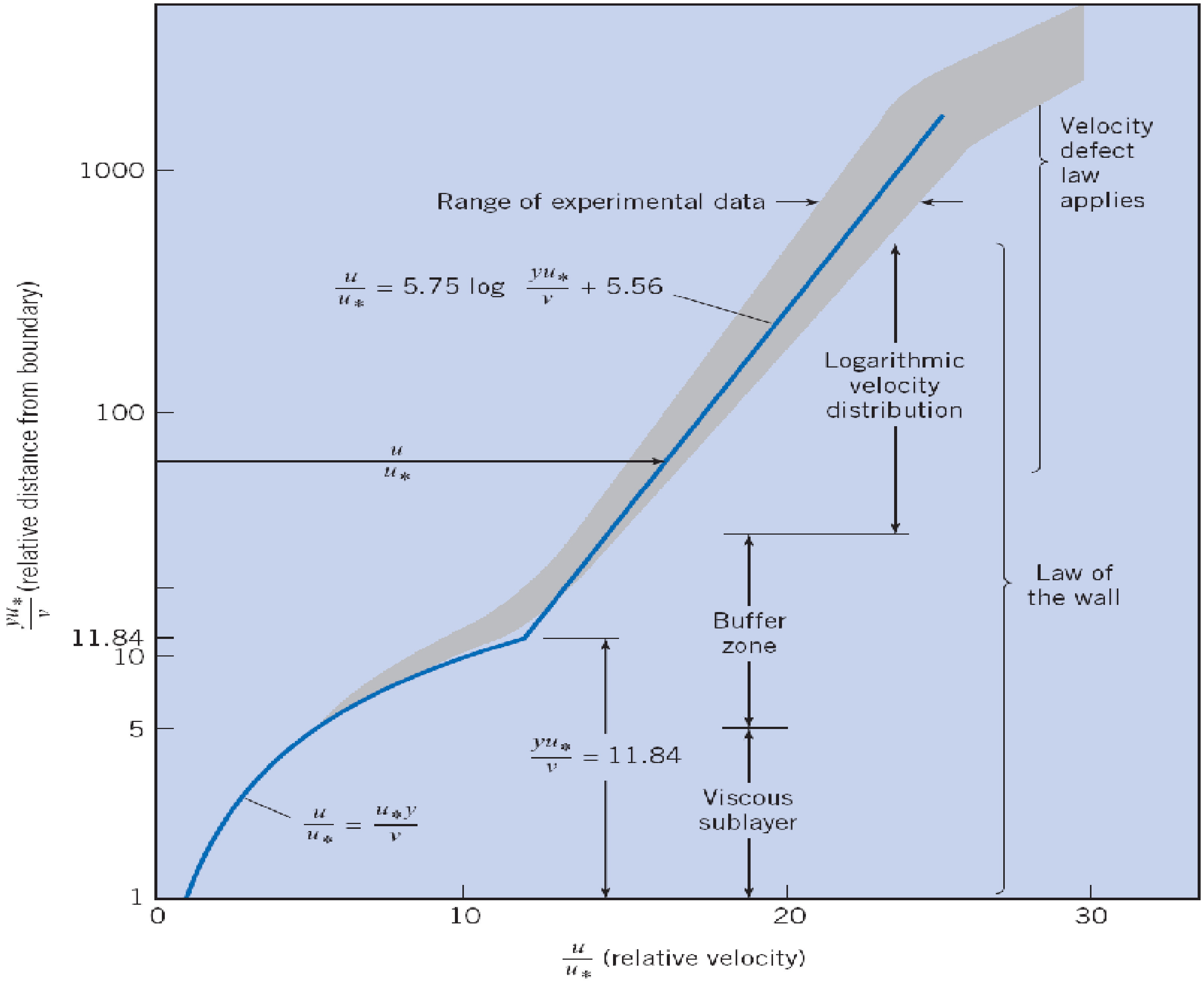
Water at 20°C flows over creating a turbulent boundary layer. Local shear stress at point is 0.1 N/m^2.

Find:

Velocity 1 cm above plate surface.

Properties:

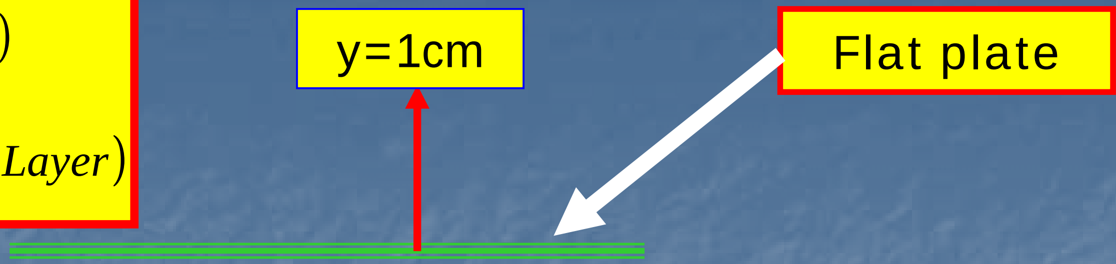
From Table A.5 $\rho = 998\ kg/m^3$; $\nu = 10^{-6}\ m^2/s$.



1000
100
11.84
10
5
1
0
10
20
30
Range of experimental data
Velocity defect law applies
$\frac{u}{u_*} = 5.75 \log \frac{yu_*}{\nu} + 5.56$
Logarithmic velocity distribution
$\frac{u}{u_*}$
Law of the wall
Buffer zone
$\frac{yu_*}{\nu} = 11.84$
Viscous sublayer
$\frac{u}{u_*} = \frac{u_* y}{\nu}$
$\frac{yu_*}{\nu}$ (relative distance from boundary)
$\frac{u}{u_*}$ (relative velocity)

$$\frac{yu_*}{n} \le 5 \; (Viscous\ Sub-Layer)$$

$$30 < \frac{yu_*}{n} < 500 \; (Logarithmic\ Layer)$$



SOLUTION

Local shear velocity and nondimensional wall distance.

$$u_* = \left(\frac{\tau_0}{\rho}\right)^{0.5} = \left(\frac{0.1 \text{ N/m}^2}{998 \text{ kg/m}^3}\right)^{0.5} = 0.01 \text{ m/s}$$

$$\frac{u_* y}{v} = \frac{(0.01 \text{ m/s})(0.01 \text{ m})}{10^{-6} \text{ m}^2\text{/s}} = 10^2$$

The point is in the law of the wall region since $11.6 < u_* y/\nu < 500$ so

$$\frac{u}{u_*} = 2.44 \ln\left(\frac{yu_*}{v}\right) + 5.56$$

$$= 2.44 \ln(100) + 5.56 = 16.8$$

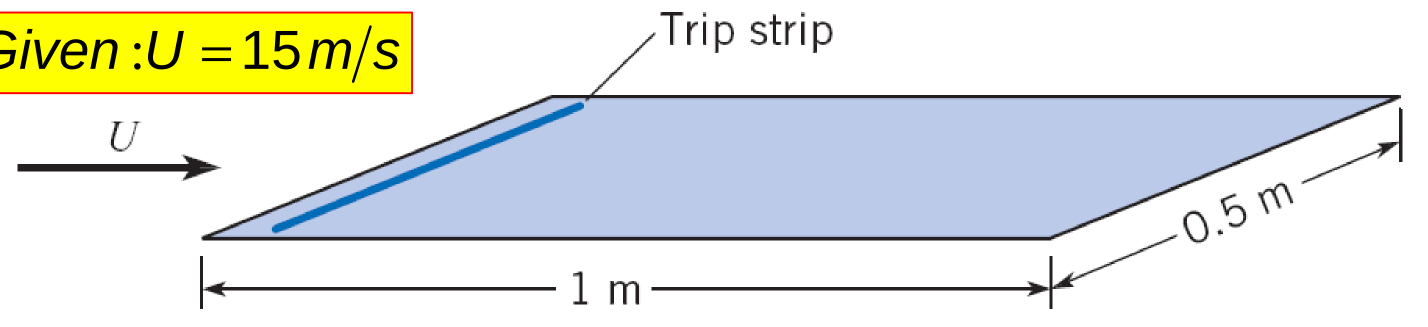
$$u = u_* \times 16.8 = 0.01 \text{ m/s} \times 16.8$$

$$\boxed{u = 0.168 \text{ m/s}}$$

Problem (9.61)

9.61 A flat plate is oriented parallel to a 15 m/s airflow at 20°C and atmospheric pressure. The plate is 1 m long in the flow direction and 0.5 m wide. On one side of the plate, the boundary layer is tripped at the leading edge, and on the other side there is no tripping device. Find the total drag force on the plate.

Given: $U = 15\,m/s$



Tripped Plate at the Leading Edge

Situation:

A 15 m/s flow in air over a flat plate 1 m long and 0.5 m wide. Boundary layer tripped on one side but not on other.

Find:

Total drag force on plate.

Properties:

From Table A.3 $\rho = 1.2\ \text{kg/m}^3$; $\nu = 1.51 \times 10^{-5}\ \text{m}^2/\text{s}$.

SOLUTION

$$C_F = \frac{0.032}{(R_e)_L^{1/7}}$$

Crowe (8) Edition

The drag force (due to shear stress) is

$$F_s = C_f \frac{1}{2} \rho U_o^2 B L$$

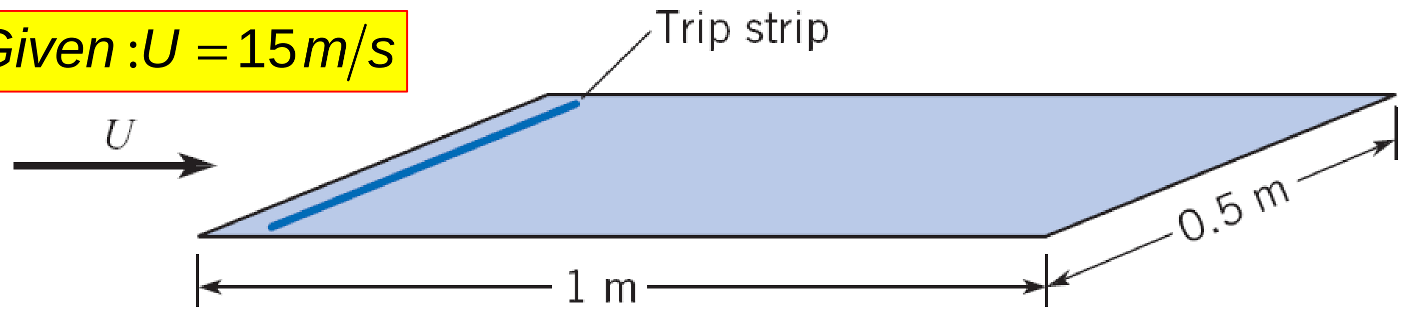
The Reynolds number based on the plate length is

$$\text{Re}_L = \frac{15\ \text{m/s} \times 1\ \text{m}}{1.51 \times 10^{-5}\ \text{m}^2/\text{s}} = 9.93 \times 10^5$$

which is less than 10^7. The average shear stress coefficient on the "tripped" side of the plate is

$$C_f = \frac{0.032}{(9.93 \times 10^5)^{1/7}} = 0.00445$$

Crowe (9) Edition



Tripped Plate at the Leading Edge

The average shear stress coefficient on the "untripped" side is

$$C_f = \frac{0.523}{\ln^2(0.06 \times 9.93 \times 10^5)} - \frac{1520}{9.93 \times 10^5} = 0.00280$$

The total force is

$$F_s = \frac{1}{2} \times 1.2\,\text{kg}/\text{m}^3 \times (15\,\text{m}/\text{s})^2 \times 1 \times 0.5 \times (0.00445 + 0.00280)$$

$$\boxed{F_s = 0.493\ \text{N}}$$

END OF SOLVED PROBLEMS