CHAPTER (14)

TURBOMACHINARY

SOLVED PROBLEMS

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Thrust Coefficient of a propeller:

$$C_T = \frac{T}{\rho n^2 D^4}$$

Power Coefficient of a propeller:

$$C_P = \frac{P}{\rho n^3 D^5}$$

Efficiency of a propeller:

$$\eta = \frac{C_T}{C_P}\left(\frac{V_0}{nD}\right)$$

Head Coefficient of an axial Pump:

$$C_H = \frac{4}{p} C_T = \frac{\Delta H}{D^2 n^2 / g}$$

Power Coefficient of an axial Pump :

$$C_P = \frac{P}{\rho D^5 n^3}$$

Discharge Coefficient of an axial Pump:

$$C_Q = \frac{Q}{nD^3}$$

Specific speed: $n_s = \frac{n\sqrt{Q}}{(gh)^{3/4}}$

$$N_{SS} = \frac{nQ^{1/2}}{(NPSH)^{3/4}}$$

$Where\ N\ (rpm),\ Q\ (gpm),\ NPSH\ (feet)$

Critical value for Cavitation to occur $(N_{SS} \leq 8500)$

Theoretical Adiabatic Power with no cooling

$$P_{theo} = \frac{k}{k-1} Q_1 p_1 \left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k} - 1\right]$$

Efficiency of a compressor with no water cooling= $h_{Comp} = \frac{P_{theo}}{(P_{actual})_{SHAFT}}$

Specific speed for Turbines

$$n_s = \frac{nP^{1/2}}{g^{3/4}\gamma^{1/2}h_t^{5/4}}$$

Wind turbine max. theoretical power produced by a

$$P_{max} = \frac{16}{27}\left(\frac{1}{2}\rho U^3 A\right)$$

Theoretical Isothermal Power with cooling

$$P_{theo} = p_1 Q_1 \ln \frac{p_2}{p_1}$$

Then: Max. Power of the Turbine

$$P = rQ\frac{V_J^{\,2}}{2}$$

Specific speed for Turbines

$$n_s = \frac{nP^{1/2}}{g^{3/4}\gamma^{1/2}h_t^{5/4}}$$

Power for wind Turbines

$$P_{max} = \frac{16}{27}\left(\frac{1}{2}\rho U^3 A\right)$$

Torque for Reaction Turbines

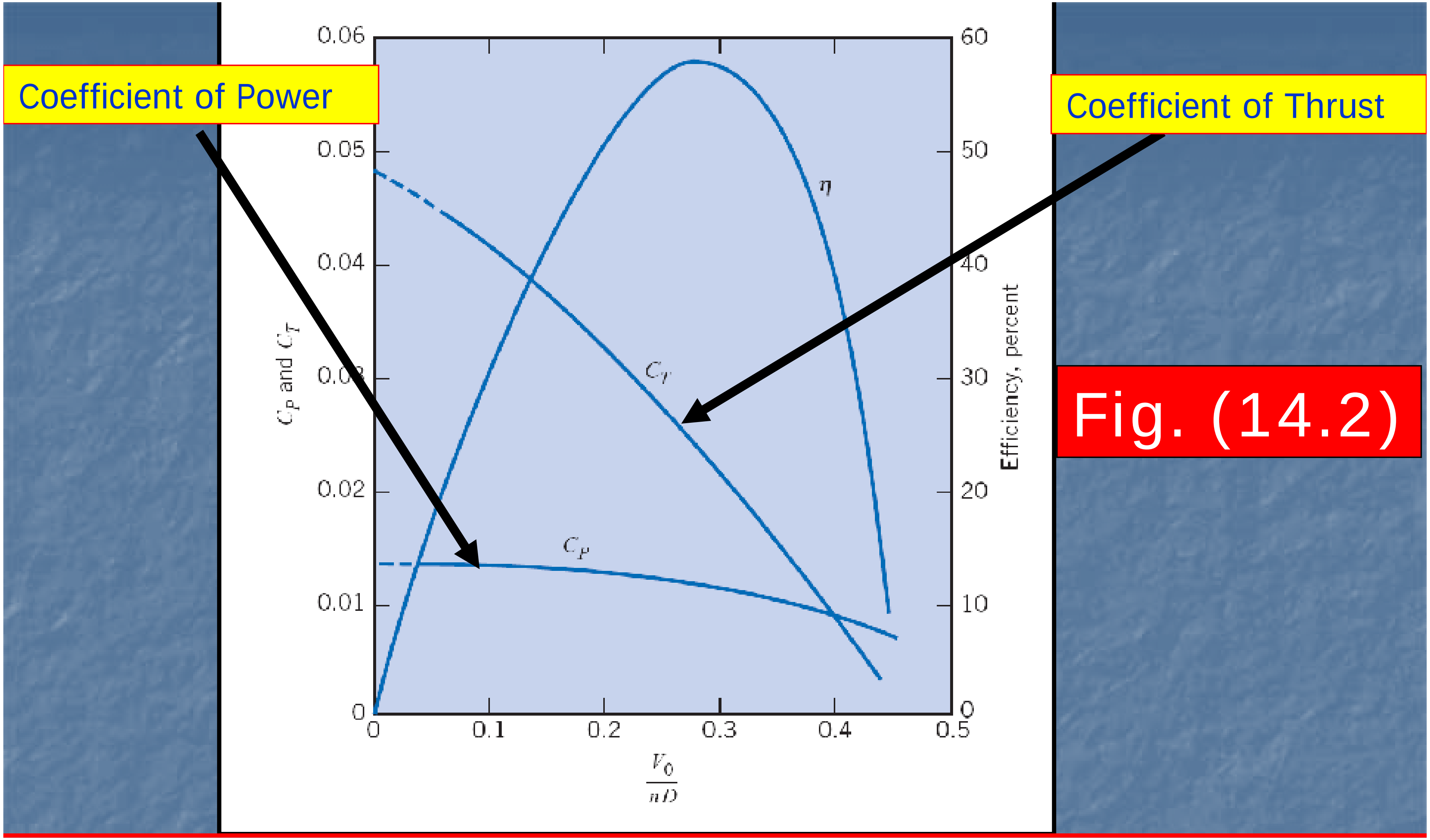
$$T = \dot{m}(-r_2 V_2 \cos\alpha_2) - \dot{m}(-r_1 V_1 \cos\alpha_1)$$

Power for Reaction Turbines

$$P = \rho Q \omega (r_1 V_1 \cos\alpha_1 - r_2 V_2 \cos\alpha_2)$$

PROBLEMS ON

PROPELLERS



Performing a dimensional analysis for the power, P, shows

$$\frac{P}{\rho n^3 D^5} = f\left(\frac{V_0}{nD}, \frac{\rho D^2 n}{\mu}\right)$$

Problem (14.1)

What thrust is obtained from a propeller 3 m in diameter that has the characteristics given in Fig. 14.2 when the propeller is operated at an angular speed of 1400 rpm and an advance velocity of zero? Assume $\rho = 1.05\ \text{kg/m}^3$.

PROBLEM 14.1

Situation: A propeller is described in the problem statement.

Find: Thrust force. From rest

ANALYSIS

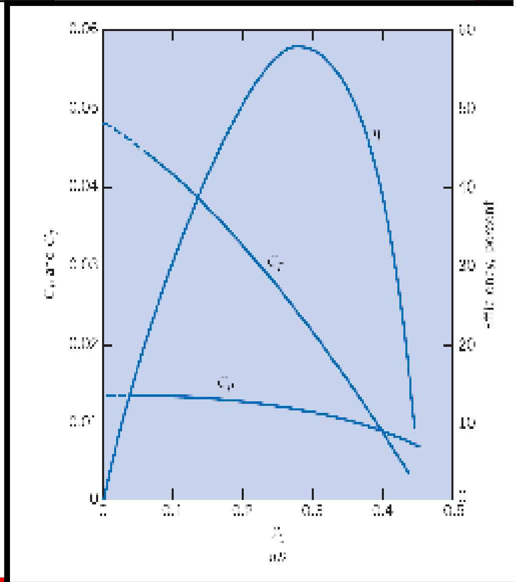
$D = 3\,m,\ n = 1400\,rpm,\quad r = 1.05\,kg/m^3$

From Fig. 14.2 $V_0 = 0$ $\quad \dfrac{V_0}{nD} = 0$

$C_T = 0.048.$ From rest

Propeller thrust force equation

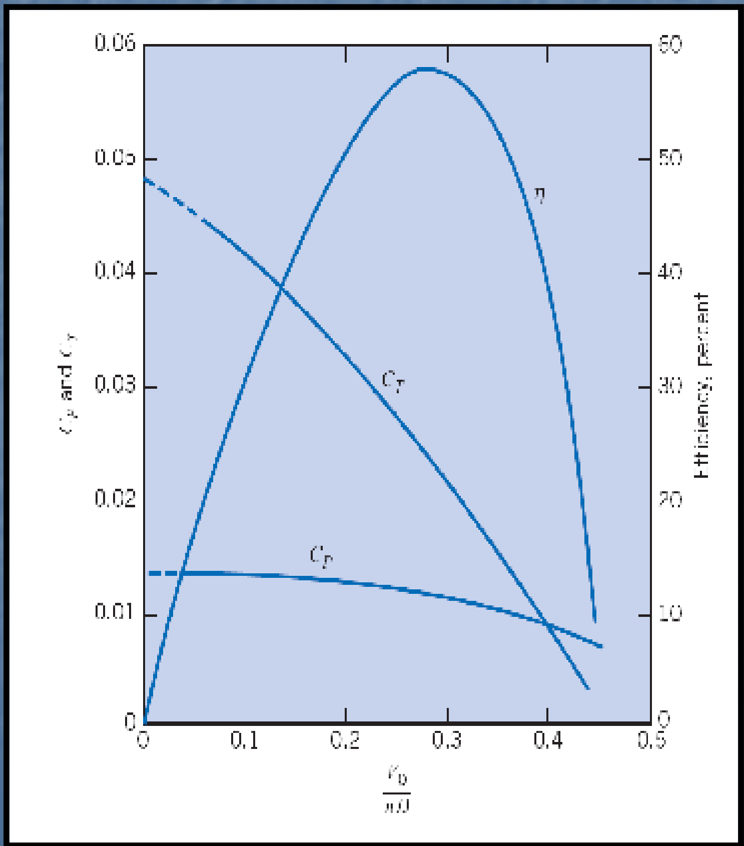
$$
\begin{aligned}
F_T &= C_T \rho D^4 n^2 \\
&= 0.048 \times 1.05 \times 3^4 \times (1,400/60)^2 \\
& \boxed{F_T = 2223\ \text{N}}
\end{aligned}
$$



Problem (14.8)

A propeller 2 m in diameter, like the one for which characteristics are given in Fig. 14.2, is to be used on a swamp boat and is to operate at maximum efficiency when cruising. If the cruising speed is to be 40 km/h, what should the angular speed of the propeller be?

$$w = ?\ At\ h_{\max}$$



Problem (14.8)

PROBLEM 14.8

Situation: A propeller is described in the problem statement.

Find: Angular speed of propeller.

$$w = ?\ At\ \eta_{max}$$

$$D = 2m$$

$$V_0 = 40\ km/h$$

APPROACH

Use Fig 14.2 to find the advance diameter ratio at maximum efficiency.

ANALYSIS

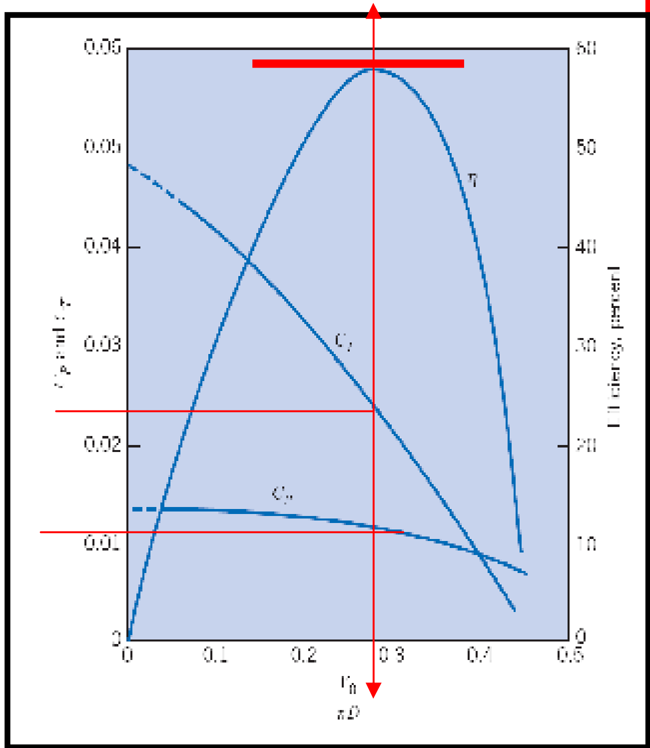
Advance ratio (from Fig. 14.2)

$$V_0/(nD) = 0.285$$

Rotation speed

$$\begin{aligned} n &= V_0/(0.285D) \\ &= (40,000/3,600)/(0.285 \times 2) \\ &= 19.5 \text{ rev/s} \\ N &= 19.5 \times 60 \end{aligned}$$

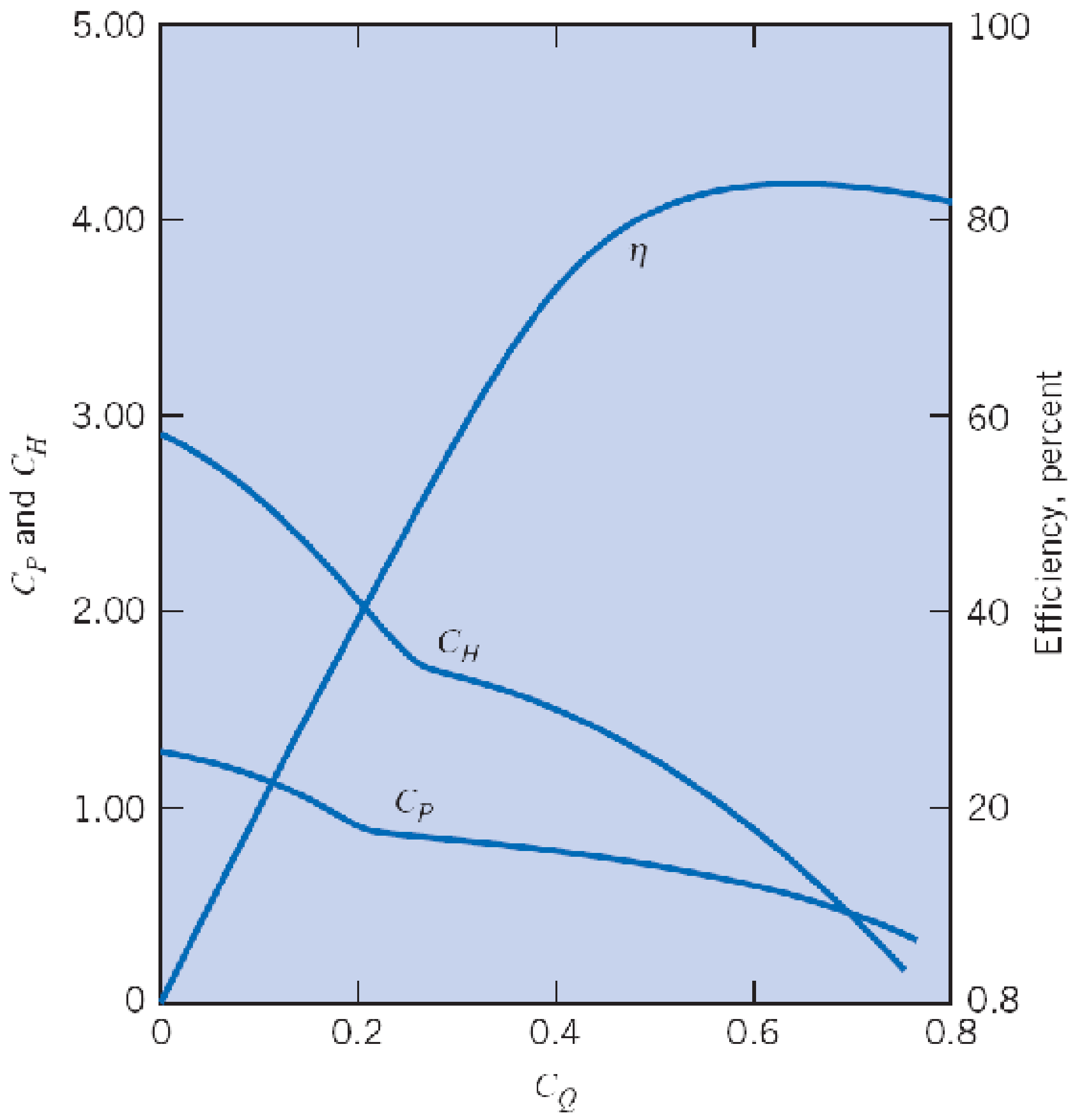
$$\boxed{N = 1170 \text{ rpm}}$$



PROBLEMS ON

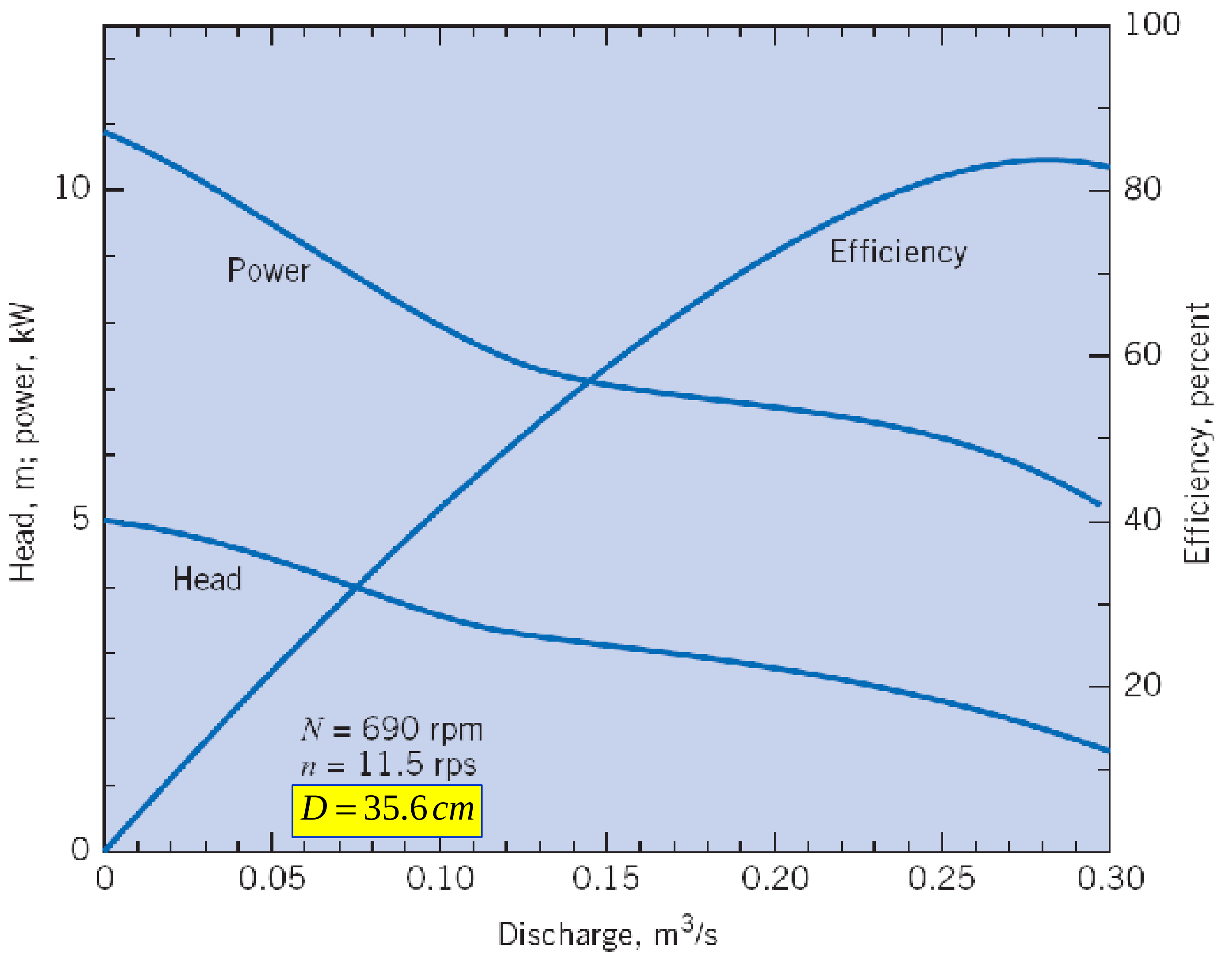
AXIAL PUMPS

Fig. (14.6)



Dimensionless performance curves for a typical axial-flow pump

Fig. (14.7)

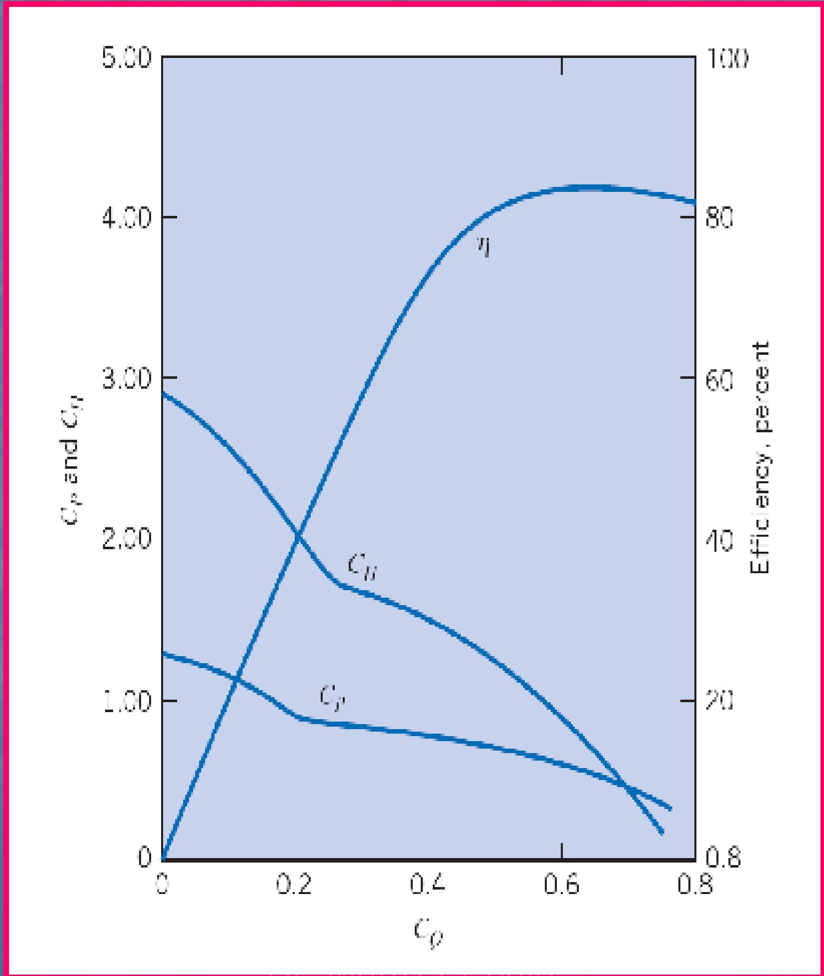


Performance curves for a typical axial-flow pump

Problem (14.11)

If a pump having the characteristics shown in Fig. 14.6 has a diameter of 40 cm and is operated at a speed of 1000 rpm, what will be the discharge when the head is 3 m?

$$\dot{Q} = ?$$



Problem (14.11)

Find: Discharge.

$$\dot{Q} = ?$$

APPROACH

Apply discharge coefficient. Calculate the head coefficient to find the corresponding discharge coefficient from Fig. 14.6.

ANALYSIS

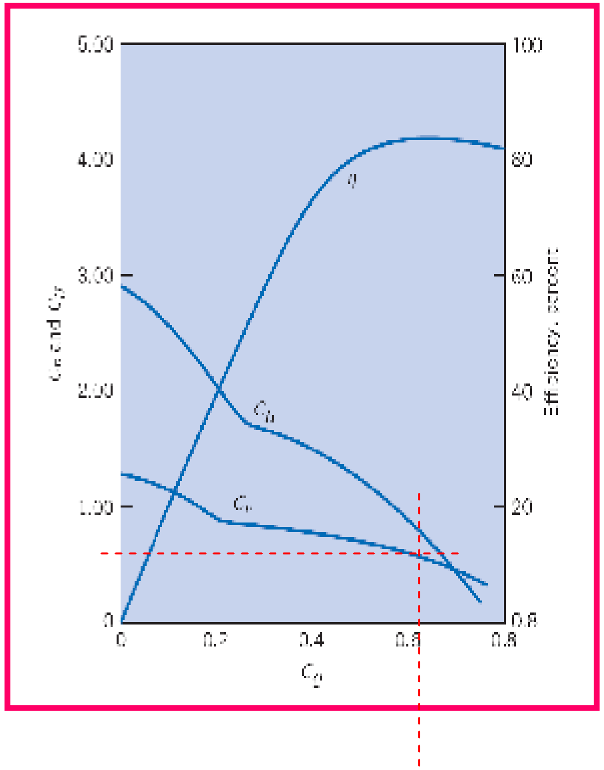
$$Given: n = 1000\,rpm,\; D = 40\,cm,\; \Delta H = 3\,m$$

$$C_H = \frac{4}{p} C_T = \frac{\Delta H}{D^2 n^2 / g}$$

$$\begin{aligned} n &= 1,000/60 \\ &= 16.67 \text{ rev/s} \end{aligned}$$

Head coefficient

$$\begin{aligned} C_H &= \Delta h g / D^2 n^2 \\ &= 3 \times 9.81 / ((0.4)^2 \times (16.67)^2) \\ &= 0.662 \end{aligned}$$



From Fig. 14.6, $C_Q = Q/(nD^3) = 0.625$.

Discharge coefficient

$$Q = 0.625 \times 16.67 \times (0.4)^3$$

$$Q = 0.667 \text{ m}^3/\text{s}$$

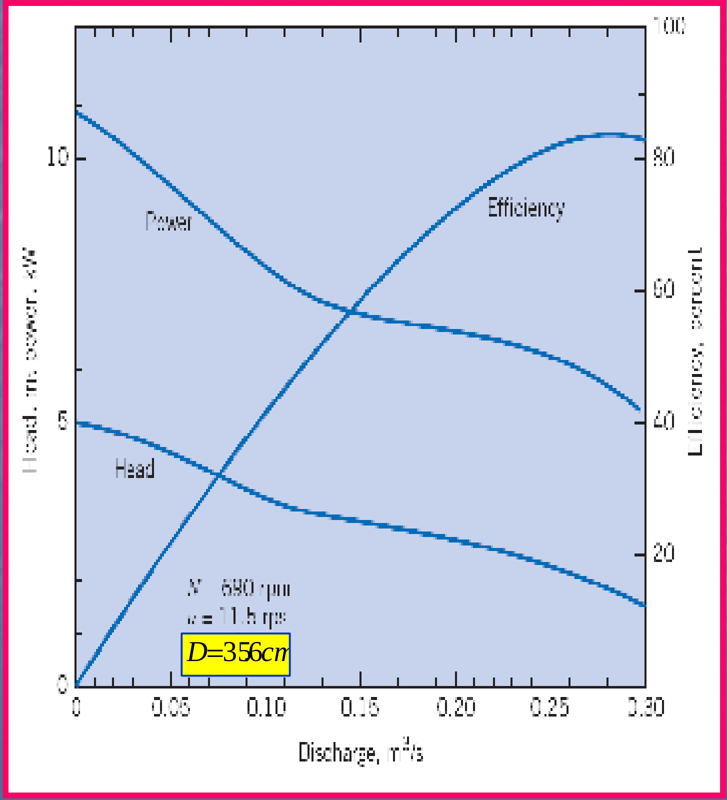
Problem (14.15)

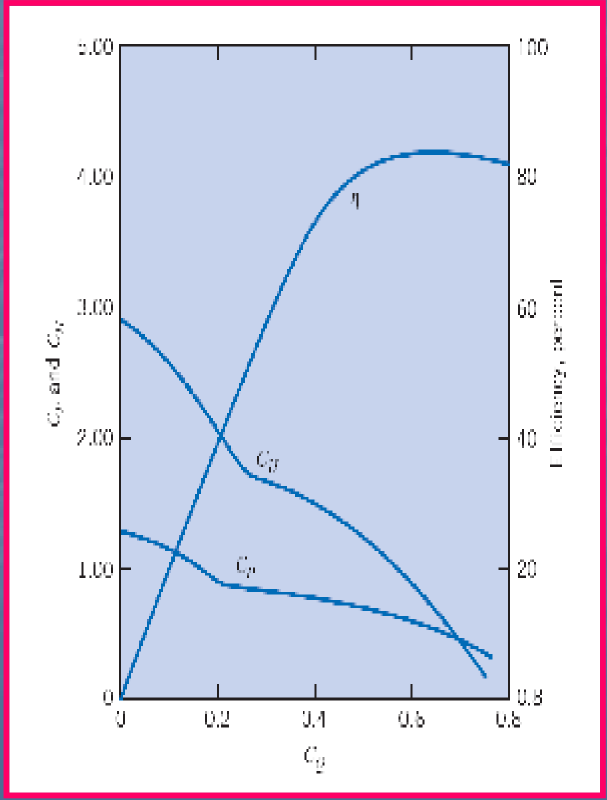
For a pump having the characteristics given in Fig. 14.6 or 14.7, what water discharge and head will be produced at maximum efficiency if the pump diameter is 20 in. and the angular speed is 1100 rpm? What power is required under these conditions?

$\dot{Q} = ?$

$H = ?$

$Power(P) = ?$





Problem (14.15)

Find: (a) Discharge

For a pump having the characteristics given in Fig. 14.6 or 14.7, what water discharge and head will be produced at maximum efficiency if the pump diameter is 20 in. and the angular speed is 1100 rpm? What power is required under these conditions?

$H = ?$ $Power(P) = ?$

APPROACH

Apply discharge, head, and power coefficients. Use Fig. 14.6 to find the discharge, power, and head coefficients at maximum efficiency.

ANALYSIS

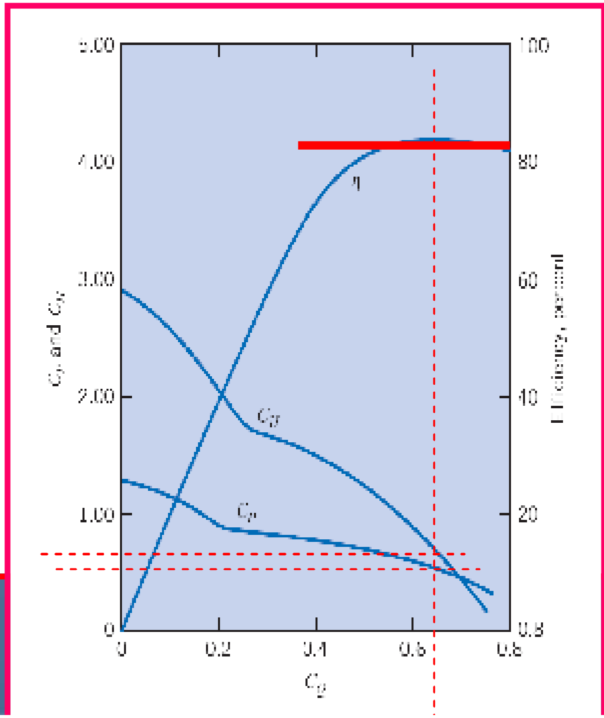
$Given: n = 1100\, rpm, D = 20\, in, \max(\eta)$

From Fig. 14.6, $C_Q = 0.64; C_p = 0.60;$ and $C_H = 0.75$

$$D = 1.67 \text{ ft}$$
$$n = 1,100/60 = 18.33 \text{ rev/s}$$

Discharge coefficient

$$Q = C_Q n D^3$$
$$= 0.64 \times 18.33 \times 1.67^3$$
$$Q = 54.6 \text{ cfs}$$



Head coefficient

$$\begin{aligned} \Delta h &= C_H n^2 D^2 / g \\ &= 0.75 \times 18.33^2 \times 1.67^2 / 32.2 \end{aligned}$$

$$\boxed{\Delta h = 21.8\,\text{ft}}$$

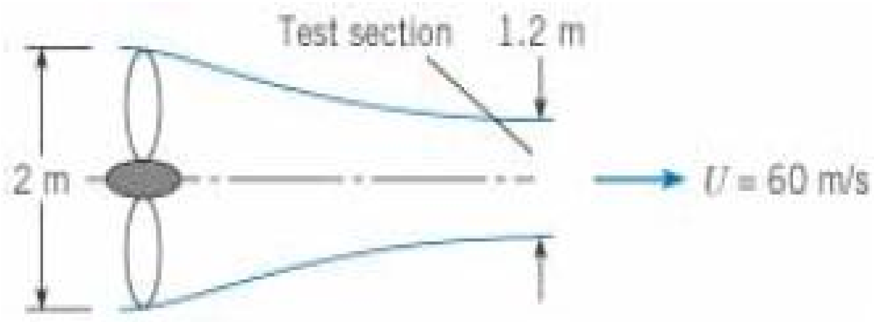
Power coefficient

$$\begin{aligned} P &= C_p \rho D^5 n^3 \\ &= 0.60 \times 1.94 \times 1.67^5 \times 18.33^3 \\ &= 93{,}116 \text{ ft-lbf/sec} \end{aligned}$$

$$\boxed{P = 169.3\,\text{hp}}$$

Problem (14.20)

An axial fan 2 m in diameter is used in a wind tunnel as shown (test section 1.2 m in diameter; test section velocity of 60 m/s). The rotational speed of the fan is 1800 rpm. Assume the density of the air is constant at 1.2 kg/m^3. There are negligible losses in the tunnel. The performance curve of the fan is identical to that shown in Fig. 14.6. Calculate the power needed to operate the fan.



$$Power(P) = ?$$

Problem (14.20)

Find: Power needed to operate fan.

$Power(P) = ?$

APPROACH

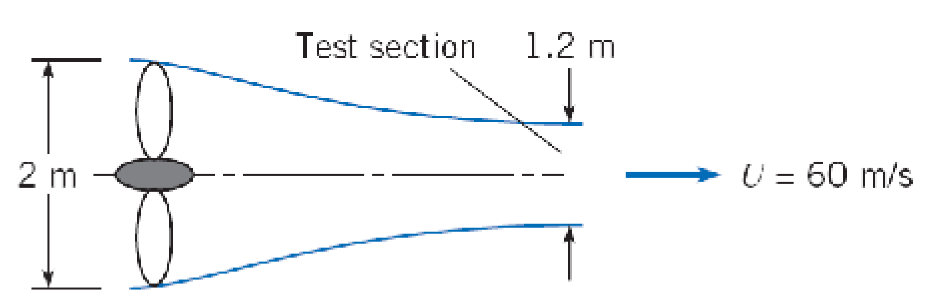
Apply power coefficient. Calculate the discharge coefficient (apply the flow rate equation to find Q) to find the corresponding power coefficient from Fig. 14.16.

ANALYSIS

$n = 1800\ rpm$

Flow rate equation

$$\begin{aligned} Q &= VA \\ &= (60)(\pi/4)(1.2)^2 \\ &= 67.8\ \text{m}^3/\text{s} \end{aligned}$$



Discharge coefficient

$$\begin{aligned} C_Q &= Q/(nD^3) \\ &= (67.8)/((1,800/60)(2)^3 \\ &= 0.282 \end{aligned}$$

$$C_P = \frac{P}{\rho D^5 n^3}$$

From Fig. 14.16 $C_p = 2.6$. Then

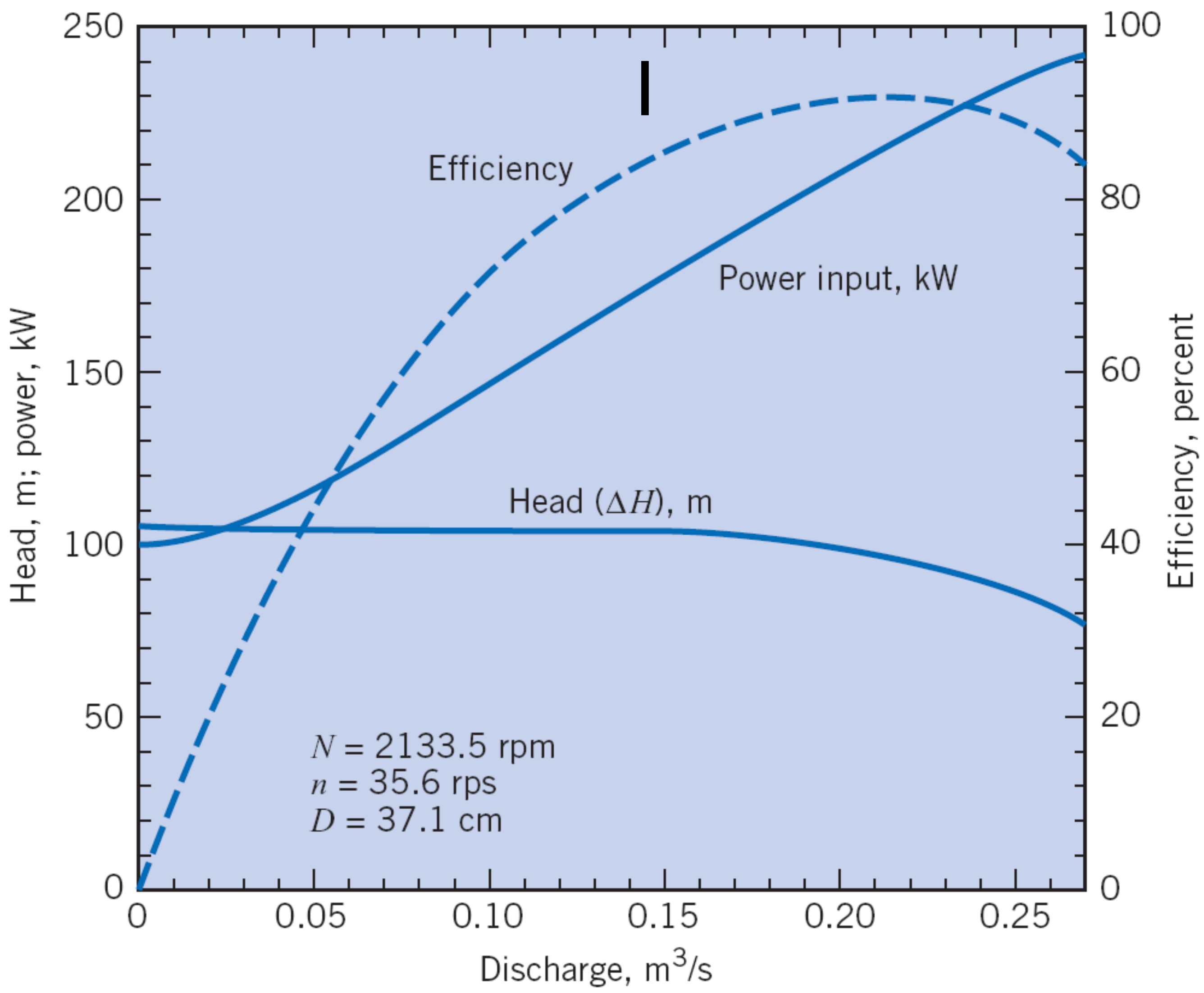
Power coefficient

$$\begin{aligned} P &= C_p \rho D^5 n^3 \\ &= (2.6)(1.2)(2)^5(30)^3) \end{aligned}$$

$$\boxed{P = 2.70\ \text{MW}}$$

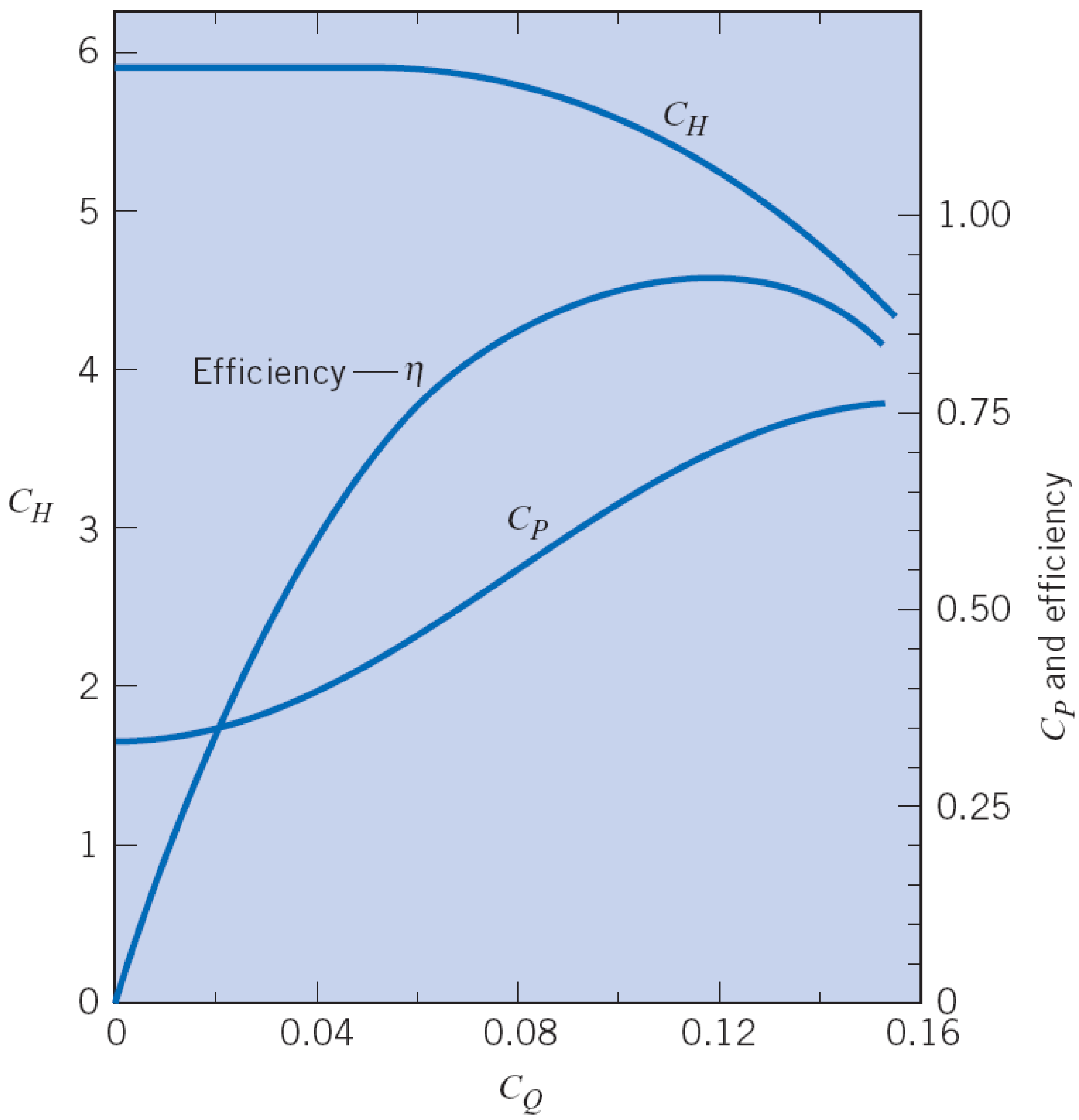
PROBLEMS ON CENTRIFUGAL PUMPS

Fig. (14.9)



Performance curves for a typical centrifugal pump; *D* = 37.1 cm

Fig. (14.10)



Dimensionless performance curves for a typical centrifugal pump from data given in Fig. 14.9

Problem (14.22)

If a pump having the characteristics given in Fig. 14.9 or 14.10 is operated at a speed of 1600 rpm, what will be the discharge when the head is 150 ft?

$Given: n = 1600\ rpm,\ \Delta h = 150\ ft$

$\dot{Q} = ?$

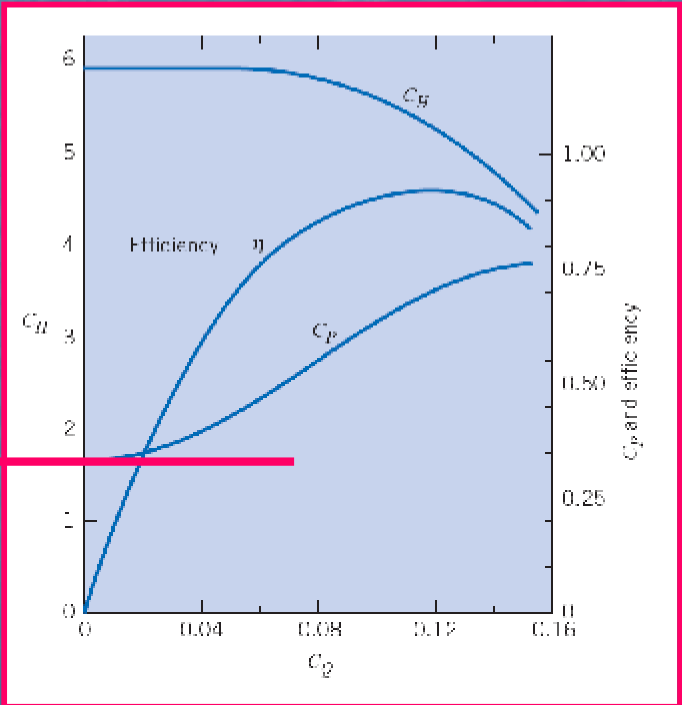


Fig. (14.10)

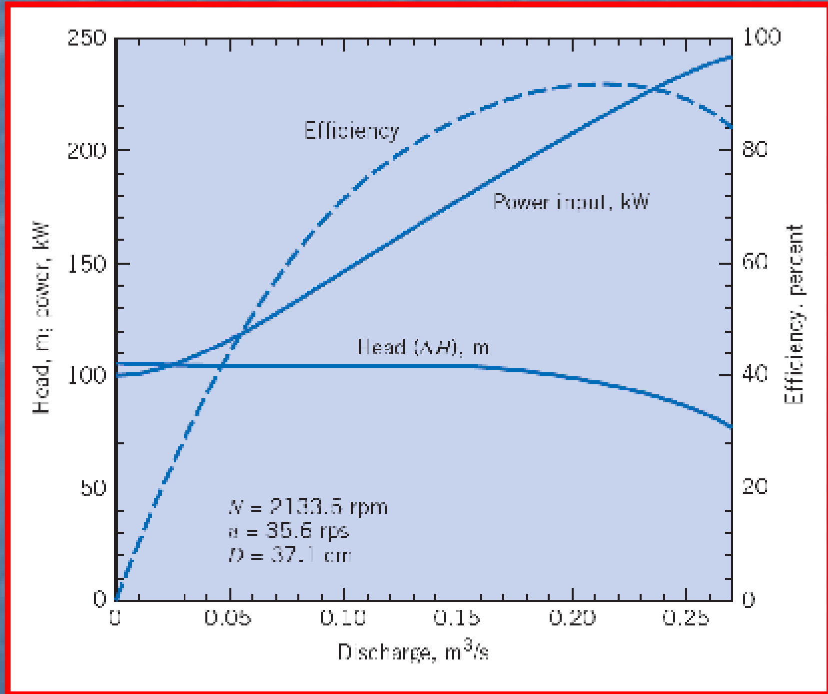


Fig. (14.9)

Problem (14.22)

Find: Discharge.

$Q = ?$

APPROACH

Apply discharge coefficient. Calculate the head coefficient to find the corresponding discharge coefficient from Fig. 14.10.

ANALYSIS

$$Given: n = 1600\ rpm,\ \Delta h = 150\ ft$$

$$\begin{aligned} D &= 0.371\text{ m} = 1.217\text{ ft} \\ n &= 1500/60 = 25\text{ rps} \end{aligned}$$

Head coefficient

$$\begin{aligned} \Delta h &= C_H n^2 D^2 / g \\ C_H &= 150(32.2)/[(25)^2(1.217)^2] \\ &= 5.217 \end{aligned}$$

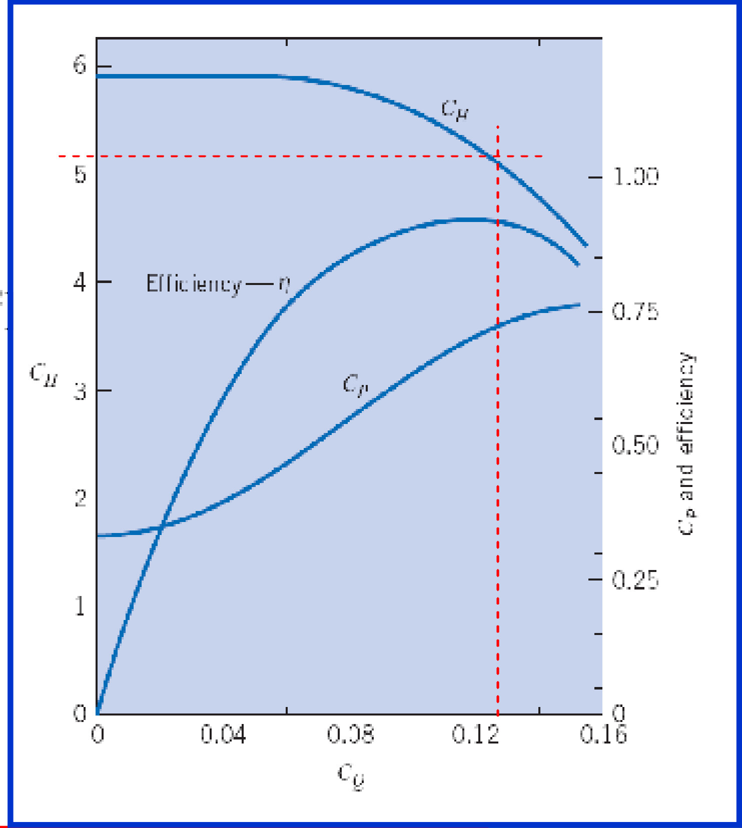
from Fig. 14.10

$$C_Q = 0.122$$

Discharge coefficient

$$\begin{aligned} Q &= C_Q n D^3 \\ &= 0.122(25)(1.217)^3 \end{aligned}$$

$$\boxed{Q = 5.50\text{ cfs}}$$



Problem (14.24)

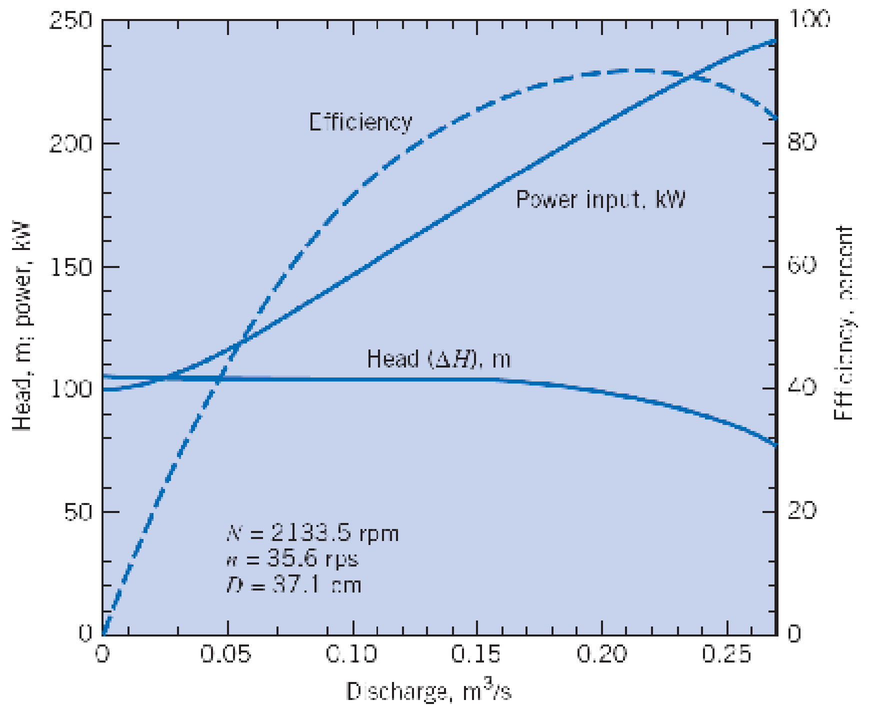
If a pump having the characteristics given in Fig. 14.9 is operated at a speed of 30 rps, what will be the shutoff head?

Head coefficient

$$C_H = \Delta H g / D^2 n^2$$

Since C_H will be the same for the maximum head condition, then

$$\Delta H \quad \alpha \quad n^2$$



Problem (14.24)

Head coefficient

$$C_H = \Delta H g / D^2 n^2$$

Since C_H will be the same for the maximum head condition, then

$$\Delta H \quad \alpha \quad n^2$$

Find: Shutoff head.

Given : $n = 30\, rps$

Pump Characteristics (Fig.14.9)

APPROACH

Apply head coefficient.

For $(n) = 35.6\, rps$, the shut off Head $= 104$

ANALYSIS

$$H \;\alpha\; n^2$$

so

$$H_{30}/H_{35.6} = (30/35.6)^2$$

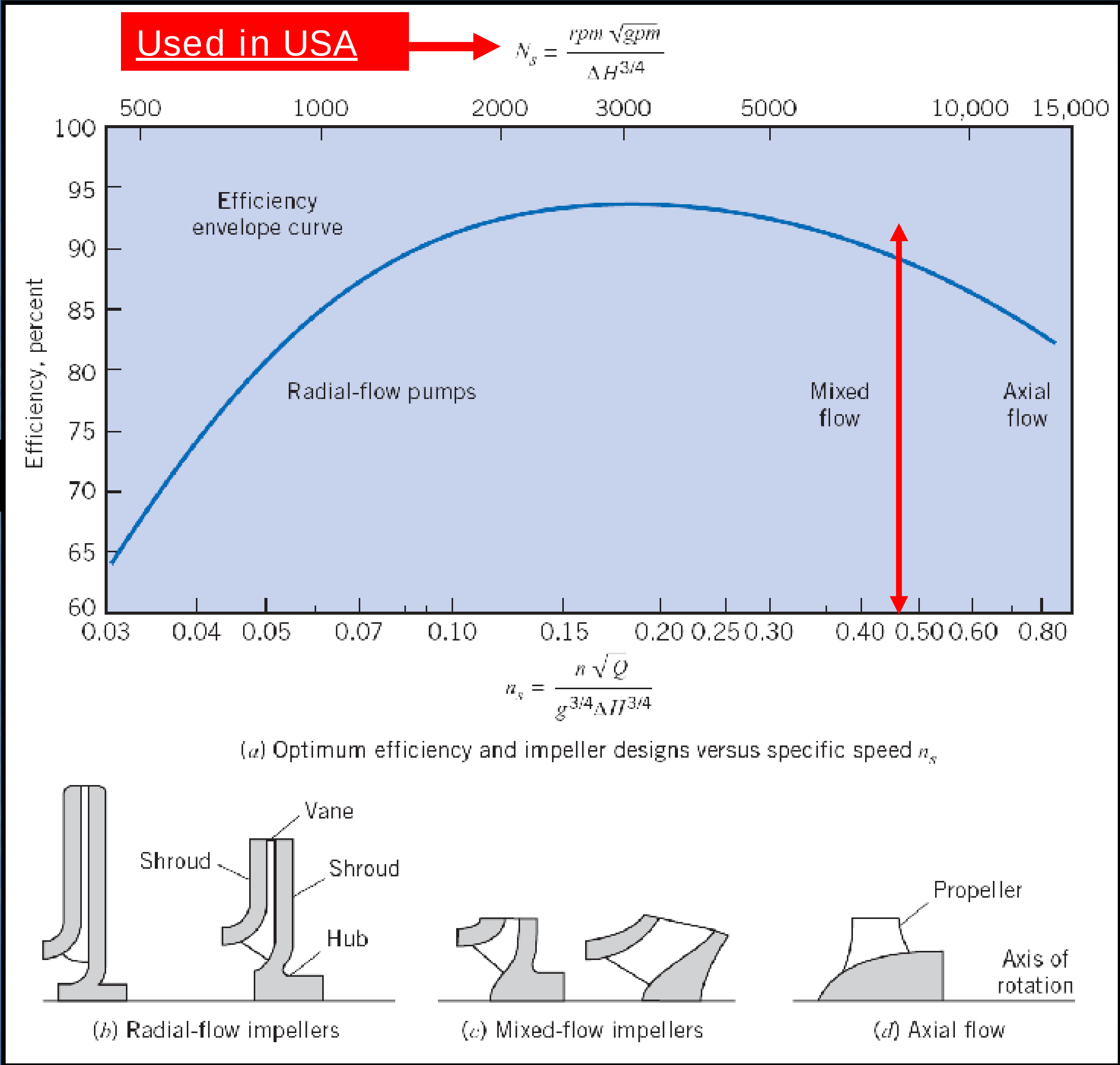
or

$$H_{30} = 104 \times (30/35.6)^2$$

$$\boxed{H_{30} = 73.8 \text{ m}}$$

PROBLEMS ON SPECIFIC SPEED

Fig. (14.14)



Used in USA
$N_s = \frac{rpm\sqrt{gpm}}{\Delta H^{3/4}}$
500
1000
2000
3000
5000
10,000
15,000
100
95
90
85
80
75
70
65
60
Efficiency, percent
Efficiency
envelope curve
Radial-flow pumps
Mixed
flow
Axial
flow
0.03
0.04
0.05
0.07
0.10
0.15
0.20
0.25
0.30
0.40
0.50
0.60
0.80
$n_s = \frac{n\sqrt{Q}}{g^{3/4}\Delta H^{3/4}}$
(a) Optimum efficiency and impeller designs versus specific speed n_s
Vane
Shroud
Shroud
Hub
Propeller
Axis of
rotation
(b) Radial-flow impellers
(c) Mixed-flow impellers
(d) Axial flow

Problem (14.30)

14.30 What type of pump should be used to pump water at a rate of 10 cfs and under a head of 30 ft? Assume $N = 1500$ rpm.

PROBLEM 14.30

Situation: A pump system is described in the problem statement.
$N = 1,500$ rpm so $n = 25$ rps; $Q = 10$ cfs; $h = 30$ ft

Find: Type of water pump.

APPROACH

Calculate the specific speed and use figure 14.14 to find the pump range to which it corresponds.

ANALYSIS

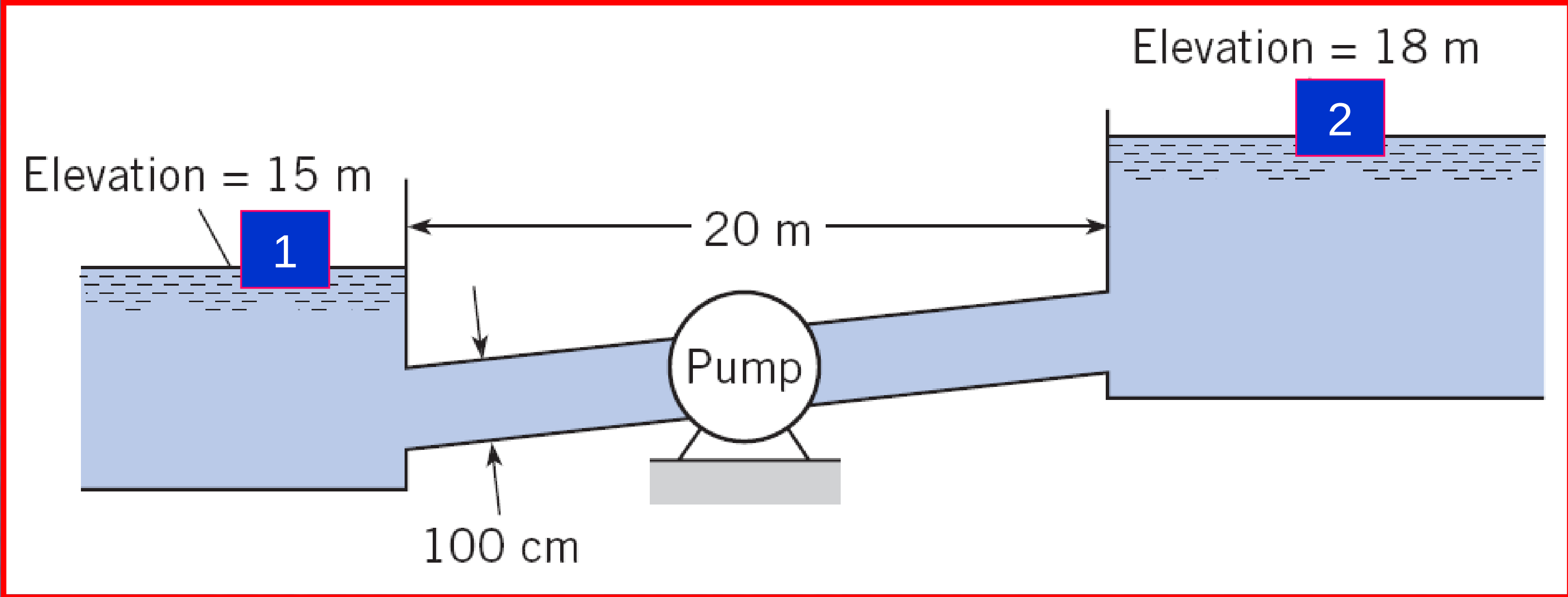
Specific speed

$$Specific\ Speed = n_S = \frac{n\sqrt{Q}}{(g \times \Delta H^{3/4})}$$

$$\begin{aligned} n_s &= n\sqrt{Q}/[g^{3/4}h^{3/4}] \\ &= (25)(10)^{1/2}/[(32.2)^{3/4}(30)^{3/4}] \\ &= 0.46 \end{aligned}$$

Then from Fig. 14.14, $n_s > 0.60$, so use a mixed flow pump.

Problem (14.34)



Find type of pump?

$$Specific\ Speed = n_S = \frac{n\sqrt{Q}}{(g\times \Delta H^{3/4})}$$

$Given: N = 600\,rpm, Q = 1.0\,m^3/s$

Apply Energy Equation between (1) & (2)

$$\left(h_P + \frac{p_1}{rg} + z_1 + a_1\frac{\overline{V}_1^{\,2}}{2g}\right)_{Mech.part} = \left(h_T + \frac{p_2}{rg} + gz_2 + a_2\frac{\overline{V}_2^{\,2}}{2g}\right)_{Mech.part} + h_{Loss}$$

APPROACH

Calculate the specific speed and use figure 14.14 to find the pump range to which it corresponds.

ANALYSIS

$Given: N = 600\,rpm, Q = 1.0\,m^3/s$

$f = 0.01$

Specific speed

$$n_s = n\sqrt{Q}/(g^{3/4}h^{3/4})$$
$$n = 10 \text{ rps}$$
$$Q = 1.0 \text{ m}^3/\text{s}$$
$$h = 3 + (1.5 + fL/D)V^2/(2g);$$
$$V = 1.27 \text{ m/s}$$

$$V = \frac{Q}{pD^2/4}$$

Assume $f = 0.01$, so

$$h = 3 + (1.5 + 0.01 \times 20/1)(1.27)^2/(2 \times 9.81)$$
$$= 3.14 \text{ m}$$

Then

$$n_s = 10 \times \sqrt{1}/(9.81 \times 3.14)^{3/4}$$
$$= 0.76$$

From Fig. 14.14, use axial flow pump.

PROBLEMS ON

NPSH

Problem (14.33)

[redacted]

14.33 An axial-flow pump is to be used to lift water against a head (friction and static) of 15 ft. If the discharge is to be 5000 gpm, what maximum speed in revolutions per minute is allowed if the suction head is 5 ft?

$$N_{max} = ?$$

$$NPSH = Suction\ pressure - vapour\ pressure\ @\ given\ liquid\ temp.$$

$$= p_2 - p_{Vapour\ @\ temp}$$

$$N_{SS} = \frac{NQ^{1/2}}{g^{3/4}(NPSH)^{3/4}}$$

Problem (14.33)

Find: Maximum speed.

$N_{max} = ?$

Axial Flow Pump

APPROACH

Apply the suction specific speed equation setting the critical value for N_{ss} proposed by the Hydraulic Institute to 8500.

T=60 F

ANALYSIS

$NPSH = Suction\ pressure - vapour\ pressure @ given\ liquid\ temp.$

$= p_2 - p_{Vapour @ temp}$

Suction specific speed

$$8500 = NQ^{1/2}/(NPSH)^{3/4}$$

$Q = 5000\, gpm$

The suction head is given as 5 ft. Then assuming that the atmospheric pressure is 14.7 psia, and the vapor pressure is 0.256 psi, the net positive suction head $(NPSH)$ is

$p_{suction} = 5\, ft + P_{atmos}$

$$NPSH = 14.7 \text{ psi } \times 2.31 \text{ ft/psi} + 5 \text{ ft} - h_{vap.press.} = 38.4 \text{ ft}$$

$h_{vapour\ pressure} = 0.256 \times \frac{144}{62.4} = 0.591$

Then

$$N = \frac{8500 \times (NPSH)^{3/4}}{Q^{1/2}} = \frac{8500 \times (38.4)^{3/4}}{5000^{1/2}}$$

$$\boxed{N = 1850 \text{ rpm}}$$

Problem (14.38)

A 12 kW (shaft output) motor is available to run a noncooled compressor for carbon dioxide. The pressure is to be increased from 90 kPa to 140 kPa. If the compressor is 60% efficient, calculate the volume flow rate into the compressor.

$$P_{\text{theo}} = \frac{k}{k-1} Q_1 p_1 \left[\left(\frac{p_2}{p_1} \right)^{(k-1)/k} - 1 \right]$$

$$\eta_{Comp} = \frac{P_{theo}}{(P_{actual})_{SHAFT}}$$

Problem (14.38)

PROBLEM 14.38

Situation: A compressor is described in the problem statement.

Find: Volume flow rate into the compressor.

APPROACH

Apply equation 14.17.

$$P_{theo} = \frac{k}{k-1} Q_1 p_1 \left[\left(\frac{p_2}{p_1} \right)^{(k-1)/k} - 1 \right]$$

ANALYSIS

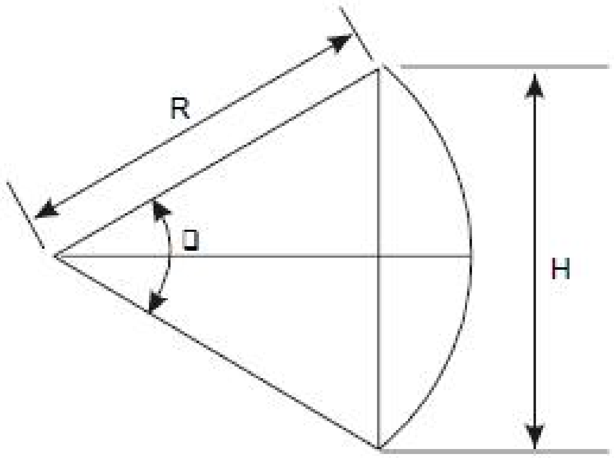
$$\eta_{Comp} = \frac{P_{theo}}{(P_{actual})_{SHAFT}}$$

$$\begin{aligned} P_{th} &= 12 \text{ kW} \times 0.6 = 7.2 \text{ kW} \\ P_{th} &= (k/(k-1)) Q p_1 [(p_2/p_1)^{(k-1)/k} - 1] \\ &= (1.3/0.3) Q \times 9 \times 10^4 [(140/90)^{0.3/1.3} - 1] \\ &= 4.18 \times 10^4 Q \\ Q &= 7.2/41.8 \end{aligned}$$

$$\boxed{Q = 0.172 \text{ m}^3/\text{s}}$$

Problem (14.38)

Consider the figure for the section of a circle.



The area of a sector is given by

$$A_s = \frac{1}{2}\theta R^2 - \frac{1}{2}RH\cos(\theta/2)$$

where θ is the angle subtended by the arc and H is the distance between the edges of the arc. But

$$R = \frac{H}{2\sin(\theta/2)}$$

Problem (14.38)

$$A = 2A_s = \frac{H^2}{4}\left[\frac{\theta}{\sin^2(\theta/2)} - 2\frac{\cos(\theta/2)}{\sin(\theta/2)}\right]$$

$$= 56.2 \times \left[\frac{\theta}{\sin^2(\theta/2)} - 2\frac{\cos(\theta/2)}{\sin(\theta/2)}\right]$$

$$1300$$

Solving graphically gives $\theta = 52^o$. The width of the windmill is

$$W = H\left[\frac{1}{\sin(\theta/2)} - \frac{1}{\tan(\theta/2)}\right]$$

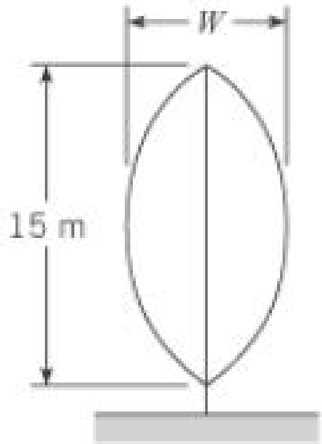
Substituting in the numbers gives W=3.45 m.

PROBLEMS ON

WIND TURBINES

Problem (14.48)

A wind "farm" consists of 20 Darrieus turbines, each 15 m high. The total output from the turbines is to be 2 MW in a wind of 20 m/s and an air density of 1.2 kg/m^3. The Darrieus turbine shown has the shape of an arc of a circle. Find the minimum width, W, of the turbine needed to provide this power output.



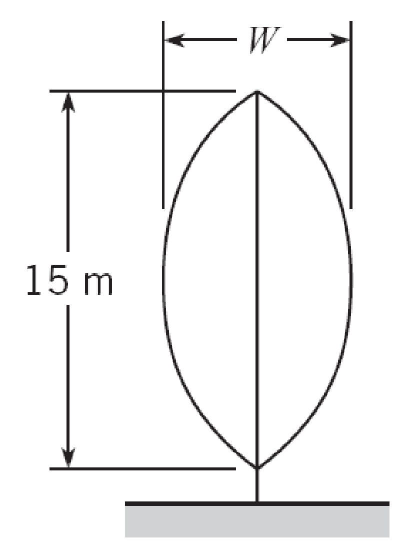
$$P_{max} = \frac{16}{27}\left(\frac{1}{2}\rho U^3 A\right)$$

U: Wind speed

A: Area captured by the wind turbine

Problem (14.48)

$$A_{Capture} = ?$$



ANALYSIS

Each windmill must produce 2 MW/20 = 100,000 W.

Wind turbine maximum power

$$P_{max} = \frac{16}{54}\rho V_o^3 A$$

In a 20 m/s wind with a density of 1.2 kg/m^3, the capture area is

$$A = \frac{54}{16}\frac{100000}{1.2 \times 20^3} = 35.16 \text{ m}^2$$

END OF SOLVED PROBLEMS