MOMENTUM PRINCIPLE

1. The Momentum Equation for Cartesian Coordinates

X-direction:

$$\sum F_X = \frac{d}{dt}\int_{cv} v_X \rho dQ + \sum_{CS} (\dot{m}v)_{outX} - \sum_{CS} (\dot{m}v)_{inX}$$

Y-direction:

$$\sum F_Y = \frac{d}{dt}\int_{cv} v_Y \rho dQ + \sum_{CS} (\dot{m}v)_{outY} - \sum_{CS} (\dot{m}v)_{inY}$$

Z-direction:

$$\sum F_Z = \frac{d}{dt}\int_{cv} v_Z \rho dQ + \sum_{CS} (\dot{m}v)_{outZ} - \sum_{CS} (\dot{m}v)_{inZ}$$

2. The pressure rise across a wave produced in a pipe (Water Hammer)

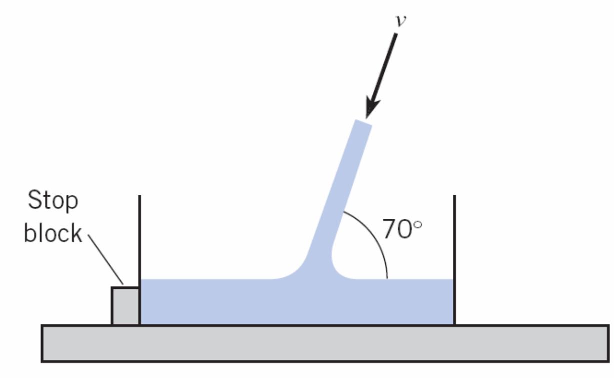
$$\Delta p = \rho v c$$

3. Momentum - of - Momentum Equation

$$\sum M = \frac{d}{dt}\int_{CV} (r \times v)\rho dQ + \sum_{CS} r \times (\dot{m}v)_{out} - \sum_{CS} r \times (\dot{m}v)_{in}$$

Problem 6.3

Situation: A water jet is filling a tank. The tank mass is 5 kg. The tank contains 20 liters of water.
Data for the jet: $d = 30$ mm, $v = 15$ m/s, $T = 15$ °C.



Find: (a) Force on the bottom of the tank: N
(b) Force acting on the stop block: F

Properties: Water–Table A.5: $\rho = 999\,\text{kg/m}^3$, $\gamma = 9800\,\text{N/m}^3$.

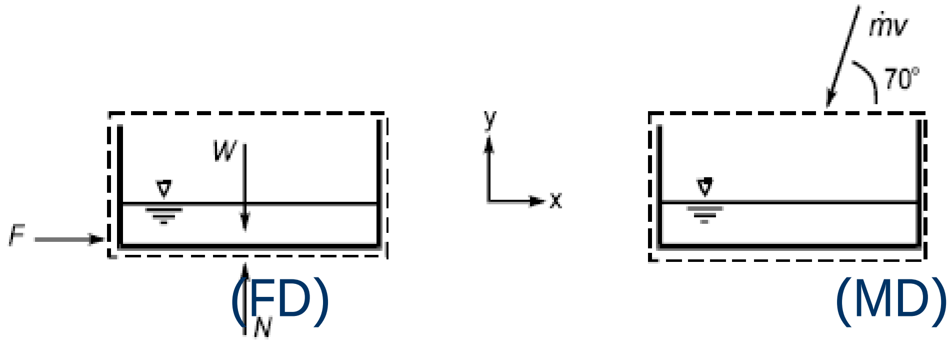
Assumptions: Steady flow.

APPROACH

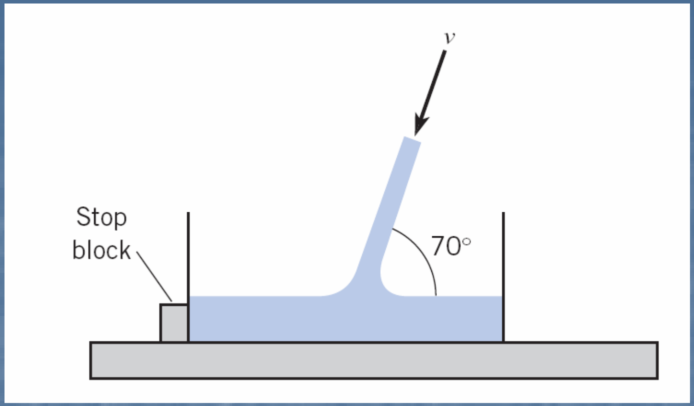
Apply the momentum principle in the x-direction and in the y-direction.

ANALYSIS

Force and momentum diagrams



Problem 6.3



Momentum principle (x-direction)

$$\begin{aligned}\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ F &= -(-\dot{m}v \cos 70^o) \\ &= \rho A v^2 \cos 70^o\end{aligned}$$

Calculations

$$\begin{aligned}\rho A v^2 &= (999)\left(\frac{\pi \times 0.03^2}{4}\right)(15^2) \\ &= 158.9 \text{ N}\end{aligned}$$

$$\begin{aligned}F &= (158.9 \text{ N})(\cos 70^o) \\ &= 54.3 \text{ N}\end{aligned}$$

$$\boxed{F = 54.3 \text{ N acting to right}}$$

y-direction

$$\begin{aligned}\sum F_y &= \sum_{cs} \dot{m}_o v_{oy} - \sum_{cs} \dot{m}_i v_{iy} \\ N - W &= -(-\dot{m}v \sin 70^o) \\ N &= W + \rho A v^2 \sin 70^o\end{aligned}$$

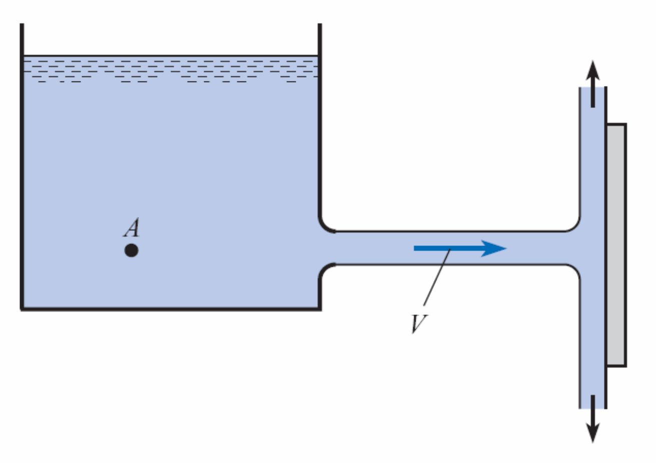
Calculations:

$$\begin{aligned}W &= W_{\text{tank}} + W_{\text{water}} \\ &= (5)(9.81) + (0.02)(9800) \\ &= 245.1 \text{ N}\end{aligned}$$

$$\begin{aligned}N &= W + \rho A v^2 \sin 70^o \\ &= (245.1 \text{ N}) + (158.9 \text{ N}) \sin 70^o\end{aligned}$$

$$\boxed{N = 149\ N \text{ acting upward}}$$

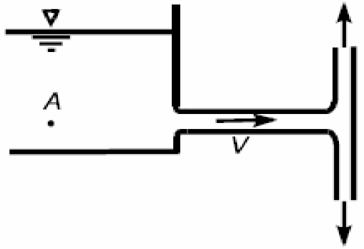
Problem 6.6



Situation: Horizontal round jet strikes a plate.

Water at 70°F, $\rho = 1.94$ slug/ft^3, $Q = 2$ cfs.

Horizontal component of force to hold plate stationary: $F_x = 200$ lbf



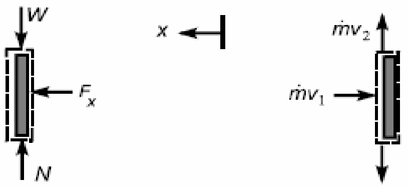
Find: Speed of water jet: v_1

APPROACH

Apply the momentum principle to a control volume surrounding the plate.

ANALYSIS

Force and momentum diagrams



Momentum principle (x-direction)

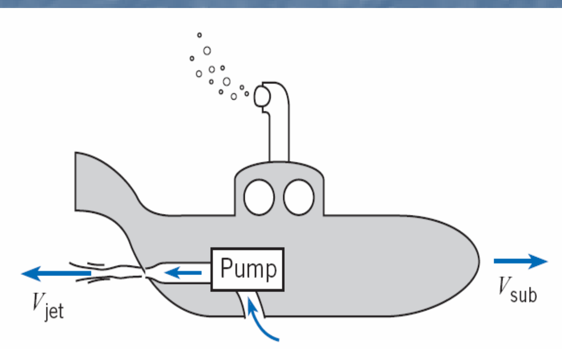
$$\sum F_x = -\dot{m}v_{1x}$$

$$F_x = -(-\dot{m}v_1) = \rho Q v_1$$

$$v_1 = \frac{F_x}{\rho Q} = \frac{200}{1.94 \times 2}$$

$$\boxed{v_1 = 51.5 \text{ ft/s}}$$

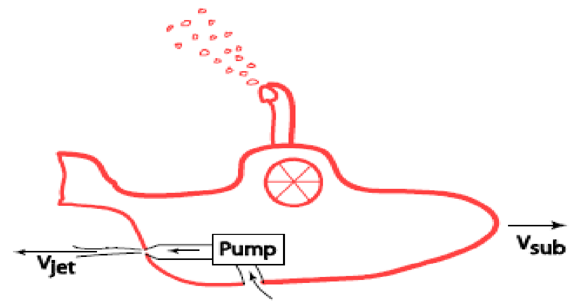
Problem 6.5



Situation: A design contest features a submarine powered by a water jet.
Speed of the sub is $V_{sub} = 1.5\,\text{m/s}$.
Inlet diameter is $D_1 = 25\,\text{mm}$. Nozzle diameter is $D_2 = 5\,\text{mm}$.
Hydrodynamic drag force (F_D) can be calculated using

$$F_D = C_D \left(\frac{\rho V_{sub}^2}{2} \right) A_p$$

Coefficient of drag is $C_D = 0.3$. Projected area is $A_p = 0.28\,\text{m}^2$.



Find: Speed of the fluid jet (V_{jet}).

Properties: Water–Table A.5: $\rho = 999\,\text{kg/m}^3$.

Assumptions: Assume steady flow so that the accumulation of momentum term is zero.

APPROACH

The speed of the fluid jet can be found from the momentum principle because the drag force will balance with the net rate of momentum outflow.

ANALYSIS

Momentum equation. Select a control volume that surrounds the sub. Select a reference frame located on the submarine. Let section 1 be the outlet (water jet) and section 2 be the inlet. The momentum equation is

$$\begin{aligned} \sum \mathbf{F} &= \sum_{cs} \dot{m}_o \mathbf{v}_o - \sum_{cs} \dot{m}_i \mathbf{v}_i \\ F_{\text{Drag}} &= \dot{m}_2 v_2 - \dot{m}_1 v_{1x} \end{aligned}$$

By continuity, $\dot{m}_1 = \dot{m}_2 = \rho A_{jet} V_{jet}$. The outlet velocity is $v_2 = V_{jet}$. The x-component of the inlet velocity is $v_{1x} = V_{sub}$. The momentum equation simplifies to

$$F_{\text{Drag}} = \rho A_{\text{jet}} V_{\text{jet}} (V_{\text{jet}} - V_{\text{sub}})$$

The drag force is

$$\begin{aligned} F_{\text{Drag}} &= C_D \left(\frac{\rho V_{\text{sub}}^2}{2} \right) A_p \\ &= 0.3 \left(\frac{(999\,\text{kg/m}^3)(1.5\,\text{m/s})^2}{2} \right) (0.28\,\text{m}^2) \\ &= 94.4\,\text{N} \end{aligned}$$

The momentum equation becomes

$$\begin{aligned} F_{\text{Drag}} &= \rho A_{\text{jet}} V_{\text{jet}} [V_{\text{jet}} - V_{\text{sub}}] \\ 94.4\,\text{N} &= (999\,\text{kg/m}^3)(1.96 \times 10^{-5}\,\text{m}^2) V_{\text{jet}} [V_{\text{jet}} - (1.5\,\text{m/s})] \end{aligned}$$

Solving for the jet speed gives

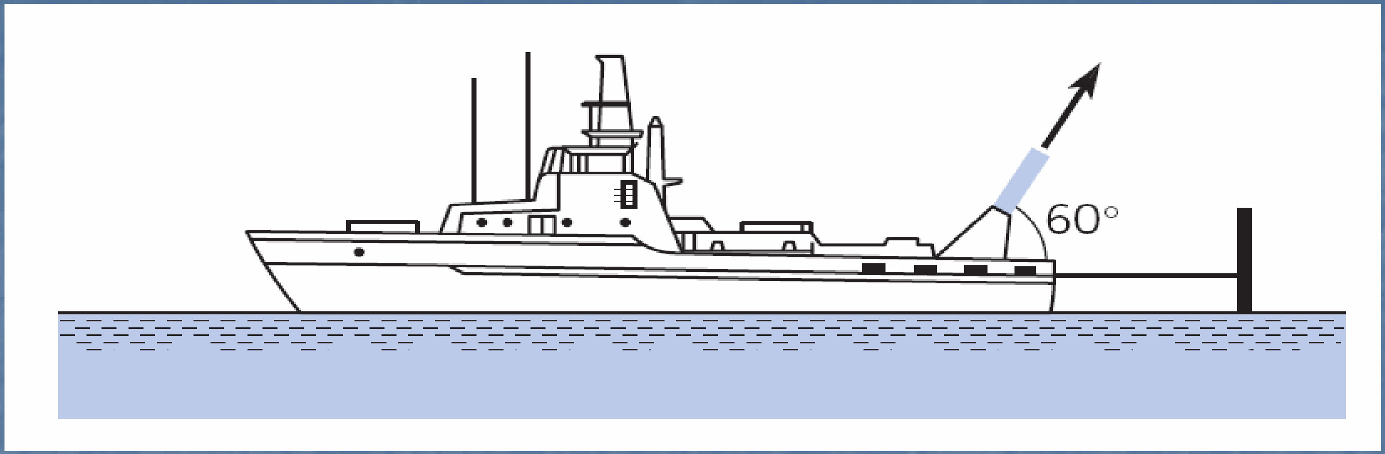
$$V_{\text{jet}} = 70.2\,\text{m/s}$$

COMMENTS

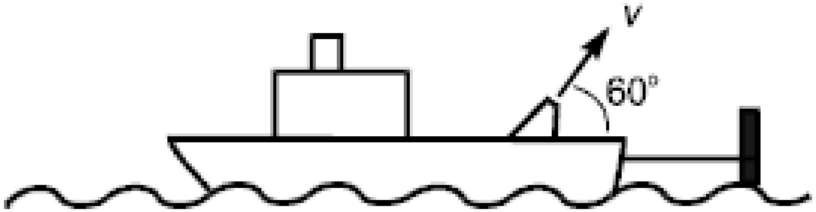
1. The jet speed (70.2 m/s) is above 150 mph. This present a safety issue. Also, this would require a pump that can produce a large pressure rise.

2. It is recommended that the design be modified to produce a lower jet velocity. One way to accomplish this goal is to increase the diameter of the jet.

Problem 6.9



Situation: Water jet from a fire hose on a boat.
Diameter of jet is $d = 3$ in., speed of jet is $V = 70$ mph $= 102.7$ ft/s.



Find: Tension in cable: T

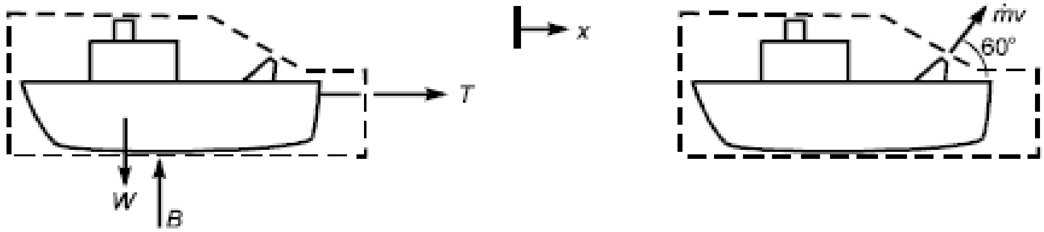
Properties: Table A.5 (water at 50 °F): $\rho = 1.94\,\text{slug/ ft}^3$.

APPROACH

Apply the momentum principle.

ANALYSIS

Force and momentum diagrams



Flow rate

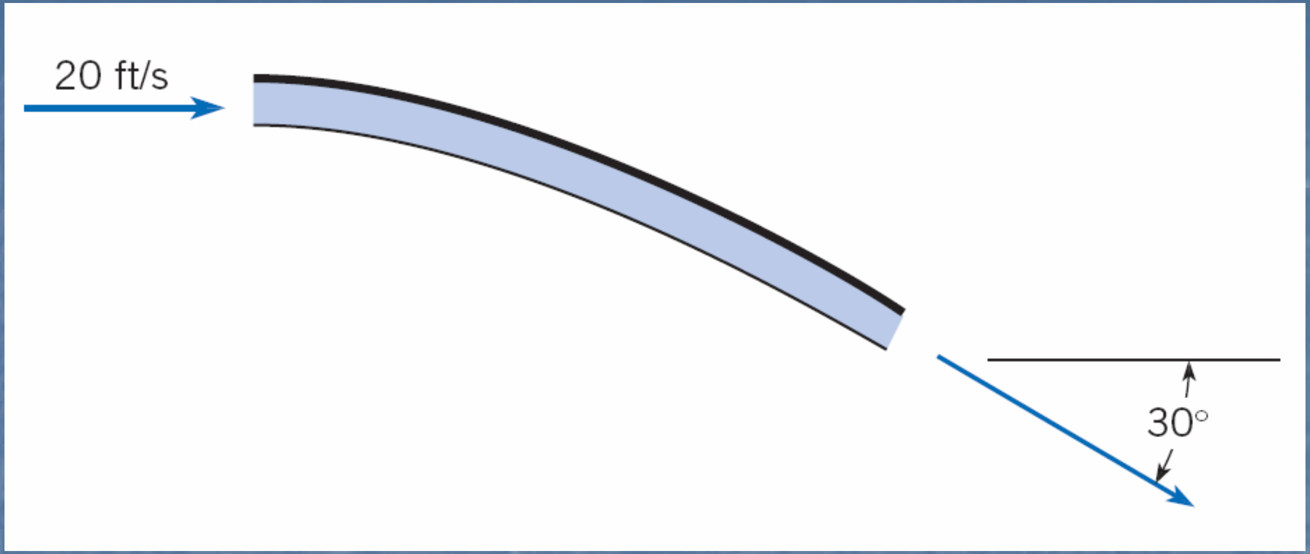
$$
\begin{aligned}
\dot{m} &= \rho A V \\
&= \left(1.94\,\mathrm{slug/\,ft^3}\right)\left(\pi \times (1.5/12\,\mathrm{ft})^2\right)\left(102.7\,\mathrm{ft/\,s}\right) \\
&= 9.78\,\mathrm{slug/\,s}
\end{aligned}
$$

Momentum principle (x-direction)

$$
\begin{aligned}
\sum F &= \dot{m}\,(v_o)_x \\
T &= \dot{m} V \cos 60^o \\
T &= (9.78\,\mathrm{slug/\,s})(102.7\,\mathrm{ft/\,s})\cos 60^o \\
&= 502.2\,\mathrm{lbf}
\end{aligned}
$$

$$\boxed{T = 502\ \ \mathrm{lbf}}$$

Problem 6.20



Situation: A water jet is deflected by a fixed vane, $\dot{m} = 25$ lbm/s $= 0.776$ slug/s.
$v_1 = 20$ ft/s

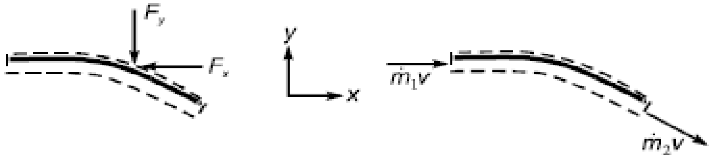
Find: Force of the water on the vane: **F**

APPROACH

Apply the Bernoulli equation, and then the momentum principle.

ANALYSIS

Force and momentum diagrams



Bernoulli equation

$$v_1 = v_2 = v = 20 \text{ ft/s}$$

Momentum principle (x-direction)

$$\begin{aligned}
\sum F_x &= \dot{m}_o (v_o)_x - \dot{m}_i (v_i)_x \\
-F_x &= \dot{m}v \cos 30 - \dot{m}v \\
F_x &= \dot{m}v(1 - \cos 30) = 0.776 \times 20 \times (1 - \cos 30) \\
F_x &= 2.08 \text{ lbf to the left}
\end{aligned}$$

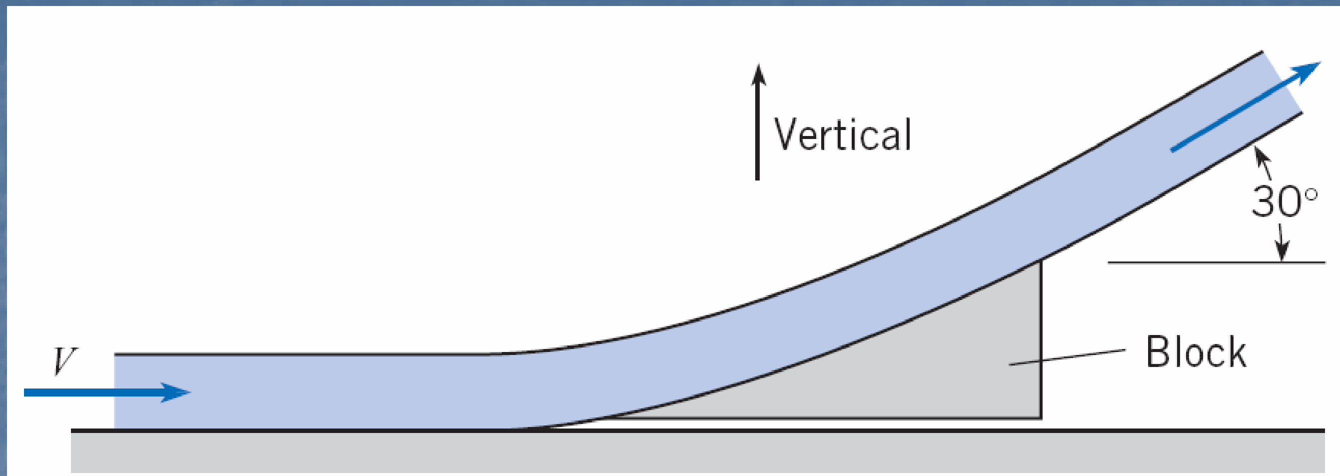
y-direction

$$\begin{aligned}
\sum F_y &= \dot{m}_o (v_o)_y \\
-F_y &= \dot{m}(-v \cos 60) = -0.776 \times 20 \times \sin 30 \\
F_y &= 7.76 \text{ lbf downward}
\end{aligned}$$

Since the forces acting on the vane represent a state of equilibrium, the force of water on the vane is equal in magnitude & opposite in direction.

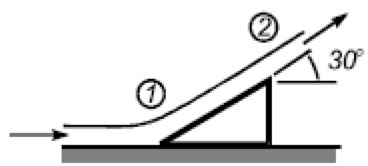
$$\begin{aligned}
\mathbf{F} &= -F_x\mathbf{i} - F_y\mathbf{j} \\
&= \boxed{(2.08 \text{ lbf})\mathbf{i} + (7.76 \text{ lbf})\mathbf{j}}
\end{aligned}$$

Problem 6.21



<u>Situation</u>: A water jet strikes a block and the block is held in place by friction–however, we do not know if the frictional force is large enough to prevent the block from sliding.

$v_1 = 10$ m/s, $\dot{m} = 1$ kg/s, $\mu = 0.1$, mass of block: $m = 1$ kg



<u>Find</u>:

(a) Will the block slip?

(b) Force of the water jet on the block: **F**

<u>Assumptions</u>:

1.) Neglect weight of water.

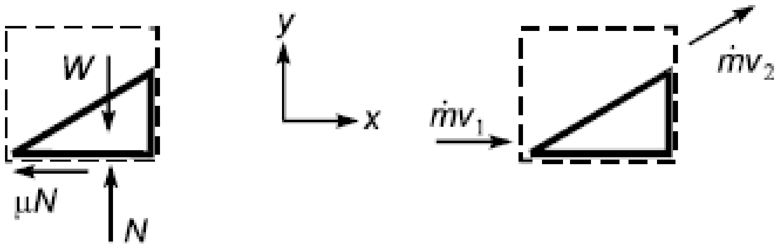
2.) As the jet passes over the block (a) neglect elevation changes and (b) neglect viscous forces.

APPROACH

Apply the Bernoulli equation, then the momentum principle.

ANALYSIS

Force and momentum diagrams



Bernoulli equation

$$v_1 = v_2 = v = 10 \text{ m/s}$$

Momentum principle (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}_o (v_o)_x - \dot{m}_i (v_i)_x \\ -F_f &= \dot{m}v \cos 30 - \dot{m}v \\ F_f &= \dot{m}v(1 - \cos 30) \\ &= 1.0 \times 10 \times (1 - \cos 30) \\ F_f &= 1.34 \text{ N} \end{aligned}$$

y-direction

$$\begin{aligned} \sum F_y &= \dot{m}_o (v_o)_y \\ N - W &= \dot{m}(v \sin 30) \\ N &= mg + \dot{m}(v \sin 30) \\ &= 1.0 \times 9.81 + 1.0 \times 10 \times \sin 30 \\ &= 14.81 \text{ N} \end{aligned}$$

Analyze friction:

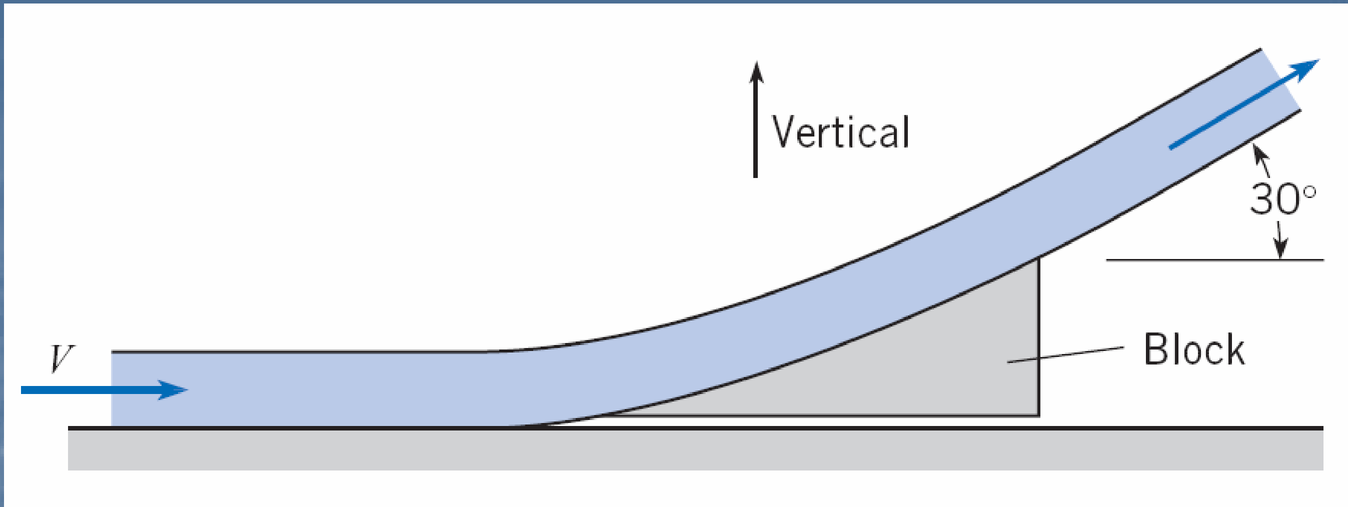
- F_f (required to prevent block from slipping) $= 1.34$ N
- F_f (maximum possible value) $= \mu N = 0.1 \times 14.81 = 1.48$ N

$$\boxed{\text{block will not slip}}$$

Equilibrium of forces acting on block gives

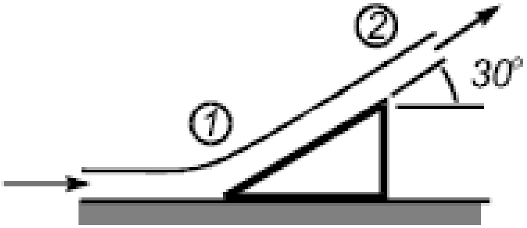
$$\begin{aligned} \mathbf{F} &= \text{(Force of the water jet on the block)} \\ &= -\text{(Force needed to hold the block stationary)} \\ &= -F_f \, \mathbf{i} + (W - N)\mathbf{j} \end{aligned}$$

Problem 6.22



Situation: A water jet strikes a block and the block is held in place by friction $\mu = 0.1$.

$\dot{m} = 1$ kg/s, mass of block: $m = 1$ kg



Find: Maximum velocity (v) such that the block will not slip.

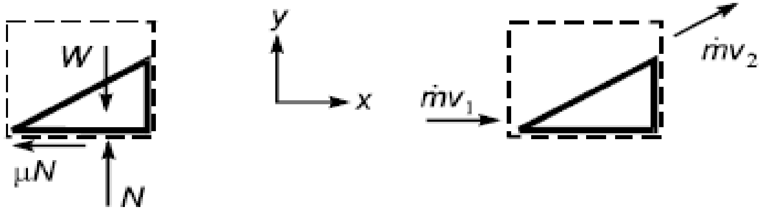
Assumptions: Neglect weight of water.

APPROACH

Apply the Bernoulli equation, then the momentum principle.

ANALYSIS

Force and momentum diagrams



Bernoulli equation

$$v_1 = v_2 = v$$

Momentum principle (x-direction)

$$\begin{aligned}\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ -\mu N &= \dot{m} v \cos 30 - \dot{m} v \\ N &= \dot{m} v \left(1 - \cos 30\right) / \mu\end{aligned}$$

y-direction

$$\begin{aligned}\sum F_y &= \sum_{cs} \dot{m}_o v_{oy} - \sum_{cs} \dot{m}_i v_{iy} \\ N - W &= \dot{m}(v \sin 30) \\ N &= mg + \dot{m}(v \sin 30)\end{aligned}$$

Combine previous two equations

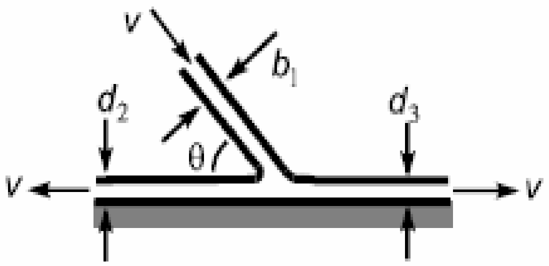
$$\begin{aligned}\dot{m} v \left(1 - \cos 30\right) / \mu &= mg + \dot{m}(v \sin 30) \\ v &= mg / \left[\dot{m} \left(1/\mu - \cos 30/\mu - \sin 30\right)\right] \\ v &= 1 \times 9.81 / \left[1 \times \left(1/0.1 - \cos 30/0.1 - \sin 30\right)\right]\end{aligned}$$

$$\boxed{v = 11.7\,\mathrm{m/s}}$$

Problem 6.24

Situation: 2D liquid jet strikes a horizontal surface.

$v_1 = v_2 = v_3 = v$



Find: Derive formulas for d_2 and d_3 as a function of b_1 and θ.

Assumptions: Force associated with shear stress is negligible; let the width of the jet in the z-direction = w.

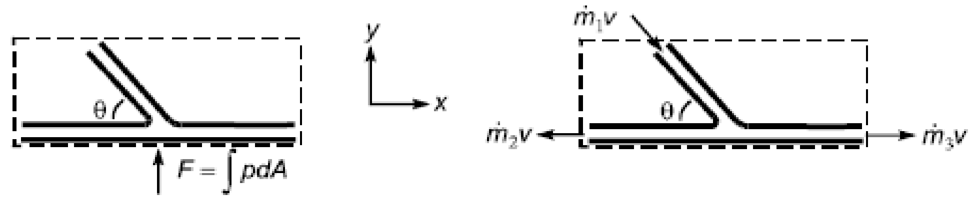
APPROACH

Apply the continuity principle, then the momentum principle.

Continuity principle

$$
\begin{aligned}
\dot{m}_1 &= \dot{m}_2 + \dot{m}_3 \\
\rho w b_1 v &= \rho w d_2 v + \rho w d_3 v \\
b_1 &= d_2 + d_3
\end{aligned}
$$

Force and momentum diagrams



Momentum principle (x-direction)

$$
\begin{aligned}
\sum F_x &= \sum_{cs} \dot{m}_o \mathbf{v}_o - \sum_{cs} \dot{m}_i \mathbf{v}_i \\
0 &= (\dot{m}_3 v + \dot{m}_2(-v)) - \dot{m}_1 v \cos\theta \\
0 &= \left(\rho w d_3 v^2 - \rho w d_2 v^2\right) - \rho w b_1 v^2 \cos\theta \\
0 &= d_3 - d_2 - b_1 \cos\theta
\end{aligned}
$$

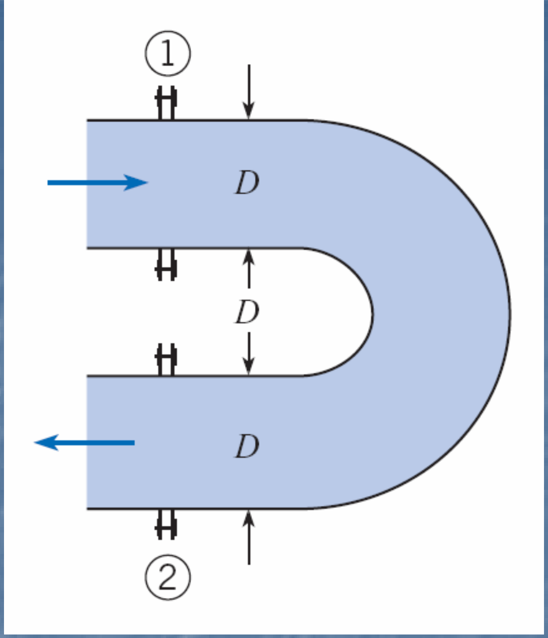
Combining x-momentum and continuity principle equations

$$
\begin{aligned}
d_3 &= d_2 + b_1 \cos\theta \\
d_3 &= b_1 - d_2
\end{aligned}
$$

$$\boxed{d_2 = b_1(1 - \cos\theta)/2}$$

$$\boxed{d_3 = b_1(1 + \cos\theta)/2}$$

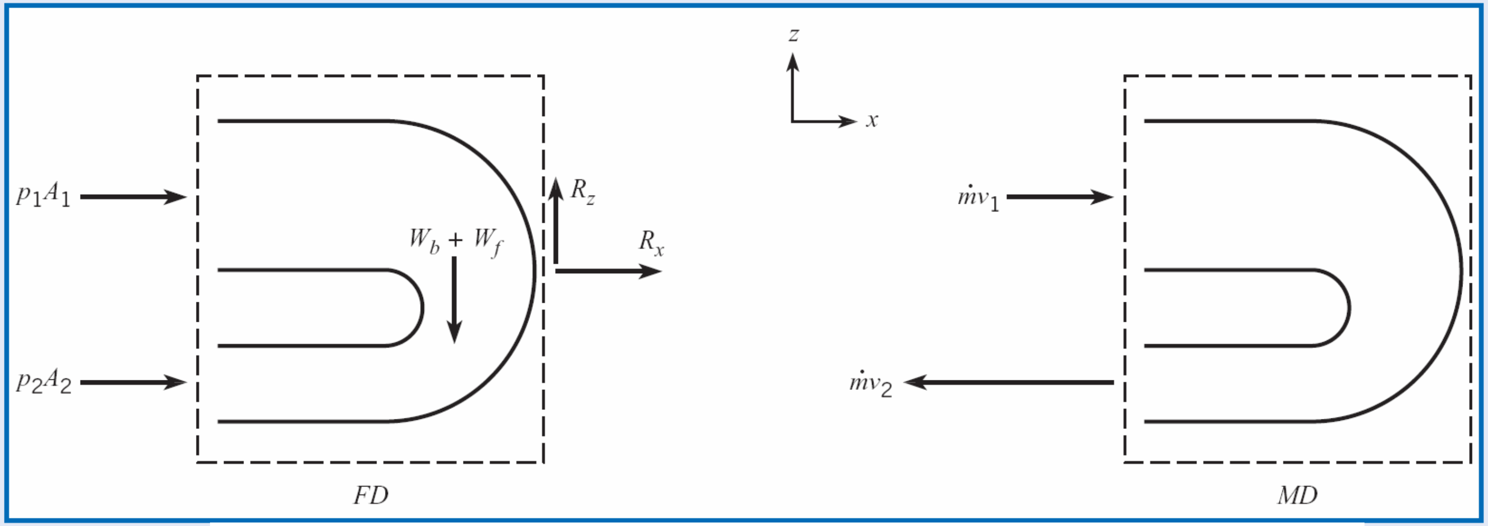
Problem 6.32



Situation: Fluid (density ρ, discharge Q, and velocity V) flows through a 180^o pipe bend—additional details are provided in the problem statement.. Cross sectional area of pipe is A.

Find: Magnitude of force required at flanges to hold the bend in place.

Assumptions: Gage pressure is same at sections 1 and 2. Neglect gravity.



APPROACH

Apply the momentum principle.

ANALYSIS

Momentum principle (x-direction)

$$\sum F_x = \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix}$$

$$p_1A_1 + p_2A_2 + F_x = \dot{m}(v_2 - v_1)$$

thus

$$F_x = -2pA - 2\dot{m}V$$

$$F_x = -2pA - 2\rho QV$$

Correct choice is (d)

Problem 6.37

Find: Vertical component of force exerted by the anchor on the bend: F_a

APPROACH

Apply the momentum principle.

ANALYSIS

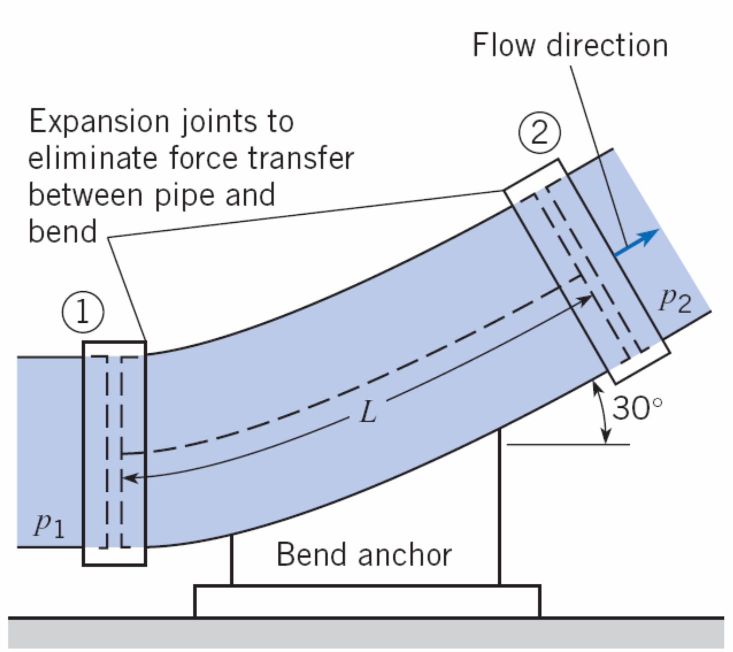
Velocity calculation

$$\begin{aligned} v &= Q/A \\ &= 31.4/(\pi \times 1 \times 1) \\ &= 9.995 \text{ ft/sec} \end{aligned}$$

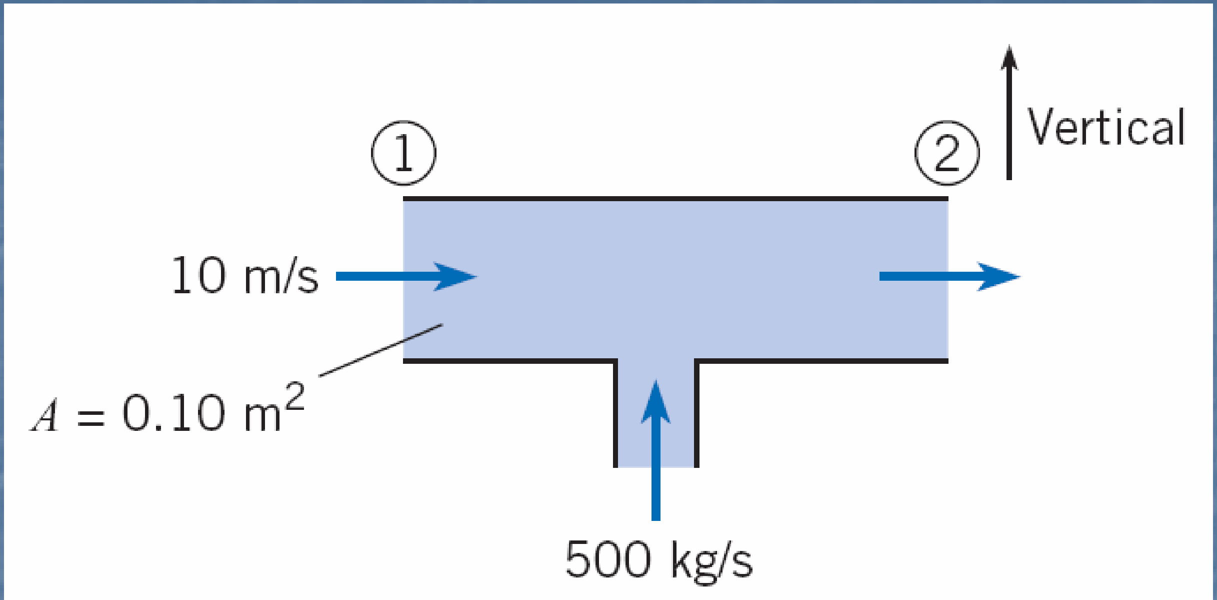
Momentum principle (y-direction)

$$\begin{aligned} \sum F_y &= \rho Q(v_{2y} - v_{1y}) \\ F_a - W_{\text{water}} - W_{\text{bend}} - p_2 A_2 \sin 30^\circ &= \rho Q(v \sin 30^\circ - v \sin 0^\circ) \\ F_a &= \pi \times 1 \times 1 \times 4 \times 62.4 + 300 \\ &\quad +8.5 \times 144 \times \pi \times 1 \times 1 \times 0.5 \\ &\quad +1.94 \times 31.4 \times (9.995 \times 0.5 - 0) \end{aligned}$$

$$\boxed{F_a = 3310 \text{ lbf}}$$



Problem 6.44

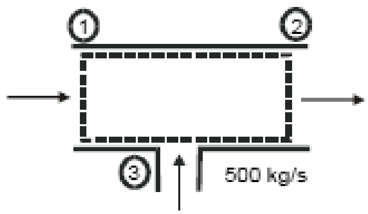


Situation: Water flows through a tee—additional details are provided in the problem statement.

Find: Pressure difference between sections 1 and 2.

APPROACH

Apply the continuity principle, then the momentum principle.



ANALYSIS

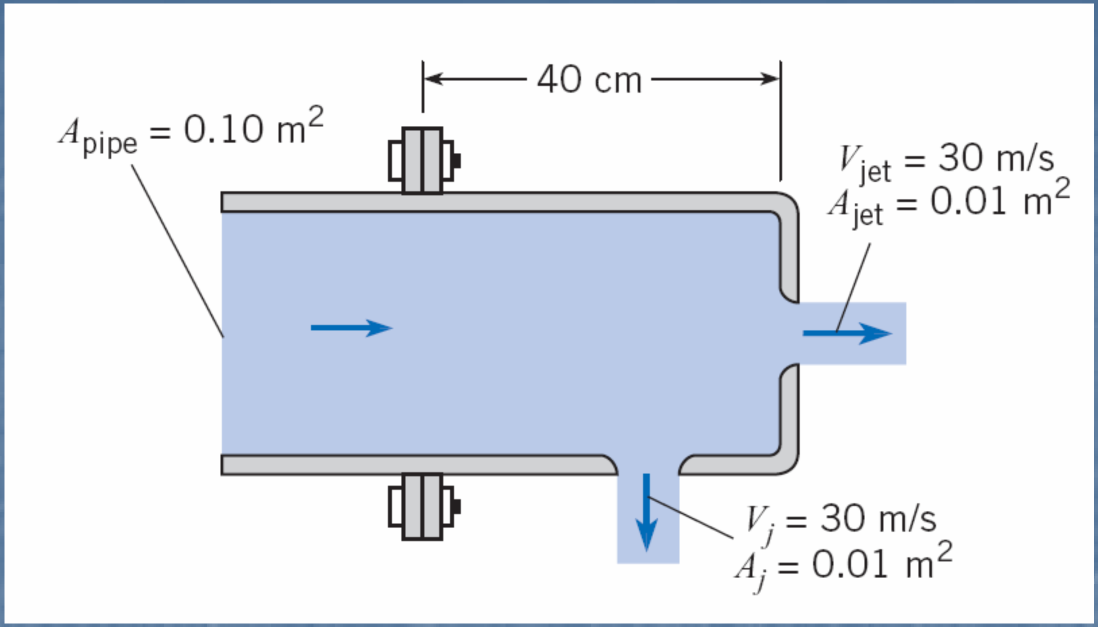
<u>Continuity principle</u>

$$
\begin{aligned}
\dot{m}_1 + 500 \text{ kg/s} &= \dot{m}_2 \\
\dot{m}_1 &= (10 \text{ m/s})(0.10 \text{ m}^2)(1000 \text{ kg/m}^3) = 1000 \text{ kg/s} \\
\dot{m}_2 &= 1000 + 500 = 1500 \text{ kg/s} \\
v_2 &= (\dot{m}_2)/(\rho A_2) = (1500)/((1000)(0.1)) = 15 \text{ m/s}
\end{aligned}
$$

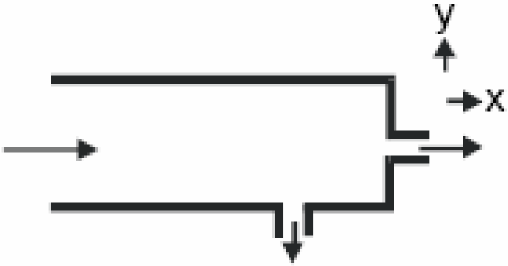
<u>Momentum principle</u> (x-direction)

$$
\begin{aligned}
\sum F_x &= \dot{m}_2 v_{2x} - \dot{m}_1 v_{1x} - \dot{m}_3 v_{3x} \\
p_1 A_1 + p_2 A_2 &= \dot{m}_2 v_2 - \dot{m}_1 v_1 - 0 \\
A(p_1 - p_2) &= (1500)(15) - (1000)(10) \\
p_1 - p_2 &= (22,500 - 10,000)/0.10 \\
&= 125,000 \text{ Pa} \\
&= \boxed{125 \text{ kPa}}
\end{aligned}
$$

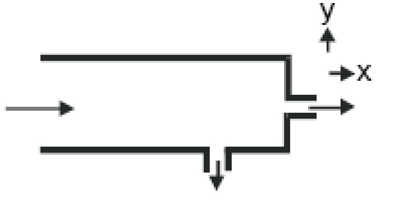
Problem 6.49



Situation: Water flows through an unusual nozzle—additional details are provided in the problem statement.



Find: Force at the flange to hold the nozzle in place: **F**



APPROACH

Apply the momentum principle.

APPROACH

Apply the continuity principle, then the Bernoulli equation, and finally the momentum principle.

ANALYSIS

Continuity principle

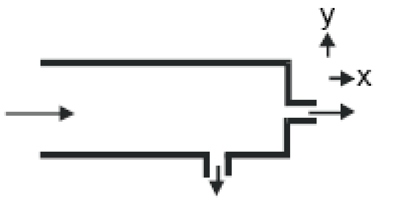
$$\begin{aligned} v_p A_p &= \sum v_j A_j \\ v_p &= 2 \times 30 \times 0.01/0.10 \\ &= 6.00 \text{ m/s} \end{aligned}$$

Bernoulli equation

$$p_{\text{pipe}}/\gamma + v_p^2/2g = p_{\text{jet}}/\gamma + v_j^2/2g$$

Then

$$\begin{aligned} p_p &= (\gamma/2g)(v_j^2 - v_p^2) \\ &= 500(900 - 36) \\ &= 432,000 \text{ Pa} \end{aligned}$$



Momentum principle (x-direction)

$$
\begin{aligned}
p_p A_p + F_x &= -v_p \rho v_p A_p + v_j \rho v_j A_j \\
F_x &= -1000 \times 6^2 \times 0.10 + 1,000 \times 30^2 \times 0.01 - 432,000 \times 0.1 \\
F_x &= -37,800 \text{ N}
\end{aligned}
$$

y-direction

$$
\begin{aligned}
F_y &= \dot{m}(-v_j) = -v_j \rho v_j A \\
&= -30 \times 1000 \times 30 \times 0.01 \\
&= -9000 \text{ N}
\end{aligned}
$$

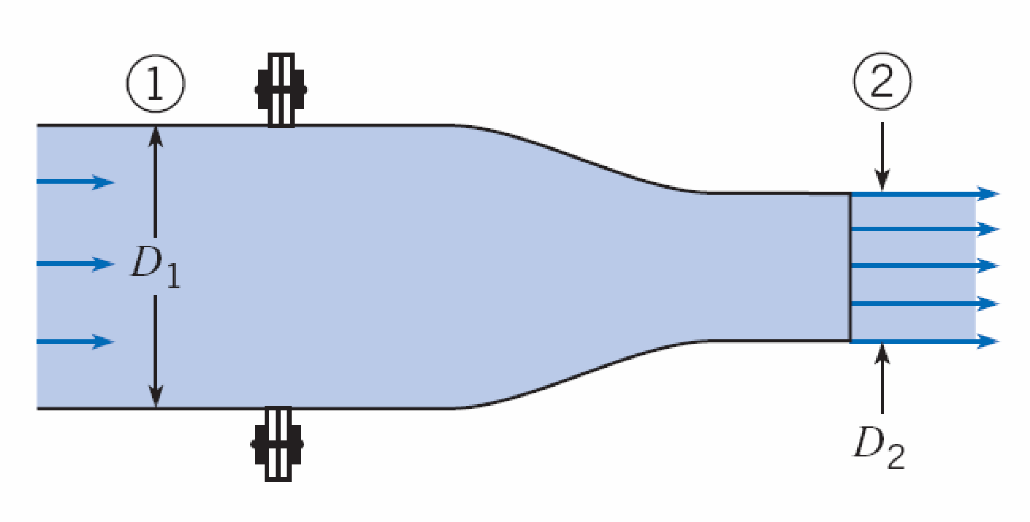
z-direction

$$
\begin{aligned}
\sum F_z &= 0 \\
-200 - \gamma \forall + F_z &= 0 \\
F_z &= 200 + 9810 \times 0.1 \times 0.4 \\
&= 592 \text{ N}
\end{aligned}
$$

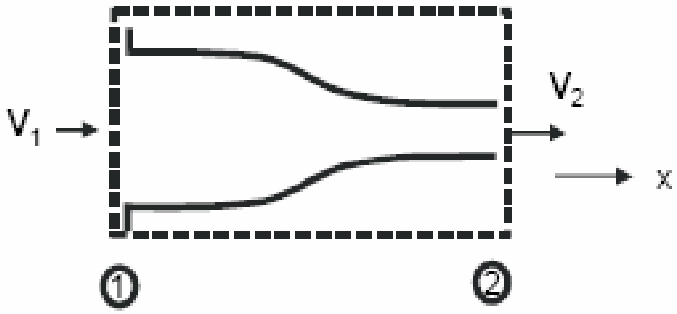
Net force

$$
\boxed{\mathbf{F} = (-37.8\mathbf{i} - 9.0\mathbf{j} + 0.59\mathbf{k}) \text{ kN}}
$$

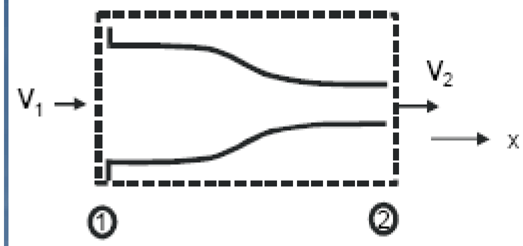
Problem 6.50



Situation: Water flows through a converging nozzle—additional details are provided in the problem statement.



Find: Force at the flange to hold the nozzle in place: F



APPROACH

Apply the Bernoulli equation to establish the pressure at section 1, and then apply the momentum principle to find the force at the flange.

ANALYSIS

Continuity equation (select a control volume that surrounds the nozzle).

$$Q_1 = Q_2 = Q = 15\,\text{ft}^3/\,\text{s}$$

Flow rate equations

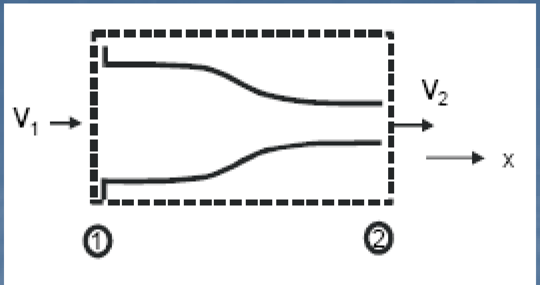
$$\begin{aligned} v_1 &= \frac{Q}{A_1} = \frac{4 \times Q}{\pi D_1^2} = \frac{4 \times \left(15\,\text{ft}^3/\,\text{s}\right)}{\pi\left(1\,\text{ft}\right)^2} \\ &= 19.099\ \text{ft}/\,\text{s} \\ v_2 &= \frac{Q}{A_2} = \frac{4 \times Q}{\pi D_2^2} = \frac{4 \times \left(15\,\text{ft}^3/\,\text{s}\right)}{\pi\left(9/12\,\text{ft}\right)^2} \\ &= 33.953\ \text{ft}/\,\text{s} \end{aligned}$$

Bernoulli equation

$$\begin{aligned} p_1 + \frac{\rho v_1^2}{2} &= p_2 + \frac{\rho v_2^2}{2} \\ p_1 &= 0 + \frac{\rho(v_2^2 - v_1^2)}{2} \\ &= \frac{1.94\,\text{slug}/\,\text{ft}^3(33.953^2 - 19.099^2)\,\text{ft}^2/\,\text{s}^2}{2} \\ &= 764.4\ \text{lbf}/\,\text{ft}^2 \end{aligned}$$

Momentum principle (x-direction)

$$p_1 A_1 + F = \dot{m} v_2 - \dot{m} v_1$$



Calculations

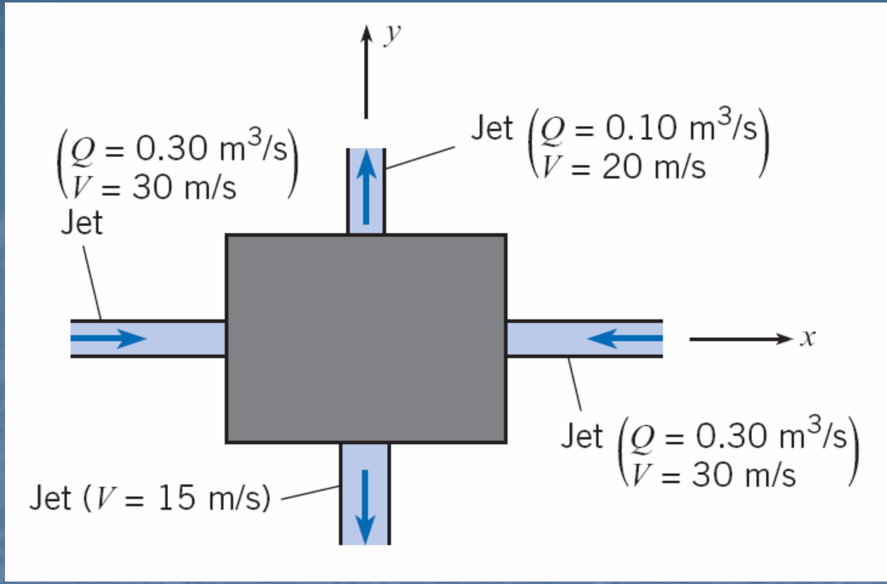
$$
\begin{aligned}
p_1A_1 &= (764.4\,\text{lbf}/\,\text{ft}^2)(\pi/4)(1\,\text{ft})^2 \\
&= 600.4\,\text{lbf} \\
\dot{m}v_2 - \dot{m}v_1 &= \rho Q\,(v_2 - v_1) \\
&= (1.94\,\text{slug}/\,\text{ft}^3)(15\,\text{ft}^3/\,\text{s})\,(33.953 - 19.098)\ \text{ft}/\,\text{s} \\
&= 432.3\,\text{lbf}
\end{aligned}
$$

Substituting numerical values into the momentum equation

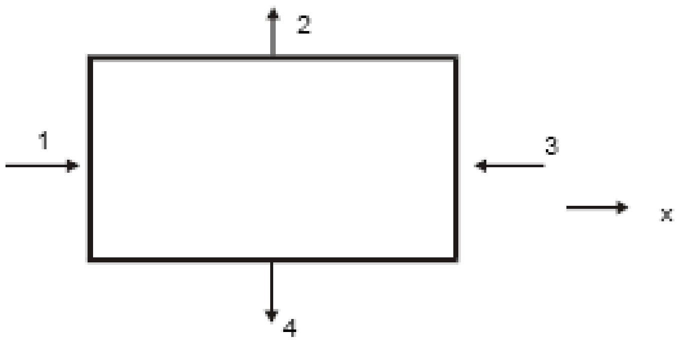
$$
\begin{aligned}
F &= -p_1A_1 + (\dot{m}v_2 - \dot{m}v_1) \\
&= -600.4\,\text{lbf} + 432.3\,\text{lbf} \\
&= -168.1\,\text{lbf}
\end{aligned}
$$

$$\boxed{F = -168\ \text{lbf (acts to left)}}$$

Problem 6.61



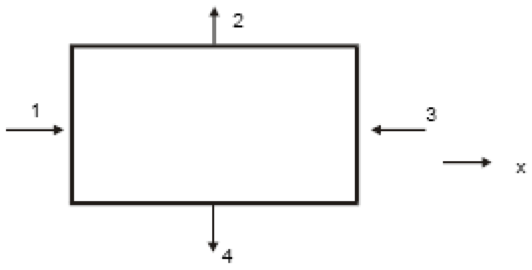
Situation: Liquid flows through a "black box"—additional details are provided in th problem statement.



Find: Force required to hold the "black box" in place: **F**

APPROACH

Apply the continuity principle, then the momentum principle.



ANALYSIS

Continuity principle

$$Q_4 = 0.6 - 0.10$$
$$= 0.50 \text{ m}^3/\text{s}$$

Momentum principle (x-direction)

$$F_x = -\dot{m}_1 v_{1_x} - \dot{m}_3 v_{3_x}$$
$$= -\dot{m}_1 v_1 + \dot{m}_3 v_3$$
$$= 0$$

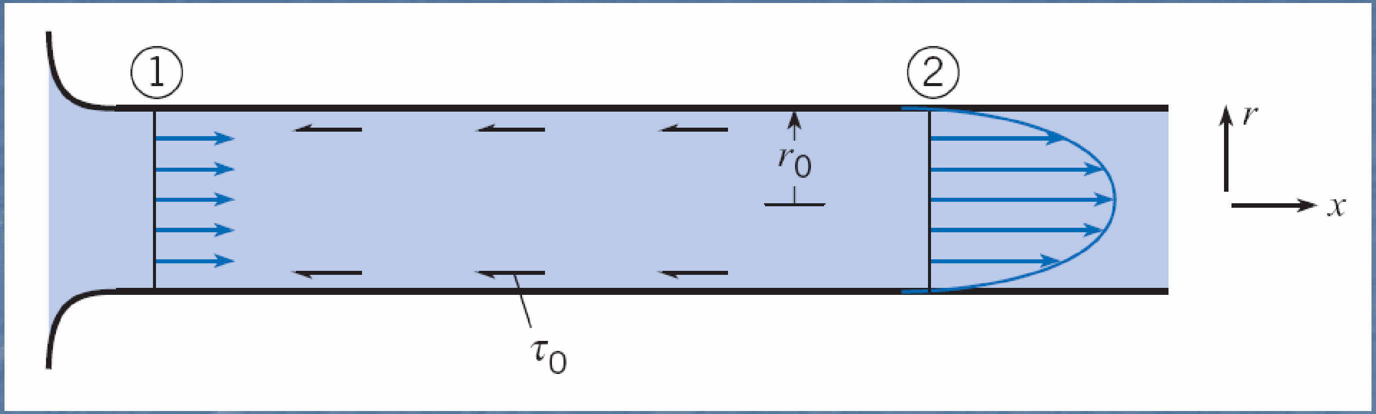
y-direction

$$F_y = \dot{m}_2 v_{2_y} + \dot{m}_4 v_{4_y}$$
$$F_y = \rho Q_2 v_2 - \rho Q_4 v_4$$
$$= (2.0 \times 1000)(0.1)(20) - (2.0 \times 1000)(0.5)(15)$$
$$= -11.0 \text{ kN}$$

Net Force

$$\boxed{\mathbf{F} = (0\mathbf{i} - 11.0\mathbf{j}) \text{ kN}}$$

Problem 6.64



Situation: A flow in a pipe is laminar and fully developed—additional details are provided in the problem statement.

Find: Derive a formula for the resisting shear force (F_τ) as a function of the parameters D, p_1, p_2, ρ, and U.

APPROACH

Apply the momentum principle, then the continuity principle.

ANALYSIS

Momentum principle (x-direction)

$$\sum F_x = \int_{cs} \rho v(v \cdot dA)$$

$$p_1A_1 - p_2A_2 - F_\tau = \int_{A_2} \rho u_2^2 dA - (\rho A u_1)u_1$$

$$p_1A - p_2A - F_\tau = -\rho u_1^2 A + \int_{A_2} \rho u_2^2 dA \qquad (1)$$

Integration of momentum outflow term

$$u_2 = u_{\max}(1-(r/r_0)^2)^2$$

$$u_2^2 = u_{\max}^2(1-(r/r_0)^2)^2$$

$$\int_{A_2} \rho u_2^2 dA = \int_0^{r_0} \rho u_{\max}^2 (1-(r/r_0)^2)^2 2\pi r dr$$

$$= -\rho u_{\max}^2 \pi r_0^2 \int_0^{r_0} (1-(r/r_0)^2)^2(-2r/r_0^2)dr$$

To solve the integral, let

$$u = 1 - \left(\frac{r}{r_o}\right)^2$$

Thus

$$du = \left(-\frac{2r}{r_o^2}\right) dr$$

The integral becomes

$$\begin{aligned}
\int_{A_2} \rho u_2^2 dA &= -\rho u_{\max}^2 \pi r_0^2 \int_1^0 u^2 du \\
&= -\rho u_{\max}^2 \pi r_0^2 \left(\frac{u^3}{3}\Big|_1^0 \right) \\
&= -\rho u_{\max}^2 \pi r_0^2 \left(0 - \frac{1}{3} \right) \\
&= \frac{+\rho u_{\max}^2 \pi r_0^2}{3} \qquad (2)
\end{aligned}$$

Continuity principle

$$\begin{aligned}
UA &= \int u dA \\
&= \int_0^{r_0} u_{\max}(1-(r/r_0)^2)2\pi \ \mathrm{rdr} \\
&= -u_{\max}\pi r_0^2 \int_0^{r_0} (1-(r/r_0)^2)(-2r/r_0^2) \ \mathrm{dr} \\
&= -u_{\max}\pi r_0^2 (1-(r/r_0)^2)^2/2|_0^{r_0} \\
&= u_{\max}\pi r_0^2/2
\end{aligned}$$

Therefore

$$u_{\max} = 2U$$

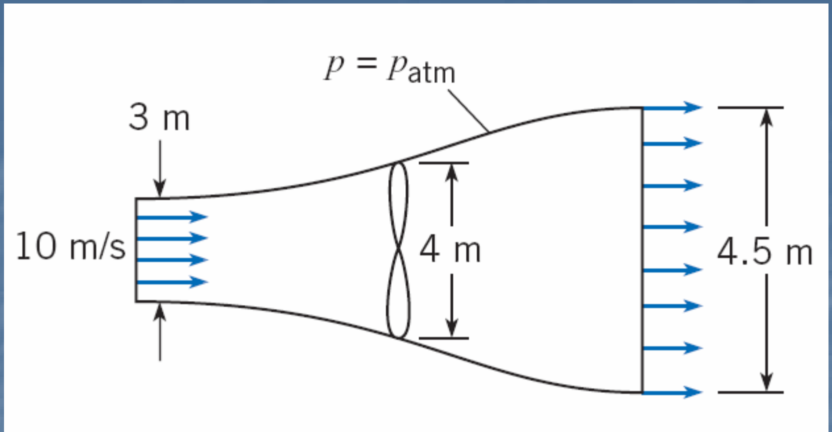
Substituting back into Eq. 2 gives

$$\int_{A_2} \rho u_2^2 dA = 4\rho U^2 \pi r_0^2/3$$

Finally substituting back into Eq. 1, and letting $u_1 = U$, the shearing force is given by

$$\boxed{F_\tau = \frac{\pi D^2}{4}\left[p_1 - p_2 - (1/3)\rho U^2\right]}$$

Problem 6.66



Situation: Air flows through a windmill—additional details are provided in the problem statement.

Find: Thrust on windmill.

APPROACH

Apply the continuity principle, then the momentum principle.

ANALYSIS

Continuity principle

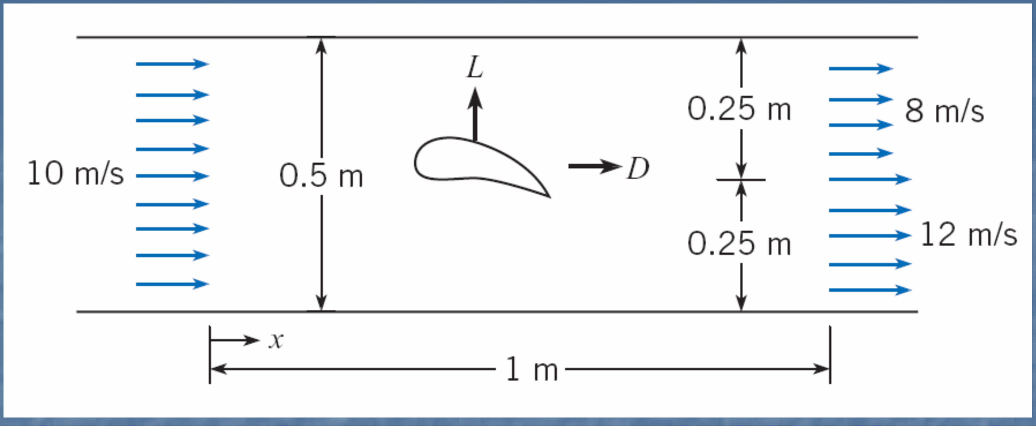
$$v_2 = 10 \times (3/4.5)^2 = 4.44 \text{ m/s}$$

Momentum principle (x-direction)

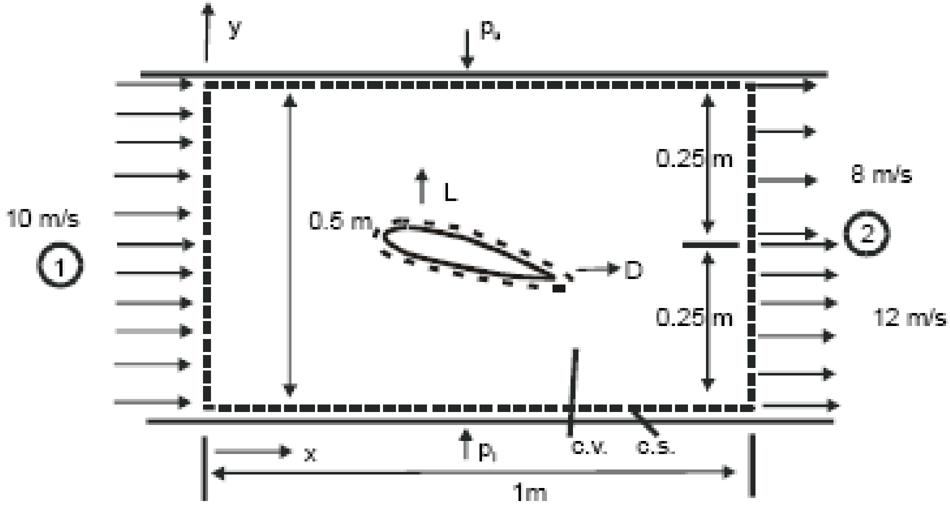
$$\begin{aligned} \sum F_x &= \dot{m}(v_2 - v_1) \\ F_x &= \dot{m}(v_2 - v_1) \\ &= (1.2)(\pi/4 \times 3^3)(10)(4.44 - 10) \\ F_x &= -472.0 \text{ N (acting to the left)} \end{aligned}$$

$$\boxed{T = 472 \text{ N (acting to the right)}}$$

Problem 6.69



Situation: Lift and drag forces are being measured on an airfoil that is situated in a wind tunnel—additional details are provided in the problem statement.



Find: (a) Lift force: L

(b) Drag force: D

APPROACH

Apply the momentum principle.

ANALYSIS

Momentum principle (x-direction)

$$\begin{aligned}\sum F_x &= \sum_{cs} \dot{m}v_0 - \dot{m}_1 v_1 \\ -D + p_1A_1 - p_2A_2 &= v_1(-\rho v_1 A) + v_a(\rho v_a A/2) + v_b(\rho v_b A/2) \\ -D/A &= p_2 - p_1 - \rho v_1^2 + \rho v_a^2/2 + \rho v_b^2/2\end{aligned}$$

where

$$\begin{aligned}p_1 &= p_u(x=0) = p_\ell(x=0) = 100 \text{ Pa, gage} \\ p_2 &= p_u(x=1) = p_\ell(x=1) = 90 \text{ Pa, gage}\end{aligned}$$

then

$$\begin{aligned}-D/A &= 90 - 100 + 1.2(-100 + 32 + 72) \\ -D/A &= -5.2 \\ D &= 5.2 \times 0.5^2 = \boxed{1.3 \text{ N}}\end{aligned}$$

Momentum principle (y-direction)

$$\sum F_y = 0$$

$$-L + \int_1^2 p_\ell B dx - \int_0^1 p_u B dx = 0 \text{ where } B \text{ is depth of tunnel}$$

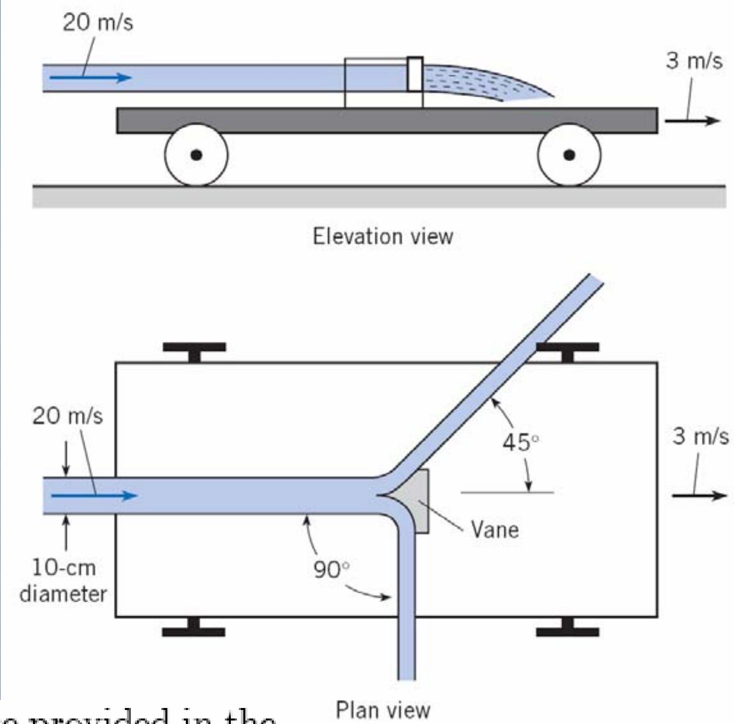
$$-L + \int_0^1 (100 - 10x + 20x(1-x))0.5dx - \int_0^1 (100 - 10x - 20x(1-x))0.5dx = 0$$

$$-L + 0.5(100x - 5x^2 + 10x^2 - (20/3)x^3)|_0^1 - 0.5(100x - 5x^2 - 10x^2 + (20/3)x^3|_0^1 = 0$$

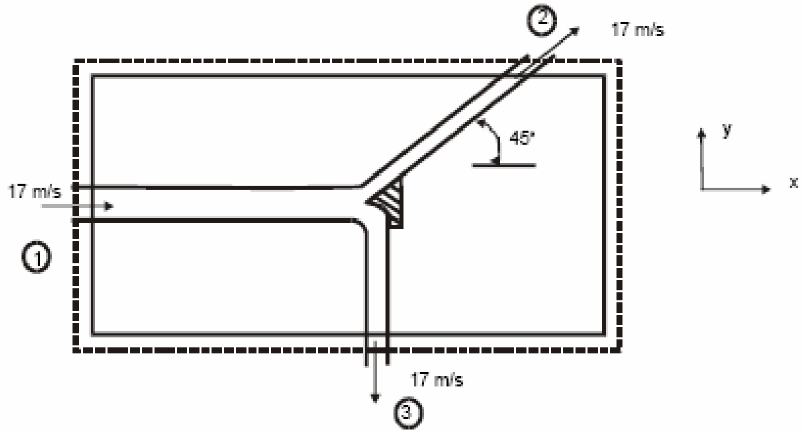
thus,

$$\begin{aligned}-L + 49.167 - 45.833 &= 0 \\ &\boxed{L = 3.334 \text{ N}}\end{aligned}$$

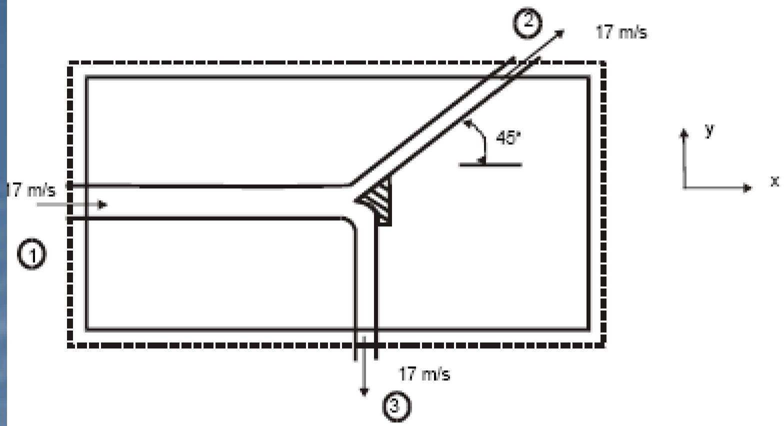
Problem 6.73



Situation: A cart is moving with steady speed—additional details are provided in the problem statement.



Find: Force exerted by the vane on the jet: **F**



APPROACH

Apply the momentum principle.

ANALYSIS

Make the flow steady by referencing all velocities to the moving vane and let the c.v. move with the vane as shown.

Momentum principle (x-direction)

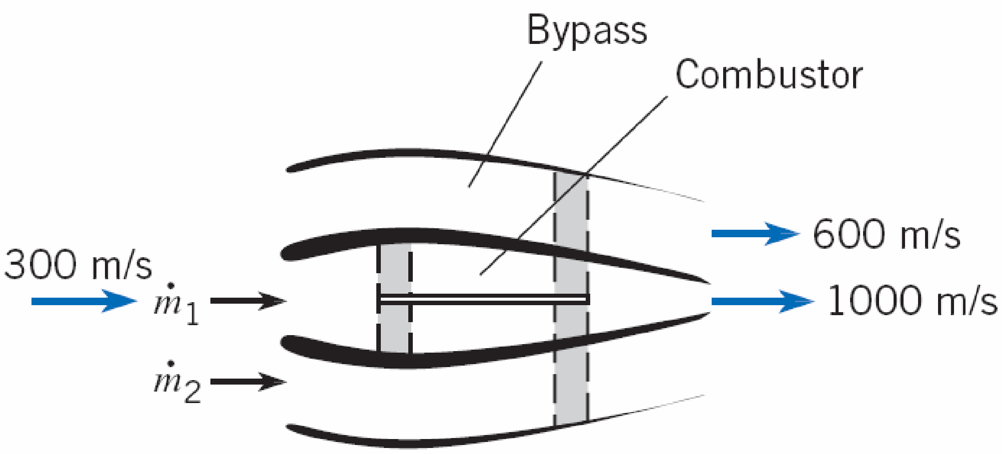
$$
\begin{aligned}
F_x &= \dot{m}_2 v_{2x} - \dot{m}_1 v_1 \\
F_x &= (17^2 \cos 45°)(1000)(\pi/4)(0.1^2)/2 - (17)(1000)(17)(\pi/4)(0.1^2) \\
&= +802 - 2270 = -1470 \text{ N}
\end{aligned}
$$

Momentum principle (y-direction)

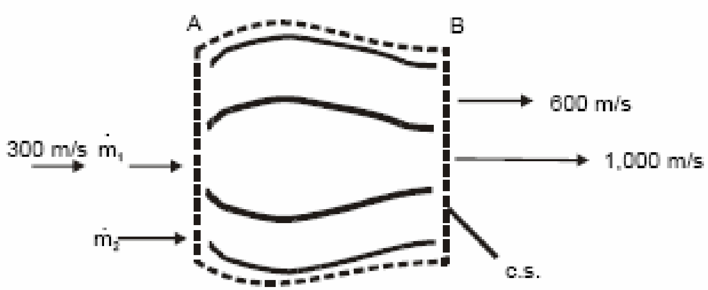
$$
\begin{aligned}
F_y &= \dot{m}_2 v_{2y} - \dot{m} v_{3y} \\
&= (17)(1,000)(\sin 45°)(17)(\pi/4)(0.1^2)/2 - (17)^2(1000)(\pi/4)(0.1^2)/2 \\
&= -333 \text{ N}
\end{aligned}
$$

$$\boxed{\mathbf{F}(\text{water on vane}) = (1470\mathbf{i} + 333\mathbf{j})\ \text{N}}$$

Problem 6.84



Situation: Air flows through a turbofan engine. Inlet mass flow is 300 kg/ s.
Bypass ratio is 2.5. Speed of bypass air is 600 m/ s.
Speed of air that passes through the combustor is 1000 m/ s.



Additional details are given in the problem statement.

Find: Thrust (T) of the turbofan engine.

Assumptions: Neglect the mass flow rate of the incoming fuel.

APPROACH

Apply the continuity and momentum equations.

ANALYSIS

Continuity equation

$$\dot{m}_A = \dot{m}_B = 300\,\mathrm{kg/s}$$

also

$$\begin{aligned}\dot{m}_B &= \dot{m}_{\text{combustor}} + \dot{m}_{\text{bypass}} \\ &= \dot{m}_{\text{combustor}} + 2.5\dot{m}_{\text{combustor}} \\ \dot{m}_B &= 3.5\dot{m}_{\text{combustor}}\end{aligned}$$

Thus

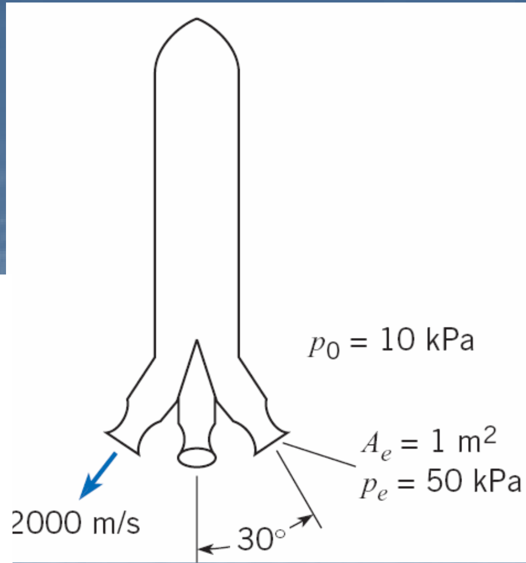
$$\begin{aligned}\dot{m}_{\text{combustor}} &= \frac{\dot{m}_B}{3.5} = \frac{300\,\mathrm{kg/s}}{3.5} \\ &= 85.71\,\mathrm{kg/s} \\ \dot{m}_{\text{bypass}} &= \dot{m}_B - \dot{m}_{\text{combustor}} \\ &= 300\,\mathrm{kg/s} - 85.71\,\mathrm{kg/s} \\ &= 214.3\,\mathrm{kg/s}\end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned}\sum F_x &= \sum \dot{m}v_{\text{out}} - \dot{m}v_{\text{in}} \\ F_x &= [\dot{m}_{\text{bypass}}V_{\text{bypass}} + \dot{m}_{\text{combustor}}V_{\text{combustor}}] - \dot{m}_A V_A \\ &= [(214.3\,\mathrm{kg/s})(600\,\mathrm{m/s}) + (85.71\,\mathrm{kg/s})(1000\,\mathrm{m/s})] - (300\,\mathrm{kg/s})(300\,\mathrm{m/s}) \\ &= 124,290\,\mathrm{N}\end{aligned}$$

$$\boxed{T = 124,300\ \mathrm{N}}$$

Problem 6.87



Situation: A rocket with four nozzles is described in the problem statement.

Find: Thrust of the rocket (all four nozzles).

APPROACH

Apply the momentum principle.

ANALYSIS

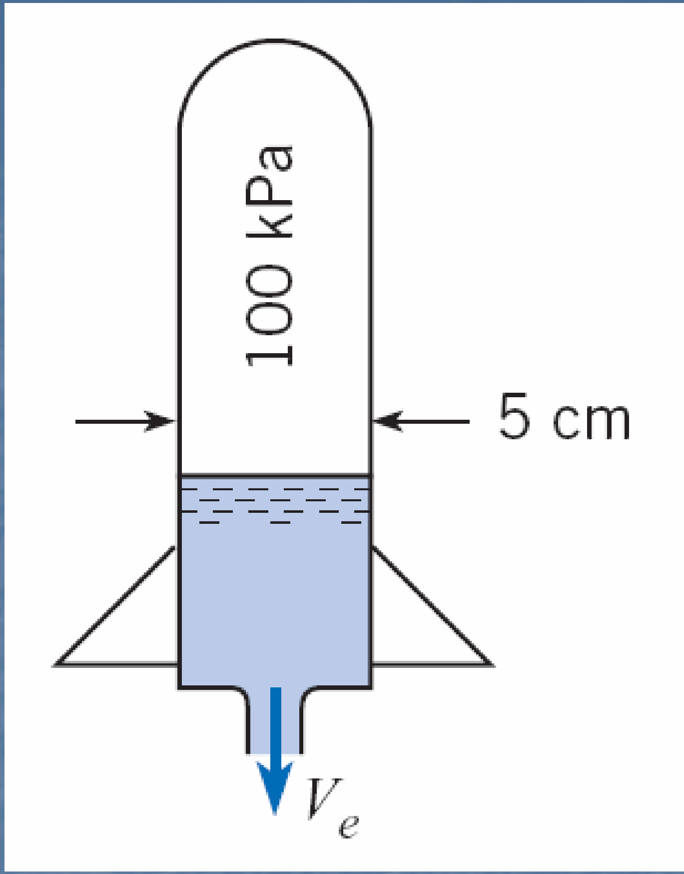
Momentum principle (z-direction)

$$\begin{aligned}\sum F_z &= \dot{m}v_z[\text{per engine}] \\ T - p_a A_e \cos 30^\circ + p_e A_e \cos 30^\circ &= -v_e \cos 30^\circ \rho v_e A_e \\ T &= -1 \times 0.866 \\ &\quad \times(50,000 - 10,000 + 0.3 \times 2000 \quad \times 2000) \\ &= -1.074 \times 10^6 \text{ N}\end{aligned}$$

Thrust of four engines

$$\begin{aligned}T_{\text{total}} &= 4 \times 1.074 \times 10^6 \\ &= 4.3 \times 10^6 N \\ &= \boxed{4.3 \text{ MN}}\end{aligned}$$

Problem 6.86



Situation: A toy rocket is powered by a jet of water—additional details are provided in the problem statement.

Find: Maximum velocity of the rocket.

Assumptions: Neglect hydrostatic pressure; Inlet kinetic pressure is negligible.

ANALYSIS

Newtons 2^{nd} law.

$$\begin{aligned} \sum F &= ma \\ T - W &= ma \end{aligned}$$

where T =thrust and W =weight

$$\begin{aligned} T &= \dot{m}v_e \\ \dot{m}v_e - mg &= m dv_R/dt \\ dv_R/dt &= (T/m) - g \\ &= (T/(m_i - \dot{m}t)) - g \\ dv_R &= ((T dt)/(m_i - \dot{m}t)) - g dt \\ v_R &= (-T/\dot{m})\ell n(m_i - \dot{m}t) - gt + \text{const.} \end{aligned}$$

where $v_R = 0$ when $t = 0$. Then

$$\begin{aligned} \text{const.} &= (T/\dot{m})\ln(m_i) \\ v_R &= (T/\dot{m})\ln((m_i)/(m_i - \dot{m}t)) - gt \\ v_{R\,\text{max}} &= (T/\dot{m})\ln(m_i/m_f) - gt_f \\ T/\dot{m} &= \dot{m}v_e/\dot{m} = v_e \end{aligned}$$

Bernoulli equation
(neglecting hydrostatic pressure)

$$p_i + \rho_f v_i^2/2 = p_e + \rho_f v_e^2/2$$

The exit pressure is zero (gage) and the inlet kinetic pressure is negligible. So

$$\begin{aligned} v_e^2 &= 2p_i/\rho_f \\ &= 2 \times 100 \times 10^3/998 \\ &= 200 \text{ m}^2/\text{s}^2 \\ v_e &= 14.14 \text{ m/s} \\ \dot{m} &= \rho_e v_e A_e \\ &= 1000 \times 14.14 \times 0.1 \times 0.05^2 \times \pi/4 \\ &= 2.77 \text{ kg/s} \end{aligned}$$

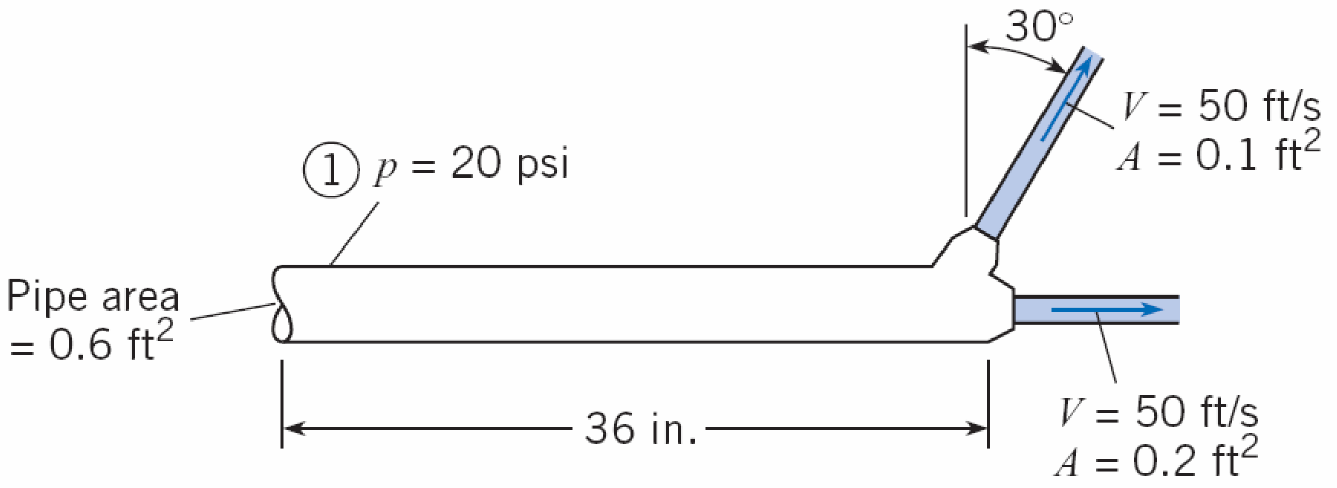
Time for the water to exhaust:

$$\begin{aligned} t &= m_w/\dot{m} \\ &= 0.10/2.77 \\ &= 0.036s \end{aligned}$$

Thus

$$\begin{aligned} v_{\text{max}} &= 14.14 \ln((100 + 50)/50) - (9.81)(0.036) \\ &= \boxed{15.2 \text{ m/s}} \end{aligned}$$

Problem 6.99



Situation: Water flows out a pipe with two exit nozzles—additional details are provided in the problem statement.

Find: Reaction (Force and Moment) at section 1.

ANALYSIS

Continuity principle equation

$$v_1 = (0.1 \times 50 + 0.2 \times 50)/0.6 = 25 \text{ ft/s}$$

Momentum equation (x-direction)

$$\sum F_x = \dot{m}_3 v_{3x} + \dot{m}_2 v_{2x}$$

$$F_x = -20 \times 144 \times 0.6 - 1.94 \times 25^2 \times 0.6 + 1.94 \times 50^2 \times 0.2$$
$$+1.94 \times 50^2 \times 0.1 \times \cos 60^\circ = -1{,}243 \text{ lbf}$$

Momentum equation (y-direction)

$$\sum F_y = \dot{m}_2 v_{2y}$$
$$F_y = 1.94 \times 50 \times 50 \times 0.1 \times \cos 30^\circ = 420 \text{ lbf}$$

Moment-of-momentum ($z-$direction)

$$r_2 \dot{m}_2 v_{2y} = (36/12)(1.94 \times 0.1 \times 50)50 \sin 60^\circ = 1260 \text{ ft-lbf}$$

Reaction at section 1

$$\boxed{\mathbf{F} = (1243\mathbf{i} - 420\mathbf{j})\text{lbf}}$$

$$\boxed{\mathbf{M} = (-1260\mathbf{k}) \text{ ft-lbf}}$$