

CHAPTER (4)

FLOWING FLUID AND PRESSURE VARIATIONS

SOLVED PROBLEMS

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1.a Uniform Flow

$$\frac{\partial V}{\partial s} = 0$$

Non-Uniform Flow

$$\frac{\partial V}{\partial s} \neq 0$$

1.b Steady Flow

$$\frac{\partial V}{\partial t} = 0$$

Non-Steady Flow

$$\frac{\partial V}{\partial t} \neq 0$$

2.a Acceleration:

$$a = a_t + a_n$$

$$a_t = \left(V \frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} \right)$$

$$a_n = \left(\frac{V^2}{r} \right)$$

2.b Acceleration of Liquid in a Tank:

$$\frac{dz}{dl} = \frac{a_l}{g} = \frac{a_x \cos \alpha}{g}$$

3. Eulerian Approach:

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

4. Euler Equation of motion:

$$-\frac{d}{dl}(p + \gamma z) = \rho a_l$$

5. Bernoulli Equation:

$$\left(p + \gamma z + \rho \frac{V^2}{2} \right) = C$$

$$OR \quad \frac{p}{\gamma} + z + \frac{V^2}{2g} = C$$

6. Pitot Tube Equation

$$V_2 = \left[\frac{2}{\rho} (p_{z,1} - p_{z,2}) \right]^{1/2}$$

6. Pressure Coefficient

$$C_p = \frac{p - p_0}{\frac{1}{2} \rho V_0^2}$$

$$C_p = \frac{h - h_0}{\frac{1}{2} \rho V_0^2}$$

7. The rotational velocity about x, y and z axes

$$\Omega_x = \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right)$$

$$\Omega_y = \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right)$$

$$\Omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

8. Vorticity $(\omega) = 2\Omega$

9. Free Vortex Flow

$$V = \frac{C}{r} \longrightarrow V \propto 1/r$$

10. Forced Vortex Flow $V = \omega r \longrightarrow V \propto r$

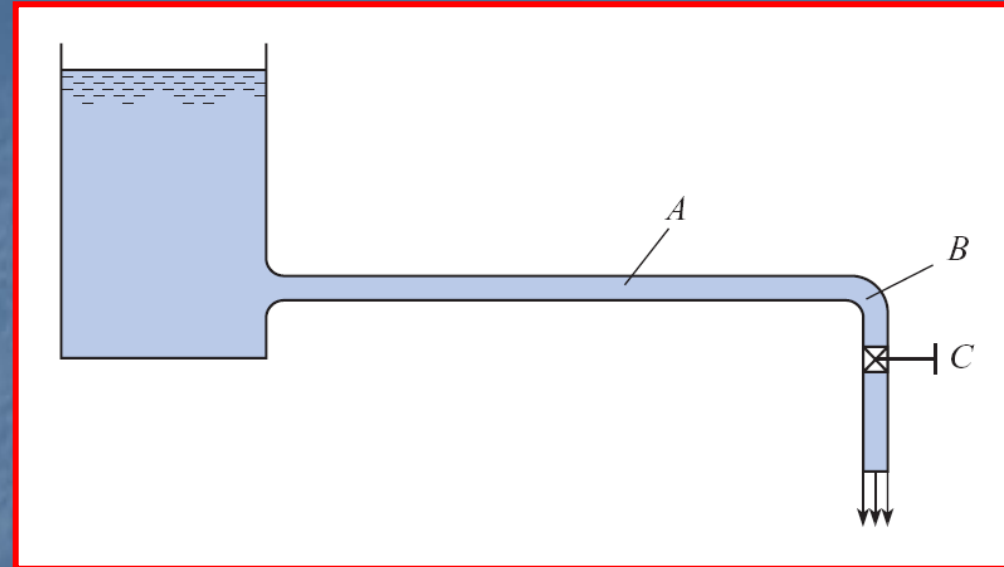
11. Pressure Variation in Rotating Flow

$$\left(\frac{p}{\gamma} + z - \frac{\omega^2 r^2}{2g} \right) = C$$

12. The Pressure Difference between point (1); i.e. the center of the tornado and the outer edge of the tornado

$$p_1 - p_0 = -\rho V_{\max}^2$$

Problem 4.1



PROBLEM 4.1

Situation: The valve in a system is gradually opened to have a constant rate of increase in discharge.

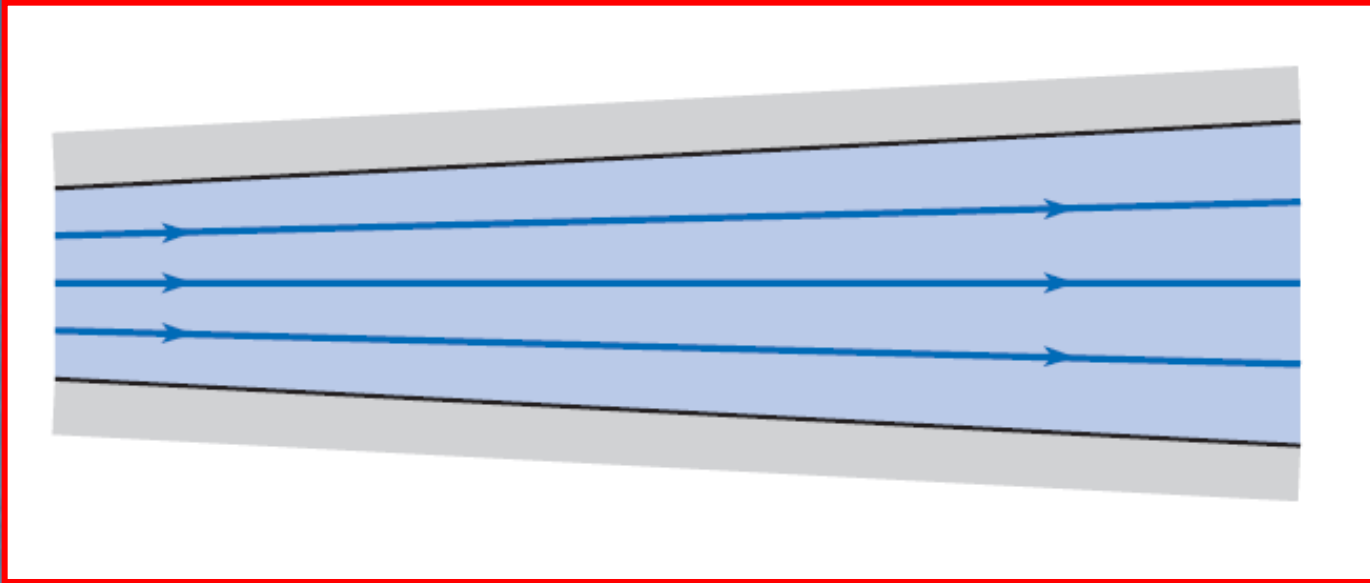
Find: Describe the flow at points A and B.

ANALYSIS

B: Non-uniform, unsteady.

A: Unsteady, uniform.

Problem 4.2



PROBLEM 4.2

Situation: Water flows in a passage with flow rate decreasing with time.

Find: Describe the flow.

ANALYSIS

(b) Unsteady and (d) non-uniform.

(a) Local and (b) convective acceleration.

**Means type of flow, also,
type of acceleration**

$$\frac{\partial V}{\partial t} \neq 0 \quad \frac{\partial V}{\partial s} \neq 0$$

$$a_t = \left(V \frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} \right)$$

Problem 4.18

PROBLEM 4.18

Situation: A flow with this velocity field: $u = xt + 2y$, $v = xt^2 - yt$, $w = 0$.

Find: Acceleration, \mathbf{a} , at location (1,2) and time $t = 3$ seconds.

ANALYSIS

Two dimensional flow (X, Y)

Acceleration in the x-direction

$$\begin{aligned} a_x &= u\partial u/\partial x + v\partial u/\partial y + w\partial u/\partial z + \partial u/\partial t \\ &= (xt + 2y)(t) + (xt^2 - yt)(2) + 0 + x \end{aligned}$$

At $x = 1$ m, $y = 2$ m and $t = 3$ s

$$a_x = (3 + 4)(3) + (9 - 6)(2) + 1 = 21 + 6 + 1 = 28 \text{ m/s}^2$$

Acceleration in the y-direction

$$\begin{aligned} a_y &= u\partial v/\partial x + v\partial v/\partial y + w\partial v/\partial z + \partial v/\partial t \\ &= (xt + 2y)(t^2) + (xt^2 - yt)(-t) + 0 + (2xt - y) \end{aligned}$$

At $x = 1$ m, $y = 2$ m and $t = 3$ s

$$a_y = (3 + 4)(9) + (9 - 6)(-3) + (6 - 2) = 63 - 9 + 4 = 58 \text{ m/s}^2$$

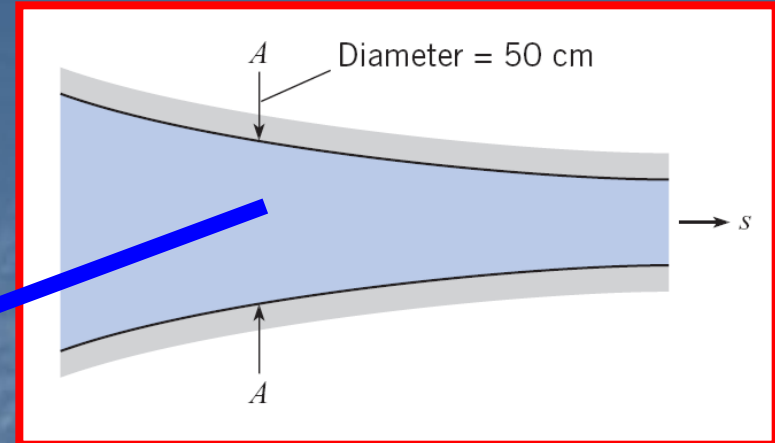
$$\mathbf{a} = 28 \mathbf{i} + 58 \mathbf{j} \text{ m/s}^2$$

Problem 4.21

Given: $Q_0 = 0.985 \text{ m}^3/\text{s}$, $Q_1 = 0.5 \text{ m}^3/\text{s}$, $t_0 = 1 \text{ s}$

$$\partial V / \partial s = 2 \text{ m/s}$$

$$d = 50 \text{ cm}$$



Situation: Flow occurs in a tapered passage. The velocity is given as

$$V = Q/A$$

and

$$Q = Q_0 - Q_1 \frac{t}{t_0}$$

Flow profile

The point of interest is section AA, where the diameter is 50 cm. The time of interest is 0.5 s.

Find: (a) Velocity at section AA: V

(b) Local acceleration at section AA: a_l

$$\frac{dV}{dt}$$

(c) Convective acceleration at section AA: a_c

$$V \frac{dV}{ds}$$

Flow is Non uniform and Unsteady

Given: $Q_0 = 0.985 \text{ m}^3/\text{s}$, $Q_1 = 0.5 \text{ m}^3/\text{s}$, $t_0 = 1 \text{ s}$

$$Q = Q_0 - Q_1 t/t_0 = 0.985 - 0.5t \quad (\text{given})$$

$$\partial V/\partial s = 2 \text{ m/s}$$

$$V = Q/A \quad (\text{given})$$

$$d = 50 \text{ cm}$$

$$\frac{\partial V}{\partial s} = +2 \frac{\text{m}}{\text{s}} \text{ per m} \quad (\text{given})$$

The velocity is

$$t = 0.5 \text{ s}$$

$$\begin{aligned} V &= Q/A \\ &= (0.985 - 0.5 \times 0.5)/(\pi/4 \times 0.5^2) \end{aligned}$$

$$V = 3.743 \text{ m/s}$$

Local acceleration

$$t = 0.5 \text{ s}$$

$$\begin{aligned} a_l &= \partial V/\partial t = \partial/\partial t(Q/A) \\ &= \partial/\partial t((0.985 - 0.5t)/(\pi/4 \times 0.5^2)) \\ &= -0.5/(\pi/4 \times 0.5^2) \end{aligned}$$

$$a_l = -2.55 \text{ m/s}^2$$

Convective acceleration

$$t = 0.5 \text{ s}$$

$$\begin{aligned} a_c &= V \partial V/\partial s \\ &= 3.743 \times 2 \end{aligned}$$

$$a_c = +7.49 \text{ m/s}^2$$

Problem 4.22 (p. 128)

$$a_t = \left(V \frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} \right)$$

PROBLEM 4.22

Situation: One-dimensional flow occurs in a nozzle. Velocity varies linearly from 1 ft/s at the base to 4 ft/s at the tip. The nozzle is 18 inches long.

Find: (a) Convective acceleration: a_c

(b) Local acceleration: a_t

At mid-point

$$V \frac{dV}{ds}$$

ANALYSIS

Velocity gradient

$$\frac{dV}{dt}$$

$$\begin{aligned} dV/ds &= (V_{\text{tip}} - V_{\text{base}})/L \\ &= (4 - 1)/1.5 \\ &= 2 \text{ s}^{-1} \end{aligned}$$

$$\begin{aligned} V &= f(X) \text{ OR } V = f(S) \\ V &= \left(\frac{4-1}{1.5} \right) (S) \end{aligned}$$

Acceleration at mid-point

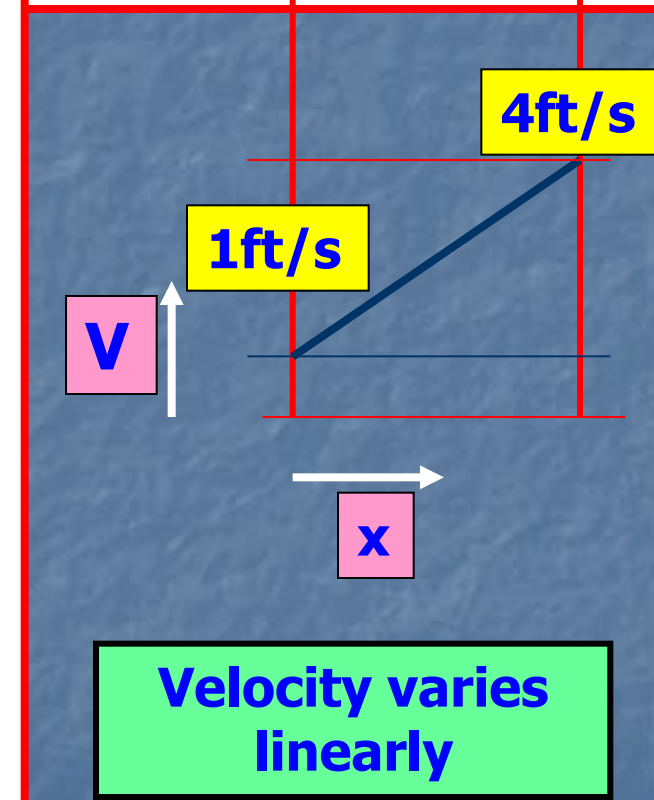
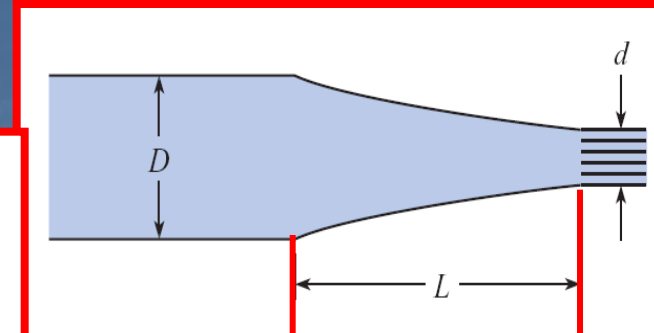
$$\begin{aligned} V &= (1 + 4)/2 \\ &= 2.5 \text{ ft/s} \end{aligned}$$

$$\begin{aligned} a_c &= V \frac{dV}{ds} \\ &= 2.5 \times 2 \end{aligned}$$

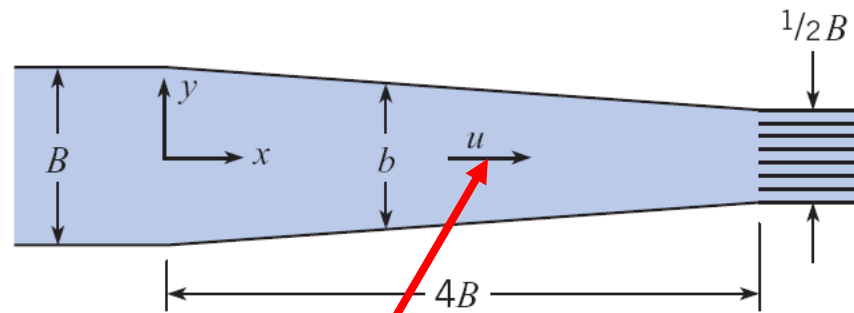
$$a_c = 5 \text{ ft/s}^2$$

Local acceleration

$$a_t = 0$$



Problem 4.24 (p. 128)



PROBLEM 4.24

Situation: Flow in a two-dimensional slot with

Flow Unsteady

$$V = 2 \left(\frac{q_o}{b} \right) \left(\frac{t}{t_o} \right)$$

Find: An expression for local acceleration midway in nozzle: a_l and $y = 0$

ANALYSIS

$$V = 2 \left(\frac{q_o}{b} \right) \left(\frac{t}{t_o} \right) \text{ but } b = B/2$$

$$V = \left(\frac{4q_o}{B} \right) \left(\frac{t}{t_o} \right)$$

$$a_l = \partial V / \partial t$$

$$a_l = 4q_o / (B t_o)$$

Problem 4.27 (p. 129)

PROBLEM 4.27

Situation: Flow through an inclined pipe at 30° from horizontal and decelerating at $0.3g$.

Find: Pressure gradient in flow direction.

APPROACH

Apply Euler's equation.

ANALYSIS

$$a_l = -0.3g$$



Euler's equation

$$\begin{aligned}\partial/\partial l(p + \gamma z) &= -\rho a_l \\ \partial p/\partial l + \gamma \partial z/\partial l &= -\rho a_l \\ \partial p/\partial l &= -\rho a_l - \gamma \partial z/\partial l \\ &= -(\gamma/g) \times (-0.30g) - \gamma \sin 30^\circ \\ &= \gamma(0.30 - 0.50)\end{aligned}$$

$$\partial p/\partial l = -0.20\gamma$$

Euler Equation

$$-\frac{d}{dl}(p + \gamma z) = \rho a_l$$

Problem 4.36 (p. 129)

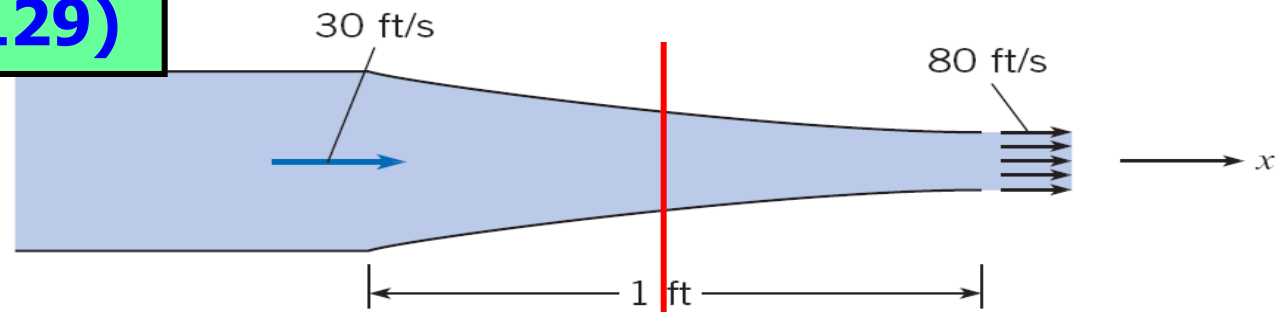
$$a_t = \left(V \frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} \right)$$

Find pressure gradient
Half-way through the
nozzle

Velocity Profile

$$V = f(X) \text{ OR } V = f(S)$$

$$V = \left(\frac{80 - 30}{1.0} \right) (X)$$



APPROACH

Apply Euler's equation.

ANALYSIS

Euler's equation

$$\frac{\partial p}{\partial x} = ?$$

$$d/dx(p + \gamma z) = -\rho a_x$$

but $z = \text{const.}$; therefore

$$\frac{dV}{dt}$$

$$V \frac{dV}{dx}$$

Finally

$$dp/dx = -\rho a_x$$

$$a_x = a_{\text{convective}} = V dV/dx$$

$$dV/dx = (80 - 30)/1 = 50 \text{ s}^{-1}$$

$$V_{\text{mid}} = (80 + 30)/2 = 55 \text{ ft/s}$$

$$= (55 \text{ ft/s})(50 \text{ ft/s/ft}) = 2,750 \text{ ft/s}^2$$

$$dp/dx = (-1.94 \text{ slug/ft}^3)(2,750 \text{ ft/s}^2)$$

$$dp/dx = -5,335 \text{ psf/ft}$$

Problem 4.44 (p. 130)

Bernoulli equation :(steady, incompressible, no viscous forces and irrotational)

PROBLEM 4.44

Situation: A water jet is described in the problem statement.

Find: Height h jet will rise. **Given**: $V_1 = 20 \text{ m/s}$

APPROACH

Apply the Bernoulli equation from the nozzle to the top of the jet. Let point 1 be in the jet at the nozzle and point 2 at the top.

ANALYSIS

Bernoulli equation

$$p_1/\gamma + V_1^2/2g + z_1 = p_2/\gamma + V_2^2/2g + z_2$$

where $p_1 = p_2 = 0$ gage

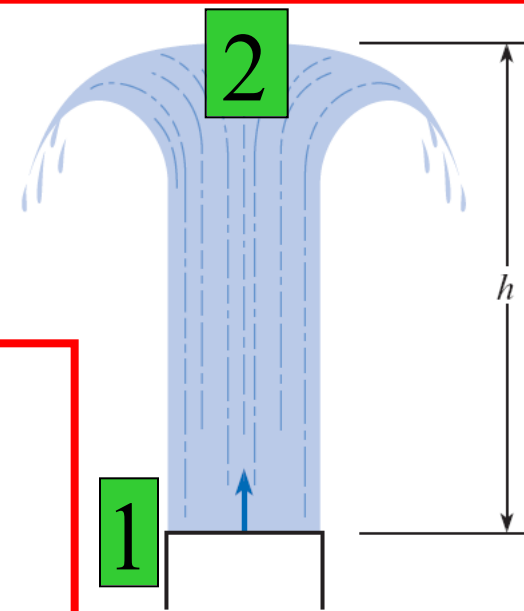
$$V_1 = 20 \text{ ft/s}$$

$$V_2 = 0$$

$$0 + (20)^2/2g + z_1 = 0 + 0 + z_2$$

$$z_2 - z_1 = h = 400/64.4$$

$$h = 6.21 \text{ ft}$$



Problem 4.47 (p. 130)

$$\left(p_1 + \gamma z_1 + \rho \frac{V_1^2}{2} \right) = \left(p_2 + \gamma z_2 + \rho \frac{V_2^2}{2} \right)$$

$$z_1 = z_2$$

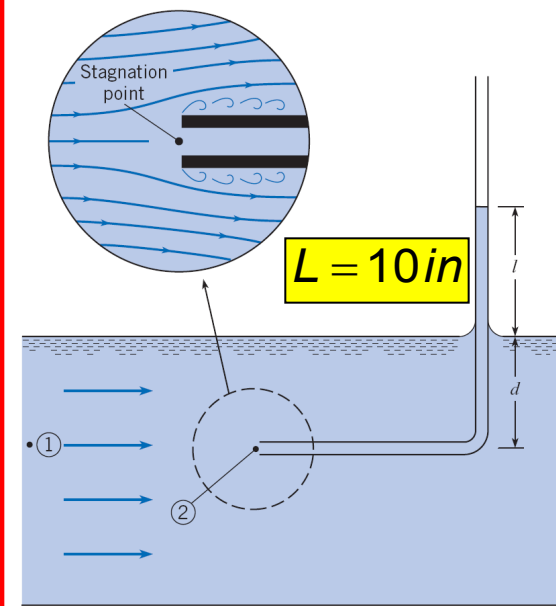
$$\left(p_1 + \rho \frac{V_1^2}{2} \right) = \left(p_2 + \rho \frac{V_2^2}{2} \right)$$

Since $V_2 = 0$

$$\left(\frac{V_1^2}{2} \right) = \frac{2}{\rho} (p_2 - p_1)$$

$$p_2 = \gamma(l + d) \quad \text{and} \quad p_1 = \gamma d$$

$$V_1 = \sqrt{2gl}$$



$$z_1 = z_2$$

Problem 4.47 (p. 130)

PROBLEM 4.47

Situation: A glass tube with 90° bend inserted into a stream of water. Water in tube rises 10 inches above water surface.

Find: Velocity.

APPROACH

Apply the Bernoulli equation.

$$V_1 = ?$$

ANALYSIS

Hydrostatic equation (between stagnation point and water surface in tube)

$$\frac{p_s}{\gamma} = h + d$$

where d is depth below surface and h is distance above water surface.

Bernoulli equation (between freestream and stagnation point)

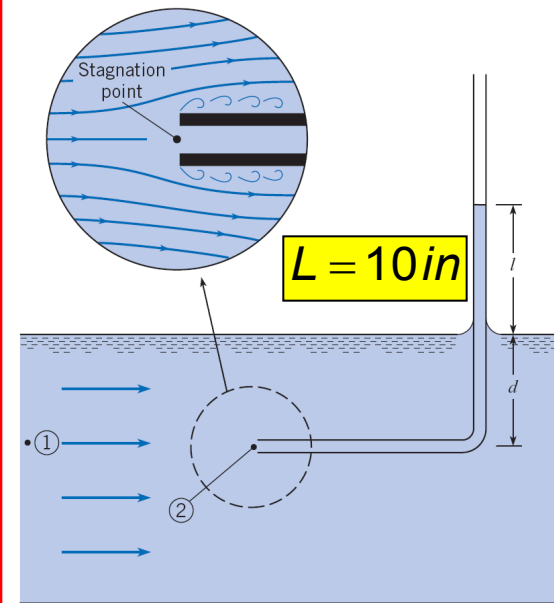
$$z_1 = z_2$$

$$\begin{aligned}\frac{p_s}{\gamma} &= d + \frac{V^2}{2g} \\ h + d &= d + \frac{V^2}{2g} \\ \frac{V^2}{2g} &= h\end{aligned}$$

$$V_1 = \sqrt{2gl}$$

$$V = (2 \times 32.2 \times 10/12)^{1/2}$$

$$V = 7.33 \text{ fps}$$



Problem 4.65 (p. 132)

$$C_p = \frac{p - p_0}{\frac{1}{2} \rho V_0^2}$$

$$C_{pA} = \frac{p_A - p_0}{\frac{1}{2} \rho V_0^2}$$

$$C_{pB} = \frac{p_B - p_0}{\frac{1}{2} \rho V_0^2}$$

PROBLEM 4.65

Situation: A spherical probe with pressure coefficients given is used to find gas velocity. The pressure difference is 4 kPa and the gas density is 1.5 kg/m^3 .

Find: Gas velocity. **Given:** $(C_p)_A = 1.0$, $(C_p)_B = -0.4$

APPROACH

Apply the definition of pressure coefficient.

ANALYSIS

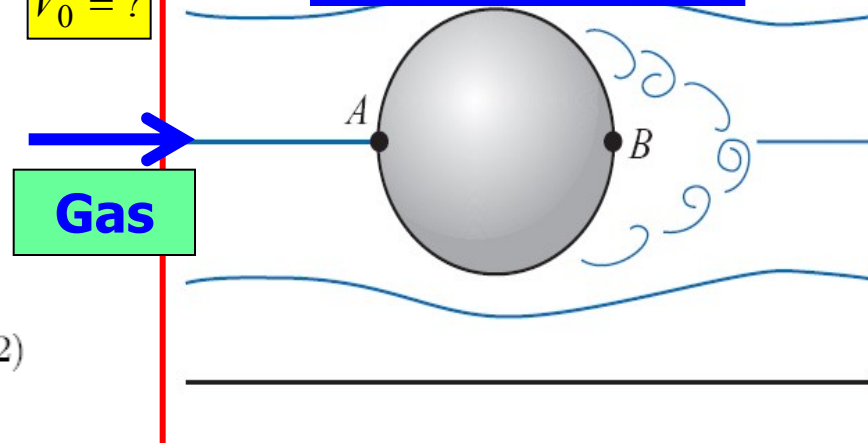
Pressure coefficient

$$\begin{aligned}\Delta C_p &= 1 - (-0.4) \\ \Delta C_p &= 1.4 = (p_A - p_B) / (\rho V_0^2 / 2) \\ V_0^2 &= 2(4,000) / (1.5 \times 1.4)\end{aligned}$$

$$V_0 = 61.7 \text{ m/s}$$

$$V_0 = ?$$

$$p_A - p_B = 4 \text{ kPa}$$



Problem 4.80(p. 135)

PROBLEM 4.80

$$p + \gamma z - \frac{\rho \omega^2 r^2}{2} = C$$

OR

$$\left(\frac{p}{\gamma} + z - \frac{\omega^2 r^2}{2g} \right) = C$$

Situation: Closed tank 4 feet in diameter with piezometer attached is rotated at 15 rad/s about a vertical axis.

Find: Pressure at bottom center of tank.

APPROACH

Apply the equation for pressure variation equation- rotating flow.

ANALYSIS

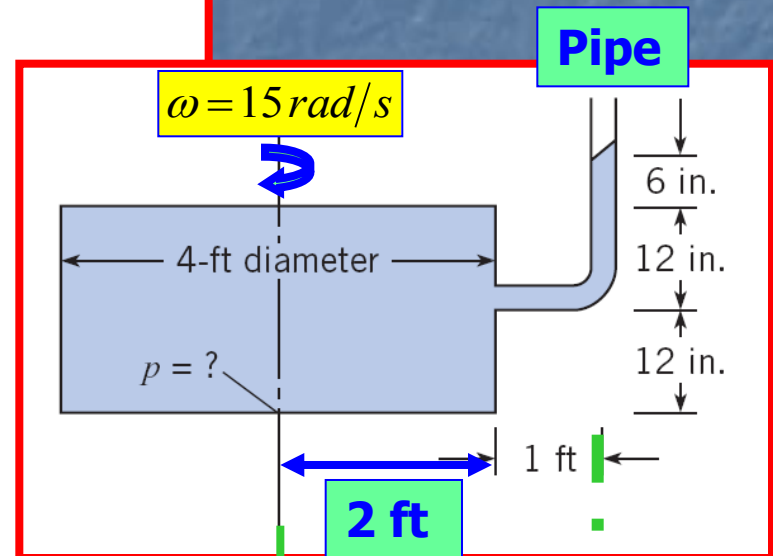
Pressure variation equation- rotating flow

$$p + \gamma z - \rho r^2 \omega^2 / 2 = p_p + \gamma z_p - \rho r_p^2 \omega^2 / 2$$

where $p_p = 0$, $r_p = 3$ ft and $r = 0$, then

$$\begin{aligned} p &= -(\rho/2)(9 \times 225) + \gamma(z_p - z) \\ &= (1.94/2)(2025) + 62.4 \times 2.5 \\ &= -1808 \text{ psfg} = -12.56 \text{ psig} \end{aligned}$$

$$p = -12.6 \text{ psig}$$



1

2

Problem 4.83(p. 135)

PROBLEM 4.83

Situation: A U-tube rotating about one leg. Before rotation, the level of liquid in each leg is 0.25 m. The length of base and length of leg is 0.5 m.

Find: Maximum rotational speed so that no liquid escapes from the leg.

$$\omega = ?$$

APPROACH

Apply the pressure variation equation for rotating flow. Let point 1 be at top of outside leg and point 2 be at surface of liquid of inside leg.

ANALYSIS

At the condition of imminent spilling, the liquid will be to the top of the outside leg and the leg on the axis of rotation will have the liquid surface at the bottom of its leg.

Pressure variation equation- rotating flow

$$p_1 + \gamma z_1 - \rho r_1^2 \omega^2 / 2 = p_2 + \gamma z_2 - \rho r_2^2 \omega^2 / 2$$

where $p_1 = p_2$, $z_1 = .5$ m and $z_2 = 0$

$$\gamma \times 0.5 - (\gamma/g) \times .5^2 \omega^2 / 2 = 0$$

$$\omega^2 = 4g$$

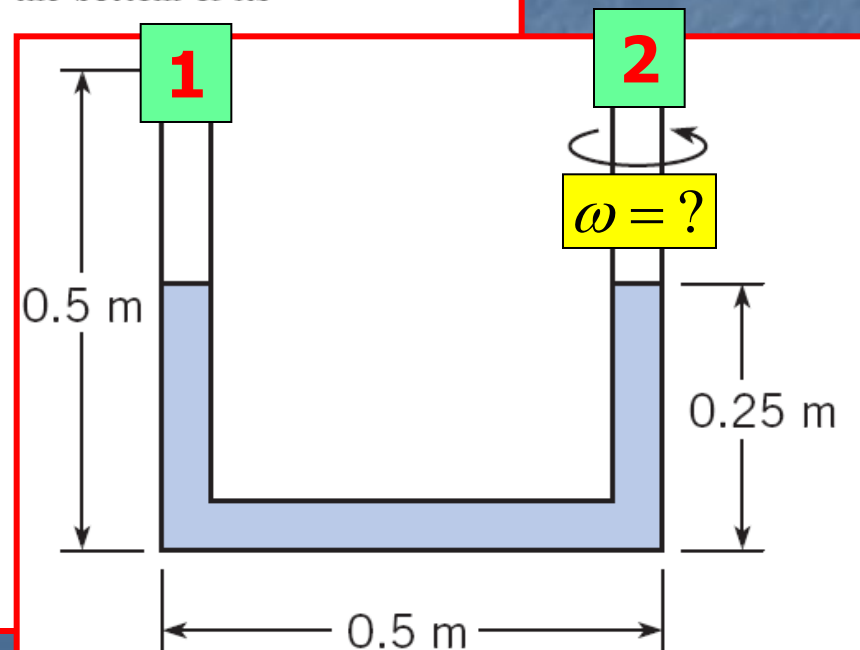
$$= 2\sqrt{g}$$

$$\omega = 6.26 \text{ rad/s}$$

$$p + \gamma z - \frac{\rho \omega^2 r^2}{2} = C$$

OR

$$\left(\frac{p}{\gamma} + z - \frac{\omega^2 r^2}{2g} \right) = C$$



Problem 4.88(p. 136)

$$p + \gamma z - \frac{\rho \omega^2 r^2}{2} = C$$

OR

$$\left(\frac{p}{\gamma} + z - \frac{\omega^2 r^2}{2g} \right) = C$$

PROBLEM 4.88

Situation: A manometer is rotated about one leg. There is a 25 cm height difference in liquid ($S=0.8$) between the legs. The length of the base is 10 cm.

Find: Acceleration in g 's in leg with greatest amount of oil.

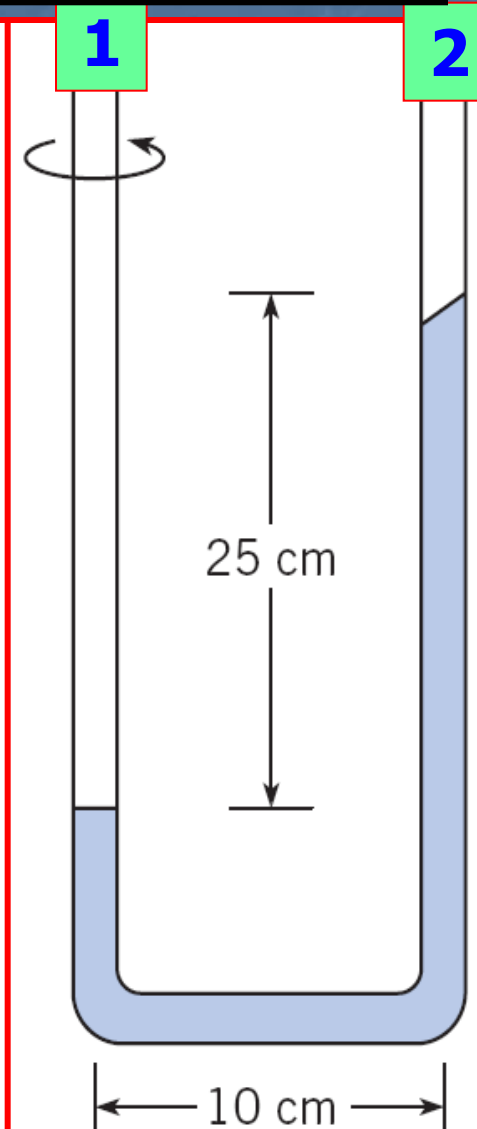
APPROACH

Apply the pressure variation equation for rotating flow between the liquid surfaces of 1 & 2. Let leg 1 be the leg on the axis of rotation. Let leg 2 be the other leg of the manometer.

ANALYSIS

Pressure variation equation- rotating flow

$$\begin{aligned} p_1 + \gamma z_1 - \rho r_1^2 \omega^2 / 2 &= p_2 + \gamma z_2 - \rho r_2^2 \omega^2 / 2 \\ 0 + \gamma z_1 - 0 &= \gamma z_2 - (\gamma/g) r_2^2 \omega^2 / 2 \\ \omega^2 r_2^2 / (2g) &= z_2 - z_1 \\ a_n &= r \omega^2 \\ &= (z_2 - z_1)(2g)/r \\ &= (0.25)(2g)/0.1 \\ &= (0.25)(2g)/0.1 \\ &= 5g \end{aligned}$$



Problem 4.103(p. 138)

PROBLEM 4.103

Situation: The velocity at outlet pipe from a reservoir is 6 m/s and reservoir height is 15 m.

Find: Pressure at point A.

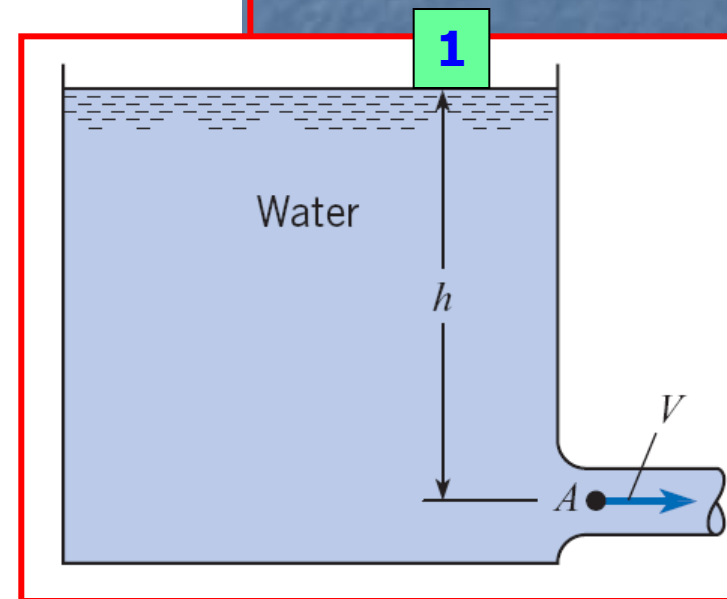
APPROACH

Apply the Bernoulli equation.

ANALYSIS

Bernoulli equation Let point 1 be at reservoir surface.

$$\begin{aligned}(p_1/\gamma) + (V_1^2/2g) + z_1 &= (p_A/\gamma) + (V_A^2/2g) + z_A \\ 0 + 0 + 15 &= p_A/9810 + 6^2/(2 \times 9.81) + 0 \\ p_A &= (15 - 1.83) \times 9810 \\ p_A &= 129,200 \text{ Pa, gage} \\ \boxed{p_A = 129.2 \text{ kPa, gage}}\end{aligned}$$



END OF
SOLVED PROBLEMS

CHAPTER (4)