

CHAPTER (5)

CONTROL VOLUME APPROACH

SOLVED PROBLEMS

DR.MUNZER EBAID

MECH. DEPT.

CONTROL VOLUME APPROACH

Volume Flow Rate

Mass Flow Rate

1. Velocity is constant:

$$\dot{Q} = AV$$

$$\dot{m} = \rho\dot{Q} = \rho AV$$

2. Velocity is variable:

$$\dot{Q} = \int_A V dA$$

$$\dot{m} = \int_A \rho V dA = \rho \int_A V dA = \rho\dot{Q}$$

3. Continuity equation

$$\frac{dM_{cv}}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

4. Continuity equation in a pipe

$$\dot{m}_1 = \dot{m}_2$$

$$(\rho AV)_1 = (\rho AV)_2$$

5. The extensive property is defined as (B)

6. The intensive property

$$\left(b = \frac{B}{mass} \right)$$

When (B) is Mass

CONTROL VOLUME APPROACH

7. Reynolds transport theorem(in general)

$$\frac{dB_{sys}}{dt} = \frac{d}{dt} \int_{cv} b\rho dQ + \int_{cs} b\rho V \cdot dA$$

8. Reynolds transport theorem (for mass)

$$\frac{dM_{sys}}{dt} = \frac{d}{dt} \int_{cv} \rho dV + \int_{cs} \rho V \cdot dA$$

9. Cavitation Index

$$\sigma = \frac{p_0 - p_v}{\frac{1}{2} \rho V_1^2}$$

10. Differential form of continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

CONTROL VOLUME APPROACH

PROBLEM 5.2

Situation: Water flows in a 16 in pipe. $V = 3 \text{ ft/s}$.

Find: Discharge in cfs and gpm.

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$\begin{aligned} Q &= VA \\ &= (3 \text{ ft/s})(\pi/4 \times 1.333^2) \\ &= 4.19 \text{ ft}^3/\text{s} \\ &= (4.17 \text{ ft}^3/\text{s})(449 \text{ gpm}/\text{ft}^3/\text{s}) \\ &= 1880 \text{ gpm} \end{aligned}$$

CONTROL VOLUME APPROACH

PROBLEM 5.4

Situation: An 8 cm. pipe carries air, $V = 20$ m/s, $T = 20^\circ\text{C}$, $p = 200$ kPa-abs.

Find: Mass flow rate: \dot{m}

ANALYSIS

Ideal gas law

$$\begin{aligned}\rho &= p/RT \\ &= 200,000/(287 \times 293) \\ \rho &= 2.378 \text{ kg/m}^3\end{aligned}$$

Flow rate equation

$$\begin{aligned}\dot{m} &= \rho VA \\ &= 2.378 \times 20 \times (\pi \times 0.08^2/4) \\ \dot{m} &= 0.239 \text{ kg/s}\end{aligned}$$

PROBLEM 5.10

Situation: Water flows in a 4 ft pipe. The velocity profile is linear. The center line velocity is $V_{\max} = 15$ ft/s. The velocity at the wall is $V_{\min} = 12$ ft/s.

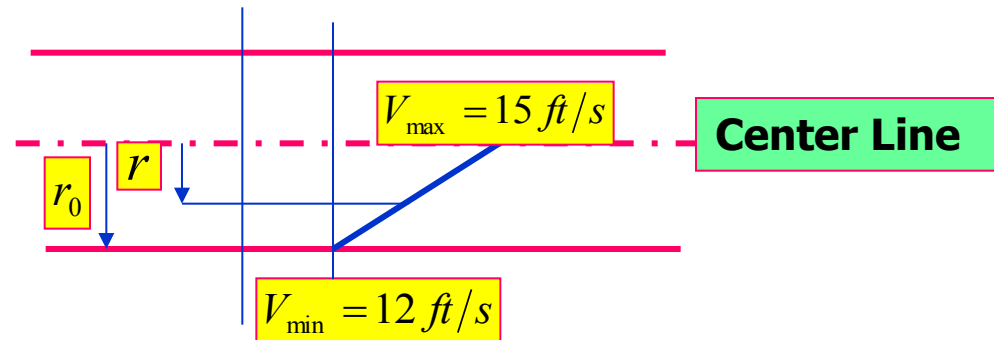
Find: Discharge in cfs and gpm.

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation



$$Q = \int_A V dA$$
$$= \int_0^{r_0} V 2\pi r dr$$

where $V = V_{\max} - 3r/r_0$.

$$\frac{V_{\max} - V_{\min}}{r_0} = \frac{V - V_{\min}}{r_0 - r}$$

$$Vr_0 - V_{\min}r_0 = V_{\max}r_0 - V_{\max}r - V_{\min}r_0 + V_{\min}r$$

$$Vr_0 = (V_{\min} - V_{\max})r + V_{\max}r_0$$

$$V = V_{\max} - \frac{3r}{r_0}$$

$$Q = \int_0^{r_0} (V_{\max} - (3r/r_0)) 2\pi r dr$$
$$= 2\pi r_0^2 ((V_{\max}/2) - (3/3))$$
$$= 2\pi \times 4.00 ((15/2) - (3/3))$$
$$Q = 163.4 \text{ cfs}$$
$$= 163.4 \times 449$$
$$Q = 73,370 \text{ gpm}$$

PROBLEM 5.54

Situation: Flows with different specific weights enter a closed tank through ports A and B and exit the tank through port C. Assume steady flow. Details are provided on figure with problem statement.

Find: At section C:

- (a) Mass flow rate.
- (b) Average velocity.
- (c) Specific gravity of the mixture.

Assumptions: Steady state.

APPROACH

Apply the continuity principle and the flow rate equation.

ANALYSIS

Continuity principle

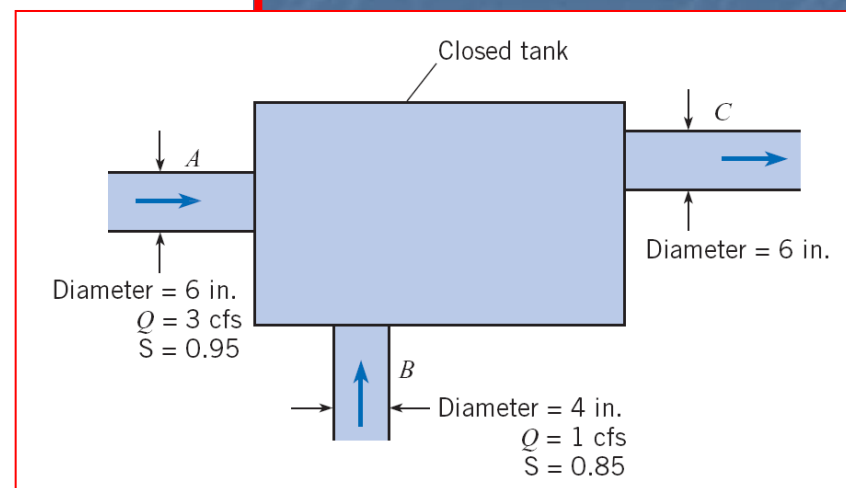
$$\begin{aligned}\sum \dot{m}_i - \sum \dot{m}_o &= 0 \\ -\rho_A V_A A_A - \rho_B V_B A_B + \rho_C V_C A_C &= 0 \\ \rho_C V_C A_C &= 0.95 \times 1.94 \times 3 + 0.85 \times 1.94 \times 1 \\ \dot{m} &= 7.18 \text{ slugs/s}\end{aligned}$$

Continuity principle, assuming incompressible flow

$$\begin{aligned}V_C A_C &= V_A A_A + V_B A_B \\ &= 3 + 1 = 4 \text{ cfs}\end{aligned}$$

Flow rate equation

$$\begin{aligned}V_C &= Q/A = 4/[\pi/4(1/2)^2] \\ &= 20.4 \text{ ft/s} \\ \rho_C &= 7.18/4 = 1.795 \text{ slugs/ft}^3 \\ S &= 1.795/1.94 \\ S &= 0.925\end{aligned}$$



PROBLEM 5.55

Situation: O_2 and CH_4 enter a mixer, each with a velocity of 5 m/s. Mixer conditions: 200 kPa-abs., 100 °C. Outlet density: $\rho = 2.2 \text{ kg/m}^3$. Flow areas: 1 cm² for the CH_4 , 3 cm² for the O_2 , and 3 cm² for the exit mixture.

Find: Exit velocity of the gas mixture: V_{exit}

APPROACH

Apply the ideal gas law to find inlet density. Then apply the continuity principle.

ANALYSIS

Ideal gas law

$$\begin{aligned}\rho_{O_2} &= p/RT \\ &= 200,000/(260 \times 373) \\ &= 2.06 \text{ kg/m}^3\end{aligned}$$

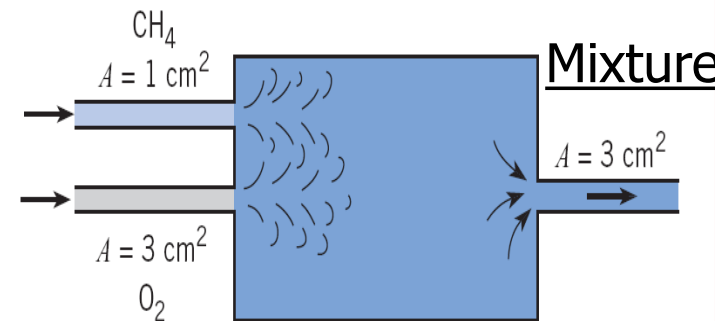
$$\begin{aligned}\rho_{CH_4} &= 200,000/(518 \times 373) \\ &= 1.03 \text{ kg/m}^3\end{aligned}$$

Continuity principle

$$\begin{aligned}\sum \dot{m}_i &= \sum \dot{m}_o \\ \rho_e V_e A_e &= \rho_{O_2} V_{O_2} A_{O_2} + \rho_{CH_4} V_{CH_4} A_{CH_4} \\ V_e &= (2.06 \times 5 \times 3 + 1.03 \times 5 \times 1)/(2.2 \times 3)\end{aligned}$$

$$V_e = 5.46 \text{ m/s}$$

Problem 5.55



PROBLEM 5.75

Situation: Cavitation in a venturi section with inlet diameter of 40 cm and throat diameter of 10 cm. Upstream pressure is 120 kPa gage and atmospheric pressure is 100 kPa. Water temperature is 10°C.

Find: Discharge for incipient cavitation.

APPROACH

Apply the continuity principle and the Bernoulli equation.

ANALYSIS

Cavitation will occur when the pressure reaches the vapor pressure of the liquid ($p_V = 1,230$ Pa abs).

Table (5)

Bernoulli equation

$$p_A + \rho V_A^2/2 = p_{\text{throat}} + \rho V_{\text{throat}}^2/2$$

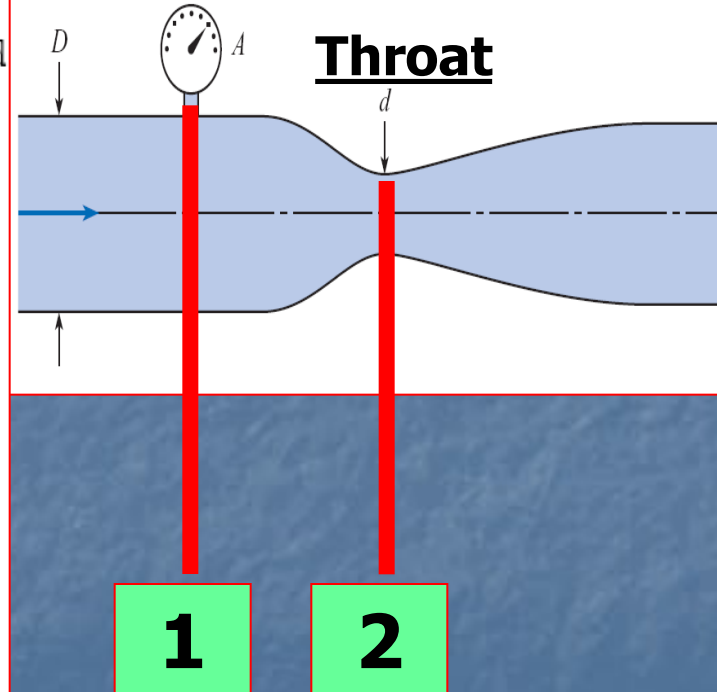
where $V_A = Q/A_A = Q/((\pi/4) \times 0.40^2)$

Continuity principle

$$\begin{aligned} V_{\text{throat}} &= Q/A_{\text{throat}} = Q/((\pi/4) \times 0.10^2) \\ \rho/2(V_{\text{throat}}^2 - V_A^2) &= p_A - p_{\text{throat}} \\ (\rho Q^2/2)[1/((\pi/4) \times 0.10^2)^2 - 1/((\pi/4) \times 0.40^2)^2] &= 220,000 - 1,230 \\ 500Q^2(16,211 - 63) &= 218,770 \end{aligned}$$

$$Q = 0.165 \text{ m}^3/\text{s}$$

Problem 5.75



PROBLEM 5.76

Situation: Air with density 0.0644 lbf/ft^3 flows upward in a vertical venturi with area ratio of 0.5. Inlet velocity is 100 ft/s . Two pressure taps connected to manometer with fluid specific weight of 120 lbf/ft^3 .

Find: Deflection of manometer.

Assumptions: Uniform air density.

APPROACH

Apply the Bernoulli equation from 1 to 2 and then the continuity principle. Let section 1 be in the large duct where the manometer pipe is connected and section 2 in the smaller duct at the level where the upper manometer pipe is connected.

ANALYSIS

Continuity principle

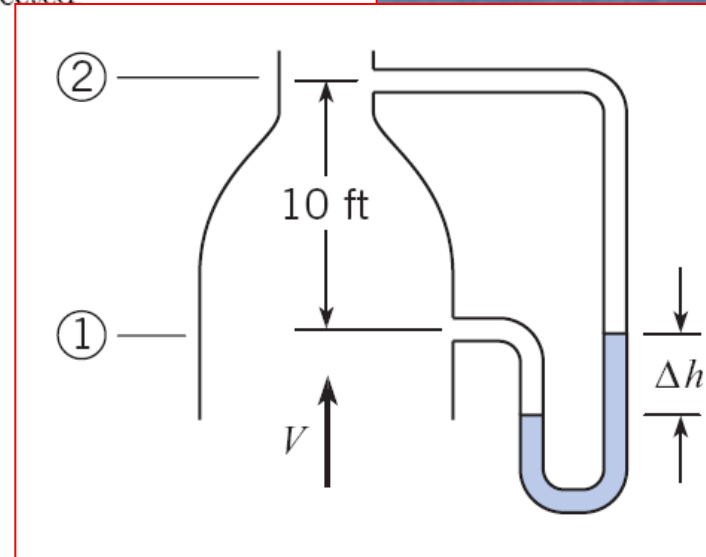
$$\begin{aligned}V_1 A_1 &= V_2 A_2 \\V_2 &= V_1 (A_1/A_2) \\&= 100(2) \\&= 200 \text{ ft/s}\end{aligned}$$

Bernoulli equation

$$\begin{aligned}p_{z1} + \rho V_1^2/2 &= p_{z2} + \rho V_2^2/2 \\p_{z1} - p_{z2} &= (1/2)\rho(V_2^2 - V_1^2) \\&= (1/2)(0.0644/32.2)(40,000 - 10,000) \\&= 30 \text{ psf}\end{aligned}$$

Manometer equation

$$\begin{aligned}p_{z1} - p_{z2} &= \Delta h(\gamma_{\text{liquid}} - \gamma_{\text{air}}) \\30 &= \Delta h(120 - .0644) \\&\quad \boxed{\Delta h = 0.25 \text{ ft.}}\end{aligned}$$



CONTROL VOLUME APPROACH

THE END