CHAPTER (12)

COMPRESSIBLE FLOW

SOLVED PROBLEMS

DR. MUNZER EBAID

SUMMARY

1. Speed of Sound:

$$c = \sqrt{kRT}$$

$$c^2 = \left(\frac{\partial p}{\partial r}\right)_s$$

Isentropic Process (Adiabatic reversible)

2. Mach Number:

$$\mathrm{M} = \frac{V}{c}$$

$\mathrm{M} < 1$		Subsonic flow
$\mathrm{M} \approx 1$		Transonic flow
$\mathrm{M} > 1$		Supersonic flow

3. Total Temperature:

$$T_t = T\left(1 + \frac{k-1}{2}M^2\right)$$

(Adiabatic Process)

4. Total Pressure:

$$p_t = p\left(1 + \frac{k-1}{2}\mathrm{M}^2\right)^{k/(k-1)}$$

5. Total Density:

$$p_t = p\left(1 + \frac{k-1}{2}\mathrm{M}^2\right)^{k/(k-1)}$$

5. Kinetic Pressure:

$$q = \frac{1}{2}\frac{pV^2}{RT}$$

$$q = \frac{k}{2}pM^2$$

6. Static Pressure Ratio across A Normal Shock Waves:

$$\frac{p_2}{p_1} = \frac{(1 + kM_1^2)}{(1 + kM_2^2)}$$

7. Static Temperature Ratio across A Normal Shock Waves:

$$\frac{T_2}{T_1} = \frac{\{1 + [(k-1)/2]M_1^2\}}{\{1 + [(k-1)/2]M_2^2\}}$$

8. Mach Numbers Upstream & Downstream of a Normal Shock Waves:

$$\frac{M_1}{1 + kM_1^2}\left(1 + \frac{k-1}{2}M_1^2\right)^{1/2} = \frac{M_2}{1 + kM_2^2}\left(1 + \frac{k-1}{2}M_2^2\right)^{1/2}$$

8. Mach Numbers Downstream of a Normal Shock Waves:

$$M_2^2 = \frac{(k-1)M_1^2 + 2}{2kM_1^2 - (k-1)}$$

9. Area Ratio in a Laval Nozzle:

$$\frac{A}{A_*} = \frac{1}{M}\left\{\frac{1+[(k-1)/2]M^2}{(k+1)/2}\right\}^{(k+1)/2(k-1)}$$

10. Mass Flow Rate Through a Laval Nozzle:

$$\dot{m} = 0.685\frac{p_t A_*}{\sqrt{RT_t}}$$

11\.

A Laval nozzle is classified by comparing the pressure at the exit, p_e, for supersonic flow in the nozzle with the back (ambient) pressure, p_b.

$p_e/p_b > 1$ underexpanded

$p_e/p_b = 1$ ideally expanded

$p_e/p_b < 1$ overexpanded

12. Critical Pressure Ratio :

$$\frac{p_*}{p_t} = \left(\frac{2}{k+1}\right)^{k/(k-1)}$$

$$\frac{p_*}{p_t} = 0.528$$

13. Mach No. Distribution along a Pipe with Friction

$$\frac{1-M^2}{kM^2}+\frac{k+1}{2k}\ln\left[\frac{(k+1)M^2}{2+(k-1)M^2}\right]=\frac{\bar{f}(x_*-x_M)}{D}$$

where x_M is the distance corresponding to a Mach number M.

14. Variation of Mach No. with Pressure along a Pipe with Friction:

$$\frac{p_M}{p_*}=\frac{1}{M}\left[\frac{k+1}{2+(k-1)M^2}\right]^{1/2}$$

15. Mach No. Distribution along a Pipe with Friction for Isothermal:

$$\frac{f(x_T-x_M)}{D}=\ln(DM^2)+\frac{(1-kM^2)}{kM^2}$$

16. Variation of Mach No. with Pressure for Isothermal Process :

$$\frac{p_M}{p_T}=\frac{1}{\sqrt{k}M}$$

Problem (12.4)

PROBLEM 12.4

Situation: A sound wave travels in helium and another in nitrogen both at 20 °C.

Find: Difference in speed of sound.

ANALYSIS

Speed of sound

$$\begin{aligned} c_{He} &= \sqrt{(kR)_{He}T} \\ &= \sqrt{1.66 \times 2077 \times 293} \\ &= 1005 \text{ m/s} \end{aligned}$$

Table (A-2), Appendix (II)

$$\begin{aligned} c_{N_2} &= \sqrt{(kR)_{N_2}T} \\ &= \sqrt{1.40 \times 297 \times 293} \\ &= 349 \text{ m/s} \end{aligned}$$

$$c_{He} - c_{N_2} = \boxed{656 \text{ m/s}}$$

Problem (12.5)

PROBLEM 12.5

Situation: A sound wave travels in an ideal gas.

Find: Speed of sound for an isothermal process.

ANALYSIS

$$c^2 = \partial p/\partial \rho; \; p = \rho RT$$

If isothermal, T=const.

$$\therefore \; \partial p/\partial \rho = RT$$

$$\therefore \; c^2 = RT$$

$$\boxed{c = \sqrt{RT}}$$

Problem (12.7)

PROBLEM 12.7

Situation: An aircraft flying in air at Mach 1.5 is described in the problem statement.

Find: (a) Surface temperature.
(b) Airspeed behind shock.

$Given: T_1 = -30\ ^{o}C$

Properties: (a) From Table A.1 at $M_1 = 1.5$, $T/T_t = 0.6897$; . $M_2 = 0.7011$, $T_2/T_1 = 1.320$. (b) Air (Table A.2) $k = 1.4$ and $R = 287$ J/kg/K.

ANALYSIS

Total temperature will develop at exposed surface

$Given: T_1 = 30\ ^{o}C$

$$\begin{aligned} \frac{T}{T_t} &= 0.6897 \\ T_t &= \frac{(273-30)}{0.6897} \\ &= 352.3\,\text{K} = \boxed{79.2\,^{\circ}\text{C}} \end{aligned}$$

Temperature (behind shock)

$Given: T_1 = 30\ ^{o}C$

$$\begin{aligned} \frac{T_2}{T_1} &= 1.320 \\ T_2 &= 1.320 \times (273.15-30) \\ &= 320.96\,\text{K} \end{aligned}$$

Problem (12.7)

Speed of sound (behind shock)

$$\begin{aligned} c_2 &= \sqrt{kRT_2} \\ &= \sqrt{(1.4)\,(287)\,320.96} \\ &= 359.1\,\mathrm{m/\,s} \end{aligned}$$

Mach number (behind shock)

$$\begin{aligned} M_2 &= \frac{V_2}{c_2} \\ V_2 &= c_2 M_2 \\ &= (359.1)\,(0.7011) \\ &= 251.77\,\mathrm{m/\,s} \end{aligned}$$

$$\boxed{V_2 = 252\,\mathrm{m/\,s} = 906\,\mathrm{km/\,h}}$$

Problem (12.12)

$T_t = ?$

$P_t = ?$

PROBLEM 12.12

Situation: An object immersed in airflow is described in the problem statement.

Given: $T_S = 20\ ^{o}C$

$P_S = 200\, kPa$

$V = 250\, m/s$

Find: (a) Pressure.
(b) Temperature at stagnation point.

ANALYSIS

Speed of sound

$$\begin{aligned} c &= \sqrt{kRT} \\ &= \sqrt{(1.4)(287)(293)} \\ &= 343 \text{ m/s} \end{aligned}$$

Mach number

$$\begin{aligned} M &= 250/343 \\ &= 0.729 \end{aligned}$$

Total properties

Temperature

$$T_t = T\left(1 + \frac{k-1}{2}M^2\right)$$

$$\begin{aligned} T_t &= (293)(1 + 0.2 \times (0.729)^2) \\ &= 293 \times 1.106 \\ &= 324 \text{ K} \end{aligned}$$

$$\boxed{T_t = 51\,^\circ\mathrm{C}}$$

You can use Tables

Pressure

$$p_t = p\left(1 + \frac{k-1}{2}\mathrm{M}^2\right)^{k/(k-1)}$$

$$p_t = (200)(1.106)^{3.5}$$

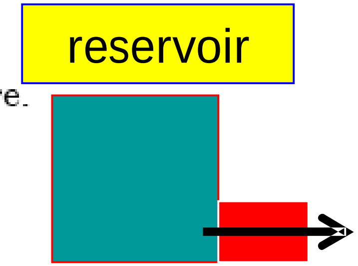
$$\boxed{p_t = 284.6\,\mathrm{kPa}}$$

Problem (12.16)

PROBLEM 12.16

Situation: Hydrogen flow from a reservoir–additional details are provided in the problem statement.

Find: (a) Temperature.
(b) Pressure.
(c) Mach number.
(d) Mass flow rate.



$Given: T_t = 20\ ^{\circ}C,\ \ P_t = 500\, kPa$

$d = 2\, cm,\ \ V = 250\, m/s$

Isentropic Flow

ANALYSIS

$$\begin{aligned} T_t &= 20^{\circ}\text{C} = 293\ \text{K} \\ P_t &= 500\ \text{kPa} \\ c_pT + V^2/2 &= c_pT_0 \\ T &= T_t - V^2/(2c_p) \\ &= 293 - (250)^2/((2)(14,223)) \\ &\boxed{T = 290.8\ \text{K}} \end{aligned}$$

$$h_t = h + \frac{1}{2}V^2 \qquad h = C_P T$$

Speed of sound

$$\begin{aligned} c &= \sqrt{kRT} \\ &= \sqrt{(1.41)(4,127)(290.8)} \\ &= 1,301\ \text{m/s} \end{aligned}$$

Problem (12.16)

Speed of sound

$$c = \sqrt{kRT} = \sqrt{(1.41)(4,127)(290.8)} = 1,301 \text{ m/s}$$

Mach number

$$M = 250/1301 = 0.192$$

Total properties (pressure)

$$P_t = P\left(1 + \frac{k-1}{2}M^2\right)^{K/K-1}$$

$$p = 500/[1 + (0.41/2) \times 0.192^2]^{(1.41/0.41)}$$

$$\boxed{p = 487.2 \text{ kPa}}$$

You can use Tables

Ideal gas law

$$\rho = p/RT = (487.2)(10^3)/(4,127 \times 290.8) = 0.406 \text{ kg/m}^3$$

Flow rate equation

$$\dot{m} = \rho AV = (0.406)(0.02)^2(\pi/4)(250)$$

$$\boxed{\dot{m} = 0.032 \text{ kg/s}}$$

Problem (12.23)

PROBLEM 12.23

Situation: A shock wave is described in the problem statement.

Find: (a)The downstream Mach number.
(b) Static pressure.
(c) Static temperature.
(d) Density.

$$Given\ Upstream: T_S = 20\ ^{o}C$$
$$P_S = 100\ kPa$$

Properties: From Table A.2 $k = 1.31$

Methane

$$M_1 = 3$$

APPROACH

Apply the Normal shock wave equations to find Mach number, pressure, and temperature. Apply the ideal gas law to find density.

ANALYSIS

Normal shock wave
Mach number

$$\begin{aligned} M_2^2 &= [(k-1)M_1^2 + 2]/[2kM_1^2 - (k-1)] \\ &= ((0.31)(9) + 2)/((2)(1.31)(9) - 0.31) = \\ &\quad 0.2058 \\ &\quad \boxed{M_2 = 0.454} \end{aligned}$$

You can use tables

Problem (12.23)

Pressure ratio

$P_1 = 100\,kPa$

You can use tables

$$
\begin{aligned}
p_2/p_1 &= (1 + kM_1^2)/(1 + kM_2^2) \\
&= (1 + 1.31 \times 9)/(1 + 1.31 \times 0.2058) = 10.07
\end{aligned}
$$

$$\boxed{p_2 = 1,007 \text{ kPa, abs}}$$

Temperature ratio

$T_1 = 20\ ^\circ C$

You can use tables

$$
\begin{aligned}
T_2/T_1 &= [1 + ((k-1)/2)M_1^2]/[1 + ((k-1)/2)M_2^2] \\
&= 2.32 \\
T_2 &= (293)(2.32)
\end{aligned}
$$

$$\boxed{T_2 = 680\,\text{K} = 407\,^\circ\text{C}}$$

<u>Ideal gas law</u>

$$
\begin{aligned}
\rho_2 &= p_2/(RT_2) \\
&= (1,007)(10^3)/((518)(680))
\end{aligned}
$$

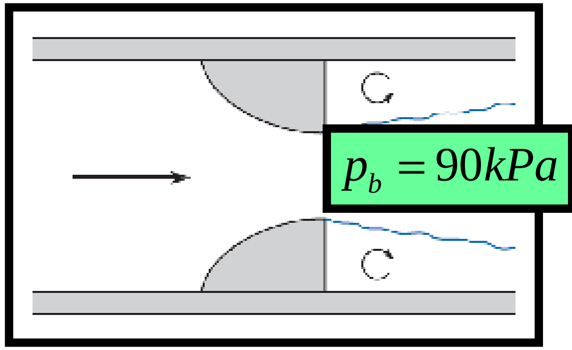
$$\boxed{\rho_2 = 2.86 \text{ kg/m}^3}$$

Problem (12.28)

PROBLEM 12.28

Situation: A truncated nozzle is described in the problem statement.

Find: Mass flow rate



ANALYSIS

$$A_T = 3\ \mathrm{cm}^2 = 3 \times 10^{-4}\ \mathrm{m}^2$$

$$p_t = 300\ \mathrm{kPa};\ T_t = 20^\circ = 293\ \mathrm{K}$$

$$p_b = 90\ \mathrm{kPa}$$

$$p_b/p_t = 90/300 = 0.3$$

$$\frac{p_*}{p_t} = 0.528$$

Because $p_b/p_t < 0.528$, sonic flow at exit.

Laval nozzle flow rate equation

$$\dot{m} = 0.685 p_t A_* / \sqrt{RT_t}$$

$$= (0.685)(3 \times 10^5)(3 \times 10^{-4}) / \sqrt{(287)(293)}$$

$$\boxed{\dot{m} = 0.212\ \mathrm{kg/s}}$$

Problem (12.36)

OBLEM 12.36

Situation: The design of a Laval nozzle is described in the problem statement.

Find: The nozzle throat area.

Properties: From Table .2 $k = 1.4$; $R = 297$ J/kgK.

Nitrogen

Laval Nozzle

Isentropic Process

ANALYSIS

Nozzle exit conditions: Given Ideally Expanded

Find Mach number

You can use Tables

$$M_e = \sqrt{(2/(k-1))[(p_t/p_e)^{(k-1)/k}1]} = \sqrt{5[(1,000/30)^{0.286} - 1]} = 2.94$$

$p_e = p_b = 30kPa$

Given :

$P_b = 30\, kPa$

$P_t = 1MPa$

$T_t = 550\ ^oC$

$\dot{m} = 5\, kg/s$

Mach number-area ratio relationship

$$A_e/A_* = (1/M)[(1+((k-1)/2)M^2)/((k+1)/2)]^{(k+1)/(2(k-1))} = (1/2.94)[(1+(0.2)(2.94)^2)/1.2]^3$$

$$\boxed{A_e/A_* = 4.00}$$

You can use tables

Flow rate equation for Laval nozzle

$$\dot{m} = 0.685 p_t A_T/\sqrt{RT_t}$$

$$A_T = \dot{m}\sqrt{RT_t}/(0.685 \times p_t) = 5 \times \sqrt{(297)(550)}/((0.685)(10^6)) = 0.00295 \text{ m}^2$$

$$\boxed{A_T = 29.5 \text{ cm}^2}$$

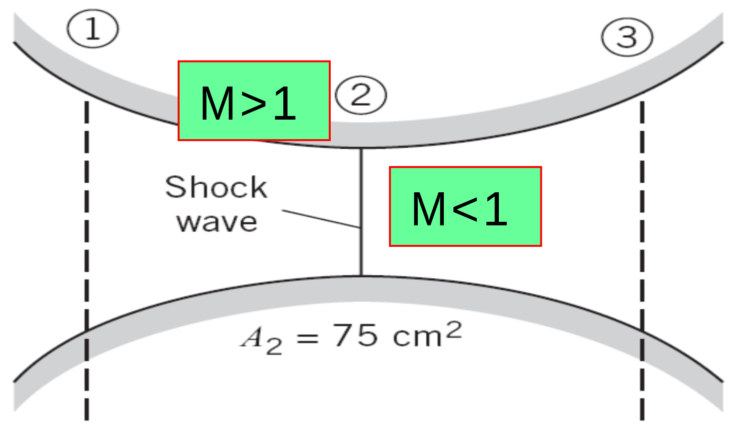
$Find : \left(\frac{A}{A^*}\right) = ?$

$\left(A^* \;\; for\ \dot{m} = 5\, kg/s\right)$

$and\ T = 550^oC$

Problem (12.46)

Assume Isentropic Flow accept across the shock waves



PROBLEM 12.46

Situation: Airflow in a channel is described in the problem statement.

Find: (a) Mach number.

(b) Static pressure.

(c) Stagnation pressure at station 3.

Properties: From Table A.1 $M = 2.1$, $A/A_* = 1.837$, $p/p_t = 0.1094$.

ANALYSIS

$$
\begin{aligned}
A_* &= 100/1.837 = 54.4 \\
p_t &= 65/0.1094 = 594 \text{ kPa} \\
A_2/A_* &= 75/54.4 = 1.379 \\
M &= 1.74 \rightarrow p_2/p_t = 0.1904 \rightarrow p_2 = 0.1904(594) = \underline{113 \text{ kPa}}
\end{aligned}
$$

Problem (12.46)

after shock, $M_2 = 0.630;\ p_2 = 3.377(113) = 382$ kPa

$$
\begin{aligned}
A_2/A_* &= (1/M)((1+0.2M^2)/1.2)^3 \\
&= 1.155 \\
p_t/p_2 &= (1+0.2M^2)^{3.5} = 1.307 \\
A_* &= 75/1.155 = 64.9;\ p_t = 382(1.307) = \boxed{499 \text{ kPa}} \\
A_3/A_* &= 120/64.9 = 1.849;\ \text{from Table A.1, } \boxed{M_3 = 0.336} \\
p_3/p_t &= 0.9245;\ p_3 = 0.9245(499) = \boxed{461 \text{ kPa}}
\end{aligned}
$$

THE END