# CHAPTER (7)

# ENERGY PRINCIPLE

# SOLVED PROBLEMS

DR. MUNZER EBAID

## 1. Steady Flow Energy Equation

$$\dot{Q} - \dot{W}_S = \sum_{CS} \dot{m}_{out}\left(\frac{V^2}{2} + gz + h\right)_{out} - \sum_{CS} \dot{m}_{in}\left(\frac{V^2}{2} + gz + h\right)_{in}$$

## 2. Energy Equation for Steady Flow of an Incompressible Fluid in a pipe with a Pump and a Turbine

$$\left(h_P + \frac{p_1}{\rho g} + z_1 + \alpha_1 \frac{\overline{V}_1^{\,2}}{2g}\right)_{Mech.\,part} = \left(h_T + \frac{p_2}{\rho g} + gz_2 + \alpha_2 \frac{\overline{V}_2^{\,2}}{2g}\right)_{Mech.\,part} + h_{Loss}$$

*The coefficients $(\alpha_1, \alpha_2)$ are called kinetic energy correction factor and can be evaluated as follows:*

$$\alpha = \frac{1}{A}\int_A \left(\frac{V}{\overline{V}}\right)^3 dA$$

# 4. The head loss due to the expansion as a function of flow velocities in the two pipes?

$$i.e.\quad h_{loss} = \left( \frac{V_1^{\,2} - V_2^{\,2}}{2g} \right)$$

*Where* $(V_1)$ *is upstream velocity and* $(V_2)$ *is downstream velocity*

$$Pump\,Head = h_p = \frac{\dot{W}_p}{\dot{m}g}$$

$$\dot{W}_p = \dot{m}gh_p = \gamma Q h_p$$

$$\dot{W}_T = \dot{m}gh_T = \gamma Q h_T$$

$$Pump\,Efficiency = \eta_P = \frac{(\dot{W}_P)}{(\dot{W}_P)_{actual}}$$

$$Turbine\,Efficiency = \eta_T = \frac{(\dot{W}_T)_{actual}}{(\dot{W}_T)}$$

# Problem 7.4

$$Given: h_1 = 300\frac{kJ}{kg}, \quad h_2 = 500\frac{kJ}{kg}, \quad V_{in} = 0, \quad V_{out} = 200\frac{m}{s}, \quad z_1 = z_2, \quad \dot{m} = 1.5\frac{kg}{\sec}$$

Situation: A compressor is described in the problem statement.

Find: Power required to operate compressor.

APPROACH

**Process is Adiabatic**

Apply the energy principle.

ANALYSIS

Energy principle

$$\dot{Q} - \dot{W}_S = \sum_{CS} \dot{m}_{out}\left(\frac{V^2}{2} + gz + h\right)_{out} - \sum_{CS} \dot{m}_{in}\left(\frac{V^2}{2} gz + h\right)_{in}$$

$$\dot{W} = \dot{Q} + \dot{m}(V_1^2/2 - V_2^2/2 + h_1 - h_2)$$

The inlet kinetic energy is negligible so

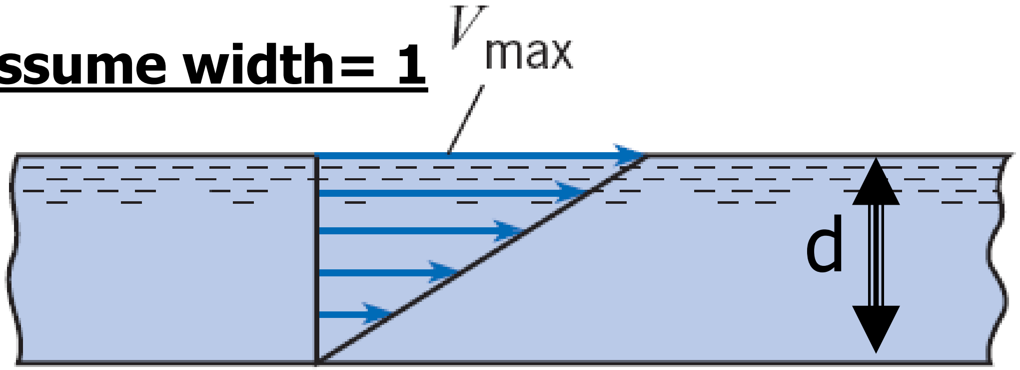
$$\begin{aligned} \dot{W} &= \dot{m}(-V_2^2/2 + h_1 - h_2) \\ &= 1.5(-200^2/2 + 300 \times 10^3 - 500 \times 10^3) \end{aligned}$$

$$\boxed{\dot{W} = \text{-330 kW}}$$

# Problem 7.7

$$V = V_{max}\left(\frac{y}{d}\right)$$

**Assume width= 1**




Situation: A hypothetical velocity distribution in a rectangular channel is described in the problem statement.

Find: Kinetic energy correction factor: $\alpha$

ANALYSIS

$$\alpha = \frac{1}{A}\int_A \left(\frac{V}{\overline{V}}\right)^3 dA$$

$$Q = \overline{V}A = \int_0^d V dA$$

$$\overline{V} = \frac{1}{A}\int_0^d V_{max}\left(\frac{y}{d}\right) dy\,(1)$$

$$\overline{V} = \frac{1}{d\times 1}\left(\frac{V_{max}}{d}\right)\left[\frac{d^2}{2}\right] = \frac{V_{max}}{2}$$

$\bar{V} = V_{max}/2$ and $V = V_{max}y/d$

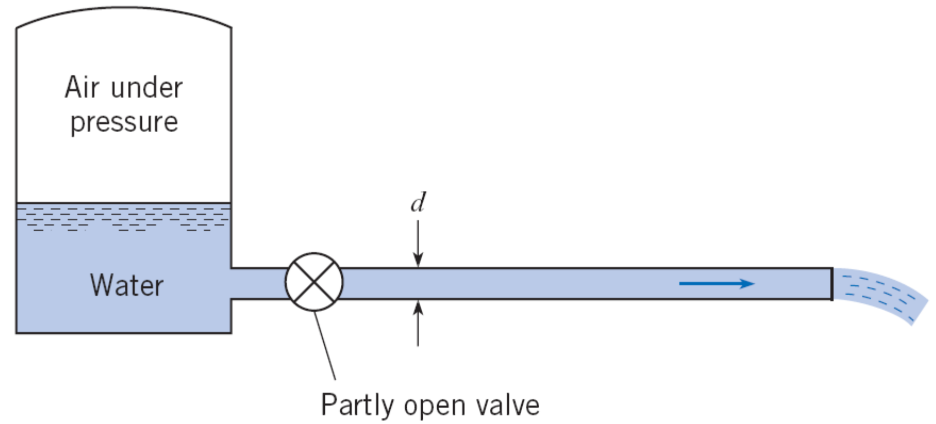
Kinetic energy correction factor

$$\begin{aligned}\alpha &= (1/d)\int_0^d (V_{max}y/((V_{max}/2)d))^3 dy \\ &= (1/d)\int_0^d (2y/d)^3 dy\end{aligned}$$

$$\boxed{\alpha = 2}$$

# Problem 7.15

## $Find: \quad K_L$




Apply the energy equation to a control volume surrounding the water. Analyze each term and then solve the resulting equation to find the minor loss coefficient.

ANALYSIS

$Given: h_L = K_L \frac{V^2}{2g}, V_{Exit} = 10\, m/s, \; p_{Tank} = 100kPa, Z_1 = 12\, m, \alpha = 1.0$ *at all locations*

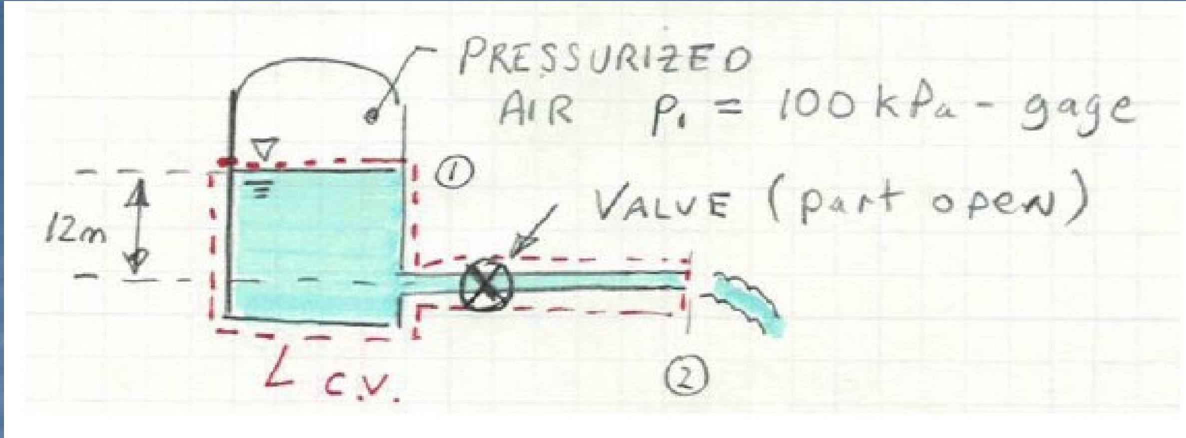
Energy equation

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \qquad (1)$$

Analyze each term:

- At the inlet. $p_1 = 100\,\text{kPa}$, $V_1 \approx 0$, $z_1 = 12\,\text{m}$
- At the exit , $p_2 = 0\,\text{kPa}$, $V_2 = 10\,\text{m/s}$, $\alpha_2 = 1.0$.
- Pumps and turbines. $h_p = h_t = 0$

# Problem 7.15




- Head loss. $h_L = K_L \frac{V^2}{2g}$

Eq. (1) simplifies to

$$\begin{aligned} \frac{p_1}{\gamma} + z_1 &= \alpha_2 \frac{V_2^2}{2g} + K_L \frac{V_2^2}{2g} \\ \frac{(100,000\,\text{Pa})}{(9800\,\text{N/m}^3)} + 12\,\text{m} &= \frac{(10\,\text{m/s})^2}{2\,(9.81\,\text{m/s}^2)} + K_L \frac{(10\,\text{m/s})^2}{2\,(9.81\,\text{m/s}^2)} \\ 22.2\,\text{m} &= (5.097\,\text{m}) + K_L\,(5.097\,\text{m}) \end{aligned}$$

Thus

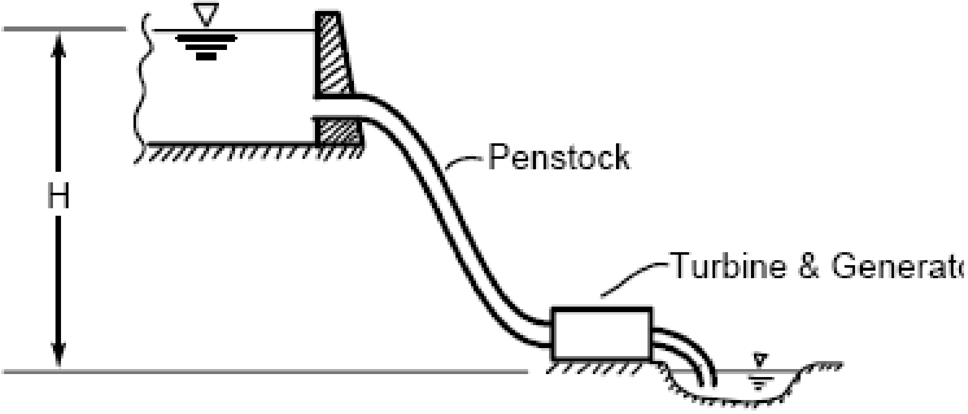
$$\boxed{K_L = 3.35}$$

# Problem 7.21

Situation: An engineer is estimating the power that can be produced by a small stream.

Stream discharge: $Q = 1.4$ cfs. Stream temperature: $T = 40\,^{\circ}\mathrm{F}$.

Stream elevation: $H = 34$ ft above the owner's residence.




Find: Estimate the maximum power in kilowatts that can be generated.

(a) The head loss is 0.0 ft, the turbine is 100% efficient and the generator is 100% efficient.

(b) The head loss is 5.5 ft, the turbine is 70% efficient and the generator is 90% efficient.

# Problem 7.21

**APPROACH**

To find the head of the turbine ($h_t$), apply the energy equation from the upper water surface (section 1) to the lower water surface (section 2). To calculation power, use $P = \eta(\dot{m}gh_t)$, where $\eta$ accounts for the combined efficiency of the turbine and generator.

**ANALYSIS**

Energy equation

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \quad (1)$$

Term by term analysis

$$\begin{aligned} p_1 &= 0; \quad V_1 \approx 0 \\ p_2 &= 0; \quad V_2 \approx 0 \\ z_1 - z_2 &= H \end{aligned}$$

Eq. (1) becomes

$$\begin{aligned} H &= h_t + h_L \\ h_t &= H - h_L \end{aligned}$$

$h_L = 0$

Flow rate

$$\begin{aligned} \dot{m}g &= \gamma Q \\ &= \left(62.4\,\mathrm{lbf/\,ft^3}\right)\left(1.4\,\mathrm{ft^3/\,s}\right) \\ &= 87.4\,\mathrm{lbf/\,s} \end{aligned}$$

# Problem 7.21

Power (case a)

$$\begin{aligned} P &= \dot{m} g h_t \\ &= \dot{m} g H \\ &= (87.4\,\text{lbf/s})\,(34\,\text{ft})\,(1.356\,\text{J/ft}\cdot\text{lbf}) \\ &= 4.02\,\text{kW} \end{aligned}$$

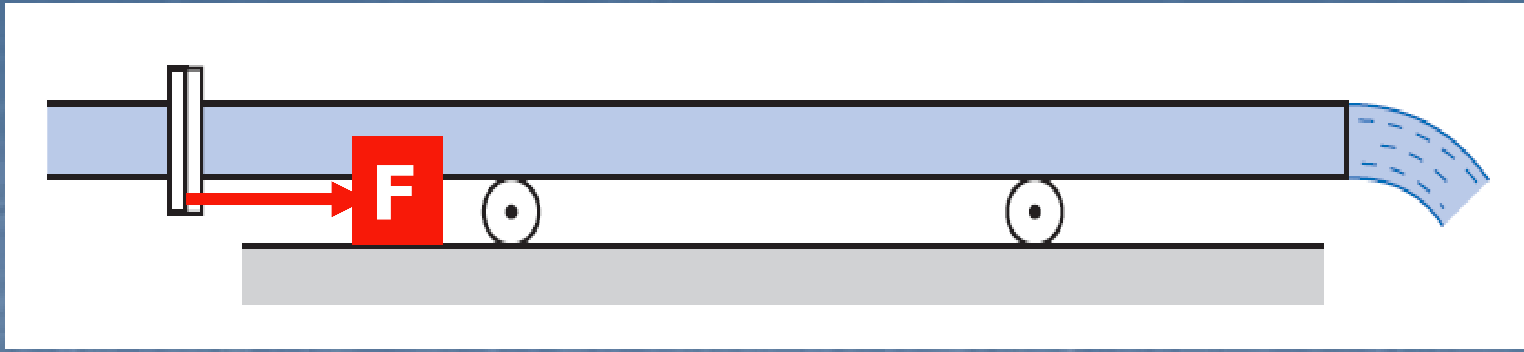
Power (case b).

$$\begin{aligned} P &= \eta \dot{m} g\,(H - h_L) \\ &= (0.7)(0.9)\,(87.4\,\text{lbf/s})\,(34\,\text{ft} - 5.5\,\text{ft})\,(1.356\,\text{J/ft}\cdot\text{lbf}) \\ &= 2.128\,\text{kW} \end{aligned}$$

$$\boxed{\text{Power (case a)} = 4.02\ \text{kW}}$$

$$\boxed{\text{Power (case b)} = 2.13\ \text{kW}}$$

# Problem 7.27




**Frictionless rollers**

$$(h_{Loss})_{pipe} = 3ft, \quad \gamma = 62.4\, lbf/ft^3, \quad A_{pipe} = 9in^2, \quad V_{pipe} = 15ft/s, \quad \alpha = 1$$

Situation: Flow though a pipe is described in the problem statement.

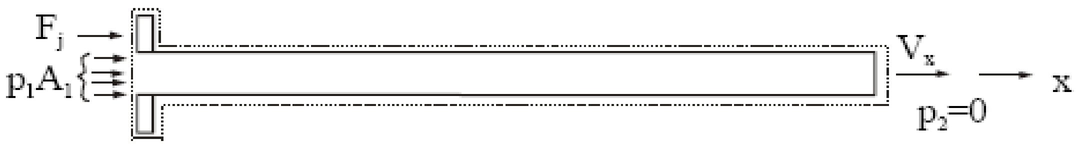
Find: Force on pipe joint.

# Problem 7.27

APPROACH

Apply the momentum principle, then the energy equation.

ANALYSIS




Momentum Equation

$$\begin{aligned} \sum F_x &= \dot{m}V_{o,x} - \dot{m}V_{i,x} \\ F_j + p_1 A_1 &= -\rho V_x^2 A + \rho V_x^2 A \\ F_j &= -p_1 A_1 \end{aligned}$$

Energy equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

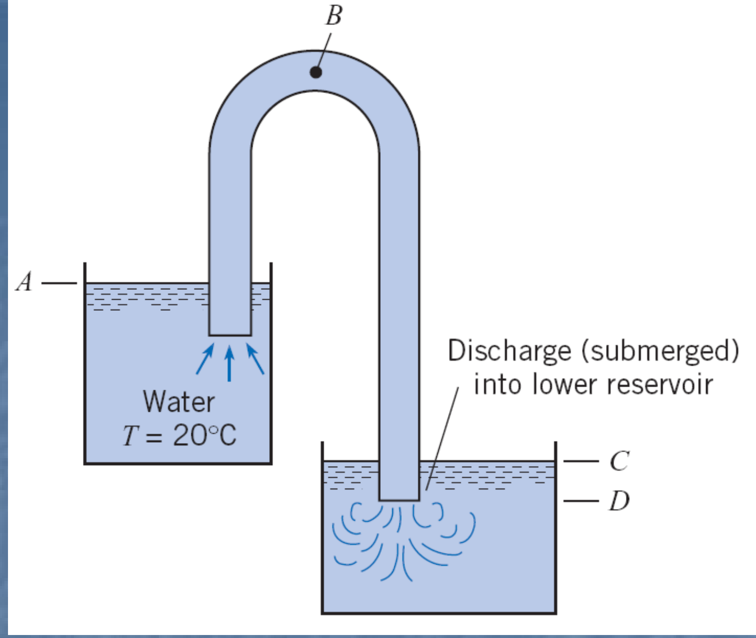
$$p_1 - p_2 = \gamma h_L$$

$$p_1 = \gamma(3) = 187.2 \text{ psfg}$$

$$F_j = -187.2 \times \left(\frac{9}{144}\right)$$

$$\boxed{F_j = -11.7 \text{ lbf}}$$

# Problem 7.28




$$Z_A = 30m, \quad Z_B = 32, \quad Z_C = 27m, \quad Z_D = 26m, \quad \alpha_1 = \alpha_2 = 1, \quad d_{pipe} = 25cm$$

$$(h_{Loss})_{A-B} = 0.75\frac{V_p^2}{2g}, \quad (h_{Loss})_{B-D} = 0.25\frac{V_p^2}{2g}$$

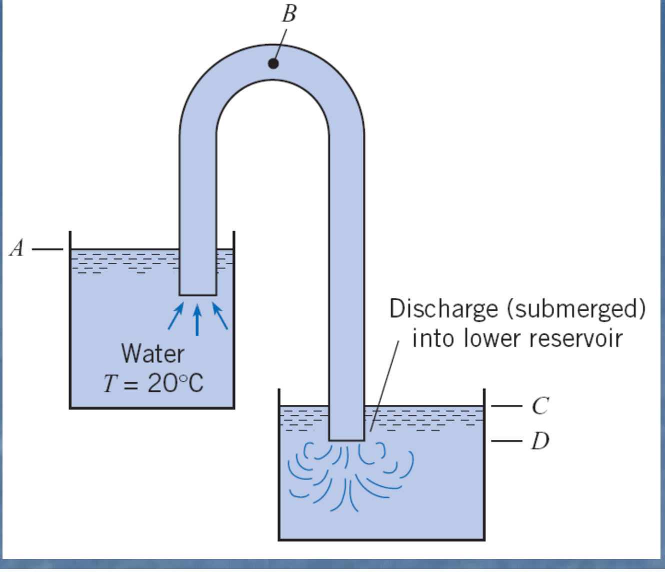
Situation: A siphon is described in the problem statement.

Find:

a.) Discharge.

b.) Pressure at point $B$.

# Problem 7.28




**APPROACH**

Apply the energy equation from A to C, then from A to B.

**ANALYSIS**

Head loss

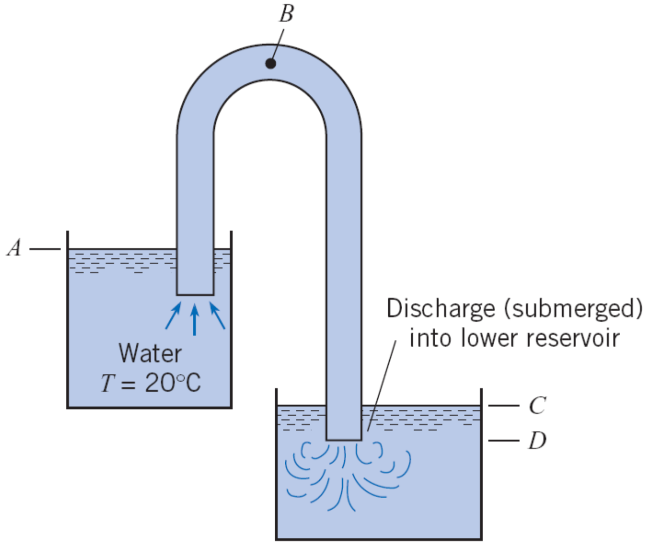
$$h_{\ell_{pipe}} = \frac{V_p^2}{2g}$$

$$\left(h_P + \frac{p_1}{\rho g} + z_1 + \alpha_1 \frac{\bar{V}_1^{\,2}}{2g}\right)_{Mech.part} = \left(h_T + \frac{p_2}{\rho g} + z_2 + \alpha_2 \frac{\bar{V}_2^{\,2}}{2g}\right)_{Mech.part} + h_{Loss}$$

## APPROACH

Apply the energy equation from A to C, then from A to B.

## ANALYSIS




Head loss

$$h_{\ell_{pipe}} = \frac{V_p^2}{2g}$$

$$h_{total} = h_{\ell_{pipe}} + h_{\ell_{outlet}} = 2\frac{V_p^2}{2g}$$

Energy equation (from A to C)

$$0 + 0 + 30 = 0 + 0 + 27 + 2\frac{V_p^2}{2g}$$

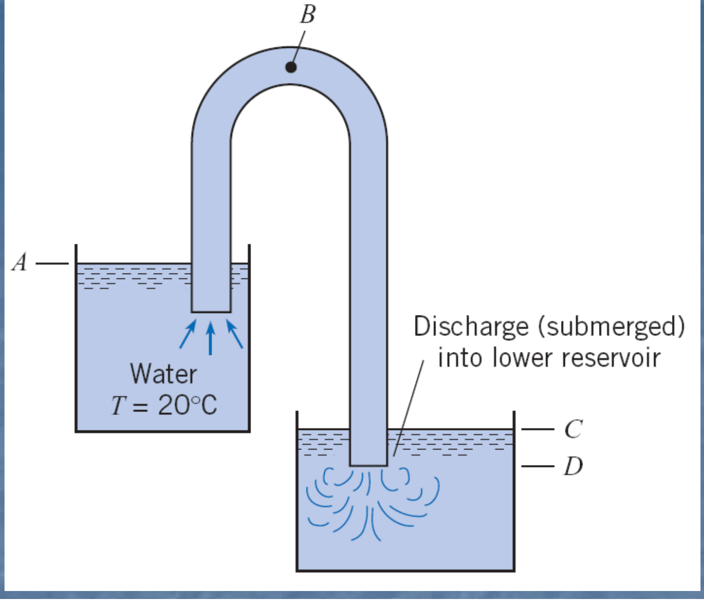
$$V_p = 5.42 \text{ m/s}$$

Flow rate equation

$$Q = V_p A_p$$

$$= 5.42 \times (\pi/4) \times 0.25^2$$

$$\boxed{Q = 0.266 \text{ m}^3/\text{s}}$$




Energy equation (from A to B)

$$30 = \frac{p_B}{\gamma} + \frac{V_p^2}{2g} + 32 + 0.75\frac{V_p^2}{2g}$$

$$\frac{p_B}{\gamma} = -2 - 1.75 \times 1.497 \text{ m}$$

$$\boxed{p_B = -45.3 \text{ kPa, gage}}$$

# Problem 7.31

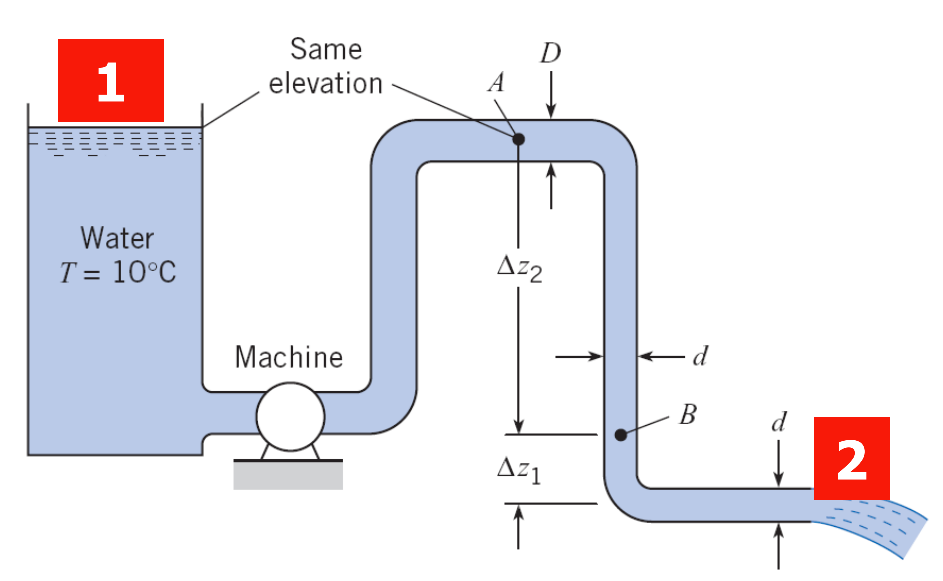
$\dot{Q} = 10 ft^3/s$ $d = 6inch$

**Neglect Head Losses**

$\alpha = 1.0$ *at all locations*

$\Delta Z_1 = 6ft$

$\Delta Z_2 = 12ft$



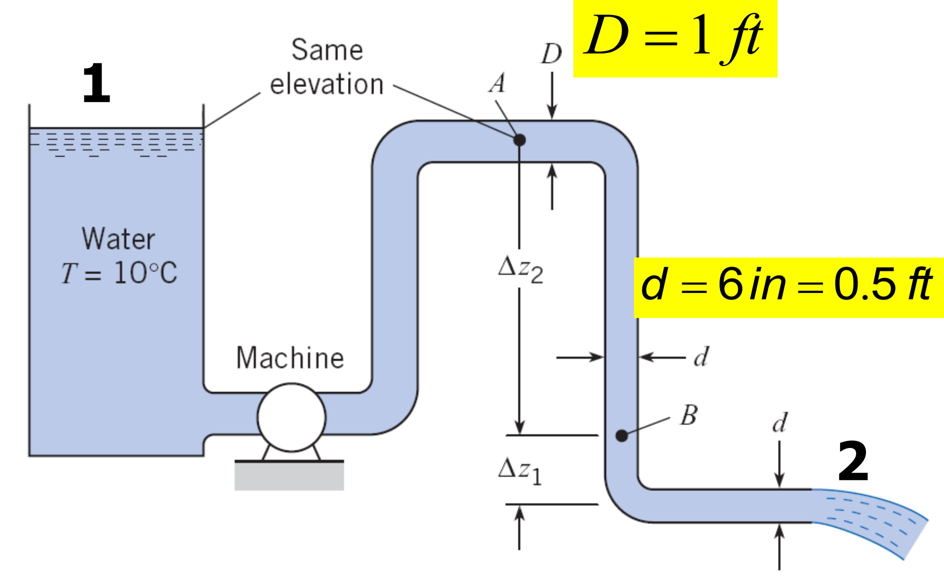

Situation: A system with a machine is described in the problem statement.

Find: Pressures at points $A$ and $B$.

Assumptions: Machine is a pump

$$\left(h_P + \frac{p_1}{\rho g} + z_1 + \alpha_1 \frac{\bar{V}_1^2}{2g}\right)_{Mech.\,part} = \left(h_T + \frac{p_2}{\rho g} + z_2 + \alpha_2 \frac{\bar{V}_2^2}{2g}\right)_{Mech.\,part} + h_{Loss}$$

$$\left(h_P + \frac{p_1}{\rho g} + z_1 + \alpha_1 \frac{\bar{V}_1^{\,2}}{2g}\right)_{Mech.part} = \left(h_T + \frac{p_2}{\rho g} + z_2 + \alpha_2 \frac{\bar{V}_2^{\,2}}{2g}\right)_{Mech.part} + h_{Loss}$$




$D = 1\ ft$

$d = 6\,in = 0.5\ ft$

Energy equation **between (1) & (2)**

$$z_1 + h_p = \frac{V_2^2}{2g} + z_2$$

$$V_2 = \frac{Q}{A_2} = \frac{10}{\frac{\pi}{4}0.5^2} = 50.93\ \text{ft/s}$$

$$h_p = \frac{50.93^2}{2 \times 32.2} - (6 + 12) = 22.3\ \text{ft}$$

Therefore the machine is a pump.

Solving for $p_B$ we have

**Neglect Head Losses**

$$p_B = \gamma(z_2 - z_B)$$
$$p_B = -6 \times 62.4 = -374 \text{ psfg}$$
$$\boxed{p_B = -2.6 \text{ psig}}$$

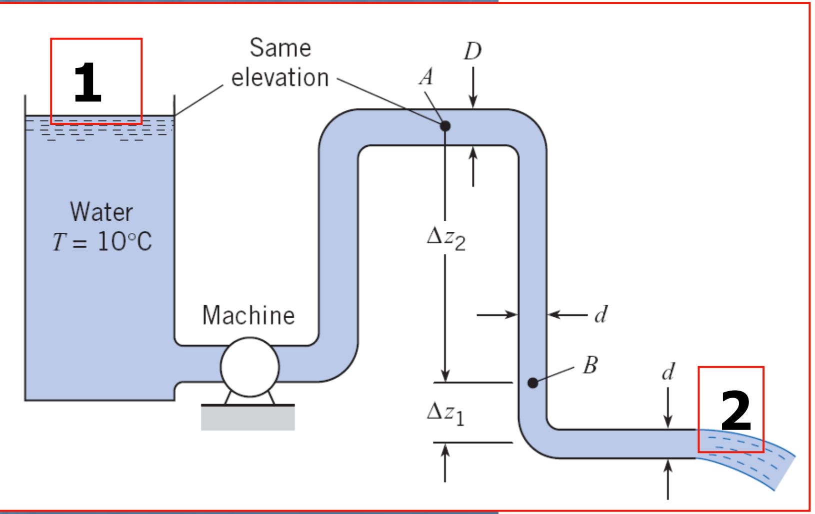
Velocity at A

$$V_A = \left(\frac{6}{12}\right)^2 \times 50.93 = 12.73 \text{ ft/s}$$

Applying the energy equation between point A and the exit gives

$$\frac{p_A}{\gamma} + z_A + \frac{V_A^2}{2g} = \frac{V_2^2}{2g}$$

$Z_2 = 0, p_2 = 0$

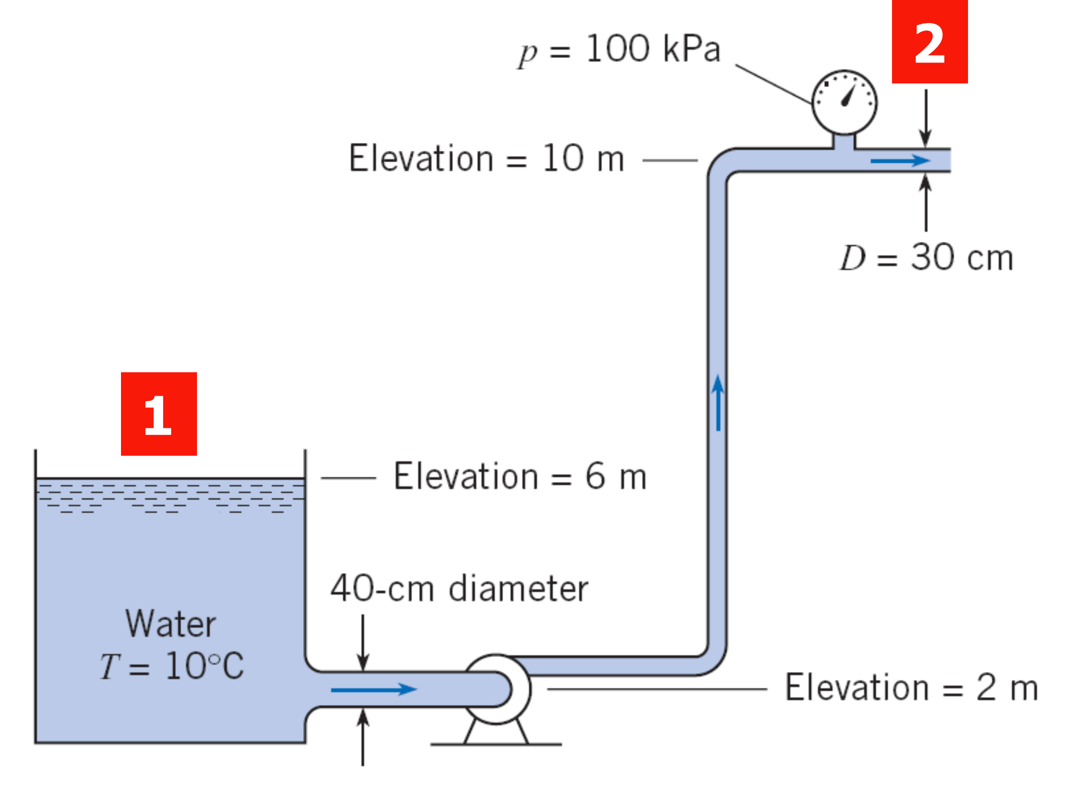



$$p_A = \gamma\left(\frac{V_2^2}{2g} - z_A - \frac{V_A^2}{2g}\right)$$
$$= 62.4 \times \left(\frac{50.93^2 - 12.73^2}{2 \times 32.2} - 18\right)$$
$$= 1233 \text{ psfg}$$
$$\boxed{p_A = 8.56 \text{ psig}}$$

# Problem 7.37

$\alpha = 1.0$ *at all locations*

$\dot{Q} = 0.25\, m^3/s$




Situation: A system with pump is described in the problem statement.

Find: Power pump must supply.

APPROACH

Apply the flow rate equation, then the energy equation from reservoir surface to the 10 m elevation. Then apply the power equation.

# Problem 7.37

$$\left(h_P + \frac{p_1}{\rho g} + z_1 + \alpha_1 \frac{\overline{V}_1^{\,2}}{2g}\right)_{Mech.part} = \left(h_T + \frac{p_2}{\rho g} + z_2 + \alpha_2 \frac{\overline{V}_2^{\,2}}{2g}\right)_{Mech.part} + h_{Loss}$$

ANALYSIS

Flow rate equation

$$\begin{aligned} V &= Q/A \\ &= 0.25/((\pi/4) \times 0.3^2) \\ &= 3.54 \text{ m/s} \\ V^2/2g &= 0.639 \text{ m} \end{aligned}$$

Energy equation **between 1 & 2**

$$h_L = 2\frac{V^2}{2g}$$

$$\begin{aligned} 0 + 0 + 6 + h_p &= 100,000/9810 + V^2/2g + 10 + 2.0V^2/2g \\ h_p &= 10.19 + 10 - 6 + 3.0 \times 0.639 \\ h_p &= 16.1 \text{ m} \end{aligned}$$

Power equation

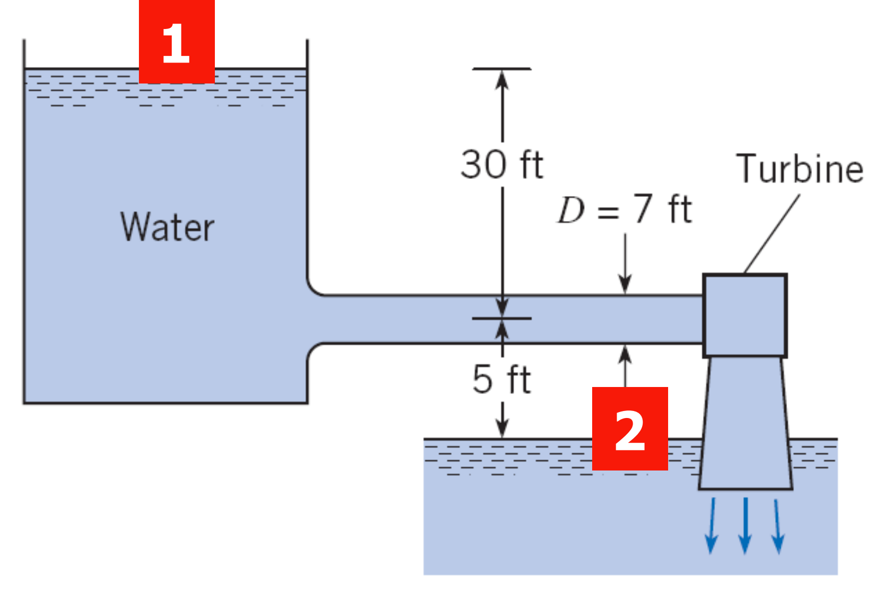
$$\begin{aligned} P &= Q\gamma h_p \\ &= 0.25 \times 9.180 \times 16.1 \end{aligned}$$

$$\boxed{P = 39.5 \text{ kW}}$$

# Problem 7.39

$\dot{Q} = 400 ft^3/s$ $\eta_{turbine} = 90\%$

$(h_{Loss})_{pipe} = (\frac{1.5V^2}{2g})_{7ft}, \quad \gamma = 62.4\, lbf/ft^3, \quad \alpha = 1$




Situation: A system with a turbine is described in the problem statement.

Find: Power output from turbine.

$$\left(h_P + \frac{p_1}{\rho g} + z_1 + \alpha_1 \frac{\overline{V}_1^2}{2g}\right)_{Mech.\,part} = \left(h_T + \frac{p_2}{\rho g} + z_2 + \alpha_2 \frac{\overline{V}_2^2}{2g}\right)_{Mech.\,part} + h_{Loss}$$

# Problem 7.39

Apply the energy equation from the upstream water surface to the downstream water surface. Then apply the power equation.

ANALYSIS

Energy equation **between 1 & 2**

$$\begin{aligned}
\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1 &= \frac{p_2}{\gamma}+\frac{V_2}{2g}+z_2+h_L+h_T \\
0+0+35 &= 0+0+0+1.5\frac{V^2}{2g}+h_T \\
V &= \frac{Q}{A}=\frac{400}{((\pi/4)\times 7^2)}=10.39\text{ ft/s} \\
\frac{V^2}{2g} &= 1.68\text{ ft} \\
h_t &= 35-2.52=32.48\text{ ft}
\end{aligned}$$

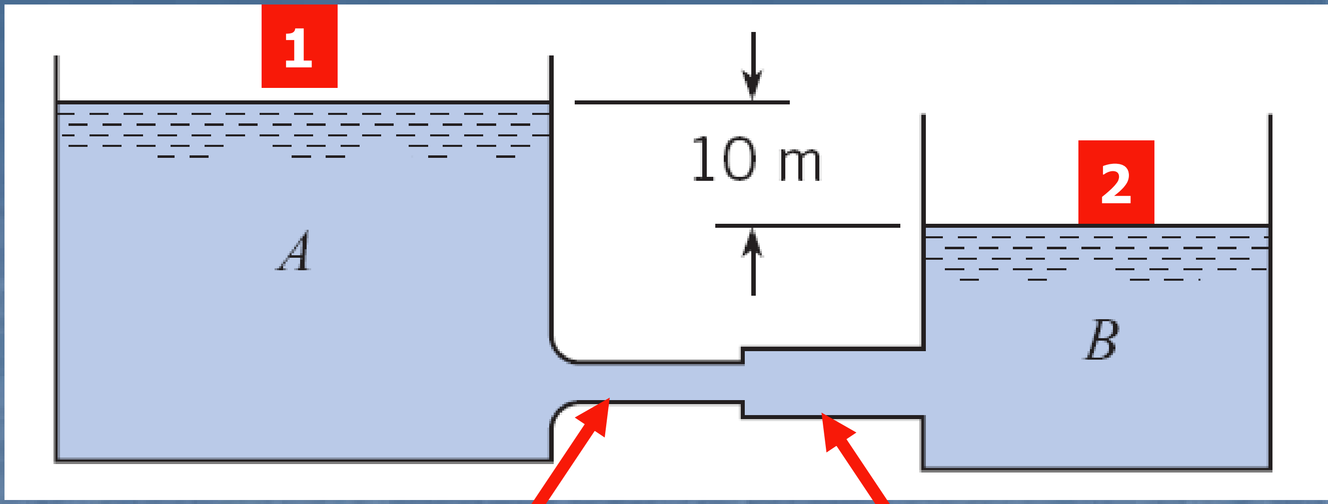
# Problem 7.39

Power equation

$$
\begin{aligned}
P(\text{hp}) &= Q\gamma h_t \times \frac{0.9}{550} \\
&= \frac{(400)(62.4)(32.48 \times 0.9)}{550} \\
&\boxed{P = 1326 \text{ hp}}
\end{aligned}
$$

# Problem 7.47

$$\dot{Q} = ?$$




$A = 10\,cm^2$

$B = 20\,cm^2$

Assuming head losses are due to sudden expansion and loss due to flow into tank (B)

Situation: A system with two tanks connected by a pipe is described in the problem statement.

Find: Discharge between two tanks: $Q$

# Problem 7.47

Apply the energy equation from water surface in A to water surface in B.

ANALYSIS

Energy equation **between surface A & surface B**

$$
\begin{aligned}
p_A/\gamma + V_A^2/2g + z_A &= p_B/\gamma + V_B^2/2g + z_B + \sum h_L \\
p_A &= p_B = p_{atm} \text{ and } V_A = V_B = 0
\end{aligned}
$$

Let the pipe from A be called pipe 1. Let the pipe from B be called pipe 2
Then

$$
\sum h_L = (V_1 - V_2)^2/2g + V_2^2/2g
$$

Continuity principle

$$
\begin{aligned}
V_1 A_1 &= V_2 A_2 \\
V_1 &= V_2(A_2/A_1)
\end{aligned}
$$

However $A_2 = 2A_1$ so $V_1 = 2V_2$. Then the energy equation gives

$$
\begin{aligned}
z_A - z_B &= (2V_2 - V_2)^2/2g + V_2^2/2g \\
&= 2V_2^2/2g \\
V_2 &= \sqrt{g(z_A - z_B)} \\
&= \sqrt{10g} \text{ m/s}
\end{aligned}
$$

# Problem 7.47

Flow rate equation

$$
\begin{aligned}
Q &= V_2 A_2 \\
&= \left(\sqrt{10g}\ \text{m/s}\right)(20\ \text{cm}^2)(10^{-4}\ \text{m}^2/\text{cm}^2)
\end{aligned}
$$

$$
\boxed{Q = 0.0198\ \text{m}^3/\text{s}}
$$

# END OF QUESTIONS