CHAPTER (3)

FLUID STATICS

SOLVED PROBLEMS

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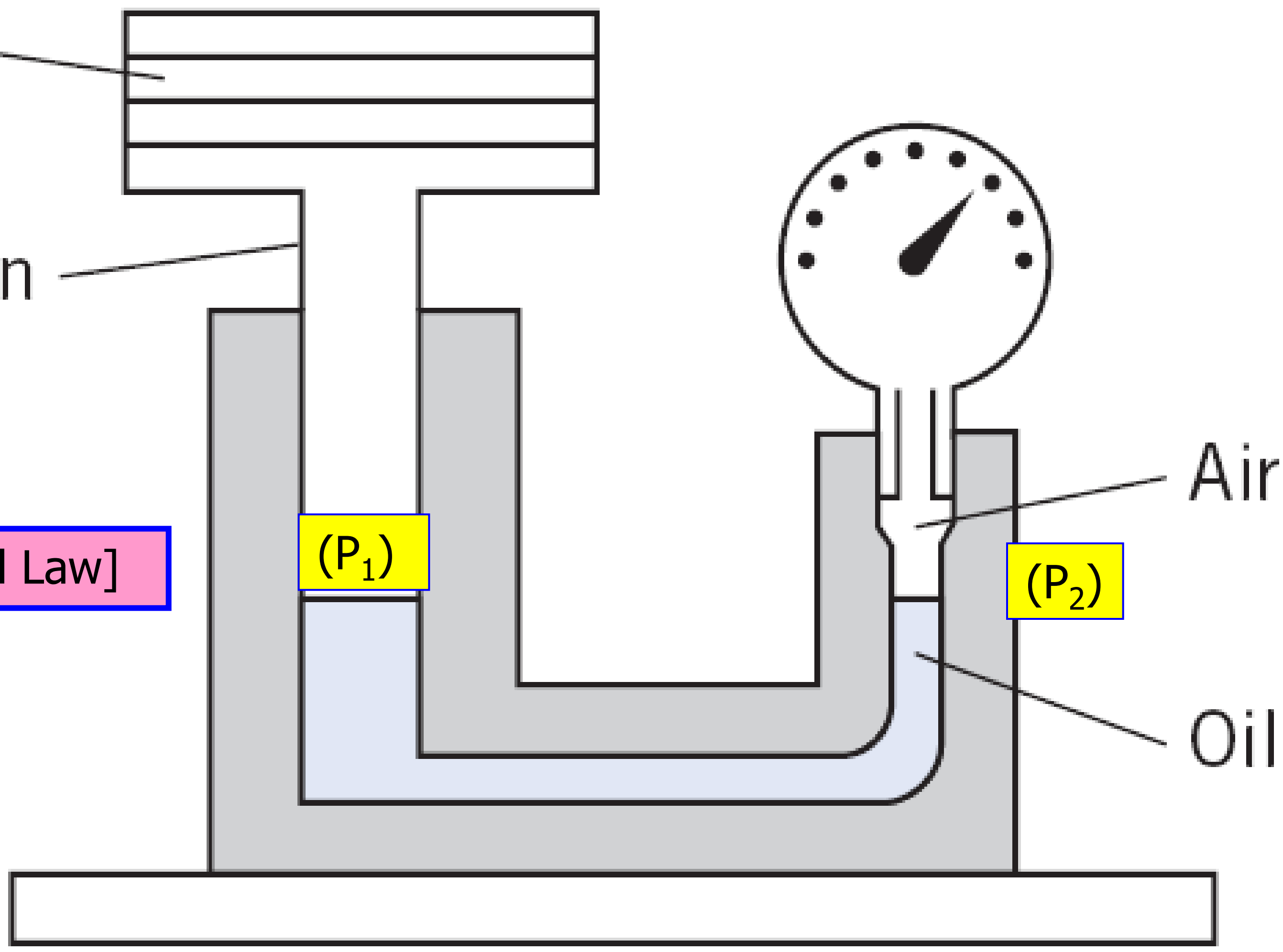
Problem (3.1)



$P_1 = P_2$ [Pascal Law]

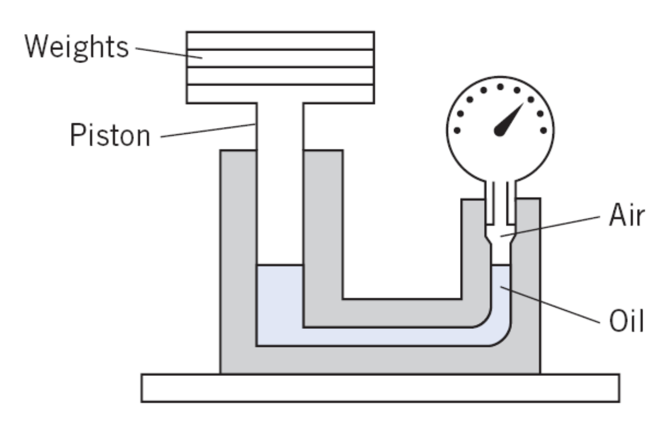
PROBLEM 3.1

Situation: A Crosby gage tester is applied to calibrate a pressure gage.
A weight of 140 N results in a reading of 200 kPa.
The piston diameter is 30 mm.

Find: Percent error in gage reading.

APPROACH

Calculate the pressure that the gage should be indicating (true pressure). Compare this true pressure with the actual pressure.



ANALYSIS

Pascal's Law Application

True pressure

$P_1 = P_2$ [Pascal Law]

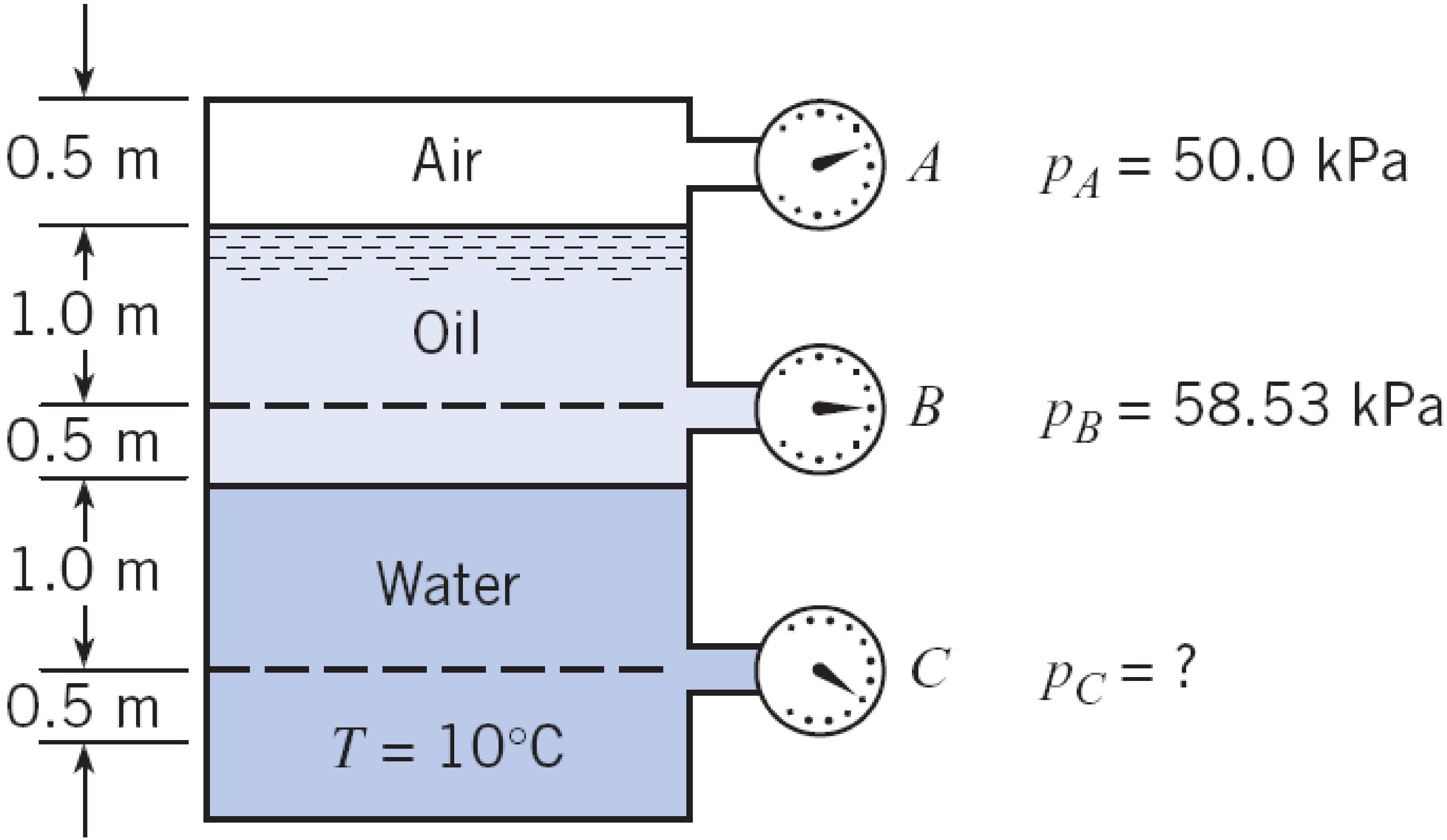
$$
\begin{aligned}
p_{\text{true}} &= \frac{F}{A} \\
&= \frac{140\,\text{N}}{(\pi/4 \times 0.03^2)\ \text{m}^2} \\
&= 198,049\,\text{kPa}
\end{aligned}
$$

Percent error

$$
\begin{aligned}
\%\ \text{Error} &= \frac{(p_{\text{recorded}} - p_{\text{true}})\,100}{p_{\text{true}}} \\
&= \frac{(200 - 198)\,100}{198} \\
&= 1.0101\%
\end{aligned}
$$

$$\boxed{\%\ \text{Error} = 1.01\%}$$

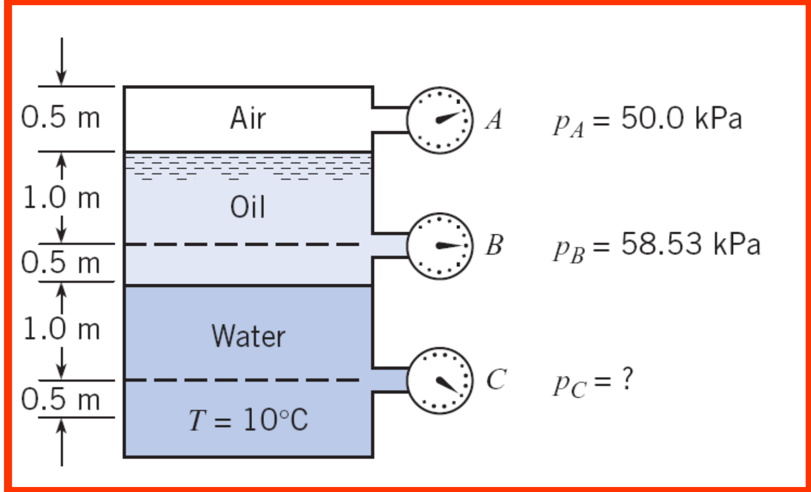
Problem (3.7)



0.5 m
1.0 m
0.5 m
1.0 m
0.5 m
Air
Oil
Water
T = 10°C
A
B
C
pA = 50.0 kPa
pB = 58.53 kPa
pC = ?

PROBLEM 3.7

Situation: A closed tank with Bourdon-tube gages tapped into it is described in the problem statement.



Find:
(a) Specific gravity of oil.
(b) Pressure at C.

APPROACH

Apply the hydrostatic equation.

ANALYSIS

Hydrostatic equation (from oil surface to elevation B)

$$\begin{aligned} p_A + \gamma z_A &= p_B + \gamma z_B \\ 50,000\ \text{N/m}^2 + \gamma_{\text{oil}}\,(1\ \text{m}) &= 58{,}530\ \text{N/m}^2 + \gamma_{\text{oil}}\,(0\ \text{m}) \\ \gamma_{\text{oil}} &= 8530\ \text{N/m}^2 \end{aligned}$$

$$p_B = p_A + \gamma_{oil}(1.0)$$
$$\gamma_{oil} = p_B - p_A$$
$$\gamma_{oil} = S.G \times \rho_w$$

Specific gravity

$$S = \frac{\gamma_{\text{oil}}}{\gamma_{\text{water}}} = \frac{8530\ \text{N/m}^2}{9810\ \text{N/m}^2}$$

$$\boxed{S_{\text{oil}} = 0.87}$$

Hydrostatic equation (in water)

$$p_c = (p_{\text{btm of oil}}) + \gamma_{\text{water}}\,(1\ \text{m})$$

Problem (3.7)

Hydrostatic equation (in oil)

$$p_{\text{btm of oil}} = (58,530\,\text{Pa} + \gamma_{\text{oil}} \times 0.5\,\text{m})$$

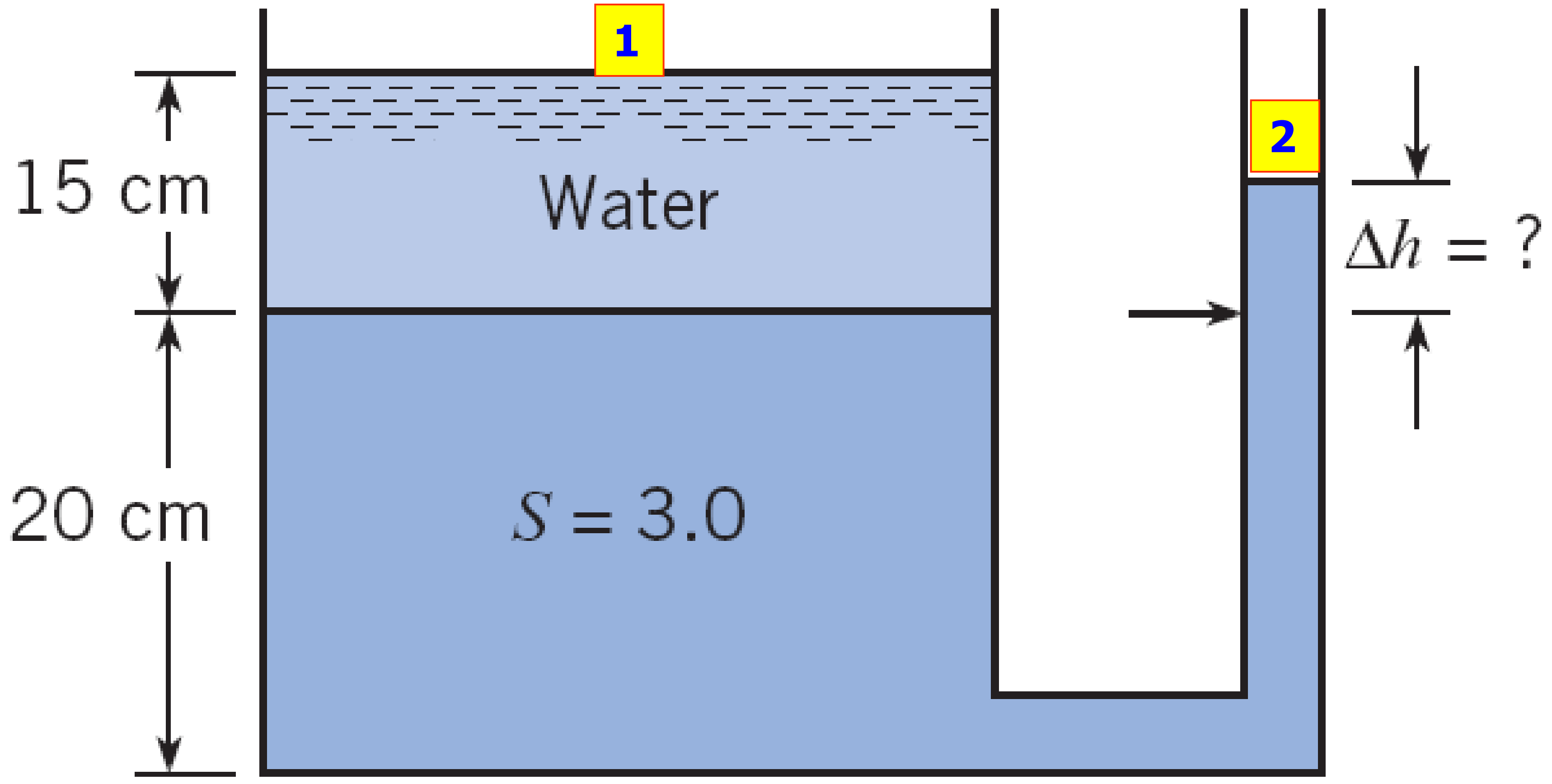
Combine equations

$$\begin{aligned} p_c &= (58,530\,\text{Pa} + \gamma_{\text{oil}} \times 0.5\,\text{m}) + \gamma_{\text{water}}\,(1\,\text{m}) \\ &= (58,530 + 8530 \times 0.5) + 9810\,(1) \\ &= 72,605\ \text{N/m}^2 \end{aligned}$$

$$\boxed{p_c = 72.6\ \text{kPa}}$$

$$p_C = p_B + \gamma_{oil}(0.5) + \gamma_w(1.0)$$

Problem (3.15)



PROBLEM 3.15

Situation: A tank fitted with a manometer is described in the problem statement.

Find: Deflection of the manometer. (Δh)

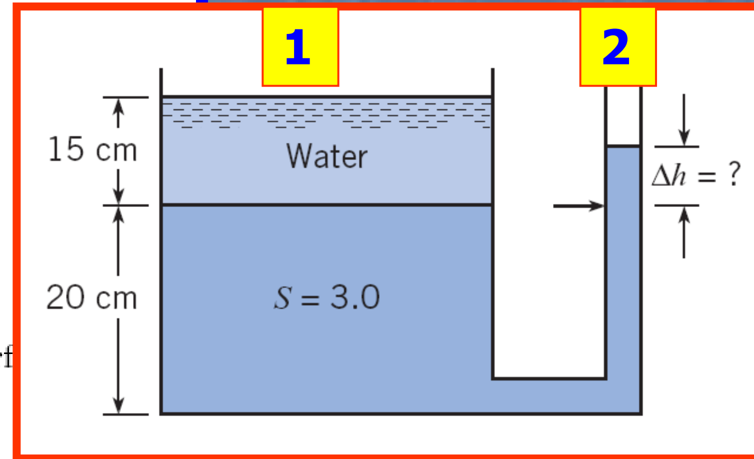
APPROACH

Apply the hydrostatic principle to the water and then to the manometer fluid.

ANALYSIS

Hydrostatic equation (location 1 is on the free surface of the water; location 2 is the interface)

$$\begin{aligned} \frac{p_1}{\gamma_{\text{water}}} + z_1 &= \frac{p_2}{\gamma_{\text{water}}} + z_2 \\ \frac{0\text{ Pa}}{9810\text{ N/m}^3} + 0.15\text{ m} &= \frac{p_2}{9810\text{ N/m}^3} + 0\text{ m} \\ p_2 &= (0.15\text{ m})\left(9810\text{ N/m}^3\right) \\ &= 1471.5\text{ Pa} \end{aligned}$$



Hydrostatic equation (manometer fluid; let location 3 be on the free surf

$$\begin{aligned} \frac{p_2}{\gamma_{\text{man. fluid}}} + z_2 &= \frac{p_3}{\gamma_{\text{man. fluid}}} + z_3 \\ \frac{1471.5\text{ Pa}}{3\left(9810\text{ N/m}^3\right)} + 0\text{ m} &= \frac{0\text{ Pa}}{\gamma_{\text{man. fluid}}} + \Delta h \end{aligned}$$

Solve for Δh

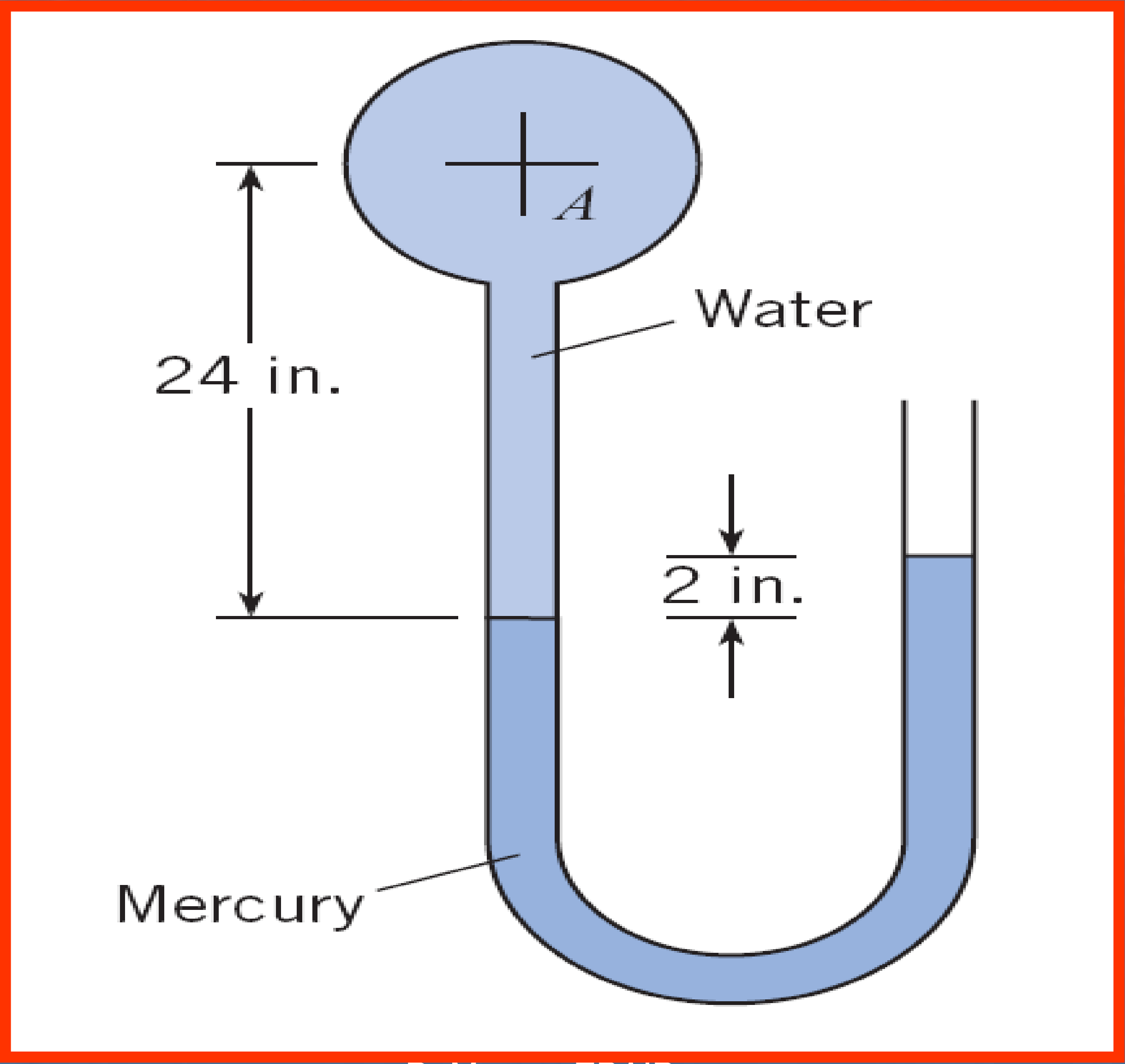
$$\begin{aligned} \Delta h &= \frac{1471.5\text{ Pa}}{3\left(9810\text{ N/m}^3\right)} \\ &= 0.0500\text{ m} \end{aligned}$$

$$\boxed{\Delta h = 5.00\text{ cm}}$$

$$p_2 = p_1 + \gamma_w(0.15) - SG \times \gamma_w(\Delta h)$$

$$\Delta h = \frac{0.15}{3} = 0.05\,m = 5\,cm$$

Problem (3.28)



Problem (3.28)

PROBLEM 3.28

Situation: A pipe system is described in the problem statement

Find: Gage pressure at pipe center.

APPROACH

Apply the manometer equation.

ANALYSIS

Manometer equation (from A to the open end of the manomete

$$p_A + (2.0 \text{ ft})(62.3 \text{ lbf/ft}^3) - (2/12 \text{ ft})(847 \text{ lbf/ft}^3$$

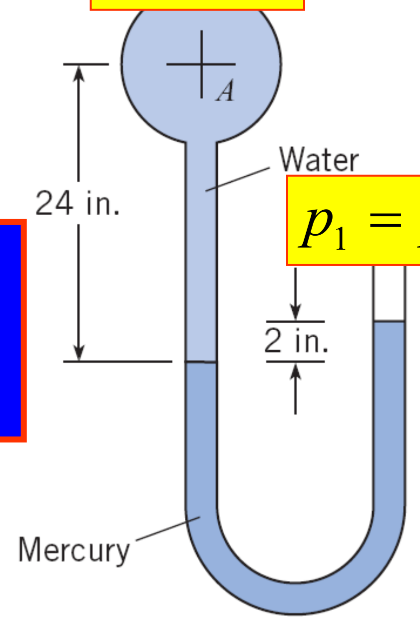
$$p_A = -124.6 \text{ lbf/ft}^2 + 141.2 \text{ lbf/ft}^2 = +16.6 \text{ lbf/ft}^2$$

$$\boxed{p_A = +0.12 \text{ psi}}$$

Find gauge pressure at center of pipe (A)=?

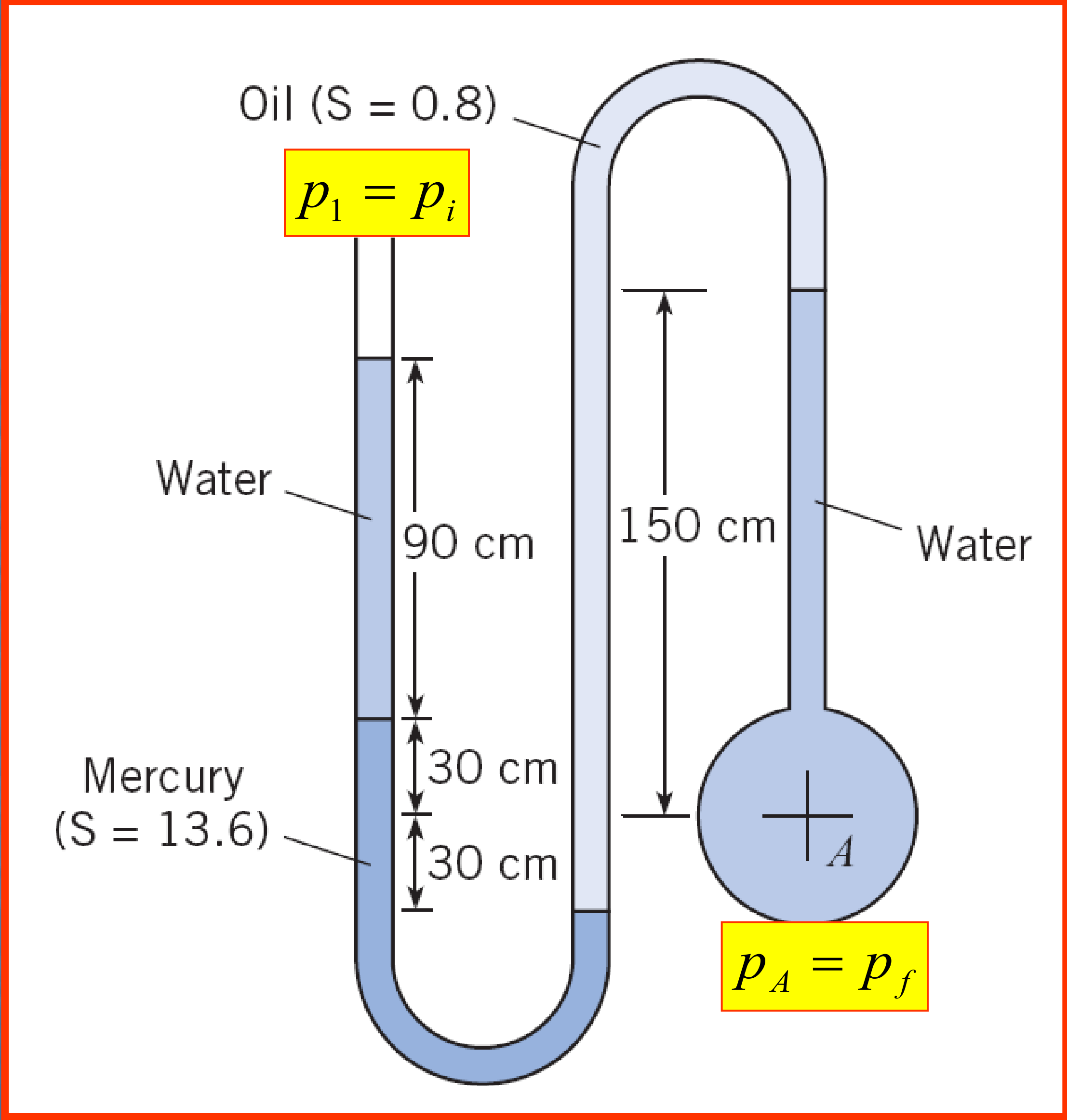
$p_A = p_f$

$p_1 = p_i$



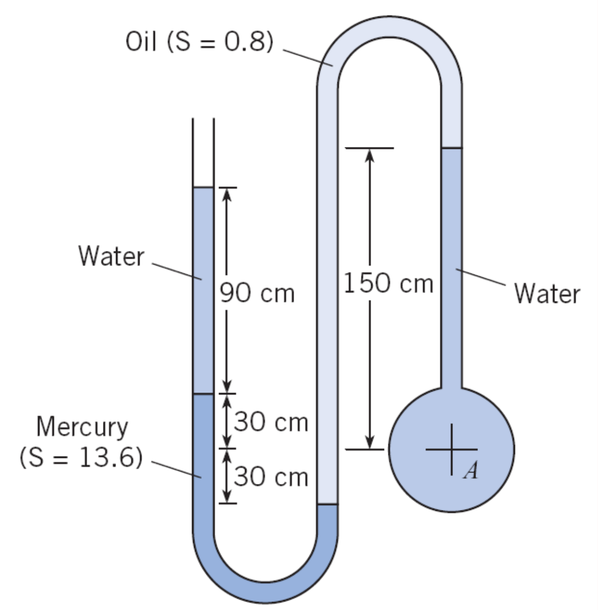
Problem (3.28)

Find gauge pressure at center of pipe (A)=?



Problem (3.28)

Find gauge pressure at center of pipe (A)=?



PROBLEM 3.39

Situation: A pipe system is described in the problem statement.

Find: Pressure at center of pipe A.

ANALYSIS

$S.G$

$\gamma_{water} = \rho g = 1000 \times 9.81$

Manometer equation

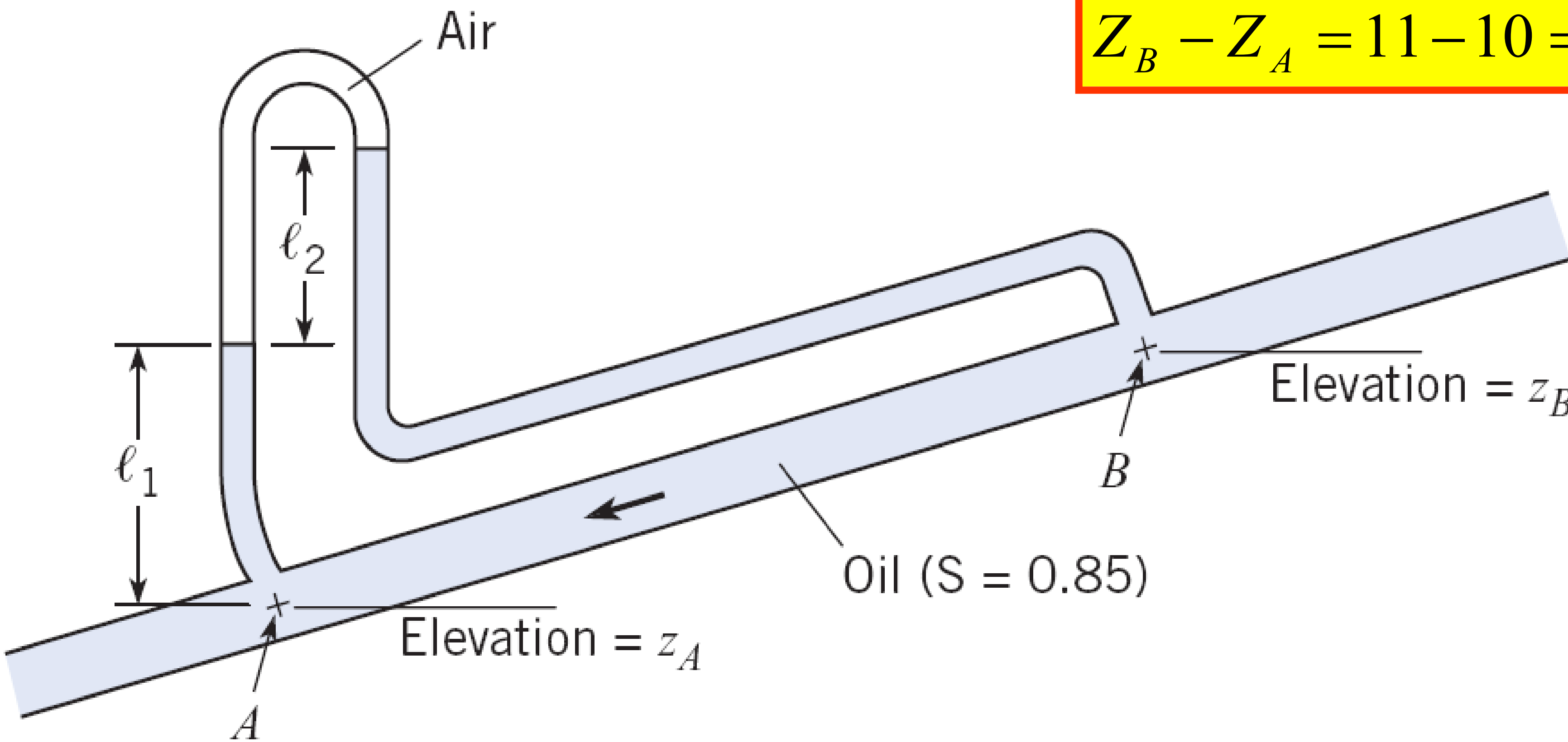
$$p_A = (0.9 + 0.6 \times 13.6 - 1.8 \times 0.8 + 1.5)9,810 = 89,467 \text{ Pa}$$

$$\boxed{p_A = 89.47 \text{ kPa}}$$

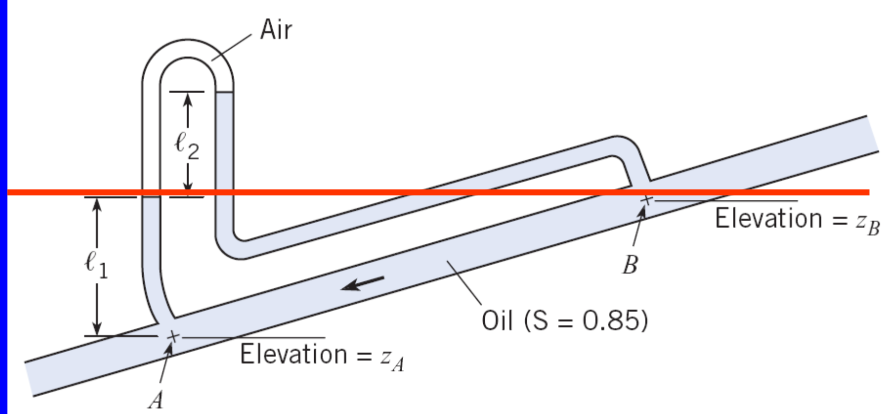
Problem (3.40)

$$P_A - P_B = ?$$
$$h_A - h_B = ?$$

$$L_1 = 1m$$
$$L_2 = 50cm$$
$$Z_B - Z_A = 11 - 10 = 1m$$



Problem (3.40)



PROBLEM 3.40

Situation: A pipe system is described in the problem statement.

Find: (a) Difference in pressure between points A and B.
(b) Difference in piezometric head between points A and B.

APPROACH

Apply the manometer equation.

$$P_A - P_B = ?$$
$$h_A - h_B = ?$$

$$L_1 = 1m$$
$$L_2 = 50cm$$
$$Z_B - Z_A = 11 - 10 = 1m$$

ANALYSIS

Manometer equation

$$p_A - (1\,\text{m})\left(0.85 \times 9810\,\text{N/m}^3\right) + (0.5\,\text{m})\left(0.85 \times 9810\,\text{N/m}^3\right) = p_B$$
$$p_A - p_B = 4169\,\text{Pa}$$

$$\boxed{p_A - p_B = 4.169 \text{ kPa}}$$

Piezometric head

$$\begin{aligned} h_A - h_B &= \left(\frac{p_A}{\gamma} + z_A\right) - \left(\frac{p_B}{\gamma} + z_B\right) \\ &= \frac{p_A - p_B}{\gamma} + (z_A - z_B) \\ &= \frac{4169\,\text{N/m}^2}{0.85 \times 9810\,\text{N/m}^3} - 1\,\text{m} \\ &= -0.5 \text{ m} \end{aligned}$$

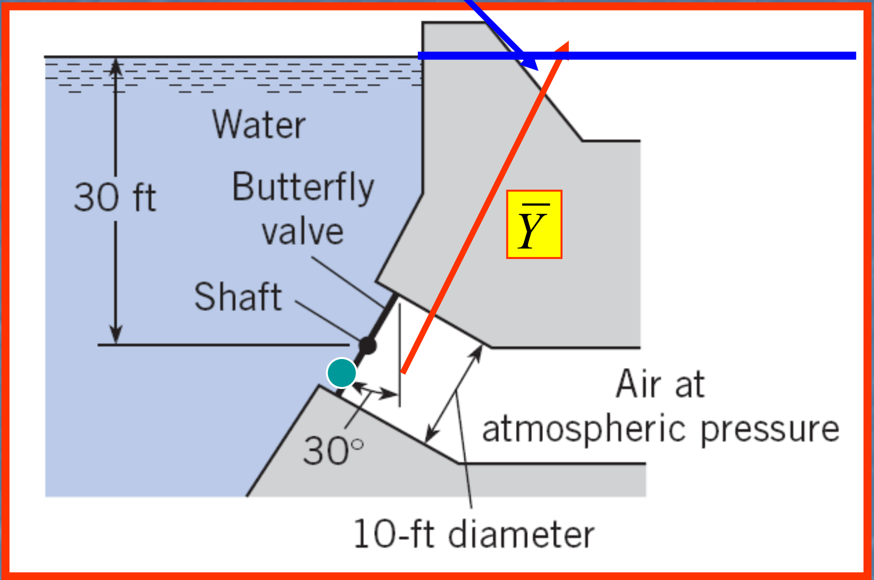
$$\boxed{h_A - h_B = -0.50 \text{ m}}$$

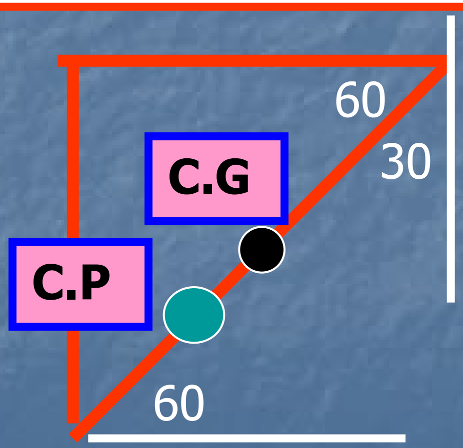
$$P_B = P_A - \gamma_f \times L_1 - \gamma_{air} \times L_2 + \gamma_f \times L_3$$

Problem (3.40)

$\alpha = 60\circ$

Butterfly valve





$\bar{y} = 30/\sin 60$

Problem (3.40)

$\alpha = 60\,\circ$

PROBLEM 3.65

Situation: A butterfly valve is described in the problem statement.

Find: Torque required to hold valve in position.

ANALYSIS

$Note: h_{C.G} = \bar{y}\sin\alpha = 30\,ft$

Hydrostatic force

$\sin\alpha$

$$\begin{aligned} F &= \bar{p}A = \bar{y}\gamma A \\ &= (30 \text{ ft} \times 62.4 \text{ lb/ft}^3)(\pi \times D^2/4) \text{ ft}^2) \\ &= (30 \times 62.4 \times \pi \times 10^2/4) \text{ lb} \\ &= 147,027 \text{ lb} \end{aligned}$$

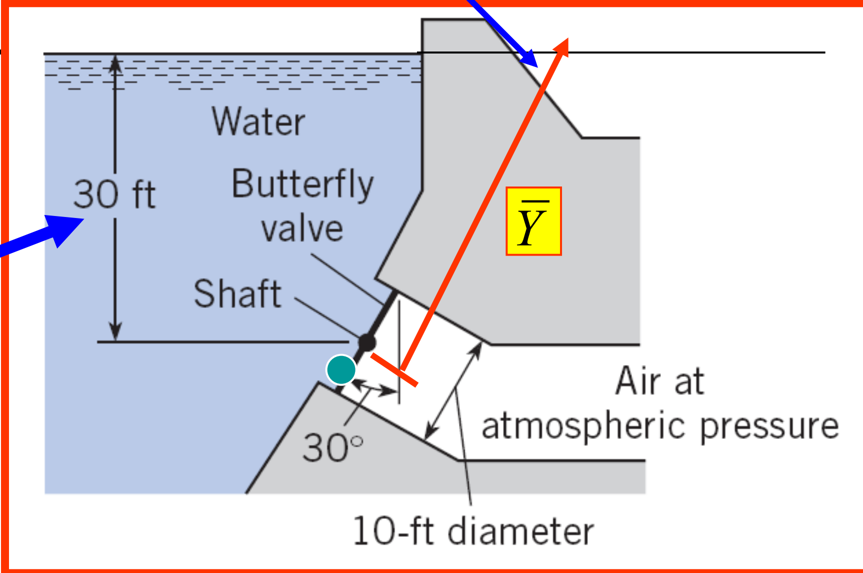
Center of pressure

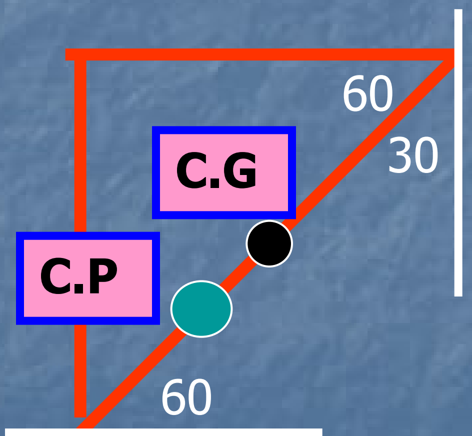
$$\begin{aligned} y_{cp} - \bar{y} &= I/\bar{y}A \\ &= (\pi r^4/4)/(\bar{y}\pi r^2) \\ &= (5^2/4)/(30/.866) \\ &= 0.1804 \text{ ft} \end{aligned}$$

Torque

$$\text{Torque} = 0.1804 \times 147,027$$

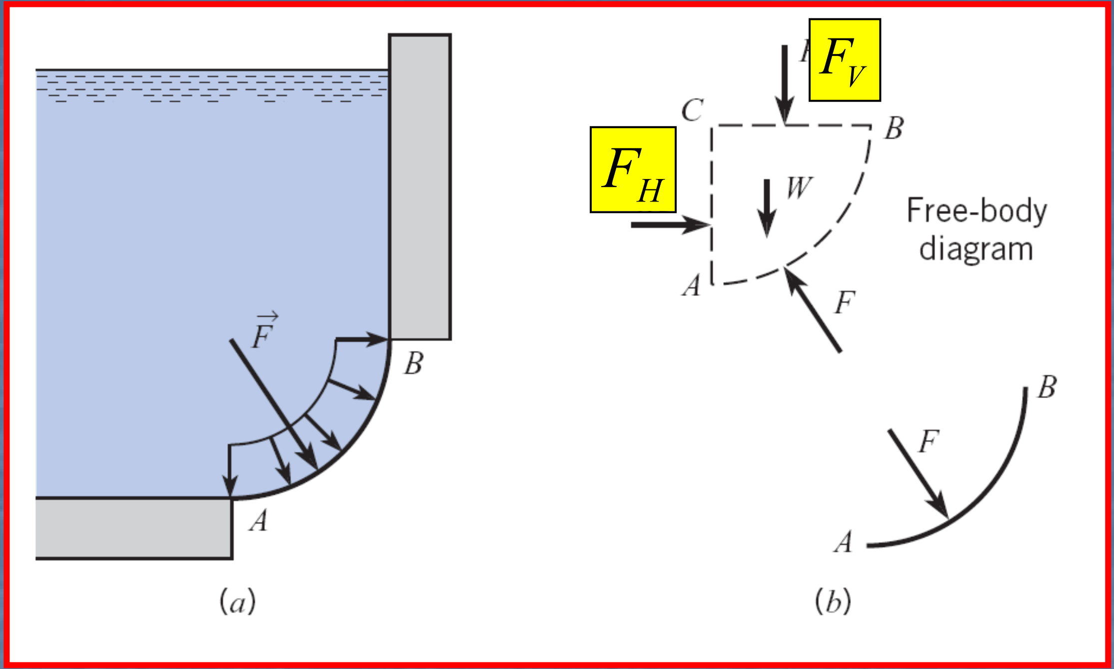
$$\boxed{T = 26,520 \text{ ft-lbf}}$$





$\bar{y} = 30/\sin 60$

HYDROSTATIC FORCES ON CURVED SURFACES



$$F_R = \sqrt{F_H{}^2 + F_Y{}^2} = \sqrt{F_H{}^2 + (F_V + W)^2}$$

$$F_{AC} = Horizontal\ component\ of\ F_r$$

$$F_{CB} = Vertical\ component\ of\ F_r$$

$$W = Weight\ of\ the\ fluid$$

Problem (3.65)

PROBLEM 3.78

Situation: A curved surface is described in the problem statement.

Find: (a) Vertical hydrostatic force.
(b) Horizontal hydrostatic force.
(c) Resultant force.

$Depth = 1m$

$F_H = ?$

$F_Y = ?$

$F_R = ?$

ANALYSIS

F_V W

$$F_Y = 1 \times 9,810 \times 1 \times +(1/4)\pi \times (1)^2 \times 1 \times 9,810$$

$$\boxed{F_V = 17,515 \text{ N}}$$

$$x = M_0/F_V$$

$$= 1 \times 1 \times 1 \times 9,810 \times 0.5 + 1 \times 9,810 \times \int_0^1 \sqrt{1-x^2}x dx / 17,515$$

$$= \boxed{0.467 \text{ m}}$$

$$F_H = \bar{p}A$$

$$= (1+0.5)9,810 \times 1 \times 1$$

$$\boxed{F_H = 14,715 \text{ N}}$$

$$y_{cp} = \bar{y} + \bar{I}/\bar{y}A$$

$$= 1.5 + (1 \times 1^3)/(12 \times 1.5 \times 1 \times 1)$$

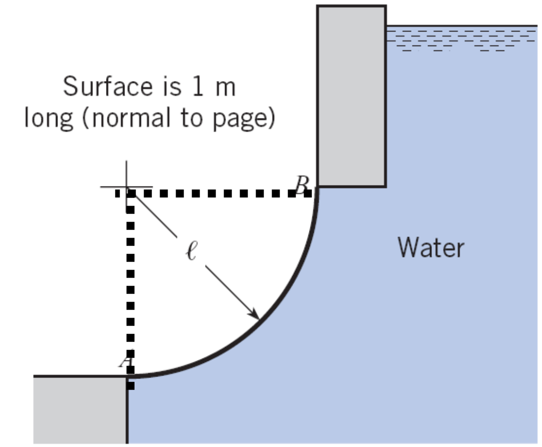
$$\boxed{y_{cp} = 1.555 \text{ m}}$$

$$F_R = \sqrt{(14,715)^2 + (17,515)^2}$$

$$\boxed{F_R = 22,876 \text{ N}}$$

$$\tan\theta = 14,715/17,515$$

$$\boxed{\theta = 40°2'}$$



$$F_v = \gamma \bar{y} \sin\alpha A = (9810)(1)(1\times1) = 9810N$$

$$W = \gamma V_{ABC} = 9810 \times \frac{1}{4}\pi \times 1^2 = 7704.76N$$

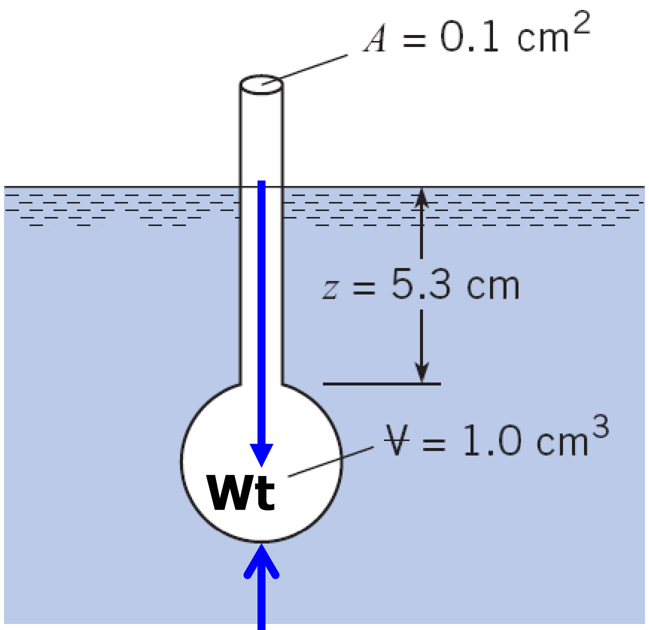
$$F_y = 9810 + 7704.76 = 17514.76N$$

$$x_{cp}F_y = (0.5 \times F_V) + (W \times \frac{4r}{3\pi})$$

$$x_{cp} = \frac{(9810 \times 0.5) + (7704.76 \times \frac{4\times1}{3\pi})}{17514.76} = 0.467$$

Problem (3.101)

Find weight of the hydrometer=?



PROBLEM 3.101

Situation: A hydrometer is described in the problem statement.

Find: Weight of hydrometer.

ANALYSIS

Volume of immersed part

$$F_{\text{buoy.}} = W$$

$$V\!\!\!/\ \gamma_W = W$$

$$(1\ \text{cm}^3 + (5.3\ \text{cm})(0.01\ \text{cm}^2))(0.1^3)\ \text{m}^3/\text{cm}^3(\gamma_W) = W$$

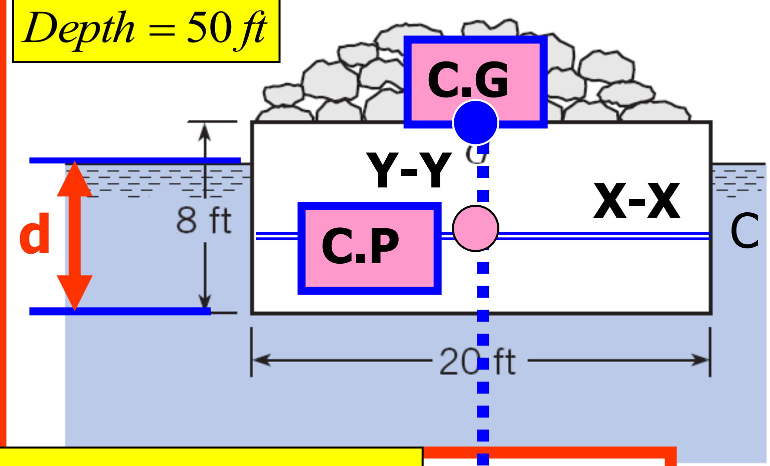
$$(1.53\ \text{cm}^3)(10^{-6}\ \text{m}^3/\text{cm}^3)(9810\ \text{N/m}^3) = W.$$

$$\boxed{W = 1.50 \times 10^{-2}\ \text{N}}$$

Problem (3.105)

$weight\ of\ bargo\ and\ rocks = 400{,}000\ lbf$

Will the bargo float upright or tip over



Axis of Symmetry

PROBLEM 3.106

Situation: A barge is described in the problem statement.

Find: Stability of barge.

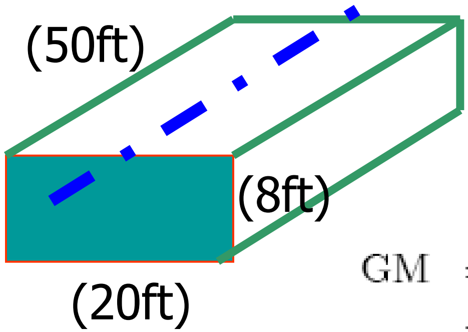
$F_B = (Wt\ of\ bargo + Rocks)_{Immersed}$

$F_B = (\gamma V)_{Immersed} = (62.4)(400{,}000 \times 50 \times 20 \times d)$

F_B

ANALYSIS

I_{00}



Wt → 400,000 ; L → 50

$$\text{Draft} = 400{,}000/(50 \times 20 \times 62.4) = 6.41\ \text{ft} < 8\ \text{ft}$$

$$\text{GM} = I_{00}/V - CG = [(50 \times 20^3/12)/(6.41 \times 50 \times 20)] - (8 - 3.205) = 0.40\ \text{ft}$$

$$CG = 8 - \frac{d}{2} = 8 - \frac{6.41}{2}$$

Will float stable

END OF QUESTIONS