CHAPTER (10)

FLOW IN CONDUITS

SOLVED PROBLEMS

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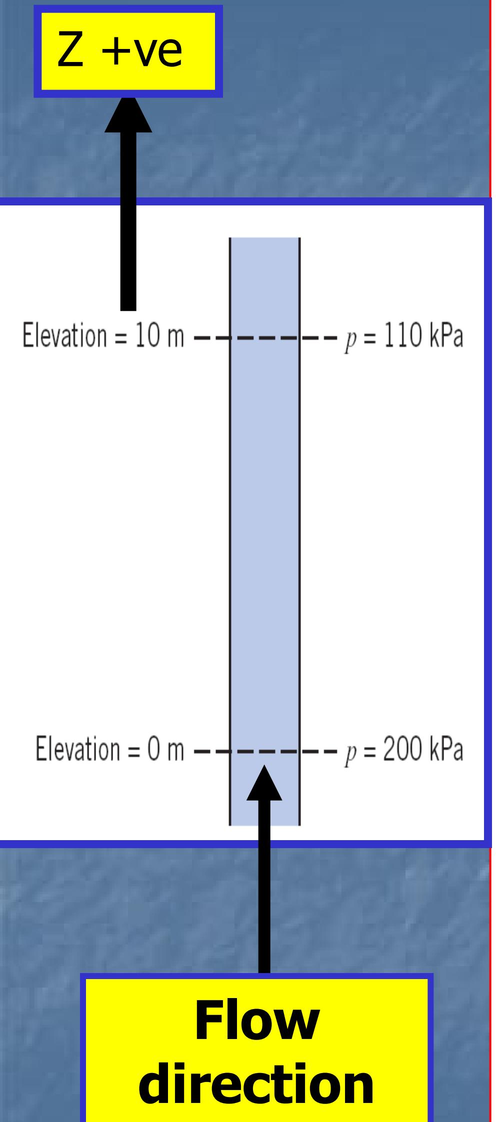
Problem (10.2)



PROBLEM 10.2

Situation: Liquid flows in a vertical pipe—details are provided in the problem statement

Find: (a) Determine the direction of flow.
(b) Calculate the mean fluid velocity in pipe.

$\gamma = 8\,kN/m^2$,
$d_{pipe} = 1cm$
$a = zero$

ANALYSIS

Energy equation

$$p_0/\gamma + \alpha_o V_0^2/2g + z_0 = p_{10}/\gamma + \alpha_{10} V_{10}^2/2g + z_{10} + h_L$$

To evaluate, note that $\alpha_o V_0^2/2g = \alpha_{10} V_{10}^2/2g$. Substituting values gives

$$200,000/8000 + 0 = 110,000/8000 + 10 + h_f$$
$$h_f = 1.25\text{ m}$$

$p + \gamma z$ is decreasing

Because h_L is positive, the flow must be upward.

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

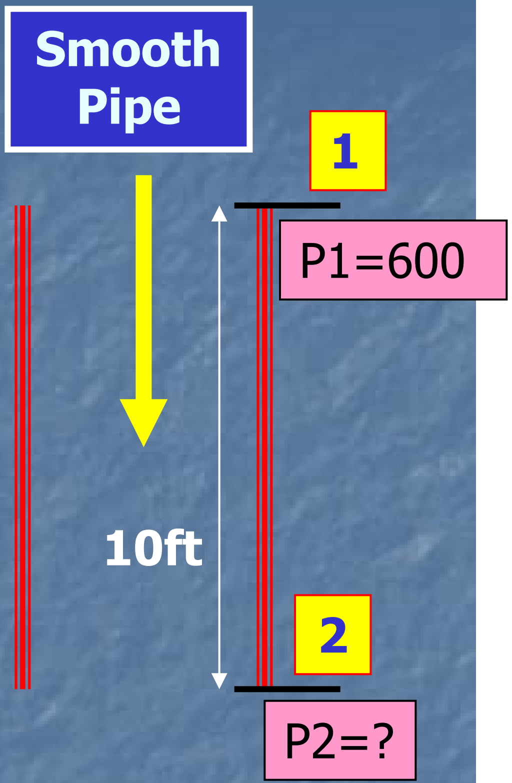
Head loss (laminar flow)

$$h_f = \frac{32\mu LV}{\gamma D^2}$$
$$V = \frac{h_f \gamma D^2}{32\mu L}$$
$$= \frac{1.25 \times 8000 \times 0.01^2}{32 \times (3.0 \times 10^{-3}) \times 10}$$
$$= 1.042\text{ m/s}$$

For Laminar Flow: $f = \frac{R_e}{64}$

$$V = 1.04\text{ m/s}$$

Problem (10.5)



PROBLEM 10.5

Situation: Liquid flows downward in a smooth vertical pipe. $D = 1\,\mathrm{cm}$ $\bar{V} = 2.0\,\mathrm{m/s}$ $p_1 = 600\,\mathrm{kPa}$

Find: Pressure at a section that is 10 feet below section 1.

Properties: $\rho = 1000\,\mathrm{kg/m^3}$ $\mu = 0.06\,\mathrm{N \cdot s/m^2}$

ANALYSIS

Reynolds number

$$\begin{aligned} \mathrm{Re} &= \frac{VD\rho}{\mu} \\ &= \frac{2 \times 0.01 \times 1000}{0.06} \\ &= 333 \end{aligned}$$

Fully developed flow

Since $\mathrm{Re} < 2000$, the flow is laminar.

Energy principle

$$p_1/\gamma + \alpha_1 V_1^2/2g + z_1 = p_2/\gamma + \alpha_1 V_2^2/2g + z_2 + h_L$$

Since $V_1 = V_2$, the velocity head terms (i.e. kinetic energy terms) cancel. The energy equation becomes

Head Loss

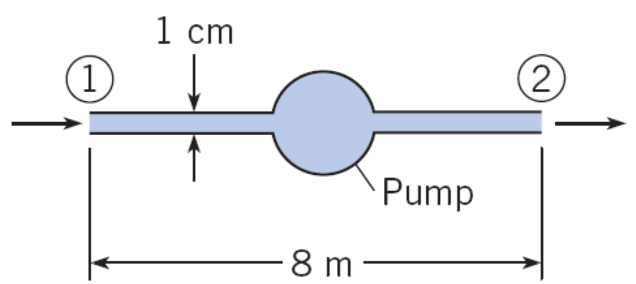
$$\begin{aligned} 600,000/(9.81 \times 1000) + 10 &= p_2/\gamma + 0 + 32\mu LV/\gamma D^2 \\ p_2/\gamma &= 600,000/\gamma + 10 - 32 \times 0.06 \times 10 \times 2/(\gamma(0.01)^2) \\ p_2 &= 600,000 + 10 \times 9810 - 384,000 \\ &= \boxed{314\ \mathrm{kPa}} \end{aligned}$$

Problem (10.10)

$Q = 7.85\times10^{-4}$

$p_1 = p_2$

$\eta_{Pump} = 100\%$



$Find : (Power)_{Pump}$

$\dot{W}_{Pump} = ?$

PROBLEM 10.10

Situation: SAE 10-W oil is pumped through a tube—other details are provided in the problem statement

Find: Power to operate the pump.

ANALYSIS

Energy equation

$$p_1/\gamma + z_1 + \alpha_1 V_1^2/2g + h_p = p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + h_L$$

Simplify

$$h_p = h_L = f(L/D)(V^2/2g)$$

Flow rate equation

$$V = Q/A = 7.85 \times 10^{-4}/((\pi/4)(0.01)^2) = 10 \text{ m/s}$$

Reynolds number

$$\text{Re} = VD/\nu = (10)(0.01)/(7.6 \times 10^{-5}) = 1316 \text{ (laminar)}$$

Friction factor (f)

$$\begin{aligned} f &= \frac{64}{\text{Re}} \\ &= \frac{64}{1316} \\ &= 0.0486 \end{aligned}$$

Head of the pump

$$\begin{aligned} h_p &= f(L/D)(V^2/2g) \\ &= 0.0486(8/0.01)(10^2/((2)(9.81)) \\ &= 198 \text{ m} \end{aligned}$$

Power equation

$$\begin{aligned} P &= h_p \gamma Q \\ &= 198 \times 8630 \times (7.85 \cdot 10^{-4}) \\ &= \boxed{1341 \text{ W}} \end{aligned}$$

Problem (10.13)

PROBLEM 10.13

Situation: Kerosene flows in a pipe.
$T = 20^oC$, $Q = 0.02\,m^3/s$, $D = 20\,cm$

Find: Determine if the flow is laminar or turbulent.

ANALYSIS

$$\begin{aligned} Re &= VD\rho/\mu \\ &= (Q/A)D/\nu \\ &= 4Q/(\pi D\nu) \\ &= 4 \times 0.04/(\pi \times 0.25 \times 2.37 \times 10^{-6}) \\ &= 85,957 \end{aligned}$$

Flow is turbulent

Problem (10.18)

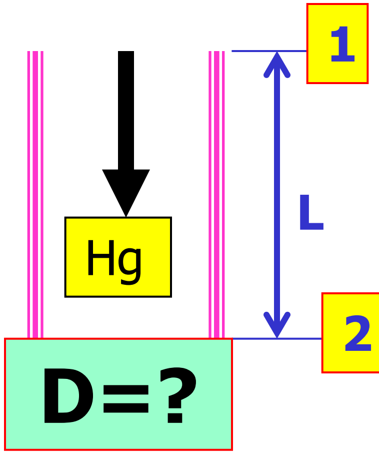
PROBLEM 10.18

Situation: Mercury flows downward through a long round tube. $T = 20°\text{C}$
The tube is oriented vertically and open at both ends.

Find: Largest tube diameter so that the flow is still laminar.

Properties: From Table A.4: $\mu = 1.5 \times 10^{-3}\ \text{N·s/m}^2$, $\nu = 1.2 \times 10^{-7}\ \text{m}^2\text{/s}$, $\gamma = 133,000\ \text{N/m}^3$

Assumptions: The tube is smooth.



ANALYSIS

Energy equation

$$p_1/\gamma + \alpha_1 V_1^2/2g + z_1 = p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + h_L$$

Term by term analysis

$$p_1 = p_2; \quad V_1 = V_2; \quad \alpha_1 = \alpha_2; \quad z_1 - z_2 = L$$

The energy equation

$$L = h_L \tag{1}$$

Head loss (laminar flow)

$$h_L = h_f = \frac{32\mu L V}{\gamma D^2} \tag{2}$$

CONT.

Combining Eqs. (1) and (2)

$$\frac{h_L \gamma D^2}{32\mu V} = h_L$$

$$\frac{\gamma D^2}{32\mu V} = 1 \qquad (3)$$

Reynolds number

$$\text{Re} = \frac{VD}{\nu} = 2000$$

$$V = \frac{2000\nu}{D} \qquad (4)$$

Combining Eqs. (3) and (4)

$$\frac{\gamma D^3}{64,000\mu\nu} = 1$$

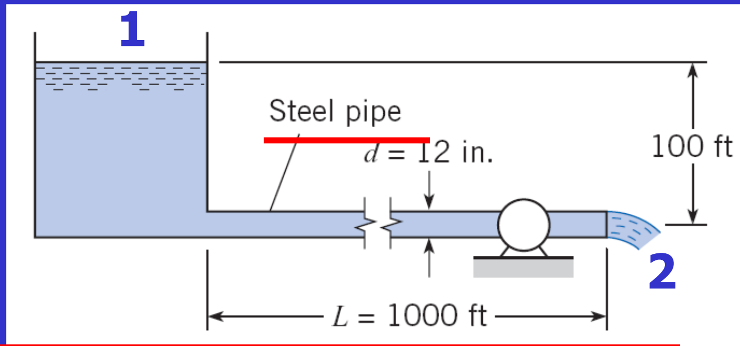
or

$$D = \sqrt[3]{\frac{64,000\mu\nu}{\gamma}}$$

$$= \sqrt[3]{\frac{(64,000)\left(1.5 \times 10^{-3}\right)\left(1.2 \times 10^{-7}\right)}{133,000}}$$

$$= \boxed{4.43 \times 10^{-4}\ \text{m}}$$

Problem (10.44)



$Q = 5\,cfs$

$\eta_{Turbine} = 80\%$

$\dot{W}_{Turbine} = ?$

Find: Power delivered by turbine.

Properties: From Table A.5 $\nu(70°F) = 1.06 \times 10^{-5}$ ft^2/s

Assumptions: turbulent flow, so $\alpha_2 \approx 1$.

APPROACH

Apply the energy equation from the reservoir water surface to the jet at the end of the pipe.

ANALYSIS

Energy equation

$$\begin{aligned}
p_1/\gamma + \alpha_1 V_1^2/2g + z_1 &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + h_T + \sum h_L \\
0 + 0 + z_1 &= 0 + \alpha_2 V_2^2/2g + z_2 + h_T + (K_e + fL/D)V^2/2g \\
z_1 - z_2 &= h_T + (1 + 0.5 + fL/D)V^2/2g \\
100 \text{ ft} &= h_T + (1.5 + fL/D)V^2/2g
\end{aligned}$$

CONT.

But

$$
\begin{aligned}
V &= Q/A = 5/((\pi/4)1^2) = 6.37 \text{ ft/s} \\
V^2/2g &= 0.629 \text{ ft} \\
\text{Re} &= VD/\nu = 6.0 \times 10^5
\end{aligned}
$$

From Fig. 10.8 $f = 0.0140$ for $k_s/D = 0.00015$. Then **Steel Pipe**

$$
\begin{aligned}
100 \text{ ft} &= h_T + (1.5 + 0.0140 \times 1,000/1)(0.629) \\
h_T &= (100 - 9.74) \text{ ft}
\end{aligned}
$$

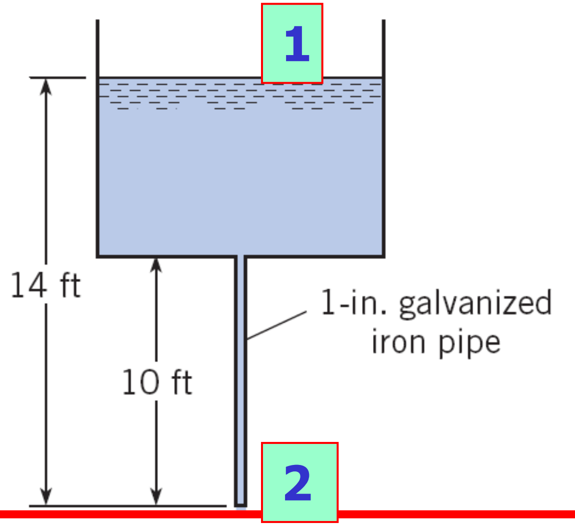
Power equation

Turbine Efficiency = 80%

$$
\begin{aligned}
P &= Q\gamma h_T \times \text{eff} \\
&= 5 \times 62.4 \times 90.26 \times 0.80 \\
&= 22,529 \text{ ft} \cdot \text{lbf/s} \\
&= \boxed{40.96 \text{ horsepower}}
\end{aligned}
$$

Problem (10.51)

$$V_{Pipe} = ?$$



Situation: Water drains from a tank through a galvanized iron pipe. $D = 1$ in. Total elevation change is 14 ft. Pipe length = 10 ft.

Find: Velocity in pipe.

Properties: Kinematic viscosity of water is 1.22×10^{-5} ft^2/s. From Table 10.3 $K_e = 0.5$. From Table 10.3, $k_s = 0.006$ inches.

Assumptions: Assume turbulent flow (check after calculations are done). Assume $\alpha_1 \approx 1.00$.

APPROACH

Apply the energy equation from the water surface in the tank to the outlet of the pipe. Use the Darcy-Weisbach equation for head loss. Assume turbulent flow and then solve the resulting equations using an iterative approach.

CONT.

Energy equation

$$
\begin{aligned}
\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \sum h_L \\
0 + 0 + 14 &= 0 + \frac{V_2^2}{2g} + 0 + (K_e + f\frac{L}{D})\frac{V_2^2}{2g} \\
14\,\text{ft} &= \left(1 + K_e + f\frac{L}{D}\right)\frac{V_2^2}{2g} \\
14\,\text{ft} &= \left(1 + 0.5 + f\frac{(120\,\text{in})}{(1\,\text{in})}\right)\frac{V_2^2}{2g} \qquad (1)
\end{aligned}
$$

Eq. (1) becomes

$$V^2 = \frac{2 \times (32.2\,\text{ft/s}^2) \times (14\,\text{ft})}{1.5 + 120 \times f}$$

Guess $f = 0.02$ and solve for V

$$
\begin{aligned}
V^2 &= \frac{2 \times (32.2\,\text{ft/s}^2) \times (14\,\text{ft})}{1.5 + 120 \times 0.02} \\
V &= 15.2\,\text{ft/s}
\end{aligned}
$$

Reynolds number (based on the guessed value of friction factor)

$$
\begin{aligned}
\text{Re} &= \frac{VD}{\nu} \\
&= \frac{(15.2\,\text{ft/s})\,(1/12\,\text{ft})}{1.22 \times 10^{-5}\,\text{ft}^2/\text{s}} \\
&= 103,856
\end{aligned}
$$

CONT.

Resistance coefficient (new value)

Swamee – Jean Eqn.

OR Moody's Diagram

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2}$$

$$= \frac{0.25}{\left[\log_{10}\left(\frac{0.006}{3.7} + \frac{5.74}{103856^{0.9}}\right)\right]^2}$$

$$= 0.0331$$

Recalculate V based on $f = 0.0331$

$$V^2 = \frac{2 \times (32.2\,\mathrm{ft/s^2}) \times (14\,\mathrm{ft})}{1.5 + 120 \times 0.0331}$$

$$V = 12.82\,\mathrm{ft/s}$$

Reynolds number (recalculate based on $V = 12.82\,\mathrm{ft/s}$)

$$\mathrm{Re} = \frac{(12.8\,\mathrm{ft/s})(1/12\,\mathrm{ft})}{1.22 \times 10^{-5}\,\mathrm{ft^2/s}}$$

$$= 874.316$$

CONT.

Recalculate f based on $\mathrm{Re} = 874,316$

$$
\begin{aligned}
f &= \frac{0.25}{\left[\log_{10}\left(\frac{0.006}{3.7} + \frac{5.74}{874316^{0.9}}\right)\right]^2} \\
&= 0.0333
\end{aligned}
$$

Recalculate V based on $f = 0.0333$

$$
\begin{aligned}
V^2 &= \frac{2 \times (32.2\,\mathrm{ft/s^2}) \times (14\,\mathrm{ft})}{1.5 + 120 \times 0.0333} \\
V &= 12.80\,\mathrm{ft/s}
\end{aligned}
$$

Since velocity is nearly unchanged, stop!

$$
\boxed{V = 12.80\,\mathrm{ft/s}}
$$

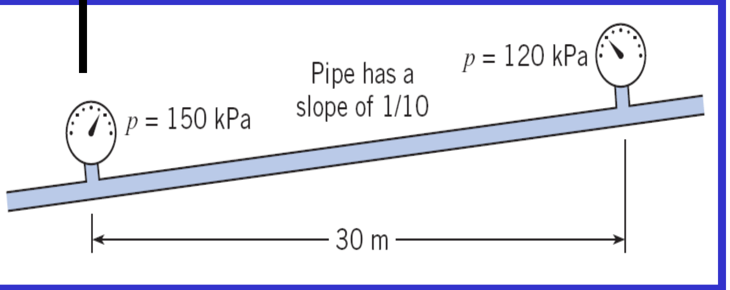
1. The Reynolds number 874,000 is much greater than 3000, so the assumption of turbulent flow is justified.
2. The solution approach, iteration with hand calculations, is straightforward. However, this problem can be solved faster by using a computer program that solves simultaneous, nonlinear equations.

Problem (10.63)

Example of case (2) where h_f is not given

1

2



PROBLEM 10.63

Situation: A fluid flows through a pipe made of galvanized iron. $D = 8\,\text{cm}$ $\nu = 10^{-6}\,\text{m}^2/\text{s}$ $\rho = 800\,\text{kg/m}^3$.
Additional details are provided in the problem statement

Find: Flow rate.

Q = ?

Properties: From Table 10.2 $k_s = 0.15$ mm.

ANALYSIS

Energy equation

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + h_f \\ 150,000/(800 \times 9.81) + V_1^2/2g + 0 &= 120,000/(800 \times 9.81) + V_2^2/2g + 3 + h_f \\ h_f &= 0.823 \\ ((D^{3/2})/(\nu)) \times (2gh_f/L)^{1/2} &= ((0.08)^{3/2}/10^{-6}) \times (2 \times 9.81 \times 0.823/30.14)^{1/2} \\ &= 1.66 \times 10^4 \end{aligned}$$

Relative roughness

$$k_s/D = 1.5 \times 10^{-4}/0.08 = 1.9 \times 10^{-3}$$

Resistance coefficient. From Fig. 10-8 $f = 0.025$. Then

$$h_f = f(L/D)(V^2/2g)$$

Solving for V

$$\begin{aligned} V &= \sqrt{(h_f/f)(D/L)2g} \\ &= \sqrt{(0.823/0.025)(0.08/30.14) \times 2 \times 9.81} = 1.312 \text{ m/s} \\ Q &= VA \\ &= 1.312 \times (\pi/4) \times (0.08)^2 \\ &= \boxed{6.59 \times 10^{-3}\ \text{m}^3/\text{s}} \end{aligned}$$

Problem (10.69)

Case (3): Iterative procedure

PROBLEM 10.69

Situation: A steel pipe will carry crude oil. $S = 0.93$ $\nu = 10^{-5}\,\mathrm{m^2/s}$ $Q = 0.1\,\mathrm{m^3/s}$.

Available pipe diameters are $D = 20,\ 22$, and $24\,\mathrm{cm}$.

Specified head loss: $h_L = 50\,\mathrm{m}$ per km of pipe length.

Find: (a) Diameter of pipe for a head loss of 50 m.

(b) Pump power.

Properties: From Table 10.2 $k_s = 0.046$ mm.

$$d_{Pipe} = ?$$

$$\dot{W}_{Pump} = ?$$

ANALYSIS

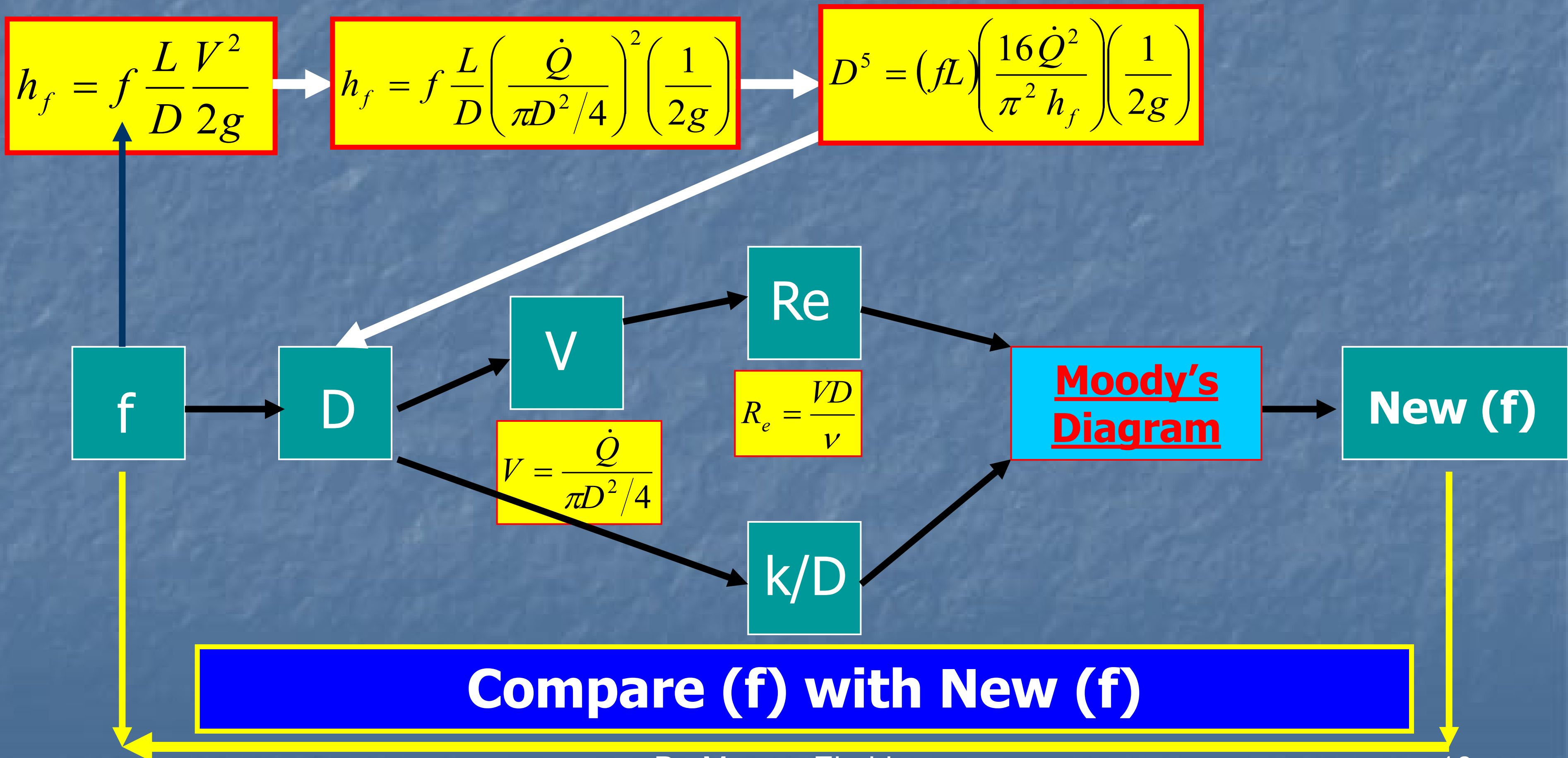
Darcy Weisbach equation

$$\begin{aligned} h_f &= f\frac{L}{D}\frac{V^2}{2g} \\ &= f\frac{L}{D}\frac{Q^2}{2gA^2} \\ &= f\frac{8LQ^2}{g\pi^2 D^5} \end{aligned}$$

Case (C): Pipe Size

Method (1): Assume an initial value for (f)

Given Values $K_{S,}\, h_f, \dot{Q}, L$



CONT.

Solve for diameter

$$D = \left(f \frac{8LQ^2}{g\pi^2 h_f} \right)^{1/5}$$

Assume $f = 0.015$

$$D = \left(0.015 \frac{8\,(1000)\,(0.1)^2}{9.81 \times \pi^2 \times 50} \right)^{1/5}$$

$$= 0.19\,\text{m}$$

Calculate a more accurate value of f

Compare

$$\text{Re} = 4Q/(\pi D \nu)$$

$$= 4 \times 0.1/(\pi \times 0.19 \times 10^{-5})$$

$$= 6.7 \times 10^4$$

Swamee – Jean Eqn.

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2}$$

OR Moody's Diagram

$$= \frac{0.25}{\left[\log_{10}\left(\frac{0.046}{3.7 \times 190} + \frac{5.74}{67000^{0.9}}\right)\right]^2}$$

$$= 0.021$$

Recalculate diameter using new value of f

CONT.

$$D = (0.021/0.015)^{1/5} \times 0.19$$
$$= 0.203 \text{ m} = 20.3 \text{ cm}$$

Use the next larger size of pipe; $\boxed{D = 22 \text{ cm.}}$

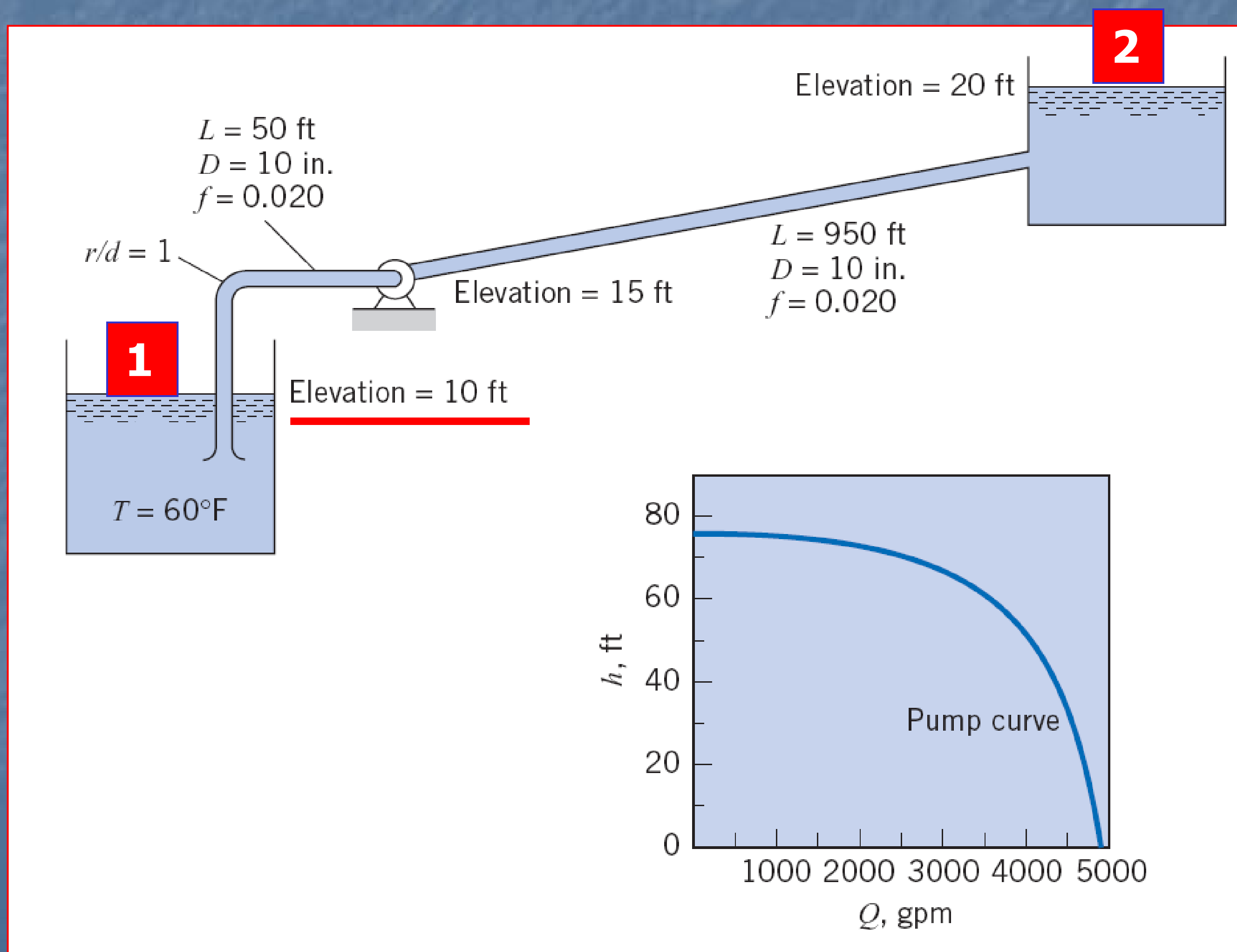
Power equation (assume the head loss is remains at $h_L \approx 50 \text{ m}/1{,}000 \text{ m}$)

$$P = Q\gamma h_f$$
$$= 0.1 \times (0.93 \times 9810) \times 50$$
$$= \boxed{45.6 \text{ kW/km}}$$

Problem (10.89)

Pump System

$$\dot{Q} = ?$$



CONT.

PROBLEM 10.89

$\dot{Q} = ?$

Situation: A pump is described in the problem statement.

Find: Discharge.

ANALYSIS

Energy equation

$$
\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\
0 + 0 + 10 + h_p &= 0 + 0 + 20 + V_2^2/2g(K_e + fL/D + k_0) \\
h_p &= 10 + (Q^2/(2gA^2))(0.1 + 0.02 \times 1,000/(10/12) + 1) \\
A &= (\pi/4) \times (10/12)^2 = 0.545 \text{ ft}^2
\end{aligned}
$$

$$
\begin{aligned}
h_p &= 10 + 1.31 Q_{\text{cfs}}^2 \\
1 \text{ cfs} &= 449 \text{ gpm} \\
h_p &= 10 + 1.31 Q_{\text{gpm}}^2/(449)^2 \\
h_p &= 10 + 6.51 \times 10^{-6} Q_{\text{gpm}}^2
\end{aligned}
$$

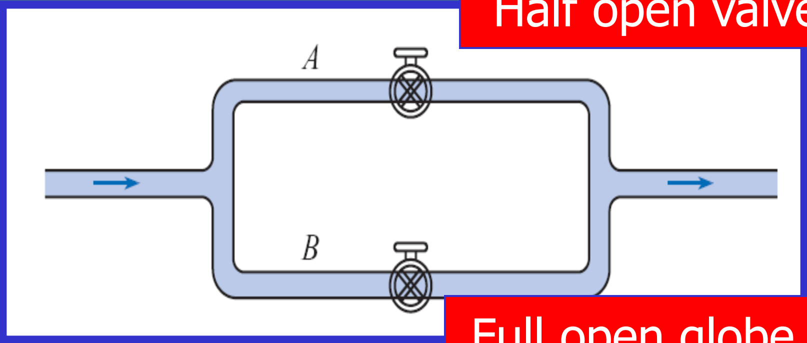
$Q \rightarrow$	1,000	2,000	3,000
$h \rightarrow$	16.5	36.0	68.6

Plotting this on pump curve figure yields $Q \approx 2,950$ gpm

Problem (10.105)

$$A_A = \frac{1}{2} A_B$$

$$K_A = 0.2, K_B = 10$$



PROBLEM 10.105

Situation: A pipe system is described in the problem statement.

Find: Ratio of discharge in line B to that in line A.

$$\frac{\dot{Q}_B}{\dot{Q}_A} = ?$$

ANALYSIS

$$\begin{aligned} h_{LA} &= h_{LB} \\ 0.2V_A^2/2g &= 10V_B^2/2g && (1) \\ V_A &= \sqrt{50}V_B \\ Q_B/Q_A &= V_B A_B / V_A A_A \\ &= V_B A_B / V_A((1/2)A_B) && (2) \\ Q_B/Q_A &= 2V_B/V_A \end{aligned}$$

Solve Eqs. (1) and (2) for Q_B/Q_A:

$$\begin{aligned} Q_B/Q_A &= 2 \times V_B/\sqrt{50}V_B \\ &= \boxed{0.283} \end{aligned}$$

THE END