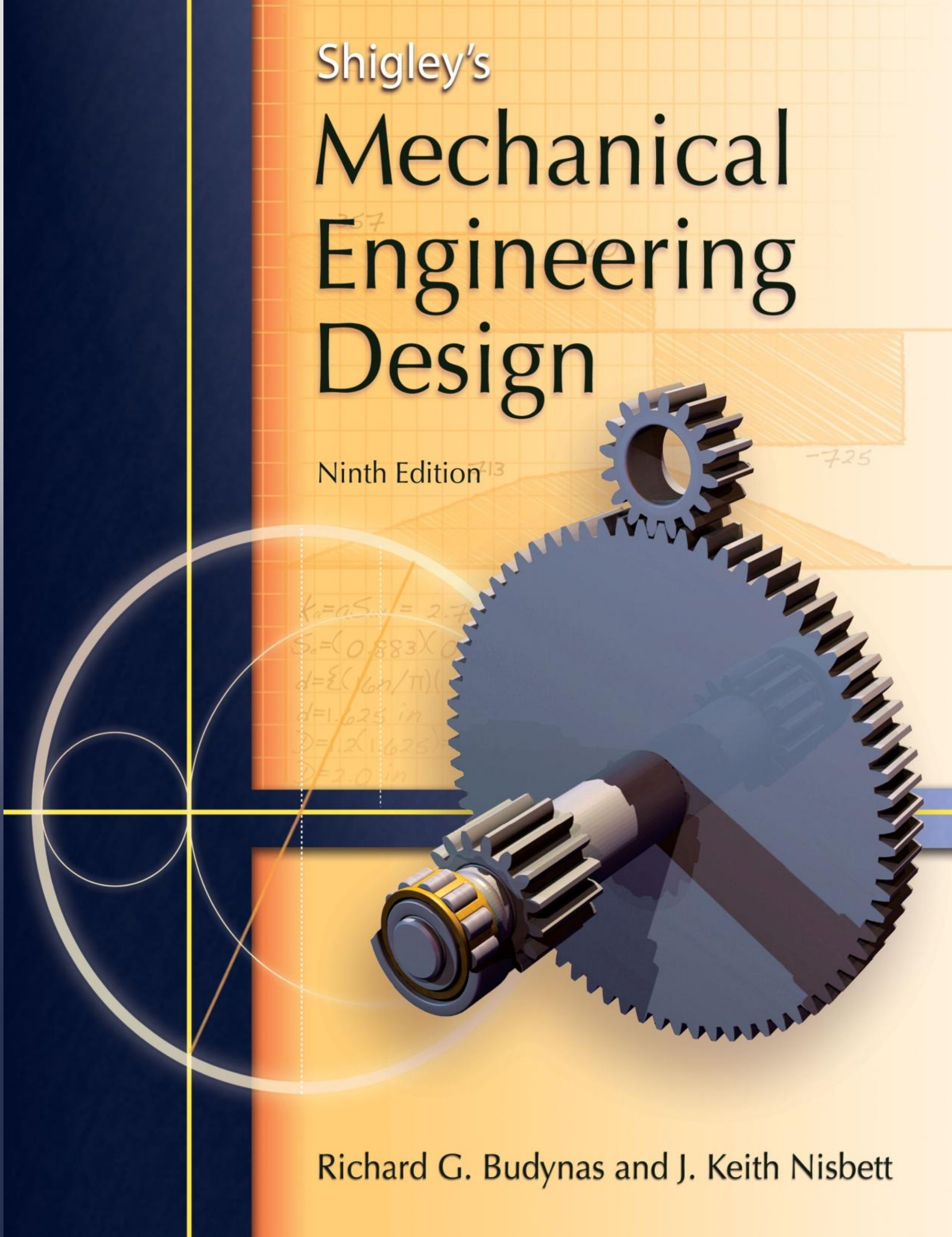
Lecture Slides

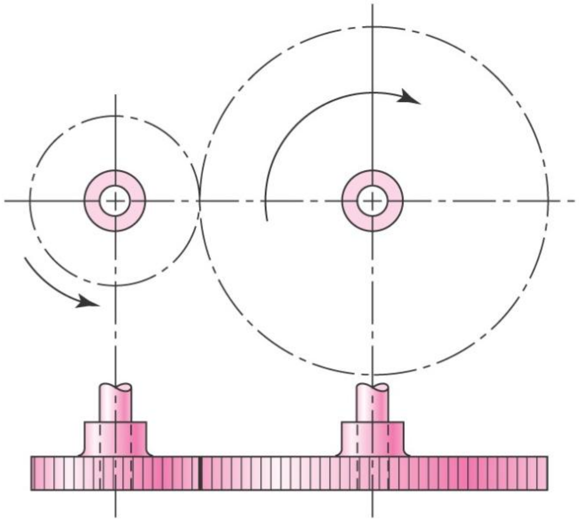
Chapter 13

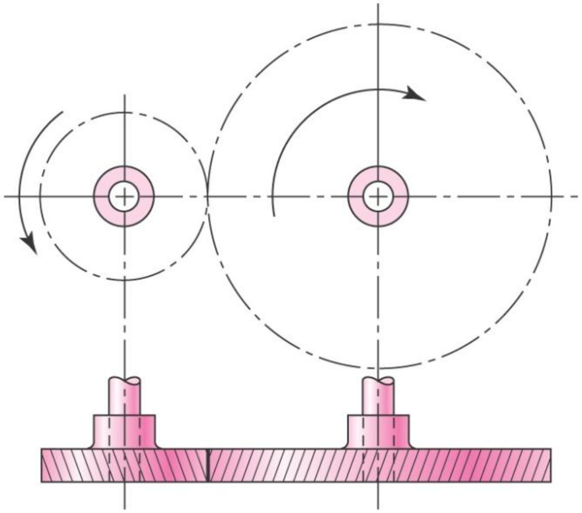
Gears – General

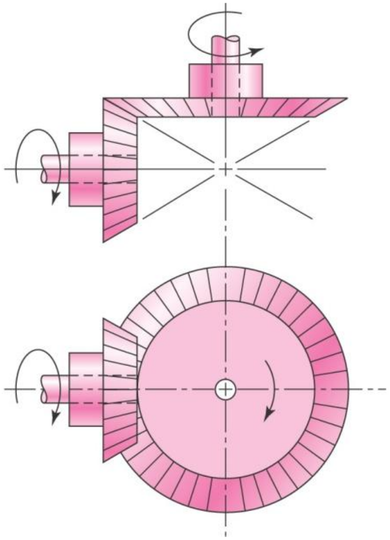


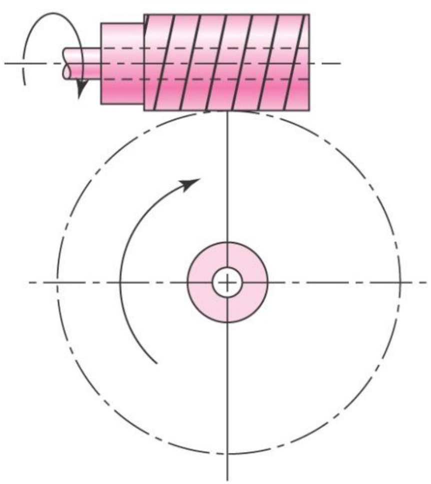
Chapter Outline

Types of Gears









Figs. 13–1 to 13–4

Nomenclature of Spur-Gear Teeth

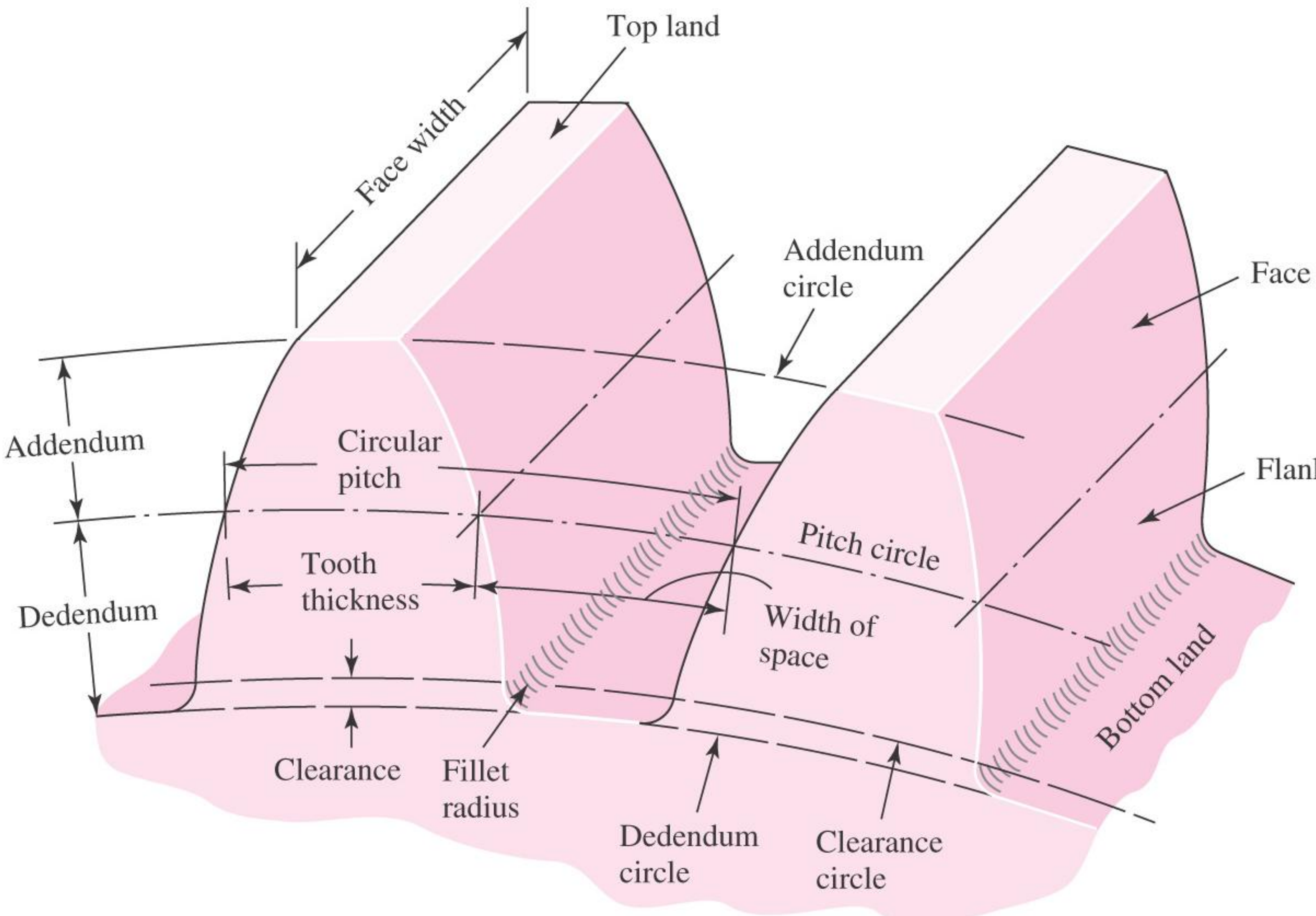


Fig. 13–5

Tooth Size

$$P = \frac{N}{d} \qquad (13\text{–}1)$$

$$m = \frac{d}{N} \qquad (13\text{–}2)$$

$$p = \frac{\pi d}{N} = \pi m \qquad (13\text{–}3)$$

$$pP = \pi \qquad (13\text{–}4)$$

where P = diametral pitch, teeth per inch
N = number of teeth
d = pitch diameter, in
m = module, mm
d = pitch diameter, mm
p = circular pitch

Tooth Sizes in General Use

Diametral Pitch	
Coarse	2, $2\frac{1}{4}$, $2\frac{1}{2}$, 3, 4, 6, 8, 10, 12, 16
Fine	20, 24, 32, 40, 48, 64, 80, 96, 120, 150, 200

Modules	
Preferred	1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
Next Choice	1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45

Table 13–2

Standardized Tooth Systems (Spur Gears)

Tooth System	Pressure Angle ϕ, deg	Addendum a	Dedendum b
Full depth	20	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
	$22\frac{1}{2}$	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
	25	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
Stub	20	$0.8/P_d$ or $0.8m$	$1/P_d$ or $1m$

Table 13–1

Standardized Tooth Systems

- Common pressure angle ϕ: 20º and 25º
- Old pressure angle: 14 ½º
- Common face width:

$$3p < F < 5p$$

$$p = \frac{\pi}{P}$$

$$\frac{3\pi}{P} < F < \frac{5\pi}{P}$$

Conjugate Action

- When surfaces roll/slide against each other and produce constant angular velocity ratio, they are said to have *conjugate action.*
- Can be accomplished if instant center of velocity between the two bodies remains stationary between the grounded instant centers.

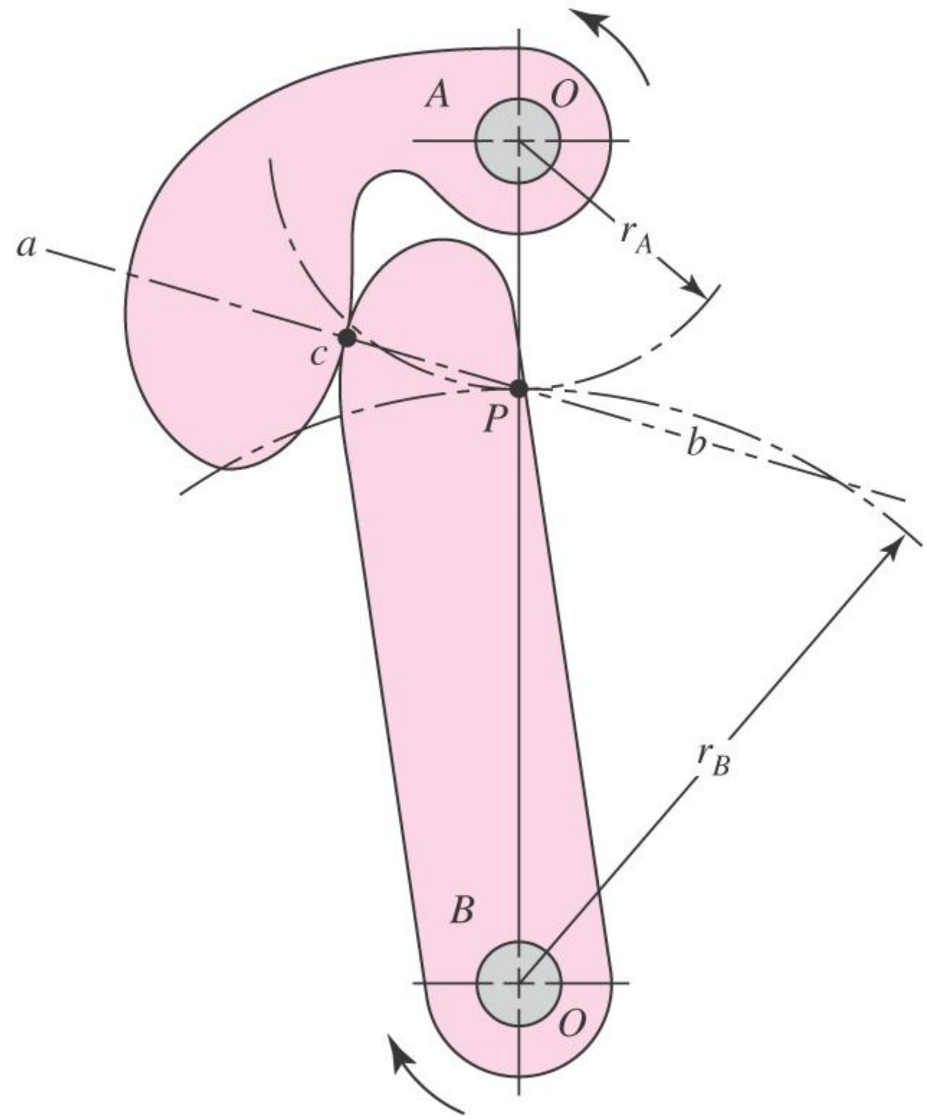


Fig. 13–6

Conjugate Action

- Forces are transmitted on *line of action* which is normal to the contacting surfaces.
- Angular velocity ratio is inversely proportional to the radii to point *P*, the *pitch point.*

$$\left|\frac{\omega_1}{\omega_2}\right| = \frac{r_2}{r_1} \qquad (13\text{–}5)$$

- Circles drawn through *P* from each fixed pivot are *pitch circles,* each with a *pitch radius.*

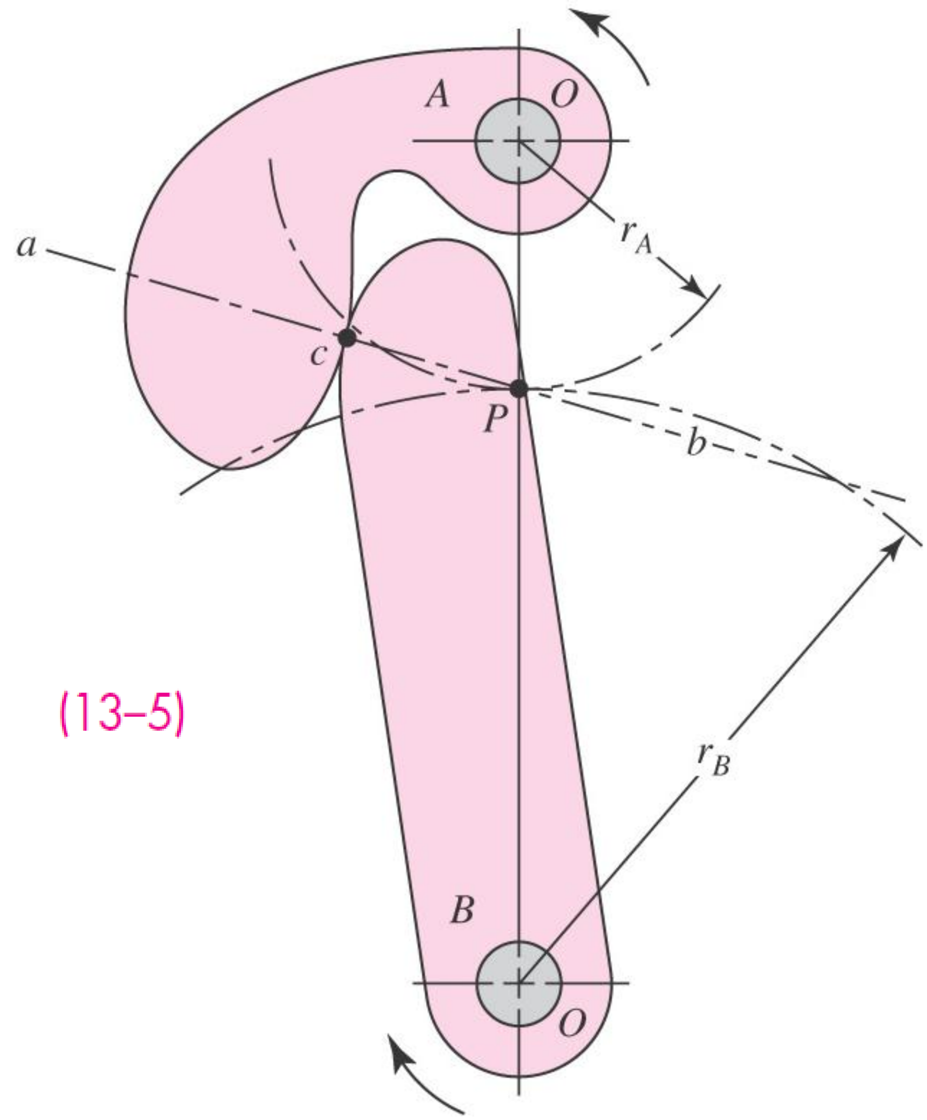


Fig. 13–6

Involute Profile

- The most common conjugate profile is the *involute* profile.
- Can be generated by unwrapping a string from a cylinder, keeping the string taut and tangent to the cylinder.
- Circle is called *base circle*.

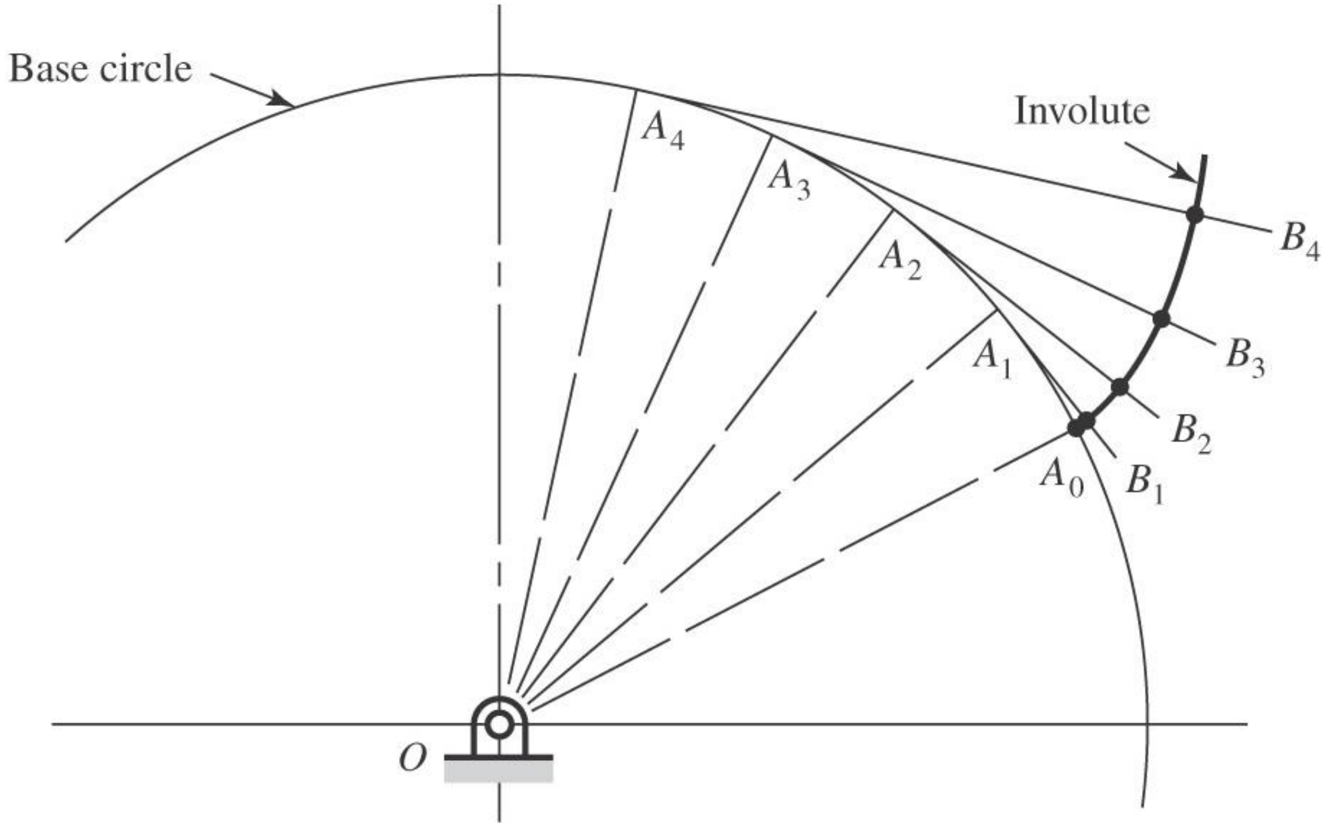


Fig. 13–8

Involute Profile Producing Conjugate Action

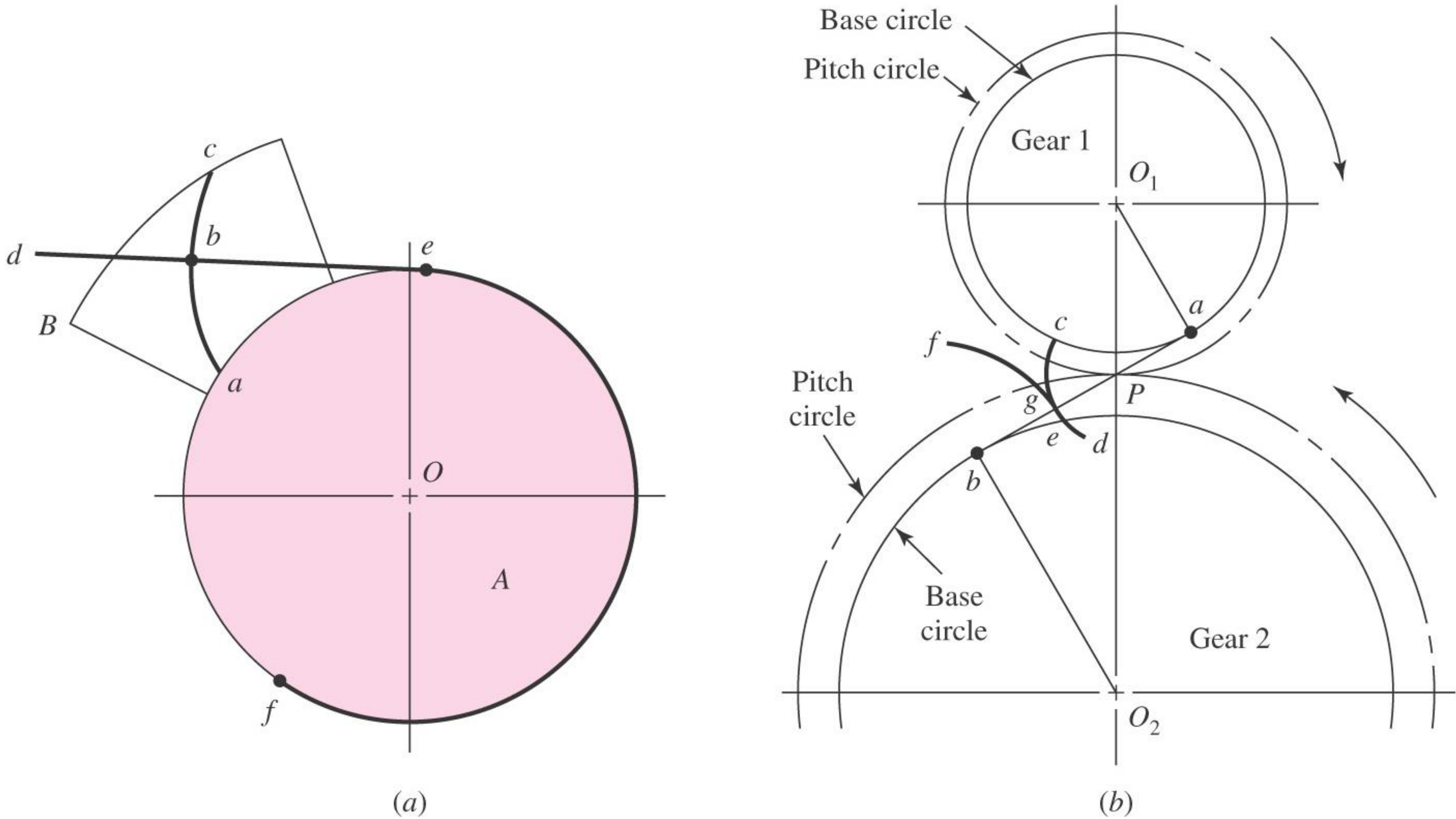


Fig. 13–7

Circles of a Gear Layout

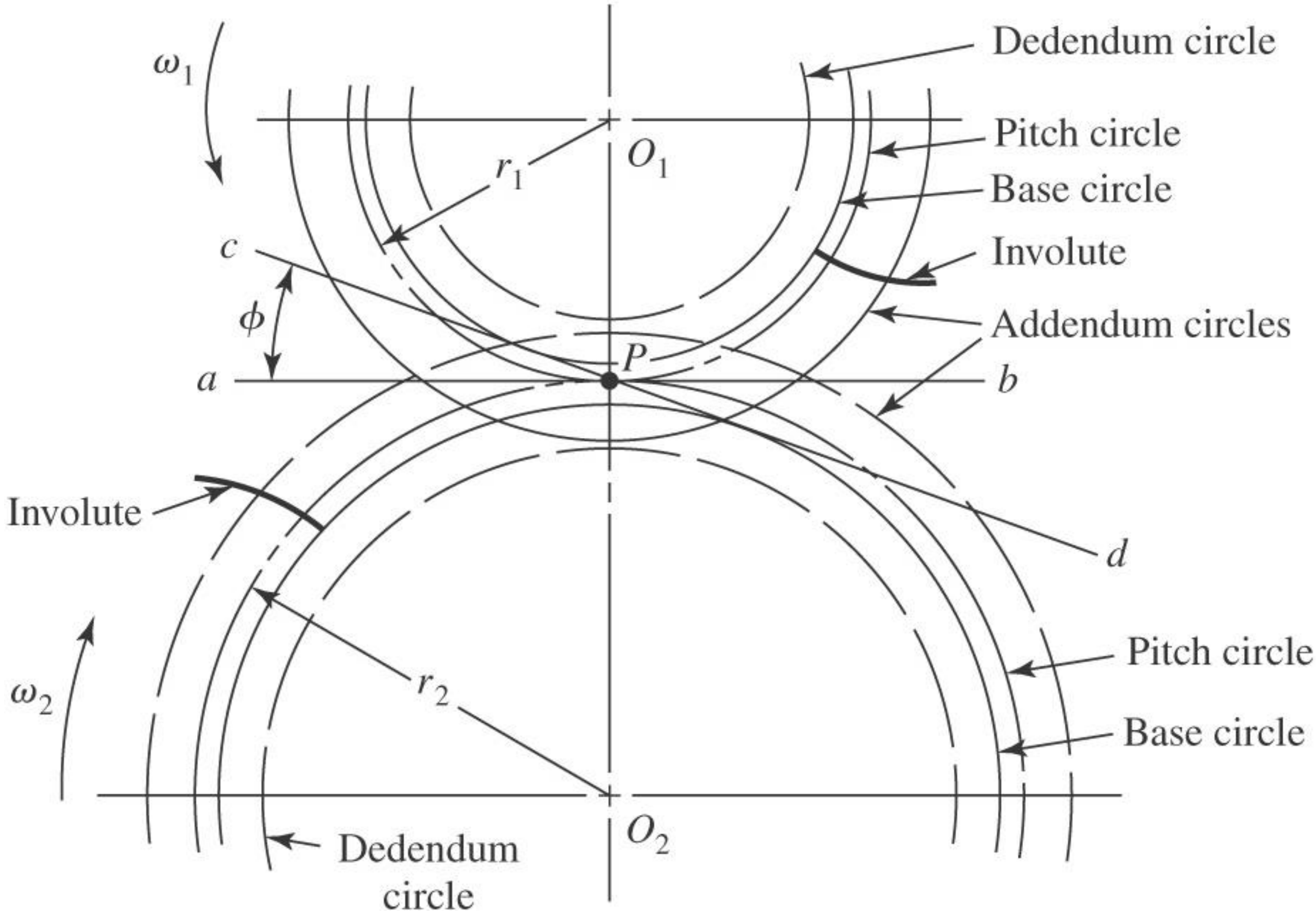


Fig. 13–9

Sequence of Gear Layout

- Pitch circles in contact
- Pressure line at desired pressure angle
- Base circles tangent to pressure line
- Involute profile from base circle
- Cap teeth at addendum circle at 1/P from pitch circle
- Root of teeth at dedendum
  circle at 1.25/P from pitch circle
- Tooth spacing from circular pitch, $p = \pi / P$

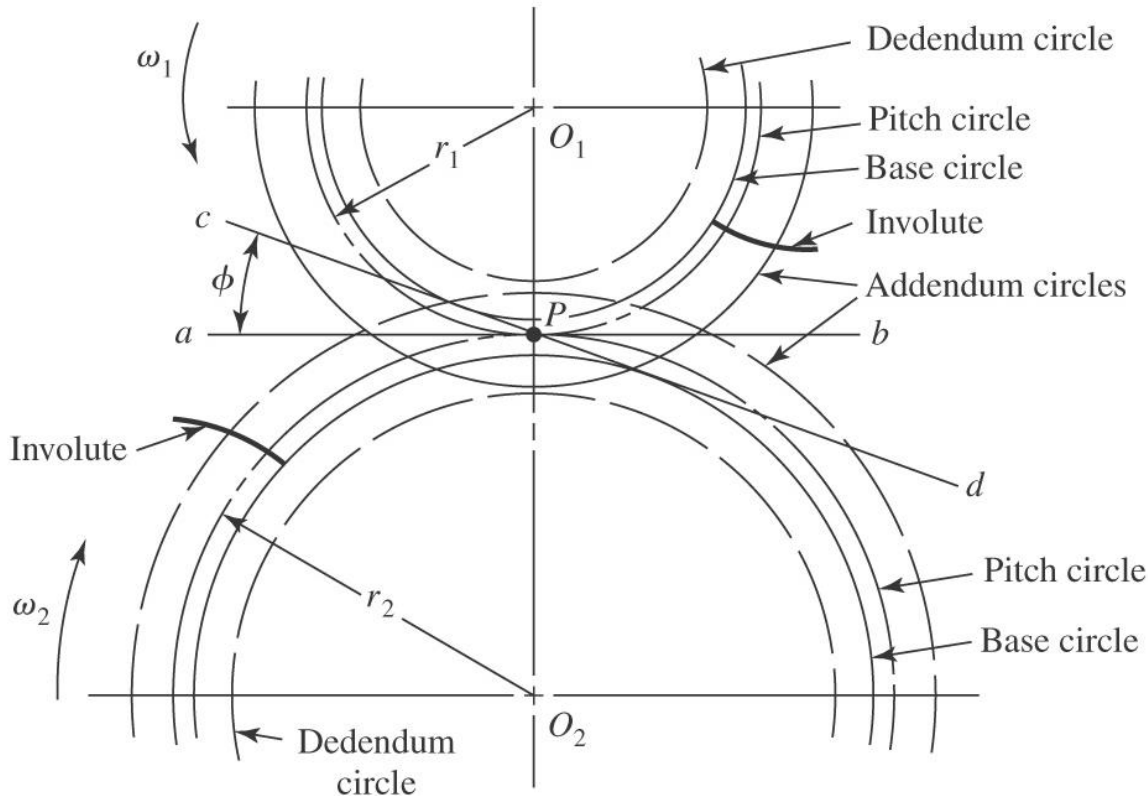


Fig. 13–9

Relation of Base Circle to Pressure Angle

$$r_b = r \cos \phi \qquad (13\text{–}6)$$

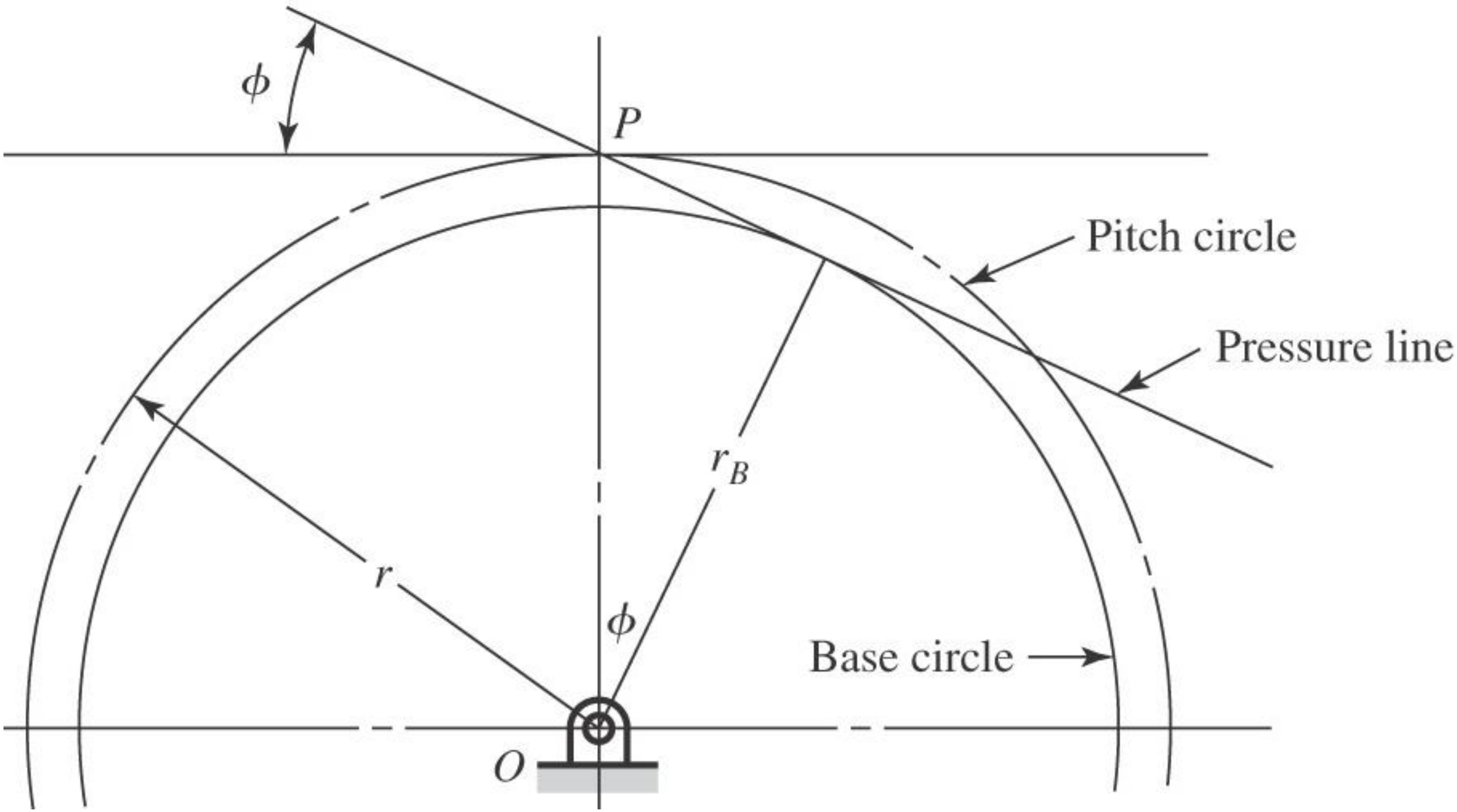


Fig. 13–10

Tooth Action

- First point of contact at *a* where flank of pinion touches tip of gear
- Last point of contact at *b* where tip of pinion touches flank of gear
- Line *ab* is *line of action*
- *Angle of action* is sum of *angle of approach* and *angle of recess*

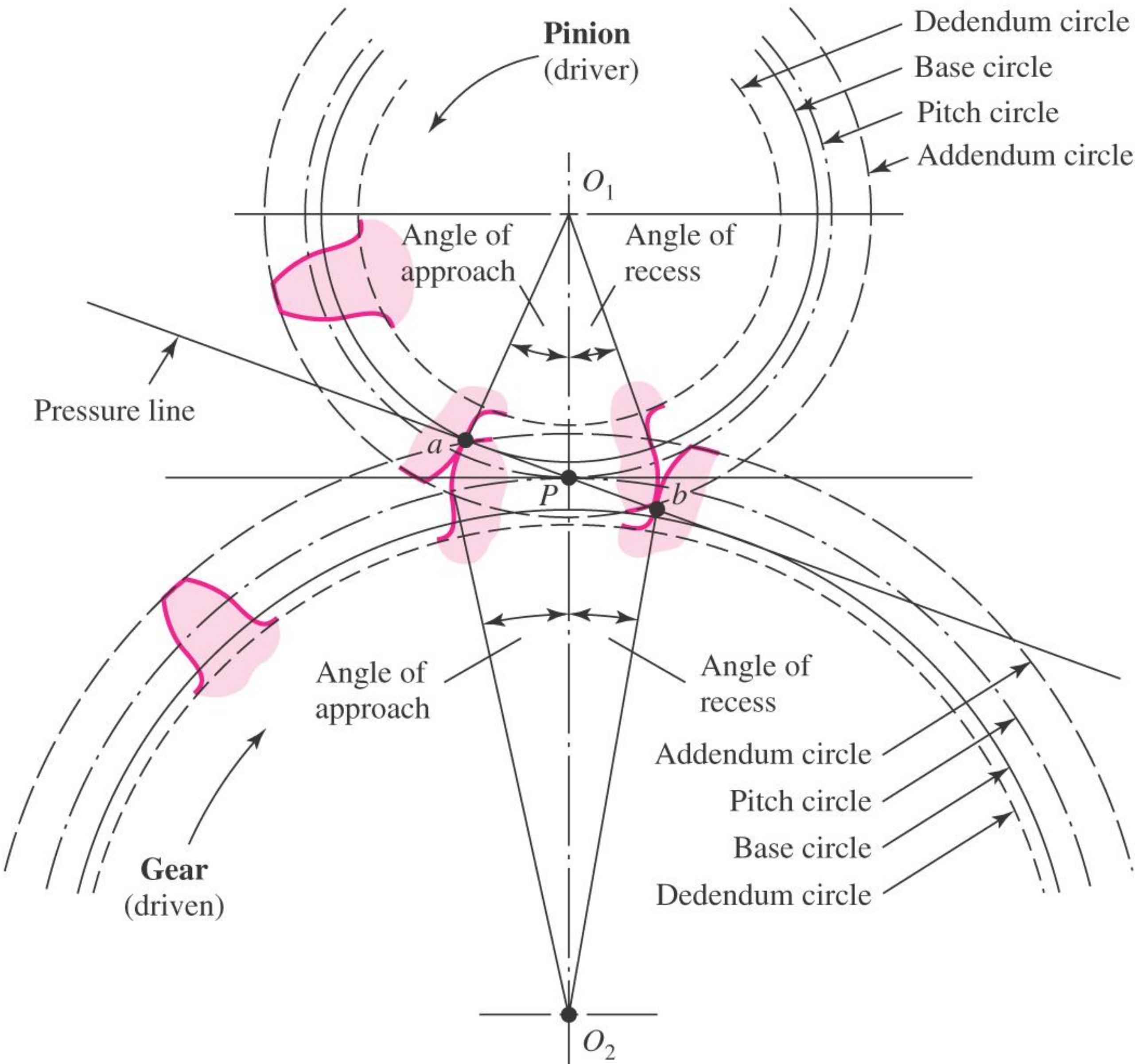


Fig. 13–12

Rack

- A *rack* is a spur gear with an pitch diameter of infinity.
- The sides of the teeth are straight lines making an angle to the line of centers equal to the pressure angle.
- The *base pitch* and *circular pitch*, shown in Fig. 13–13, are related by

$$p_b = p_c \cos\phi \qquad (13\text{–}7)$$

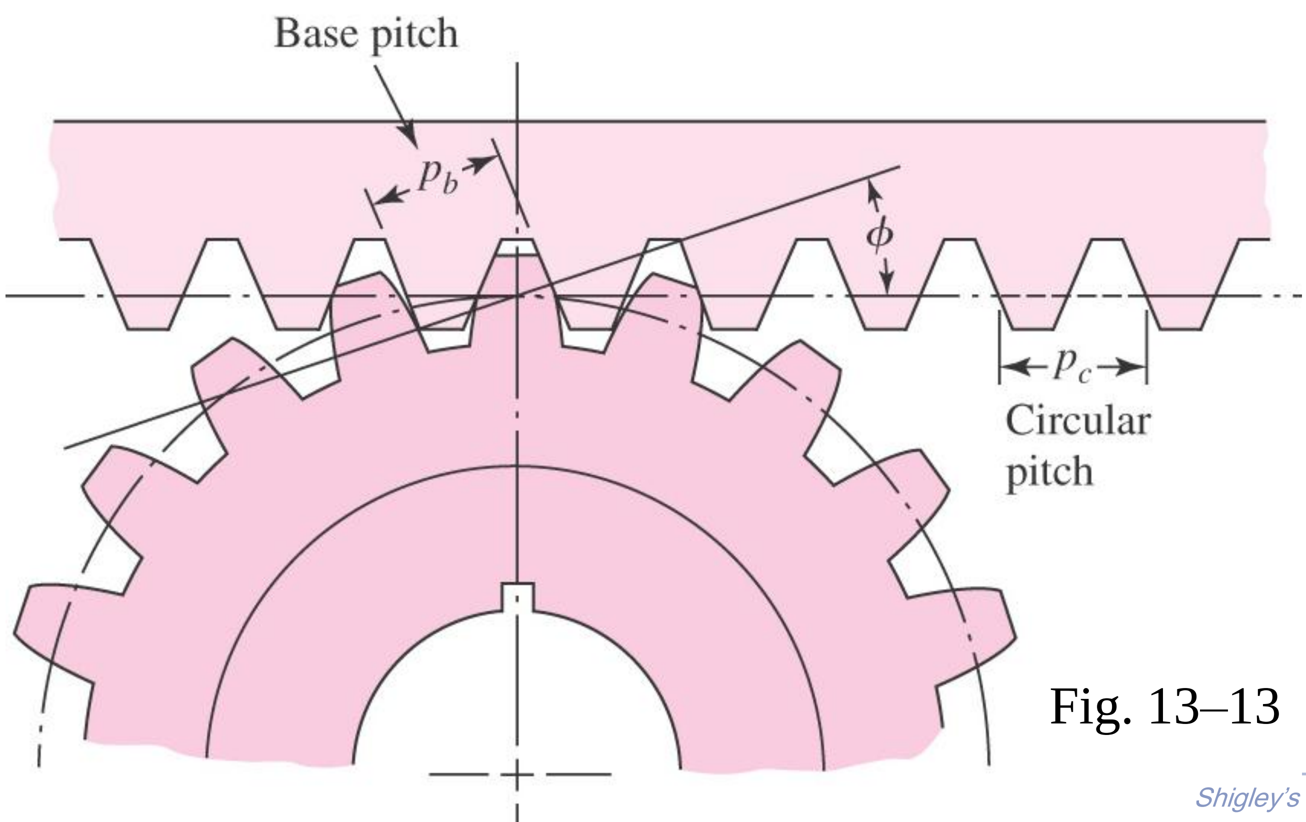


Fig. 13–13

Internal Gear

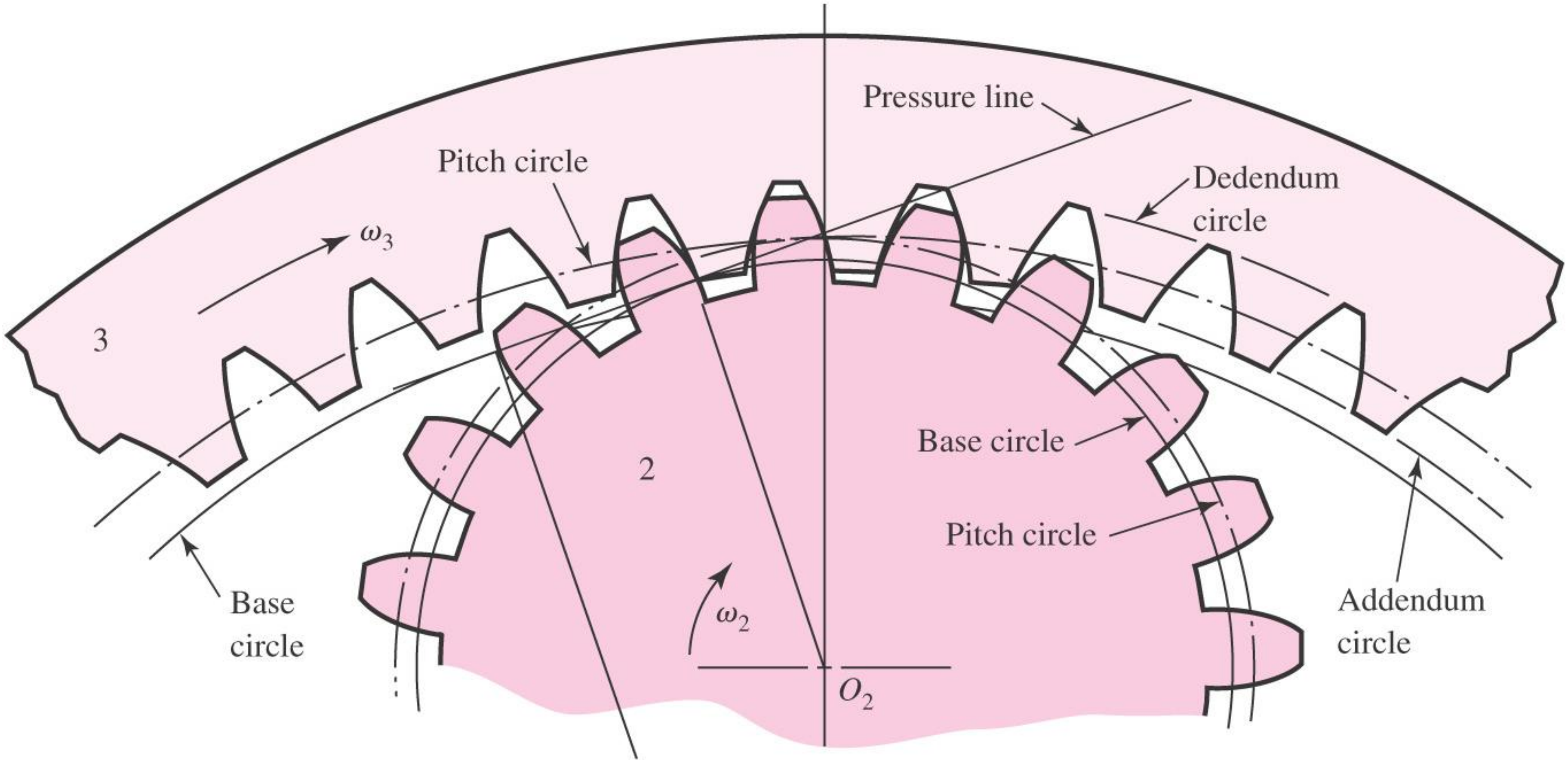


Fig. 13–14

Example 13–1

A gearset consists of a 16-tooth pinion driving a 40-tooth gear. The diametral pitch is 2, and the addendum and dedendum are $1/P$ and $1.25/P$, respectively. The gears are cut using a pressure angle of 20°.

(*a*) Compute the circular pitch, the center distance, and the radii of the base circles.

(*b*) In mounting these gears, the center distance was incorrectly made $\frac{1}{4}$ in larger. Compute the new values of the pressure angle and the pitch-circle diameters.

Solution

(*a*)

$$p = \frac{\pi}{P} = \frac{\pi}{2} = 1.57 \text{ in}$$

The pitch diameters of the pinion and gear are, respectively,

$$d_P = \frac{16}{2} = 8 \text{ in} \qquad d_G = \frac{40}{2} = 20 \text{ in}$$

Therefore the center distance is

$$\frac{d_P + d_G}{2} = \frac{8 + 20}{2} = 14 \text{ in}$$

Example 13–1

Since the teeth were cut on the 20° pressure angle, the base-circle radii are found to be, using $r_b = r \cos\phi$,

$$r_b \text{ (pinion)} = \frac{8}{2} \cos 20^\circ = 3.76 \text{ in}$$

$$r_b \text{ (gear)} = \frac{20}{2} \cos 20^\circ = 9.40 \text{ in}$$

Example 13–1

(b) Designating d'_P and d'_G as the new pitch-circle diameters, the $\frac{1}{4}$-in increase in the center distance requires that

$$\frac{d'_P + d'_G}{2} = 14.250 \tag{1}$$

Also, the velocity ratio does not change, and hence

$$\frac{d'_P}{d'_G} = \frac{16}{40} \tag{2}$$

Solving Eqs. (1) and (2) simultaneously yields

$$d'_P = 8.143 \text{ in} \qquad d'_G = 20.357 \text{ in}$$

Since $r_b = r\cos\phi$, the new pressure angle is

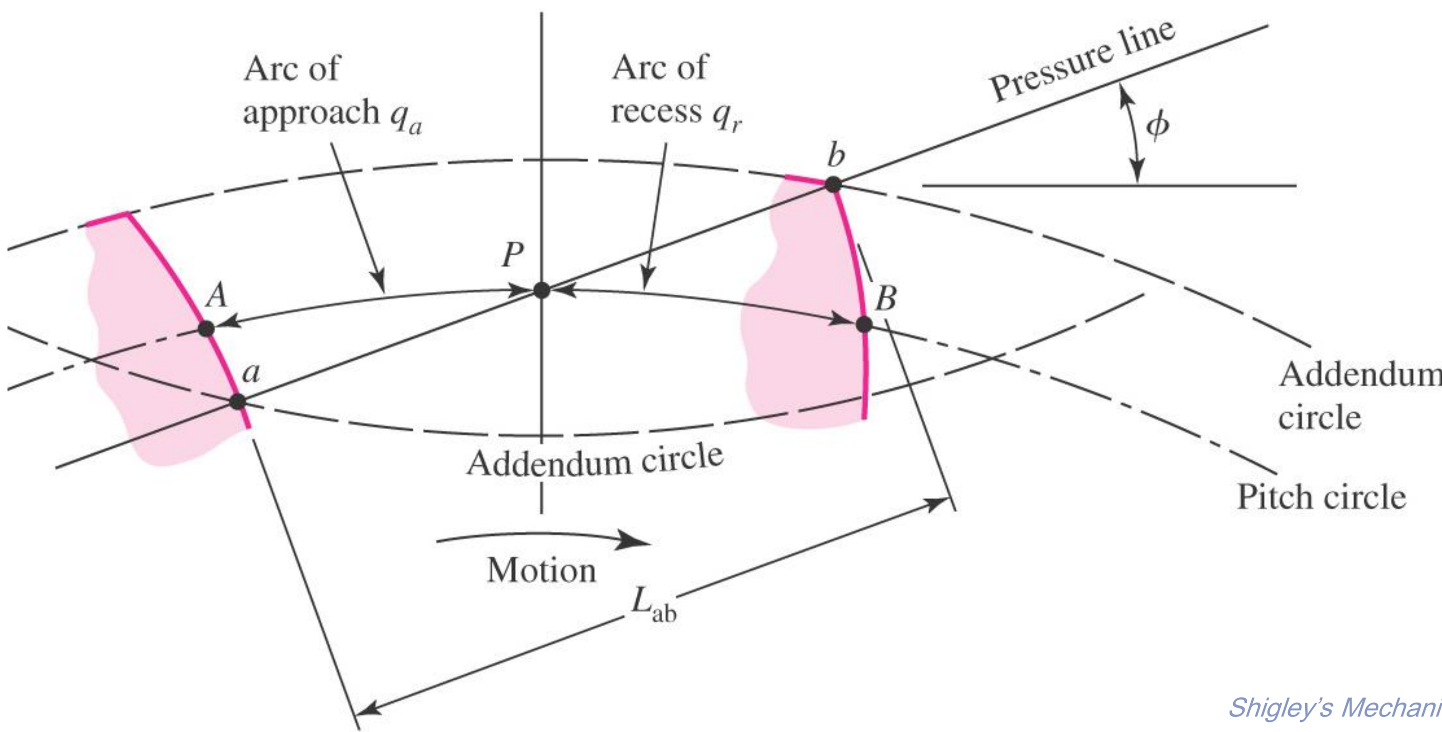
$$\phi' = \cos^{-1}\frac{r_b\,(\text{pinion})}{d'_P/2} = \cos^{-1}\frac{3.76}{8.143/2} = 22.56^\circ$$

Contact Ratio

- Arc of action q_t is the sum of the arc of approach q_a and the arc of recess q_r, that is $q_t = q_a + q_r$
- The *contact ratio* m_c is the ratio of the arc of action and the circular pitch.

$$m_c = \frac{q_t}{p} \qquad (13\text{–}8)$$

- The contact ratio is the average number of pairs of teeth in contact.



Contact Ratio

- Contact ratio can also be found from the length of the line of action

$$m_c = \frac{L_{ab}}{p \cos \phi} \qquad (13\text{–}9)$$

- The contact ratio should be at least 1.2

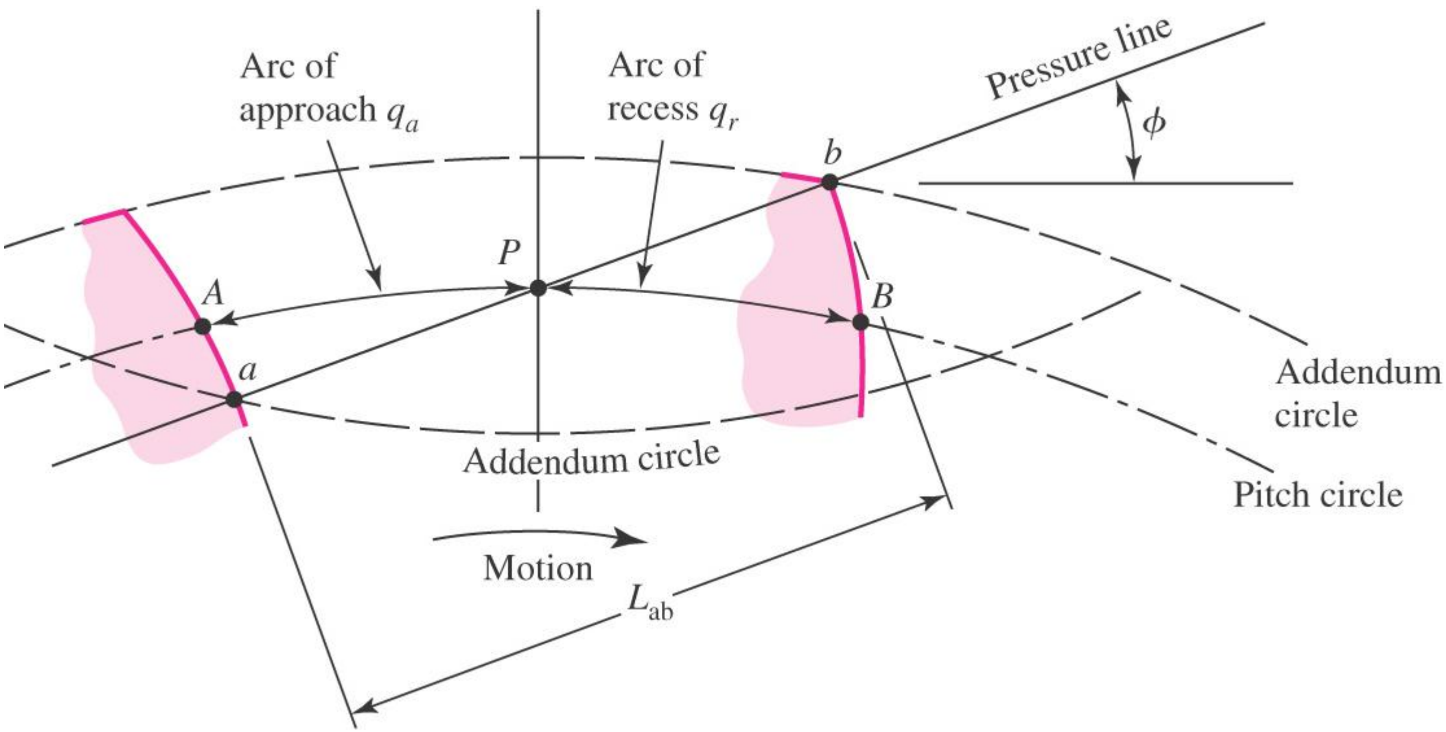


Fig. 13–15

Interference

- Contact of portions of tooth profiles that are not conjugate is called *interference*.
- Occurs when contact occurs below the base circle
- If teeth were produced by generating process (rather than stamping), then the generating process removes the interfering portion; known as *undercutting*.

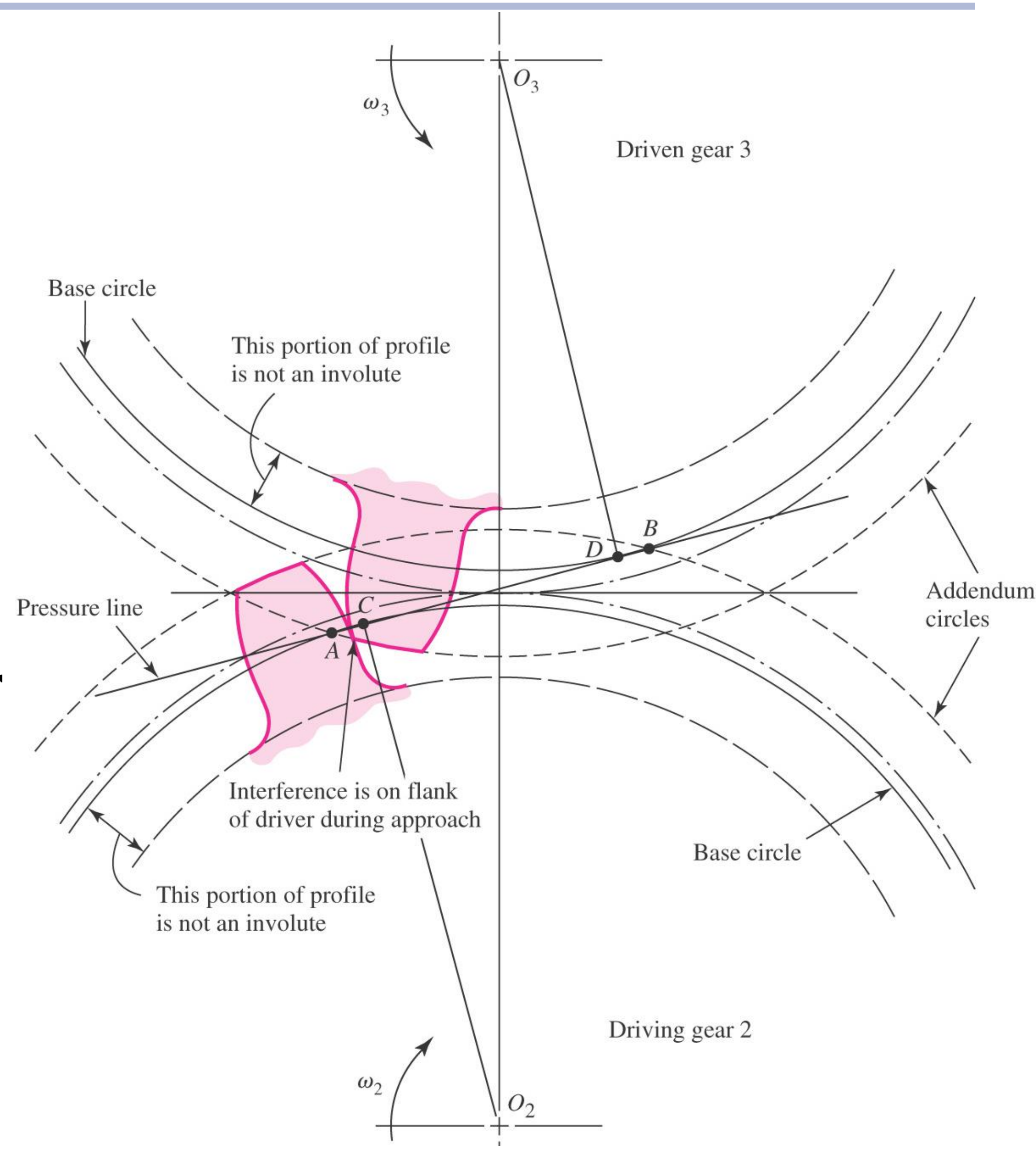


Fig. 13–16

Interference of Spur Gears

- On spur and gear with one-to-one gear ratio, smallest number of teeth which will not have interference is

$$N_P = \frac{2k}{3\sin^2\phi}\left(1 + \sqrt{1 + 3\sin^2\phi}\right) \qquad (13\text{–}10)$$

- $k = 1$ for full depth teeth. $k = 0.8$ for stub teeth
- On spur meshed with larger gear with gear ratio $m_G = N_G/N_P = m$, the smallest number of teeth which will not have interference is

$$N_P = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right) \qquad (13\text{–}11)$$

Interference of Spur Gears

- Largest gear with a specified pinion that is interference-free is

$$N_G = \frac{N_P^2 \sin^2 \phi - 4k^2}{4k - 2N_P \sin^2 \phi} \qquad (13\text{–}12)$$

- Smallest spur pinion that is interference-free with a rack is

$$N_P = \frac{2(k)}{\sin^2 \phi} \qquad (13\text{–}13)$$

Interference

- For 20º pressure angle, the most useful values from Eqs. (13–11) and (13–12) are calculated and shown in the table below.

Minimum N_P	**Max N_G**	**Integer Max N_G**	**Max Gear Ratio $m_G = N_G/N_P$**
13	16.45	16	1.23
14	26.12	26	1.86
15	45.49	45	3
16	101.07	101	6.31
17	1309.86	1309	77

Interference

- Increasing the pressure angle to 25º allows smaller numbers of teeth

Minimum N_P	**Max N_G**	**Integer Max N_G**	**Max Gear Ratio $m_G = N_G/N_P$**
9	13.33	13	1.44
10	32.39	32	3.2
11	249.23	249	22.64

Interference

- Interference can be eliminated by using more teeth on the pinion.
- However, if tooth size (that is diametral pitch P) is to be maintained, then an increase in teeth means an increase in diameter, since $P = N/d$.
- Interference can also be eliminated by using a larger pressure angle. This results in a smaller base circle, so more of the tooth profile is involute.
- This is the primary reason for larger pressure angle.
- Note that the disadvantage of a larger pressure angle is an increase in radial force for the same amount of transmitted force.

Forming of Gear Teeth

- Common ways of forming gear teeth
  - Sand casting
  - Shell molding
  - Investment casting
  - Permanent-mold casting
  - Die casting
  - Centrifugal casting
  - Powder-metallurgy
  - Extrusion
  - Injection molding (for thermoplastics)
  - Cold forming

Cutting of Gear Teeth

- Common ways of cutting gear teeth
  - Milling
  - Shaping
  - Hobbing

Shaping with Pinion Cutter

Fig. 13–17

Shaping with a Rack

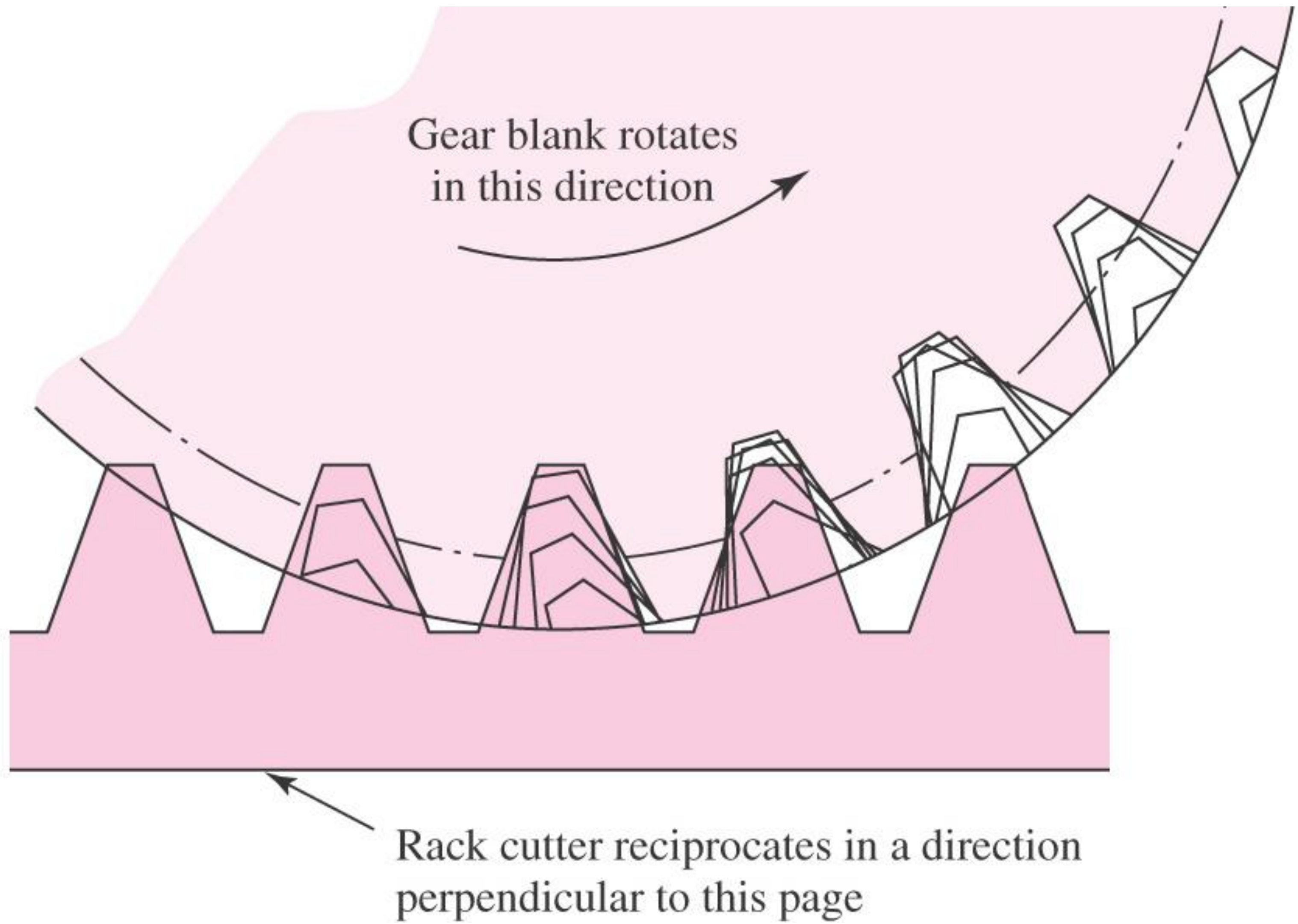


Fig. 13–18

Hobbing a Worm Gear

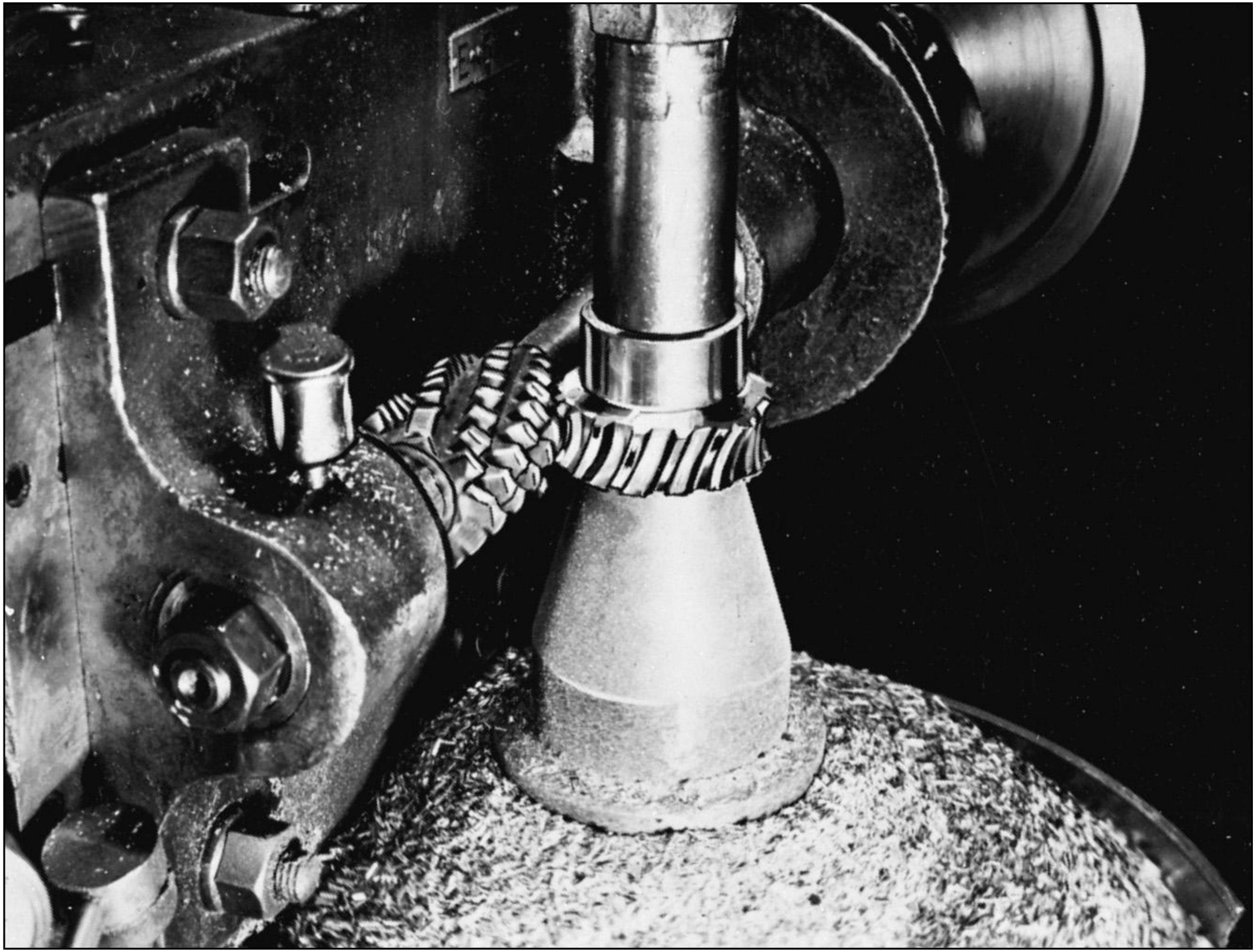


Fig. 13–19

Straight Bevel Gears

- To transmit motion between intersecting shafts

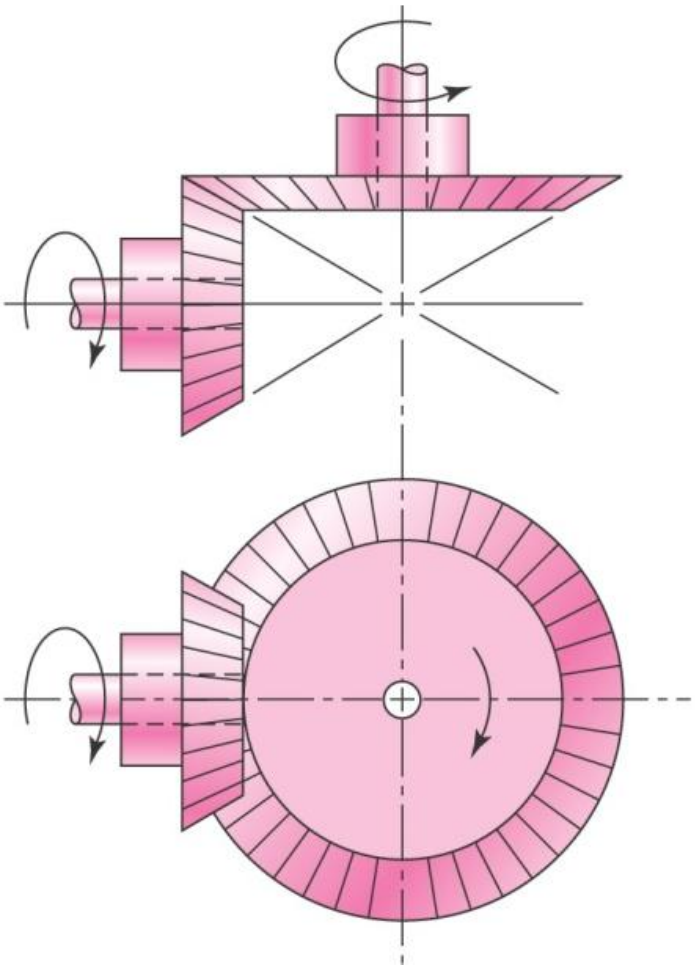


Fig. 13–3

Straight Bevel Gears

$$\tan\gamma = \frac{N_P}{N_G} \qquad \tan\Gamma = \frac{N_G}{N_P} \qquad (13\text{–}14)$$

- The shape of teeth, projected on back cone, is same as in a spur gear with radius r_b
- *Virtual number of teeth* in this virtual spur gear is

$$N' = \frac{2\pi r_b}{p} \qquad (13\text{–}15)$$

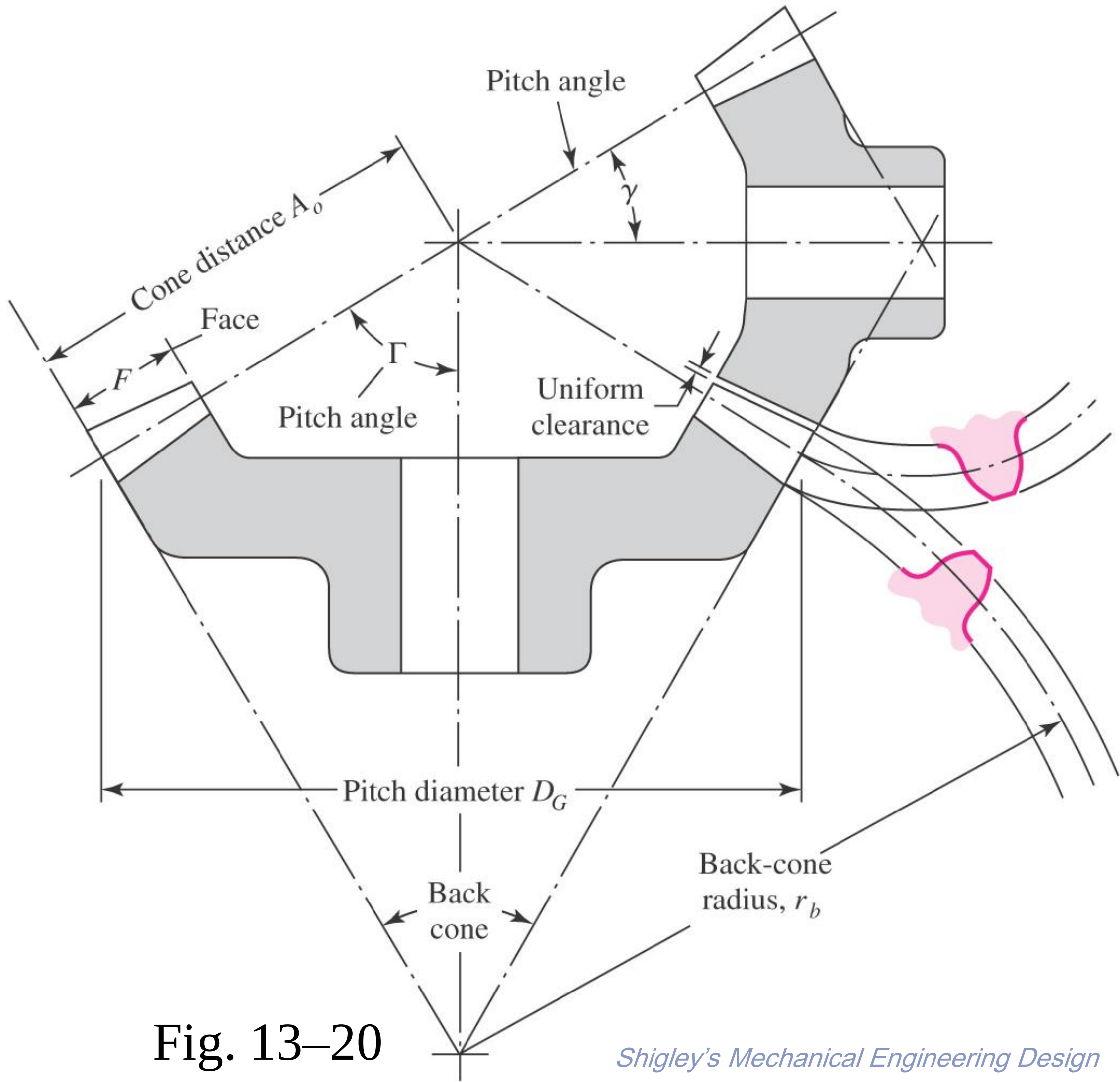


Fig. 13–20

Parallel Helical Gears

- Similar to spur gears, but with teeth making a *helix angle* with respect to the gear centerline
- Adds axial force component to shaft and bearings
- Smoother transition of force between mating teeth due to gradual engagement and disengagement

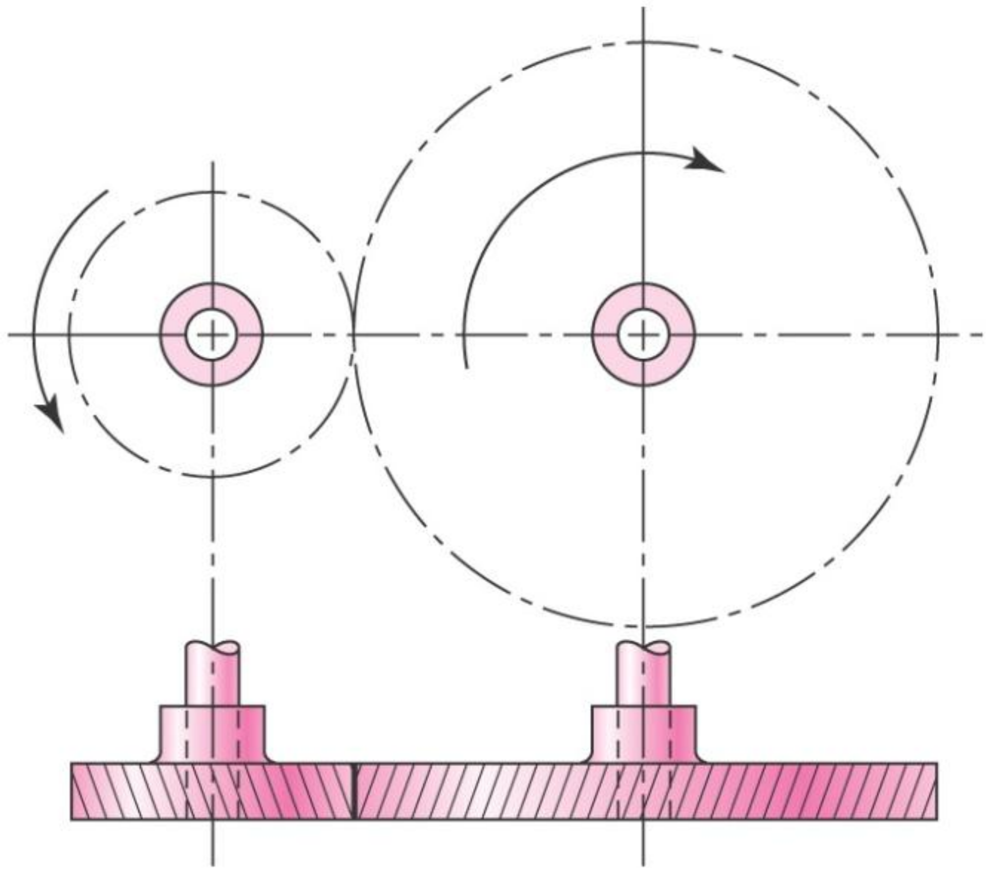


Fig. 13–2

Parallel Helical Gears

- Tooth shape is involute helicoid

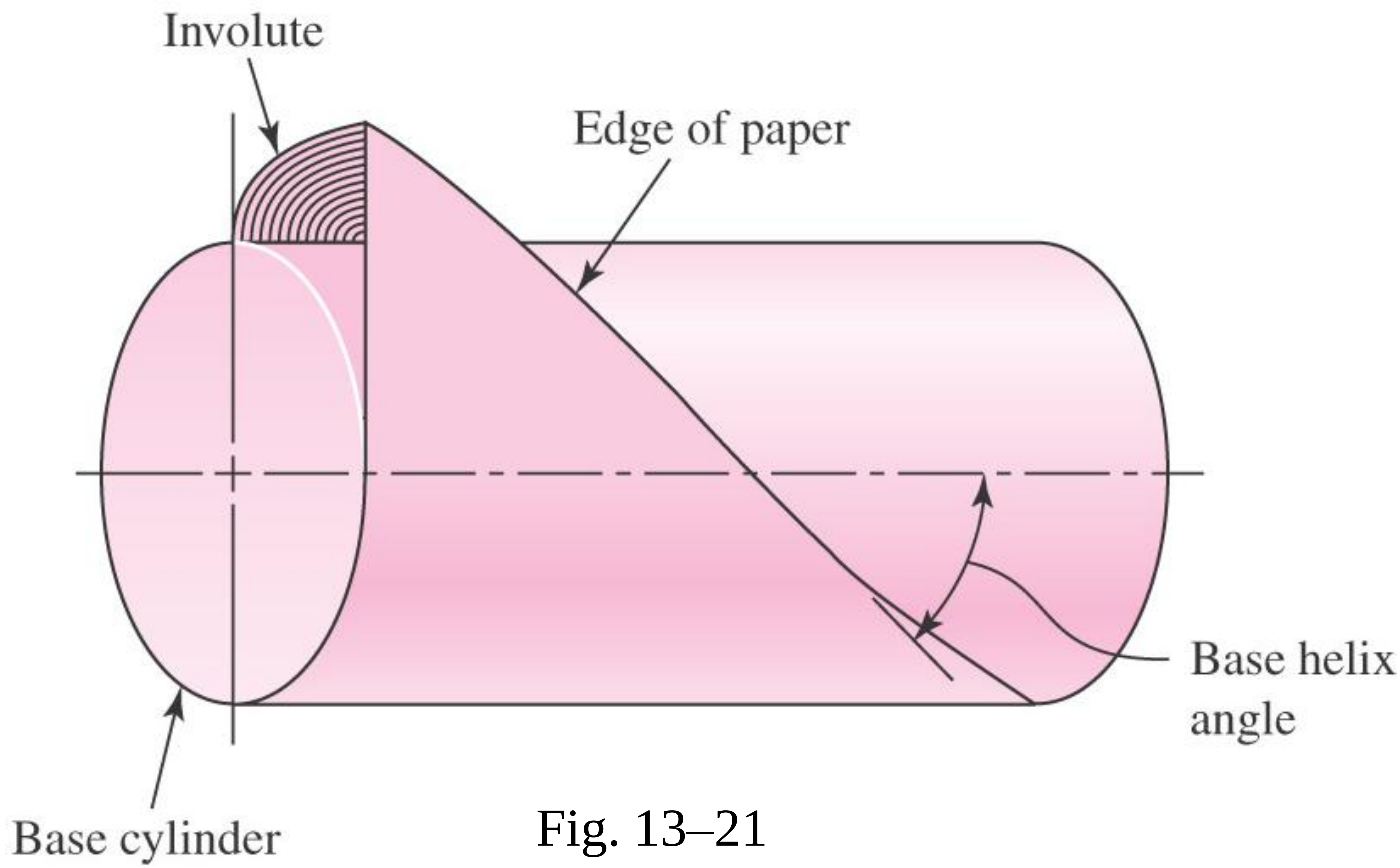


Fig. 13–21

Parallel Helical Gears

- *Transverse circular pitch* p_t is in the plane of rotation
- *Normal circular pitch* p_n is in the plane perpendicular to the teeth

$$p_n = p_t \cos\psi \qquad (13\text{–}16)$$

- *Axial pitch* p_x is along the direction of the shaft axis

$$p_x = \frac{p_t}{\tan\psi} \qquad (13\text{–}17)$$

- *Normal diametral pitch*

$$P_n = \frac{P_t}{\cos\psi} \qquad (13\text{–}18)$$

$$p_n P_n = \pi$$

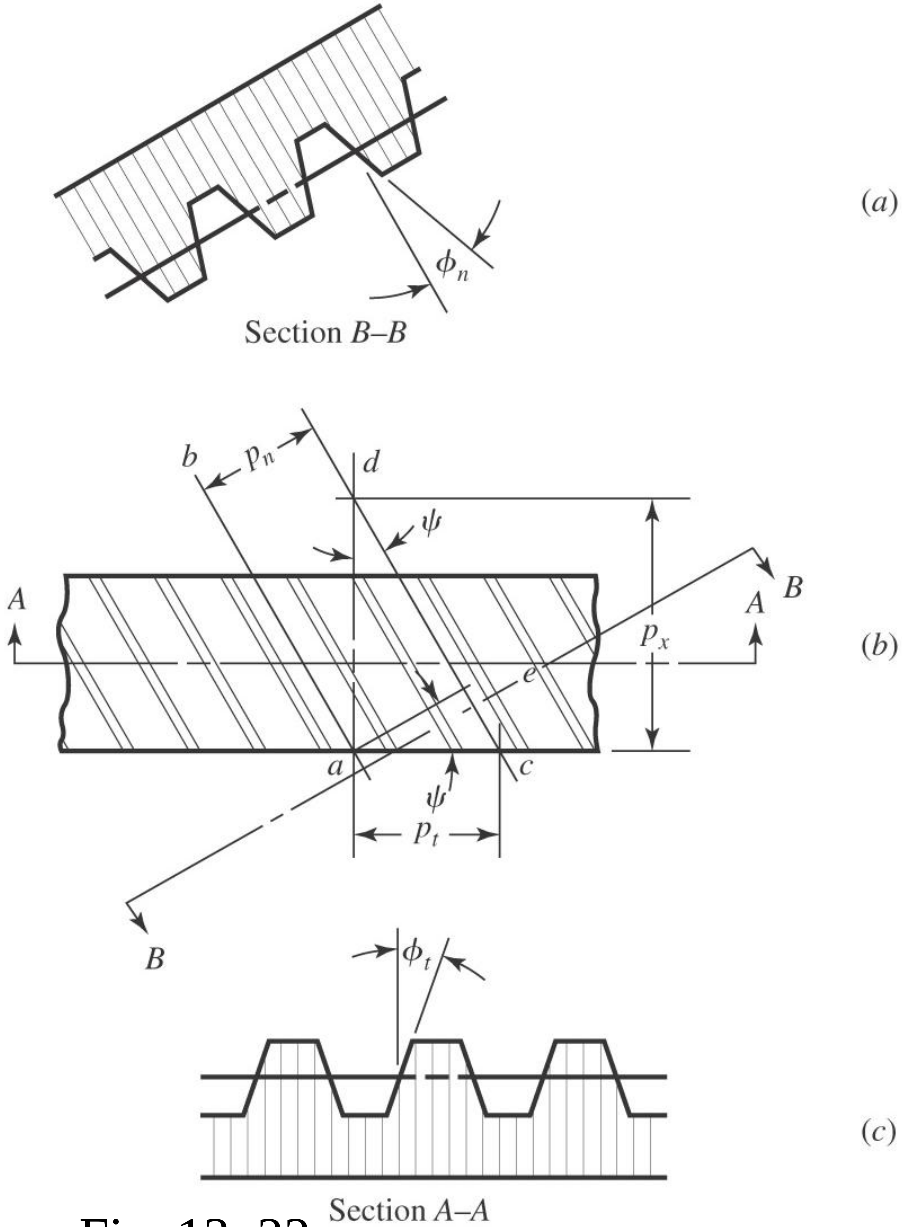


Fig. 13–22

Parallel Helical Gears

- Relationship between angles

$$\cos\psi = \frac{\tan\phi_n}{\tan\phi_t} \qquad (13\text{–}19)$$

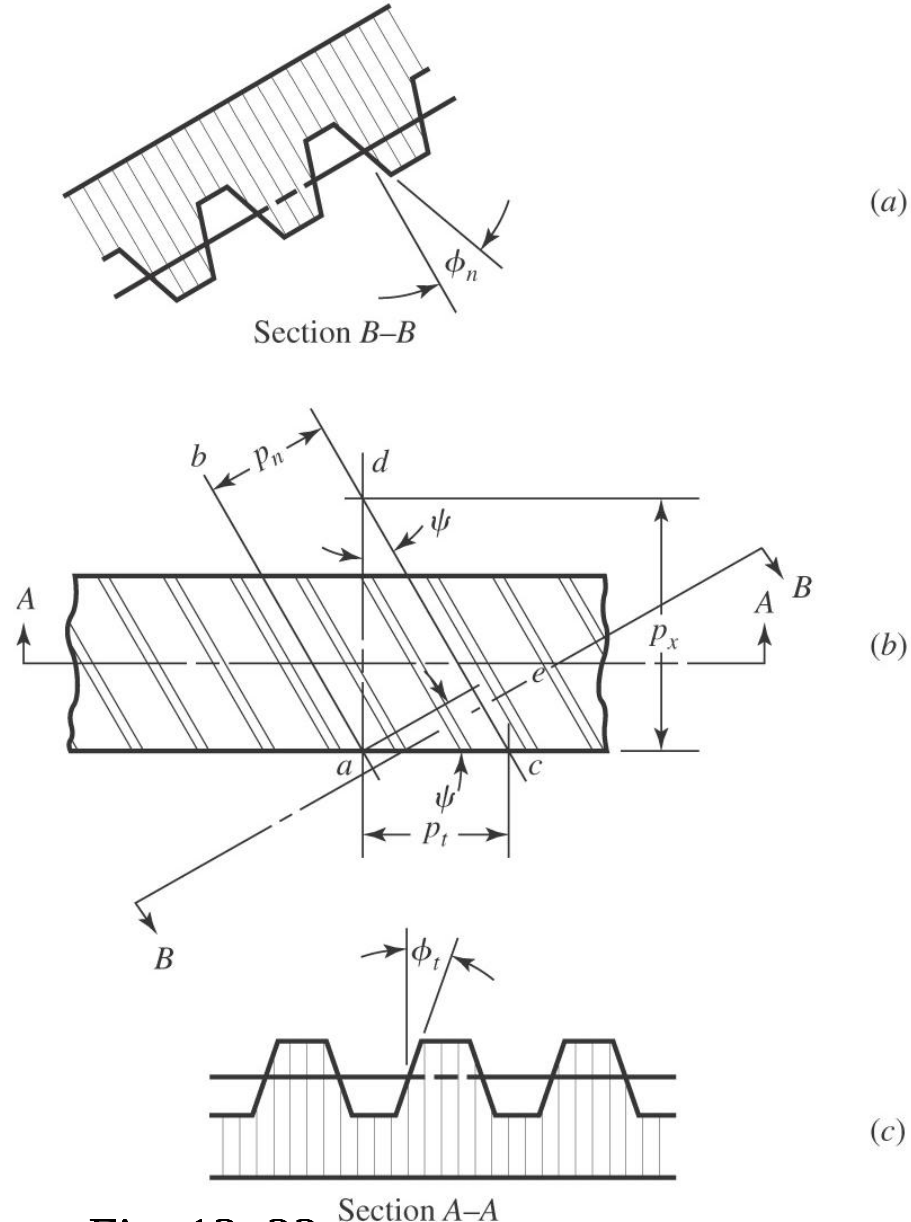


Fig. 13–22

Parallel Helical Gears

- Viewing along the teeth, the apparent pitch radius is greater than when viewed along the shaft.
- The greater virtual *R* has a greater *virtual number of teeth N′*

$$N' = \frac{N}{\cos^3 \psi} \qquad (13\text{–}20)$$

- Allows fewer teeth on helical gears without undercutting.

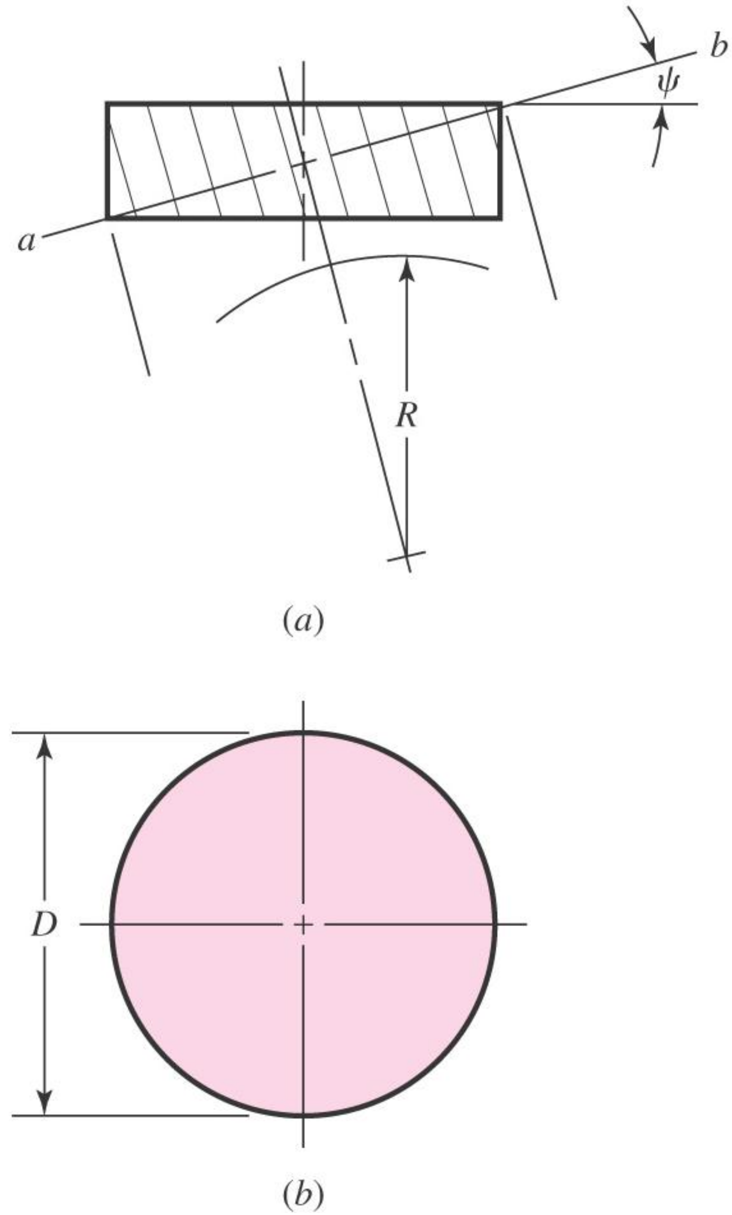


Fig. 13–23

Example 13–2

A stock helical gear has a normal pressure angle of 20°, a helix angle of 25°, and a transverse diametral pitch of 6 teeth/in, and has 18 teeth. Find:

(*a*) The pitch diameter

(*b*) The transverse, the normal, and the axial pitches

(*c*) The normal diametral pitch

(*d*) The transverse pressure angle

Example 13–2

(*a*)
$$d = \frac{N}{P_t} = \frac{18}{6} = 3 \text{ in}$$

(*b*)
$$p_t = \frac{\pi}{P_t} = \frac{\pi}{6} = 0.5236 \text{ in}$$

$$p_n = p_t \cos\psi = 0.5236 \cos 25^\circ = 0.4745 \text{ in}$$

$$p_x = \frac{p_t}{\tan\psi} = \frac{0.5236}{\tan 45^\circ} = 1.123 \text{ in}$$

(*c*)
$$P_n = \frac{P_t}{\cos\psi} = \frac{6}{\cos 25^\circ} = 6.620 \text{ teeth/in}$$

(*d*)
$$\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right) = \tan^{-1}\left(\frac{\tan 20^\circ}{\cos 25^\circ}\right) = 21.88^\circ$$

Interference with Helical Gears

- On spur and gear with one-to-one gear ratio, smallest number of teeth which will not have interference is

$$N_P = \frac{2k\cos\psi}{3\sin^2\phi_t}\left(1 + \sqrt{1 + 3\sin^2\phi_t}\right) \qquad (13\text{–}21)$$

- $k = 1$ for full depth teeth. $k = 0.8$ for stub teeth
- On spur meshed with larger gear with gear ratio $m_G = N_G/N_P = m$, the smallest number of teeth which will not have interference is

$$N_P = \frac{2k\cos\psi}{(1+2m)\sin^2\phi_t}\left[m + \sqrt{m^2 + (1+2m)\sin^2\phi_t}\right] \qquad (13\text{–}22)$$

Interference with Helical Gears

- Largest gear with a specified pinion that is interference-free is

$$N_G = \frac{N_P^2 \sin^2 \phi_t - 4k^2 \cos^2 \psi}{4k \cos \psi - 2N_P \sin^2 \phi_t} \qquad (13\text{–}23)$$

- Smallest spur pinion that is interference-free with a rack is

$$N_P = \frac{2k \cos \psi}{\sin^2 \phi_t} \qquad (13\text{–}24)$$

Worm Gears

- Common to specify lead angle λ for worm and helix angle ψ_G for gear.
- Common to specify *axial pitch* p_x for worm and *transverse circular pitch* p_t for gear.
- Pitch diameter of gear is measured on plane containing worm axis

$$d_G = \frac{N_G p_t}{\pi} \qquad (13\text{–}25)$$

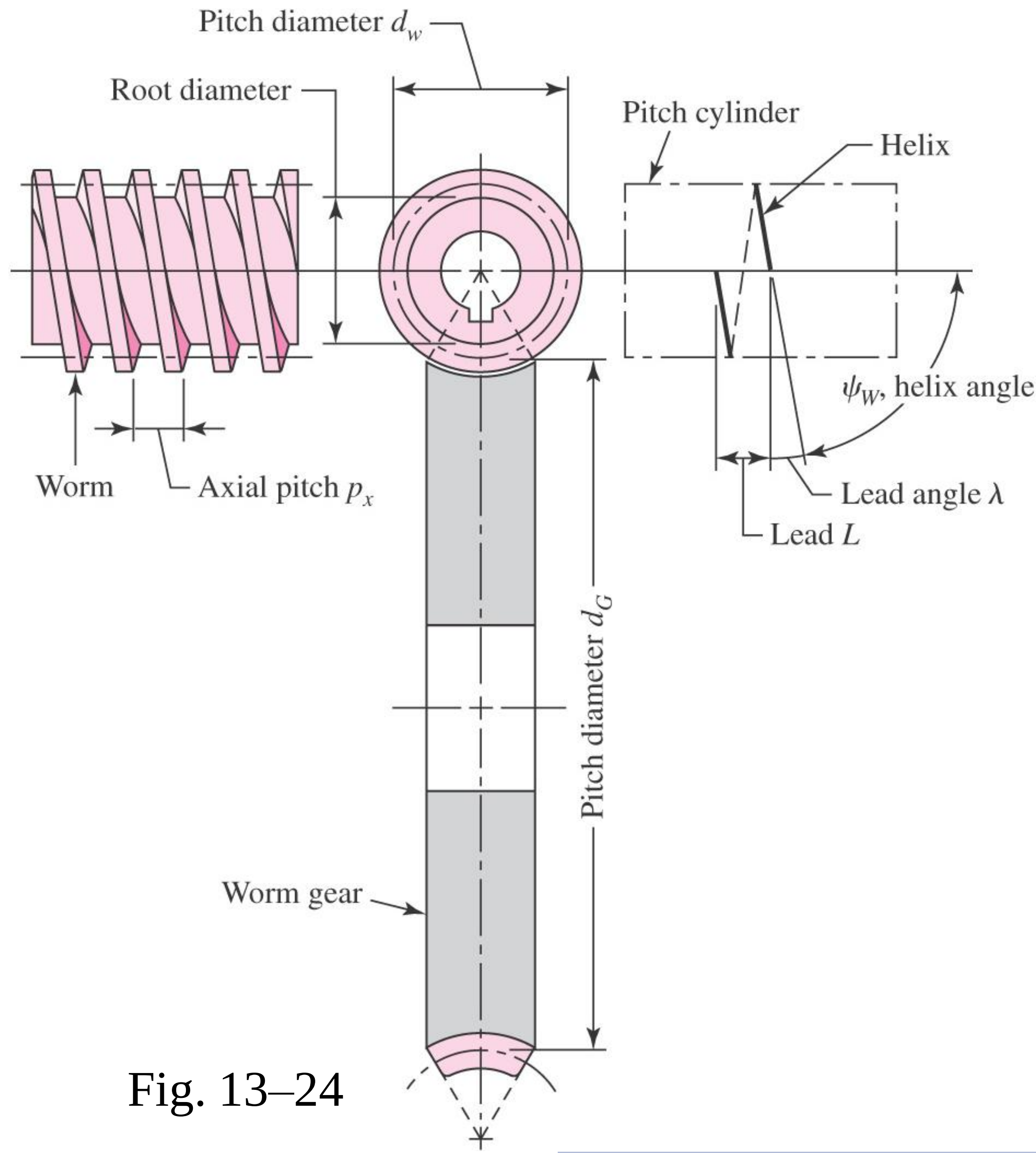


Fig. 13–24

Worm Gears

- Worm may have any pitch diameter.
- Should be same as hob used to cut the gear teeth
- Recommended range for worm pitch diameter as a function of center distance *C*,

$$\frac{C^{0.875}}{3.0} \le d_W \le \frac{C^{0.875}}{1.7} \qquad (13\text{–}26)$$

- Relation between *lead L* and *lead angle λ*,

$$L = p_x N_W \qquad (13\text{–}27)$$

$$\tan\lambda = \frac{L}{\pi d_W} \qquad (13\text{–}28)$$

Standard and Commonly Used Tooth Systems for Spur Gears

Tooth System	Pressure Angle ϕ, deg	Addendum a	Dedendum b
Full depth	20	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
	$22\frac{1}{2}$	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
	25	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
Stub	20	$0.8/P_d$ or $0.8m$	$1/P_d$ or $1m$

Table 13–1

Tooth Sizes in General Use

Diametral Pitch	
Coarse	2, $2\frac{1}{4}$, $2\frac{1}{2}$, 3, 4, 6, 8, 10, 12, 16
Fine	20, 24, 32, 40, 48, 64, 80, 96, 120, 150, 200

Modules	
Preferred	1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
Next Choice	1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45

Table 13–2

Tooth Proportions for 20º Straight Bevel-Gear Teeth

Item	Formula
Working depth	$h_k = 2.0/P$
Clearance	$c = (0.188/P) + 0.002$ in
Addendum of gear	$a_G = \dfrac{0.54}{P} + \dfrac{0.460}{P(m_{90})^2}$
Gear ratio	$m_G = N_G/N_P$
Equivalent 90° ratio	$m_{90} = m_G$ when $\Gamma = 90^\circ$
	$m_{90} = \sqrt{m_G \dfrac{\cos\gamma}{\cos\Gamma}}$ when $\Gamma \neq 90^\circ$
Face width	$F = 0.3A_0$ or $F = \dfrac{10}{P}$, whichever is smaller
Minimum number of teeth	Pinion: 16, 15, 14, 13; Gear: 16, 17, 20, 30

Pinion	16	15	14	13
Gear	16	17	20	30

Table 13–3

Standard Tooth Proportions for Helical Gears

Quantity*	Formula	Quantity*	Formula
Addendum	$\frac{1.00}{P_n}$	External gears:	
Dedendum	$\frac{1.25}{P_n}$	Standard center distance	$\frac{D+d}{2}$
Pinion pitch diameter	$\frac{N_P}{P_n \cos \psi}$	Gear outside diameter	$D+2a$
Gear pitch diameter	$\frac{N_G}{P_n \cos \psi}$	Pinion outside diameter	$d+2a$
Normal arc tooth thickness†	$\frac{\pi}{P_n} - \frac{B_n}{2}$	Gear root diameter	$D-2b$
Pinion base diameter	$d \cos \phi_t$	Pinion root diameter	$d-2b$
		Internal gears:	
Gear base diameter	$D \cos \phi_t$	Center distance	$\frac{D-d}{2}$
Base helix angle	$\tan^{-1}(\tan \psi \cos \phi_t)$	Inside diameter	$D-2a$
		Root diameter	$D+2b$

Table 13–4

Recommended Pressure Angles and Tooth Depths for Worm Gearing

Lead Angle λ, deg	Pressure Angle ϕ_n, deg	Addendum a	Dedendum b_G
0–15	$14\frac{1}{2}$	$0.3683p_x$	$0.3683p_x$
15–30	20	$0.3683p_x$	$0.3683p_x$
30–35	25	$0.2865p_x$	$0.3314p_x$
35–40	25	$0.2546p_x$	$0.2947p_x$
40–45	30	$0.2228p_x$	$0.2578p_x$

Table 13–5

Face Width of Worm Gear

- *Face width* F_G of a worm gear should be equal to the length of a tangent to the worm pitch circle between its points of intersection with the addendum circle

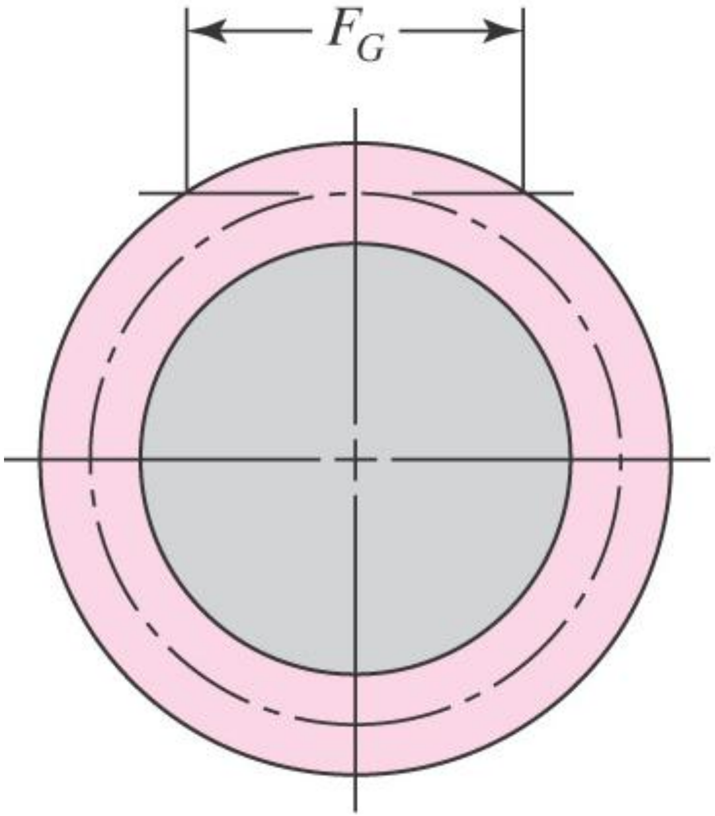


Fig. 13–25

Gear Trains

- For a pinion 2 driving a gear 3, the speed of the driven gear is

$$n_3 = \left|\frac{N_2}{N_3} n_2\right| = \left|\frac{d_2}{d_3} n_2\right| \qquad (13\text{–}29)$$

where n = revolutions or rev/min

N = number of teeth

d = pitch diameter

Relations for Crossed Helical Gears

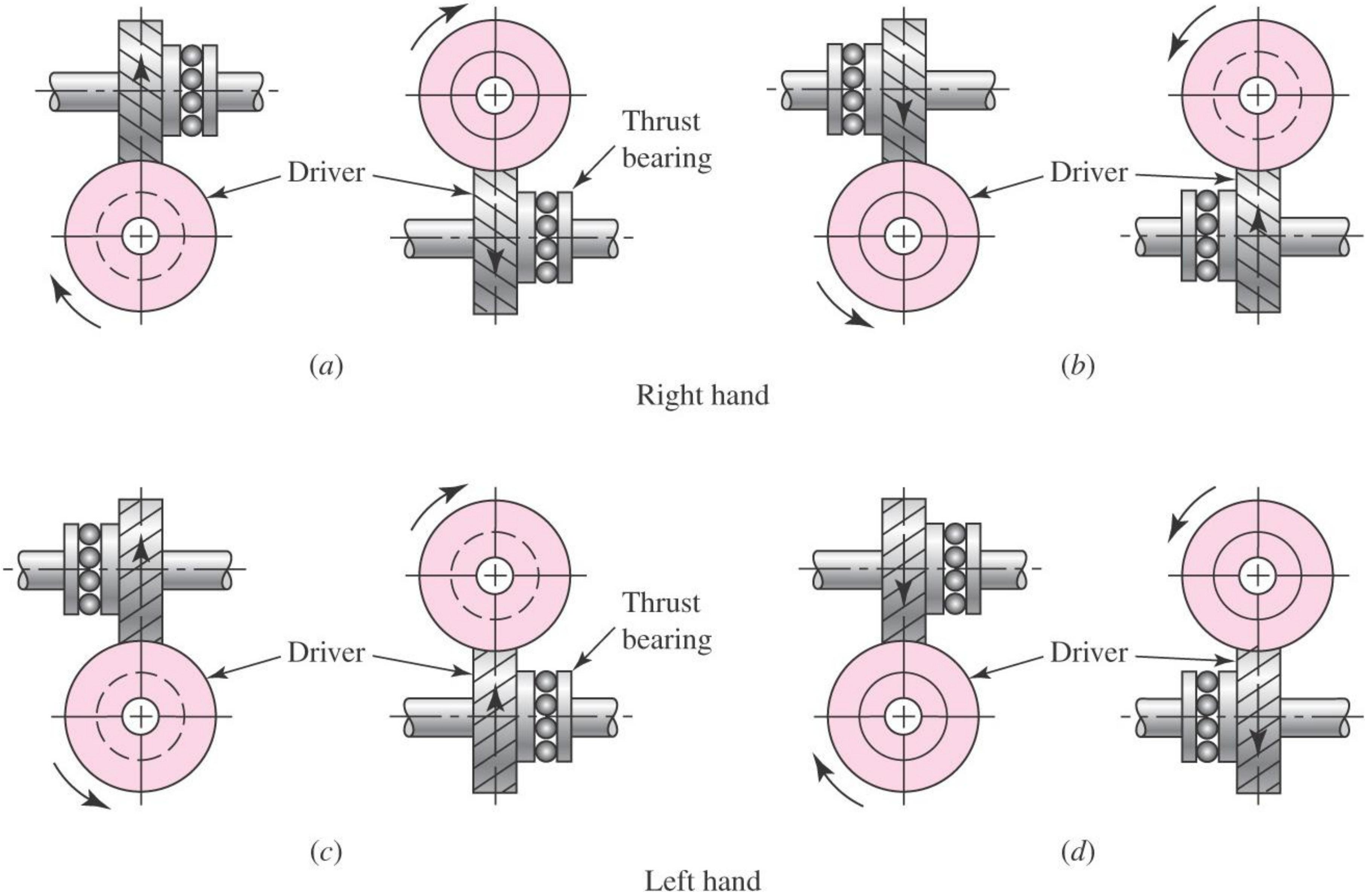


Fig. 13–26

Train Value

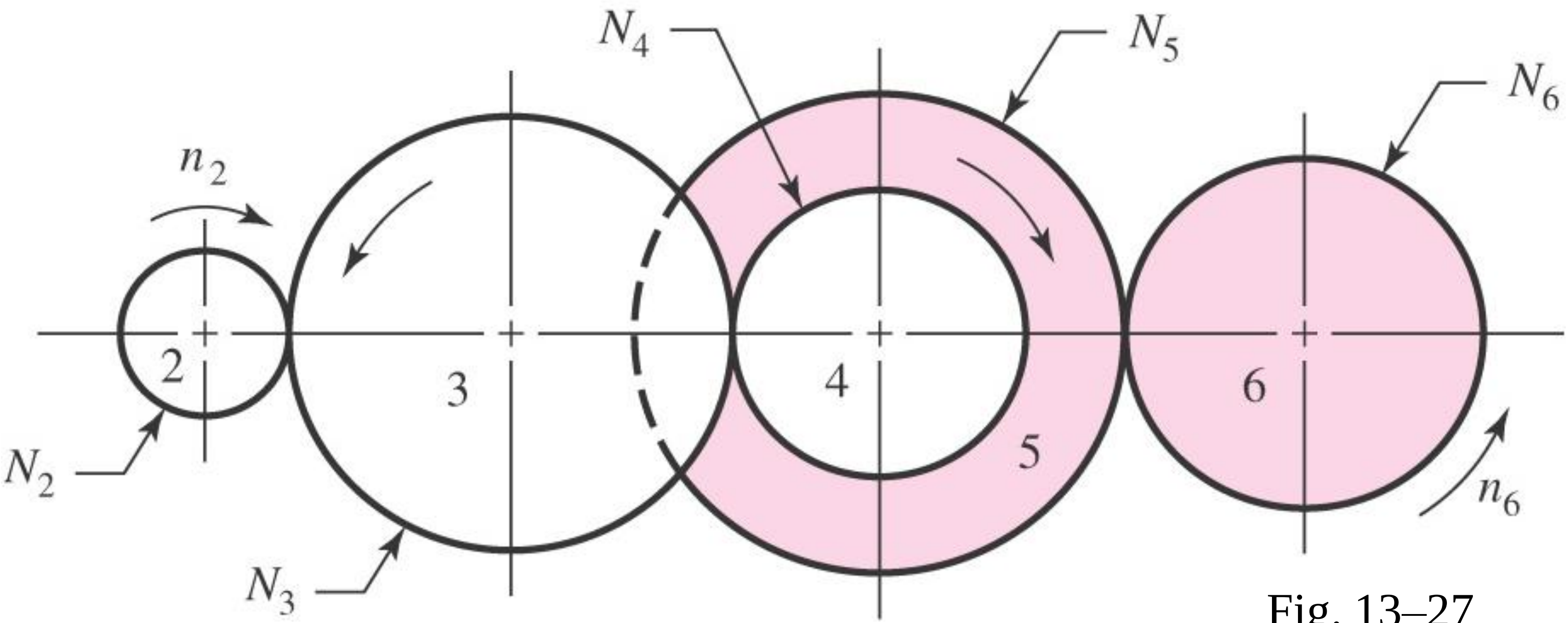


Fig. 13–27

$$n_6 = -\frac{N_2}{N_3}\frac{N_3}{N_4}\frac{N_5}{N_6}n_2$$

$$e = \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}} \qquad (13\text{–}30)$$

$$n_L = e n_F \qquad (13\text{–}31)$$

Compound Gear Train

- A practical limit on train value for one pair of gears is 10 to 1
- To obtain more, compound two gears onto the same shaft

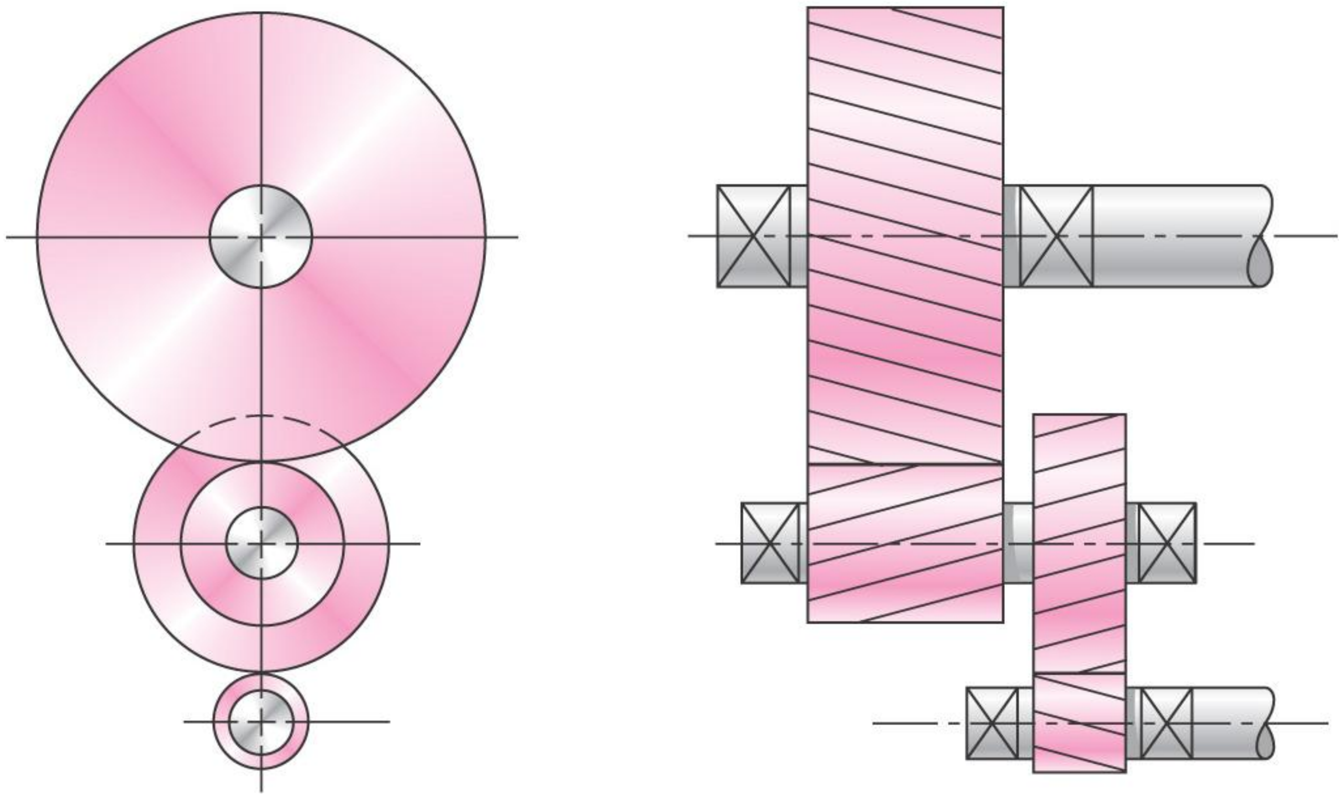


Fig. 13–28

Example 13–3

A gearbox is needed to provide a 30:1 ($\pm$ 1 percent) increase in speed, while minimizing the overall gearbox size. Specify appropriate teeth numbers.

Solution

Since the ratio is greater than 10:1, but less than 100:1, a two-stage compound gear train, such as in Figure 13–28, is needed. The portion to be accomplished in each stage is $\sqrt{30} = 5.4772$. For this ratio, assuming a typical 20° pressure angle, the minimum number of teeth to avoid interference is 16, according to Eq. (13–11). The number of teeth necessary for the mating gears is

$$16\sqrt{30} = 87.64 \doteq 88$$

From Eq. (13–30), the overall train value is

$$e = (88/16)(88/16) = 30.25$$

This is within the 1 percent tolerance. If a closer tolerance is desired, then increase the pinion size to the next integer and try again.

Example 13–4

A gearbox is needed to provide an *exact* 30:1 increase in speed, while minimizing the overall gearbox size. Specify appropriate teeth numbers.

Solution

The previous example demonstrated the difficulty with finding integer numbers of teeth to provide an exact ratio. In order to obtain integers, factor the overall ratio into two integer stages.

$$e = 30 = (6)(5)$$

$$N_2/N_3 = 6 \quad \text{and} \quad N_4/N_5 = 5$$

With two equations and four unknown numbers of teeth, two free choices are available. Choose N_3 and N_5 to be as small as possible without interference. Assuming a 20° pressure angle, Eq. (13–11) gives the minimum as 16.

Example 13–4

Then

$$N_2 = 6\,N_3 = 6\,(16) = 96$$

$$N_4 = 5\,N_5 = 5\,(16) = 80$$

The overall train value is then exact.

$$e = (96/16)(80/16) = (6)(5) = 30$$

Compound Reverted Gear Train

- A compound gear train with input and output shafts in-line
- Geometry condition must be satisfied

$$d_2/2 + d_3/2 = d_4/2 + d_5/2$$

$$P = N/d$$

$$N_2/(2P) + N_3/(2P) = N_4/(2P) + N_5/(2P)$$

$$N_2 + N_3 = N_4 + N_5$$

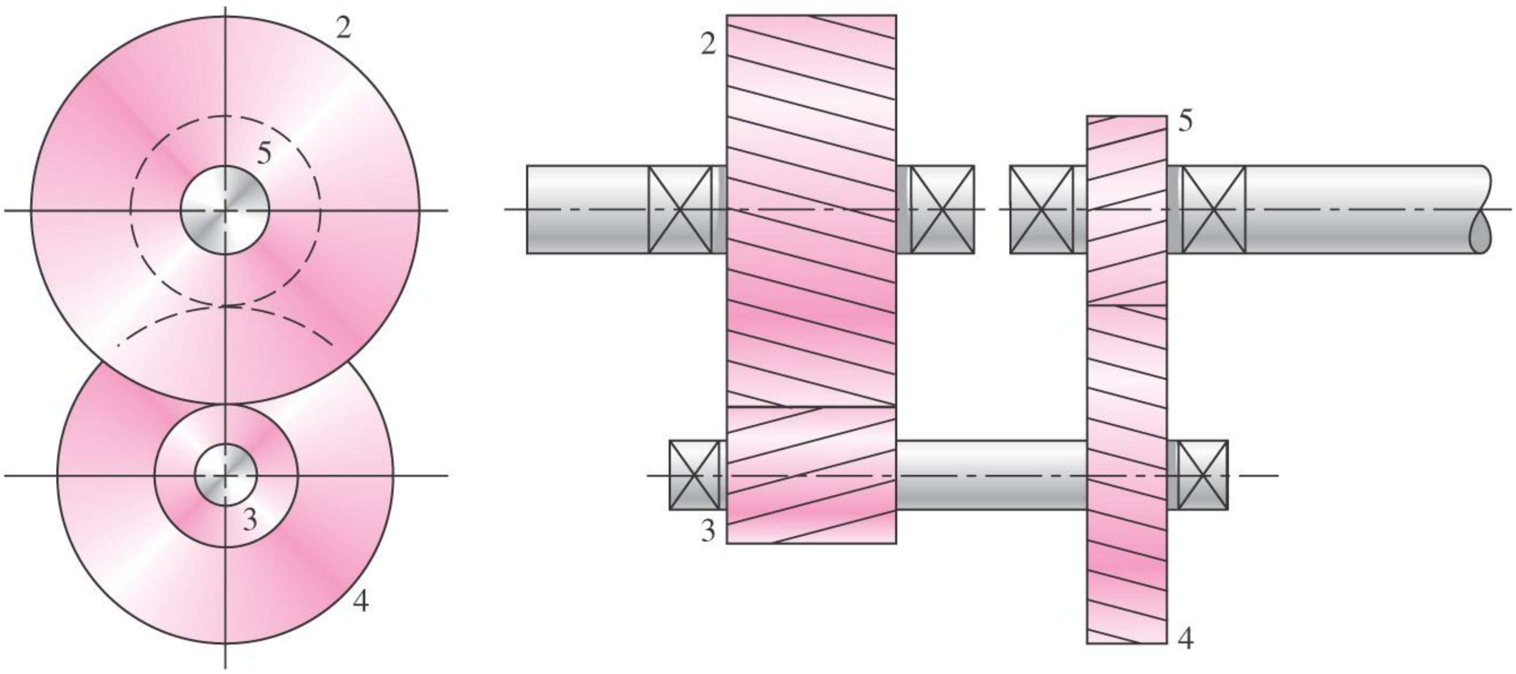


Fig. 13–29

Example 13–5

A gearbox is needed to provide an exact 30:1 increase in speed, while minimizing the overall gearbox size. The input and output shafts should be in-line. Specify appropriate teeth numbers.

Solution

The governing equations are

$$N_2/N_3 = 6$$

$$N_4/N_5 = 5$$

$$N_2 + N_3 = N_4 + N_5$$

With three equations and four unknown numbers of teeth, only one free choice is available. Of the two smaller gears, N_3 and N_5, the free choice should be used to minimize N_3 since a greater gear ratio is to be achieved in this stage. To avoid interference, the minimum for N_3 is 16.

Example 13–5

Applying the governing equations yields

$$N_2 = 6N_3 = 6(16) = 96$$

$$N_2 + N_3 = 96 + 16 = 112 = N_4 + N_5$$

Substituting $N_4 = 5N_5$ gives

$$112 = 5N_5 + N_5 = 6N_5$$

$$N_5 = 112/6 = 18.67$$

If the train value need only be approximated, then this can be rounded to the nearest integer. But for an exact solution, it is necessary to choose the initial free choice for N_3 such that solution of the rest of the teeth numbers results exactly in integers. This can be done by trial and error, letting $N_3 = 17$, then 18, etc., until it works. Or, the problem can be normalized to quickly determine the minimum free choice. Beginning again, let the free choice be $N_3 = 1$. Applying the governing equations gives

Example 13–5

$$N_2 = 6N_3 = 6(1) = 6$$

$$N_2 + N_3 = 6 + 1 = 7 = N_4 + N_5$$

Substituting $N_4 = 5N_5$, we find

$$7 = 5N_5 + N_5 = 6N_5$$

$$N_5 = 7/6$$

This fraction could be eliminated if it were multiplied by a multiple of 6. The free choice for the smallest gear N_3 should be selected as a multiple of 6 that is greater than the minimum allowed to avoid interference. This would indicate that $N_3 = 18$.

Example 13–5

Repeating the application of the governing equations for the final time yields

$$N_2 = 6N_3 = 6(18) = 108$$

$$N_2 + N_3 = 108 + 18 = 126 = N_4 + N_5$$

$$126 = 5N_5 + N_5 = 6N_5$$

$$N_5 = 126/6 = 21$$

$$N_4 = 5N_5 = 5(21) = 105$$

Thus,

$$N_2 = 108$$

$$N_3 = 18$$

$$N_4 = 105$$

$$N_5 = 21$$

Example 13–5

Checking, we calculate $e = (108/18)(105/21) = (6)(5) = 30$.

And checking the geometry constraint for the in-line requirement, we calculate

$$N_2 + N_3 = N_4 + N_5$$

$$108 + 18 = 105 + 21$$

$$126 = 126$$

Planetary Gear Train

- *Planetary,* or *epicyclic* gear trains allow the axis of some of the gears to move relative to the other axes
- *Sun gear* has fixed center axis
- *Planet gear* has moving center axis
- *Planet carrier* or *arm* carries planet axis relative to sun axis
- Allow for two degrees of freedom (i.e. two inputs)

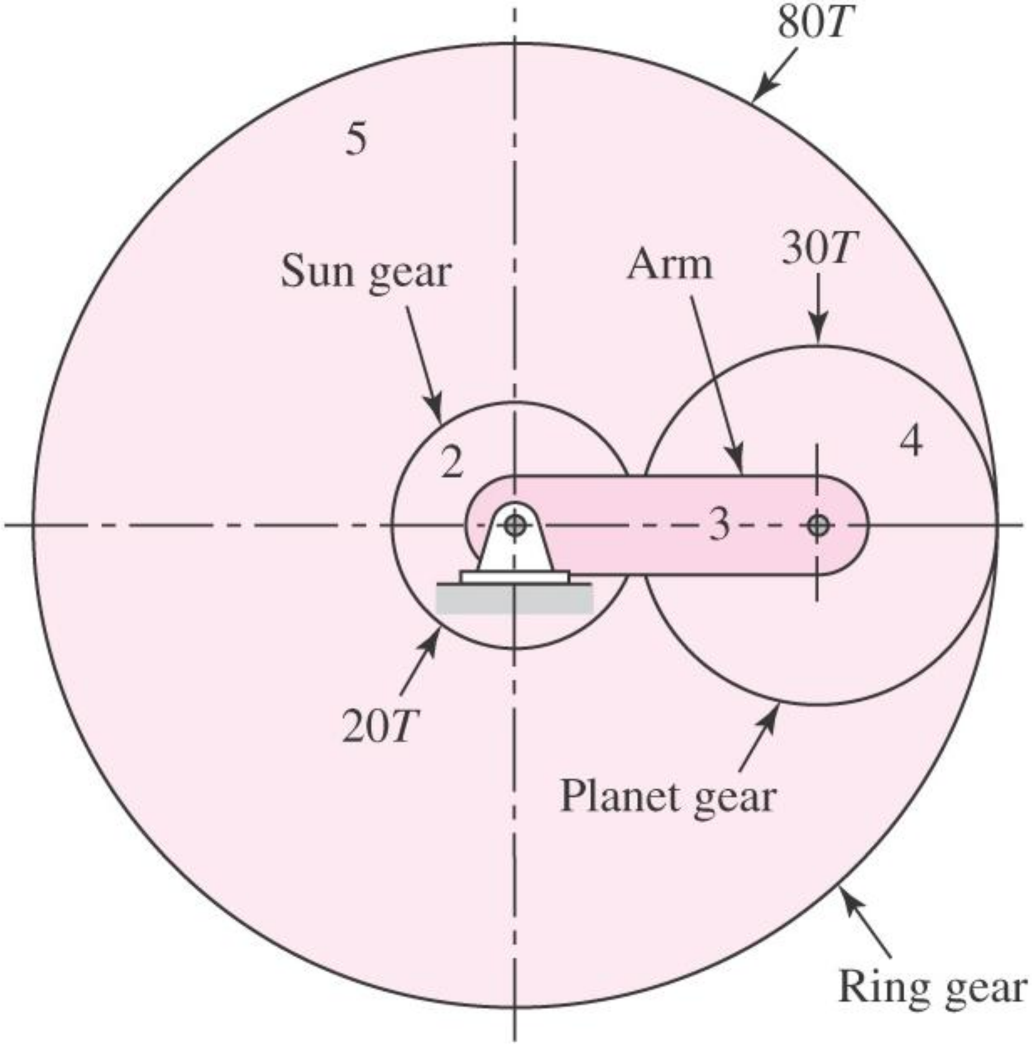


Fig. 13–30

Planetary Gear Trains

- Train value is relative to arm

$$e = \frac{n_L - n_A}{n_F - n_A} \qquad (13\text{–}32)$$

where n_F = rev/min of first gear in planetary train

n_L = rev/min of last gear in planetary train

n_A = rev/min of arm

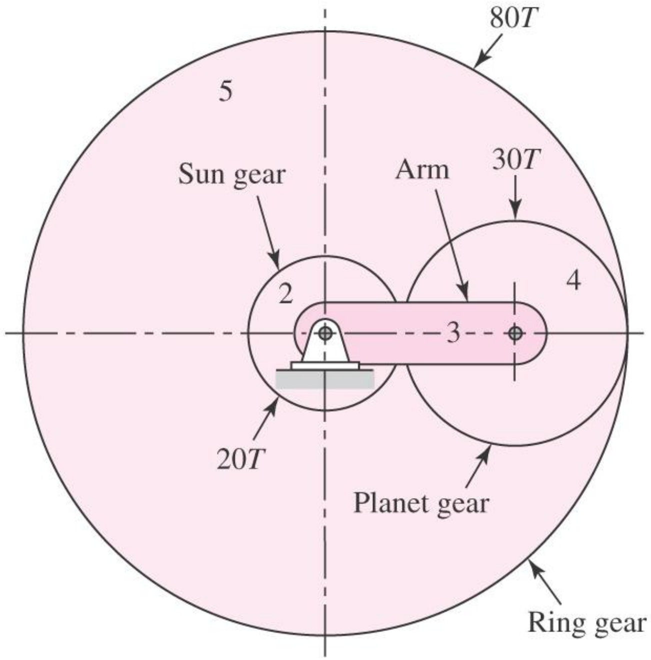


Fig. 13–30

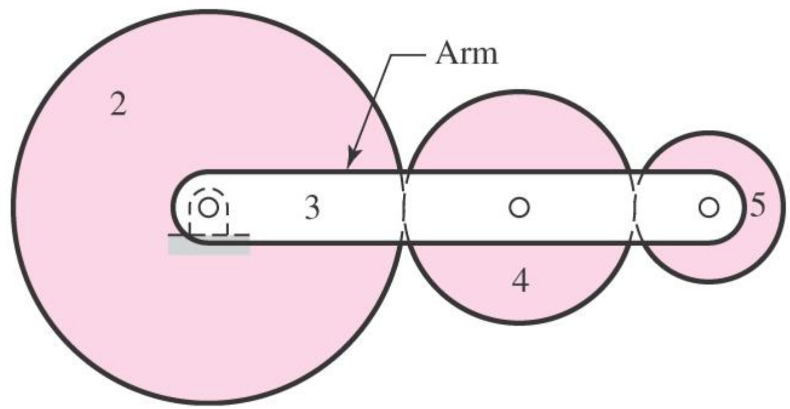


Fig. 13–31

Example 13–6

In Fig. 13–30 the sun gear is the input, and it is driven clockwise at 100 rev/min. The ring gear is held stationary by being fastened to the frame. Find the rev/min and direction of rotation of the arm and gear 4.

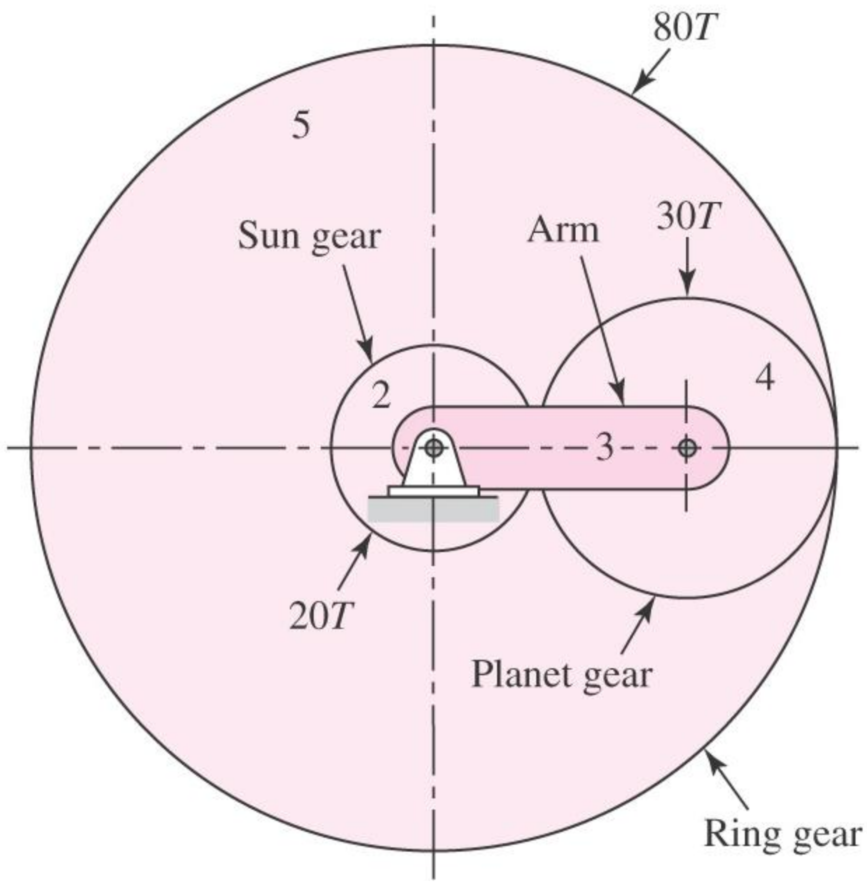


Fig. 13–30

Example 13–6

Designate $n_F = n_2 = -100$ rev/min, and $n_L = n_5 = 0$. Unlocking gear 5 and holding the arm stationary, in our imagination, we find

$$e = -\left(\frac{20}{30}\right)\left(\frac{30}{80}\right) = -0.25$$

Substituting this value in Eq. (13–32) gives

$$-0.25 = \frac{0 - n_A}{(-100) - n_A}$$

or

$$n_A = -20 \text{ rev/min}$$

Example 13–6

To obtain the speed of gear 4, we follow the procedure outlined by Eqs. (*b*), (*c*), and (*d*). Thus

$$n_{43} = n_4 - n_3 \qquad n_{23} = n_2 - n_3$$

and so

$$\frac{n_{43}}{n_{23}} = \frac{n_4 - n_3}{n_2 - n_3} \tag{1}$$

But

$$\frac{n_{43}}{n_{23}} = -\frac{20}{30} = -\frac{2}{3} \tag{2}$$

Substituting the known values in Eq. (1) gives

$$-\frac{2}{3} = \frac{n_4 - (-20)}{(-100) - (-20)}$$

Solving gives

$$n_4 = 33\tfrac{1}{3} \text{ rev/min}$$

Force Analysis – Spur Gearing

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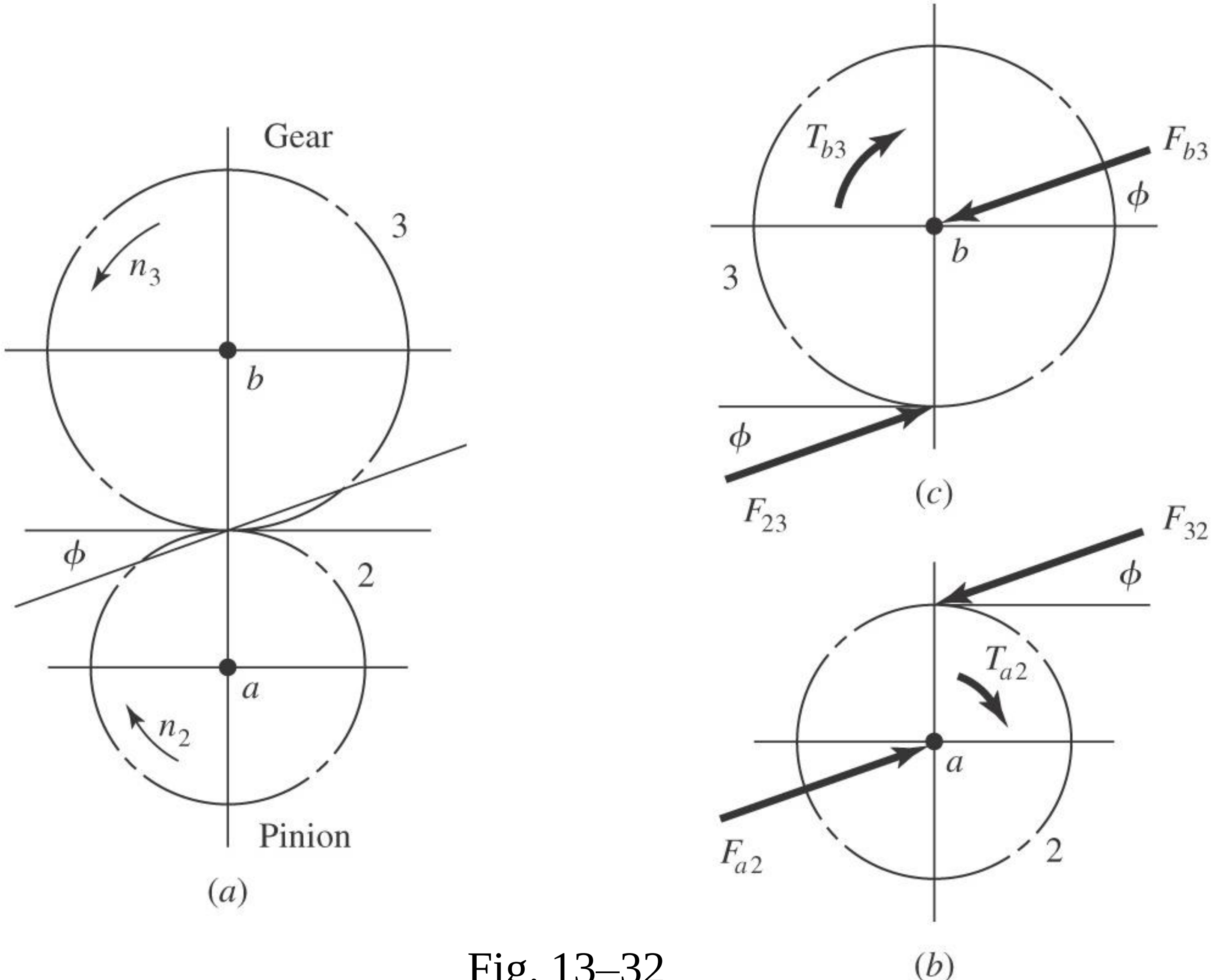

Fig. 13–32

Force Analysis – Spur Gearing

- *Transmitted load* W_t is the tangential load

$$W_t = F_{32}^t$$

- It is the useful component of force, transmitting the torque

$$T = \frac{d}{2} W_t$$

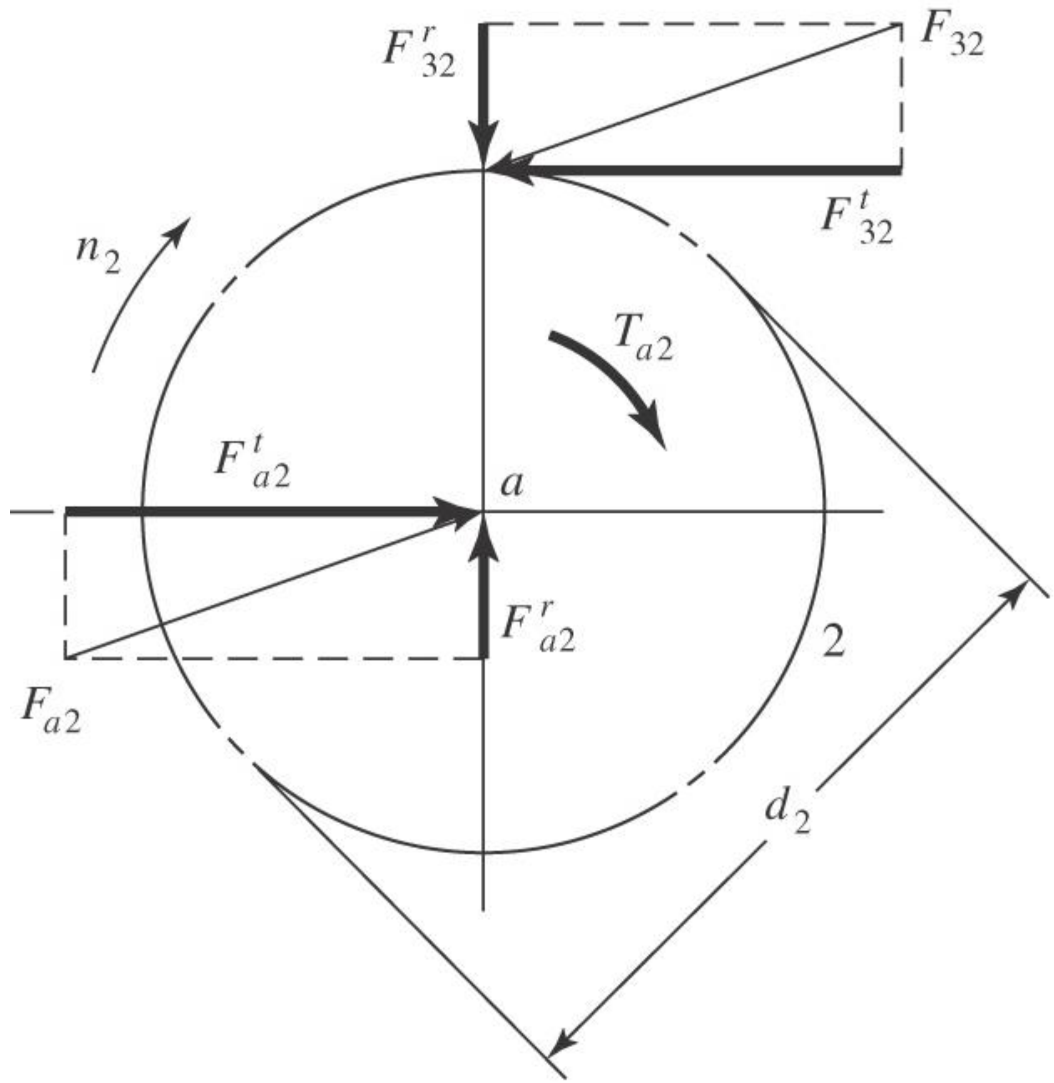

Fig. 13–33

Power in Spur Gearing

- *Transmitted power H*

$$H = T\omega = (W_t d/2)\,\omega \qquad (13\text{–}33)$$

- *Pitch-line velocity* is the linear velocity of a point on the gear at the radius of the pitch circle. It is a common term in tabulating gear data.

$$V = \pi d n/12 \qquad (13\text{–}34)$$

where V = pitch-line velocity, ft/min

d = gear diameter, in

n = gear speed, rev/min

Power in Spur Gearing

- Useful power relation in customary units,

$$W_t = 33\,000\frac{H}{V} \qquad (13\text{–}35)$$

where W_t = transmitted load, lbf

H = power, hp

V = pitch-line velocity, ft/min

- In SI units,

$$W_t = \frac{60\,000H}{\pi d n} \qquad (13\text{–}36)$$

where W_t = transmitted load, kN

H = power, kW

d = gear diameter, mm

n = speed, rev/min

Example 13–7

Pinion 2 in Fig. 13–34*a* runs at 1750 rev/min and transmits 2.5 kW to idler gear 3. The teeth are cut on the 20° full-depth system and have a module of $m = 2.5$ mm. Draw a free-body diagram of gear 3 and show all the forces that act upon it.

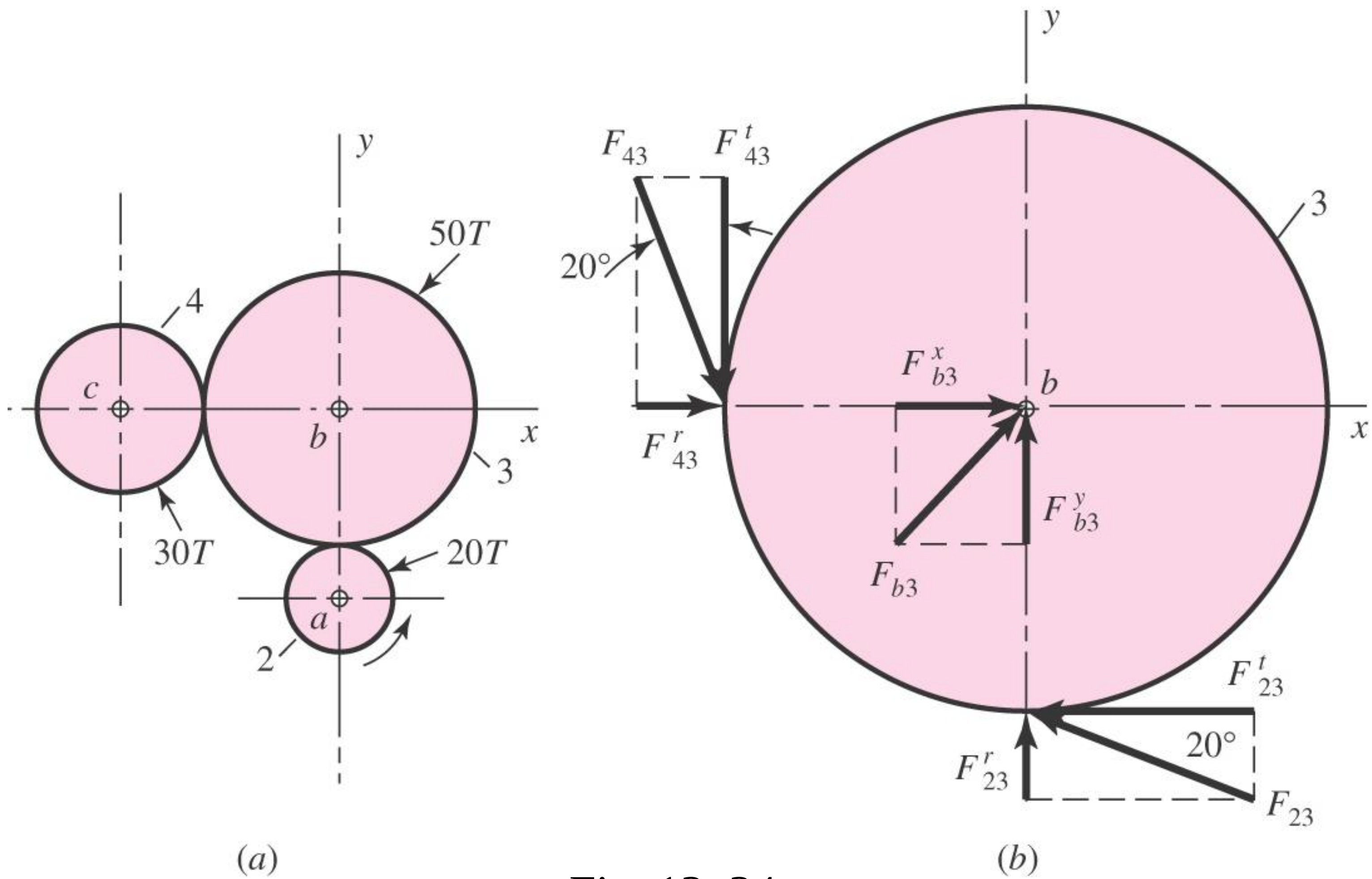

Fig. 13–34

Example 13–7

The pitch diameters of gears 2 and 3 are

$$d_2 = N_2 m = 20(2.5) = 50 \text{ mm}$$

$$d_3 = N_3 m = 50(2.5) = 125 \text{ mm}$$

From Eq. (13–36) we find the transmitted load to be

$$W_t = \frac{60\,000 H}{\pi d_2 n} = \frac{60\,000(2.5)}{\pi(50)(1750)} = 0.546 \text{ kN}$$

Thus, the tangential force of gear 2 on gear 3 is $F_{23}^t = 0.546$ kN, as shown in Fig. 13–34*b*. Therefore

$$F_{23}^r = F_{23}^t \tan 20^\circ = (0.546) \tan 20^\circ = 0.199 \text{ kN}$$

and so

$$F_{23} = \frac{F_{23}^t}{\cos 20^\circ} = \frac{0.546}{\cos 20^\circ} = 0.581 \text{ kN}$$

Example 13–7

Since gear 3 is an idler, it transmits no power (torque) to its shaft, and so the tangential reaction of gear 4 on gear 3 is also equal to W_t. Therefore

$$F_{43}^t = 0.546 \text{ kN} \qquad F_{43}^r = 0.199 \text{ kN} \qquad F_{43} = 0.581 \text{ kN}$$

and the directions are shown in Fig. 13–34*b*.

The shaft reactions in the *x* and *y* directions are

$$F_{b3}^x = -\left(F_{23}^t + F_{43}^r\right) = -(-0.546 + 0.199) = 0.347 \text{ kN}$$

$$F_{b3}^y = -\left(F_{23}^r + F_{43}^t\right) = -(0.199 - 0.546) = 0.347 \text{ kN}$$

The resultant shaft reaction is

$$F_{b3} = \sqrt{(0.347)^2 + (0.347)^2} = 0.491 \text{ kN}$$

These are shown on the figure.

Force Analysis – Bevel Gearing

$$W_t = \frac{T}{r_{av}} \quad (13\text{–}37)$$

$$W_r = W_t \tan\phi \cos\gamma$$

$$W_a = W_t \tan\phi \sin\gamma \quad (13\text{–}38)$$

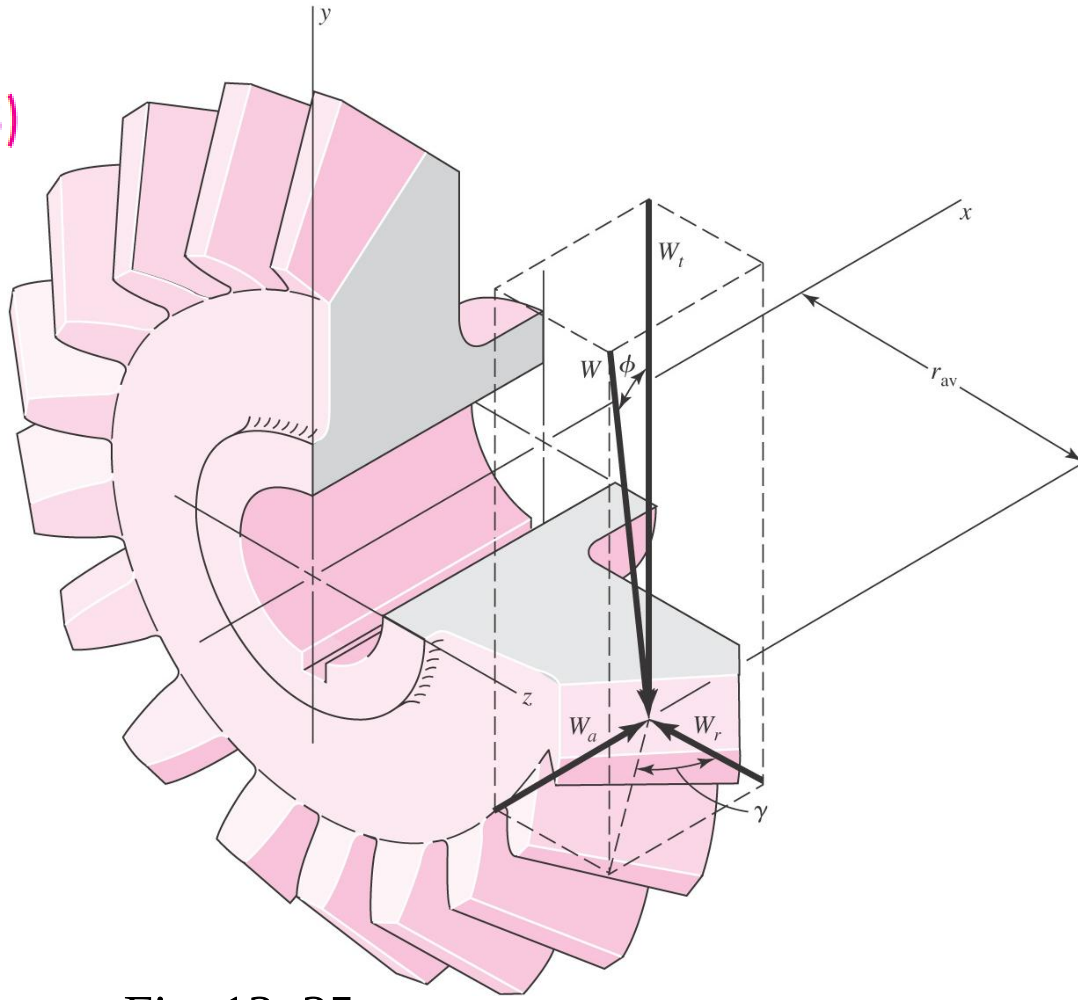

Fig. 13–35

Example 13–8

The bevel pinion in Fig. 13–36*a* rotates at 600 rev/min in the direction shown and transmits 5 hp to the gear. The mounting distances, the location of all bearings, and the average pitch radii of the pinion and gear are shown in the figure. For simplicity, the teeth have been replaced by pitch cones. Bearings *A* and *C* should take the thrust loads. Find the bearing forces on the gearshaft.

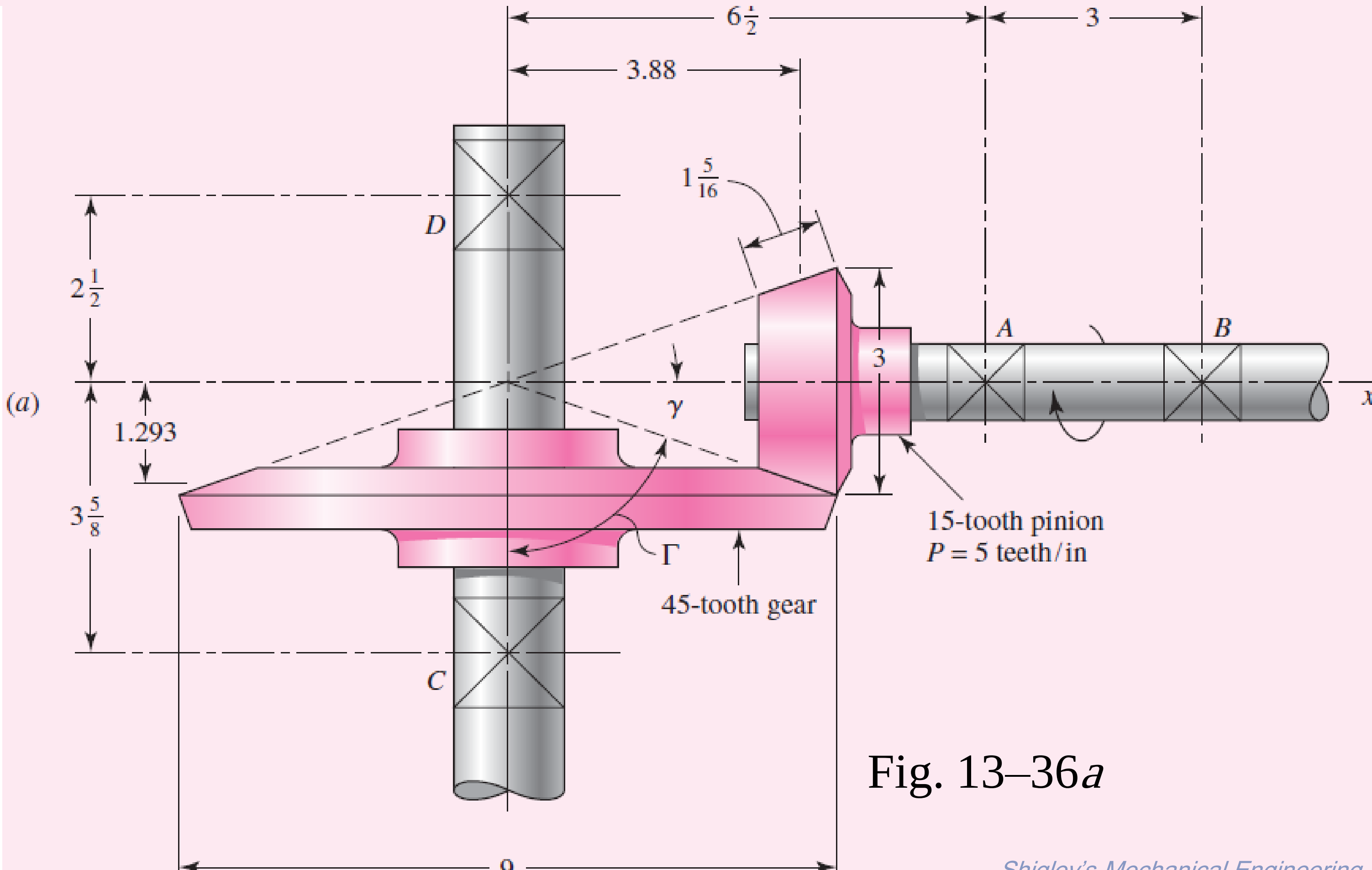

Fig. 13–36*a*

Example 13–8

The pitch angles are

$$\gamma = \tan^{-1}\left(\frac{3}{9}\right) = 18.4^\circ \qquad \Gamma = \tan^{-1}\left(\frac{9}{3}\right) = 71.6^\circ$$

The pitch-line velocity corresponding to the average pitch radius is

$$V = \frac{2\pi r_P n}{12} = \frac{2\pi(1.293)(600)}{12} = 406 \text{ ft/min}$$

Therefore the transmitted load is

$$W_t = \frac{33\,000H}{V} = \frac{(33\,000)(5)}{406} = 406 \text{ lbf}$$

which acts in the positive z direction, as shown in Fig. 13–36b. We next have

$$W_r = W_t \tan\phi \cos\Gamma = 406 \tan 20^\circ \cos 71.6^\circ = 46.6 \text{ lbf}$$

$$W_a = W_t \tan\phi \sin\Gamma = 406 \tan 20^\circ \sin 71.6^\circ = 140 \text{ lbf}$$

where W_r is in the $-x$ direction and W_a is in the $-y$ direction, as illustrated in the isometric sketch of Fig. 13–36b.

Example 13–8

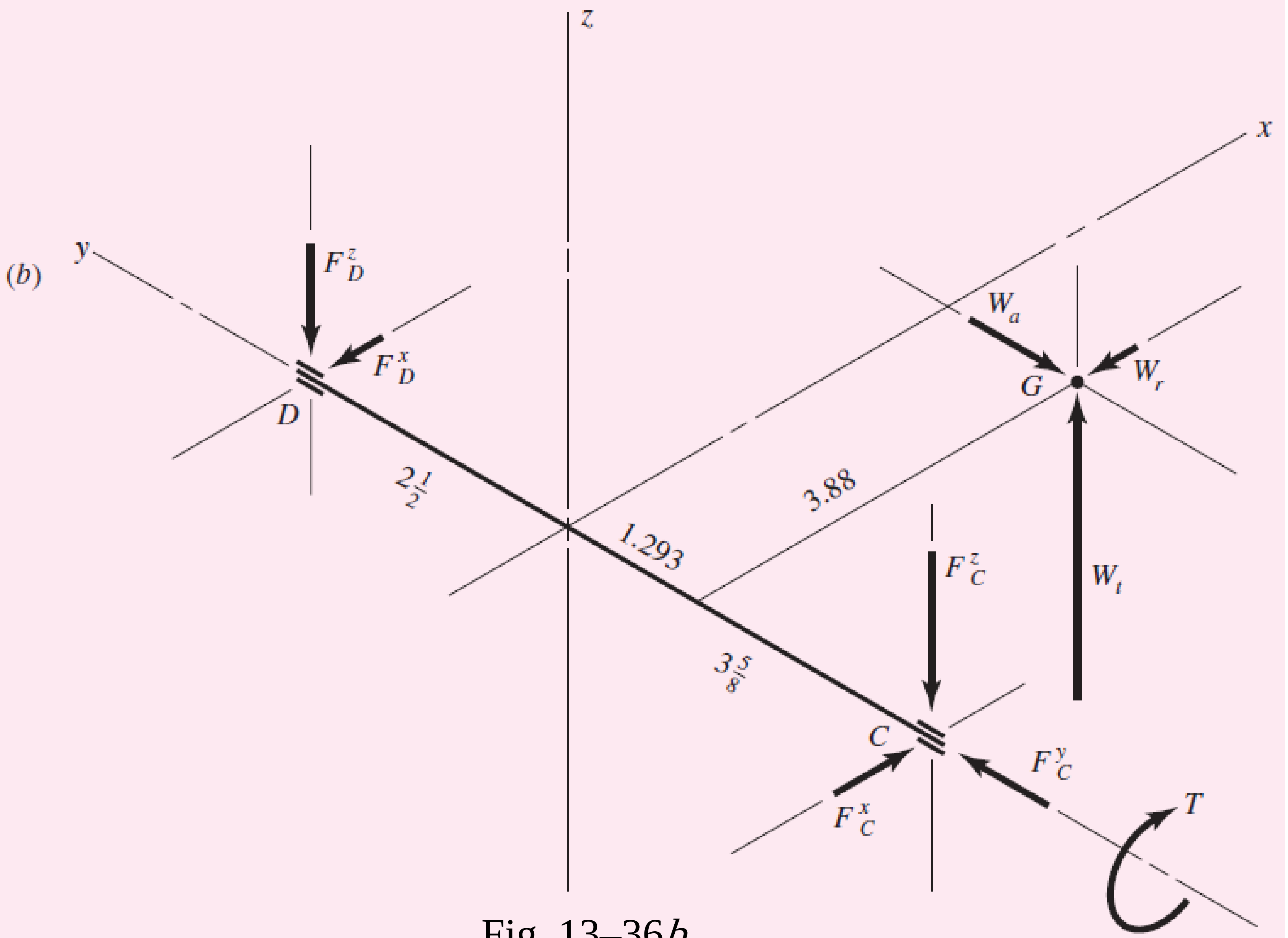

Fig. 13–36*b*

Example 13–8

In preparing to take a sum of the moments about bearing D, define the position vector from D to G as

$$\mathbf{R}_G = 3.88\mathbf{i} - (2.5 + 1.293)\mathbf{j} = 3.88\mathbf{i} - 3.793\mathbf{j}$$

We shall also require a vector from D to C:

$$\mathbf{R}_C = -(2.5 + 3.625)\mathbf{j} = -6.125\mathbf{j}$$

Then, summing moments about D gives

$$\mathbf{R}_G \times \mathbf{W} + \mathbf{R}_C \times \mathbf{F}_C + \mathbf{T} = \mathbf{0} \qquad (1)$$

When we place the details in Eq. (1), we get

$$\begin{aligned}(3.88\mathbf{i} - 3.793\mathbf{j}) \times (-46.6\mathbf{i} - 140\mathbf{j} + 406\mathbf{k})\\ + (-6.125\mathbf{j}) \times \left(F_C^x\mathbf{i} + F_C^y\mathbf{j} + F_C^z\mathbf{k}\right) + T\mathbf{j} = \mathbf{0}\end{aligned} \qquad (2)$$

Example 13–8

After the two cross products are taken, the equation becomes

$$(-1540\mathbf{i} - 1575\mathbf{j} - 720\mathbf{k}) + \left(-6.125F_C^z\mathbf{i} + 6.125F_C^x\mathbf{k}\right) + T\mathbf{j} = \mathbf{0}$$

from which

$$\mathbf{T} = 1575\mathbf{j}\ \text{lbf}\cdot\text{in} \qquad F_C^x = 118\ \text{lbf} \qquad F_C^z = -251\ \text{lbf} \tag{3}$$

Now sum the forces to zero. Thus

$$\mathbf{F}_D + \mathbf{F}_C + \mathbf{W} = \mathbf{0} \tag{4}$$

When the details are inserted, Eq. (4) becomes

$$\left(F_D^x\mathbf{i} + F_D^z\mathbf{k}\right) + \left(118\mathbf{i} + F_C^y\mathbf{j} - 251\mathbf{k}\right) + (-46.6\mathbf{i} - 140\mathbf{j} + 406\mathbf{k}) = \mathbf{0} \tag{5}$$

First we see that $F_C^y = 140$ lbf, and so

$$\mathbf{F}_C = 118\mathbf{i} + 140\mathbf{j} - 251\mathbf{k}\ \text{lbf}$$

Then, from Eq. (5),

$$\mathbf{F}_D = -71.4\mathbf{i} - 155\mathbf{k}\ \text{lbf}$$

Force Analysis – Helical Gearing

$$W_r = W \sin \phi_n$$

$$W_t = W \cos \phi_n \cos \psi \qquad (13\text{–}39)$$

$$W_a = W \cos \phi_n \sin \psi$$

$$W_r = W_t \tan \phi_t$$

$$W_a = W_t \tan \psi \qquad (13\text{–}40)$$

$$W = \frac{W_t}{\cos \phi_n \cos \psi}$$

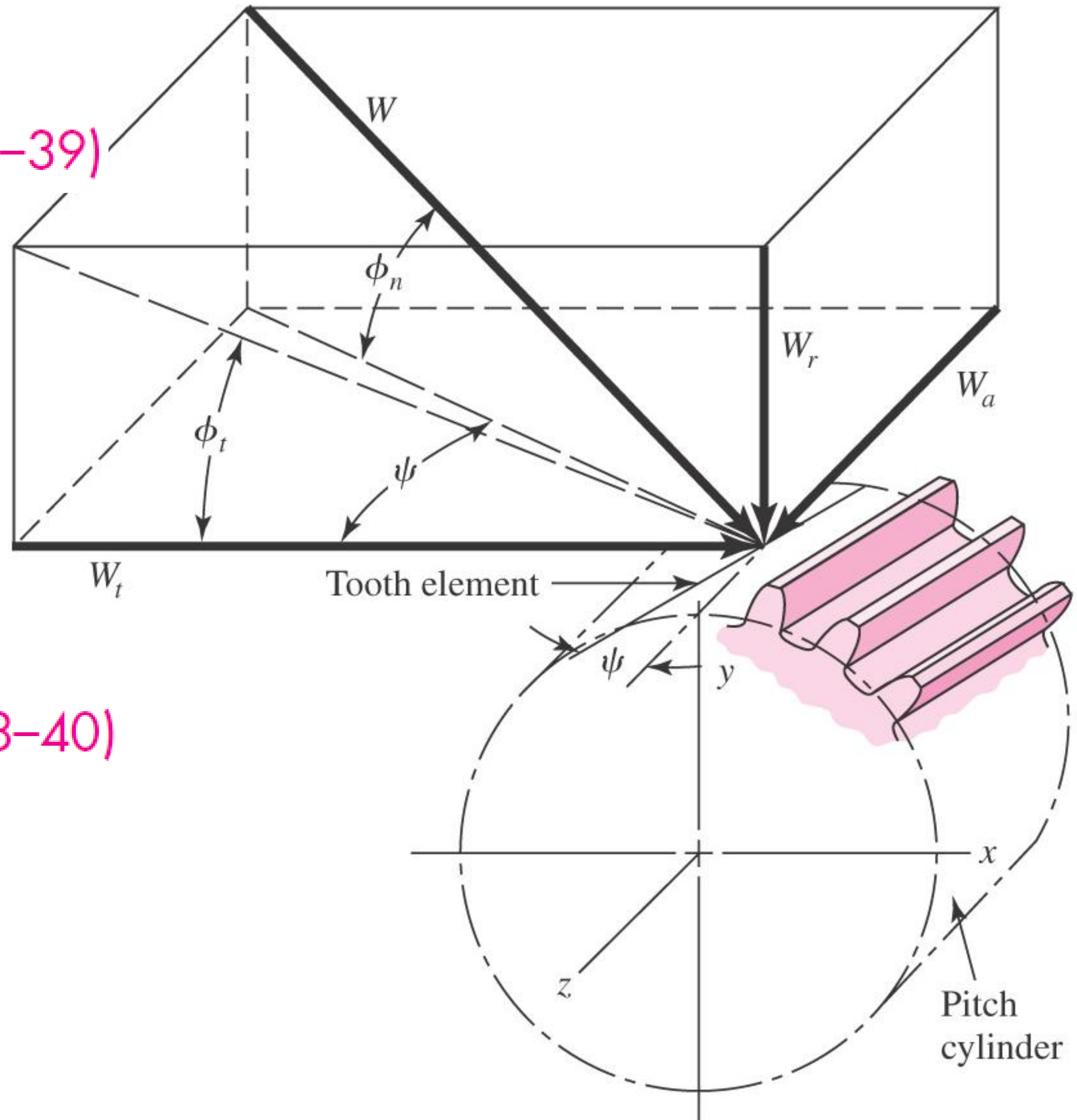

Fig. 13–37

Example 13–9

In Fig. 13–38 a 1-hp electric motor runs at 1800 rev/min in the clockwise direction, as viewed from the positive x axis. Keyed to the motor shaft is an 18-tooth helical pinion having a normal pressure angle of 20°, a helix angle of 30°, and a normal diametral pitch of 12 teeth/in. The hand of the helix is shown in the figure. Make a three-dimensional sketch of the motor shaft and pinion, and show the forces acting on the pinion and the bearing reactions at A and B. The thrust should be taken out at A.

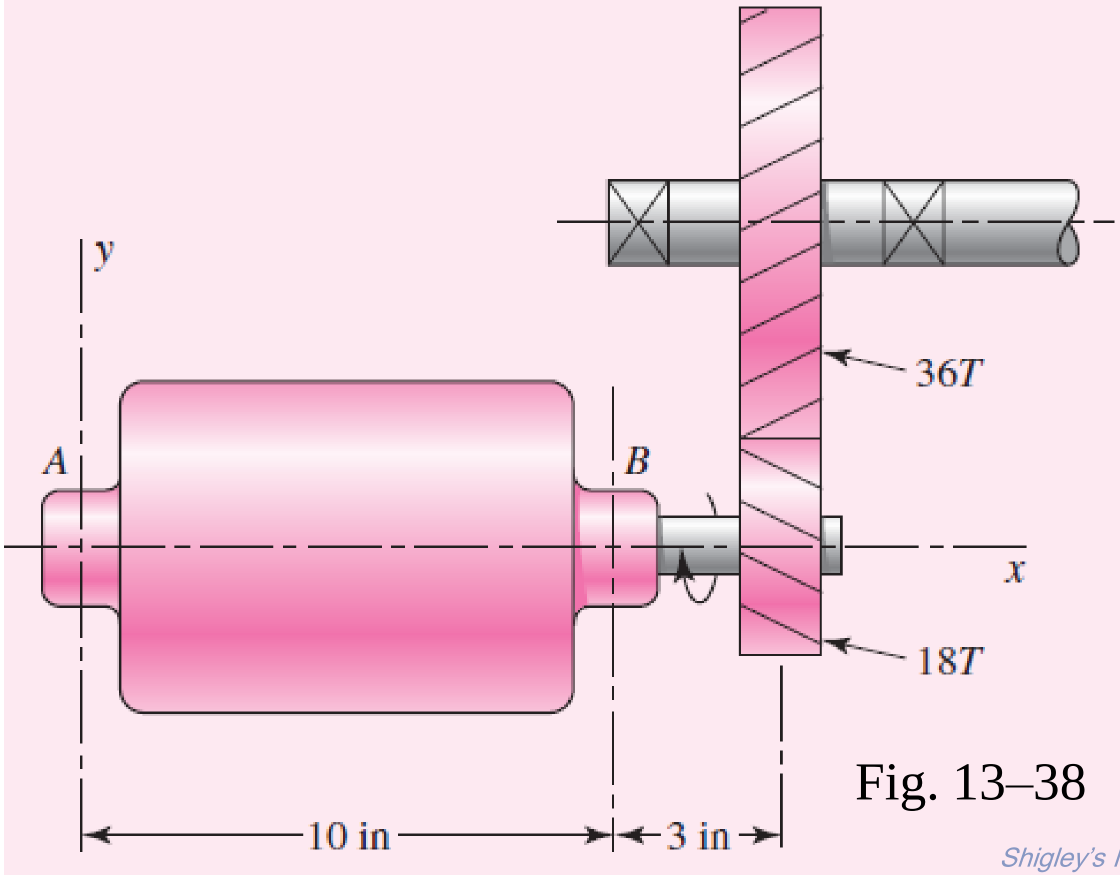

Fig. 13–38

Example 13–9

From Eq. (13–19) we find

$$\phi_t = \tan^{-1}\frac{\tan\phi_n}{\cos\psi} = \tan^{-1}\frac{\tan 20^\circ}{\cos 30^\circ} = 22.8^\circ$$

Also, $P_t = P_n \cos\psi = 12\cos 30^\circ = 10.39$ teeth/in. Therefore the pitch diameter of the pinion is $d_p = 18/10.39 = 1.732$ in. The pitch-line velocity is

$$V = \frac{\pi dn}{12} = \frac{\pi(1.732)(1800)}{12} = 816 \text{ ft/min}$$

The transmitted load is

$$W_t = \frac{33\,000H}{V} = \frac{(33\,000)(1)}{816} = 40.4 \text{ lbf}$$

Example 13–9

From Eq. (13–40) we find

$$W_r = W_t \tan \phi_t = (40.4) \tan 22.8^\circ = 17.0 \text{ lbf}$$

$$W_a = W_t \tan \psi = (40.4) \tan 30^\circ = 23.3 \text{ lbf}$$

$$W = \frac{W_t}{\cos \phi_n \cos \psi} = \frac{40.4}{\cos 20^\circ \cos 30^\circ} = 49.6 \text{ lbf}$$

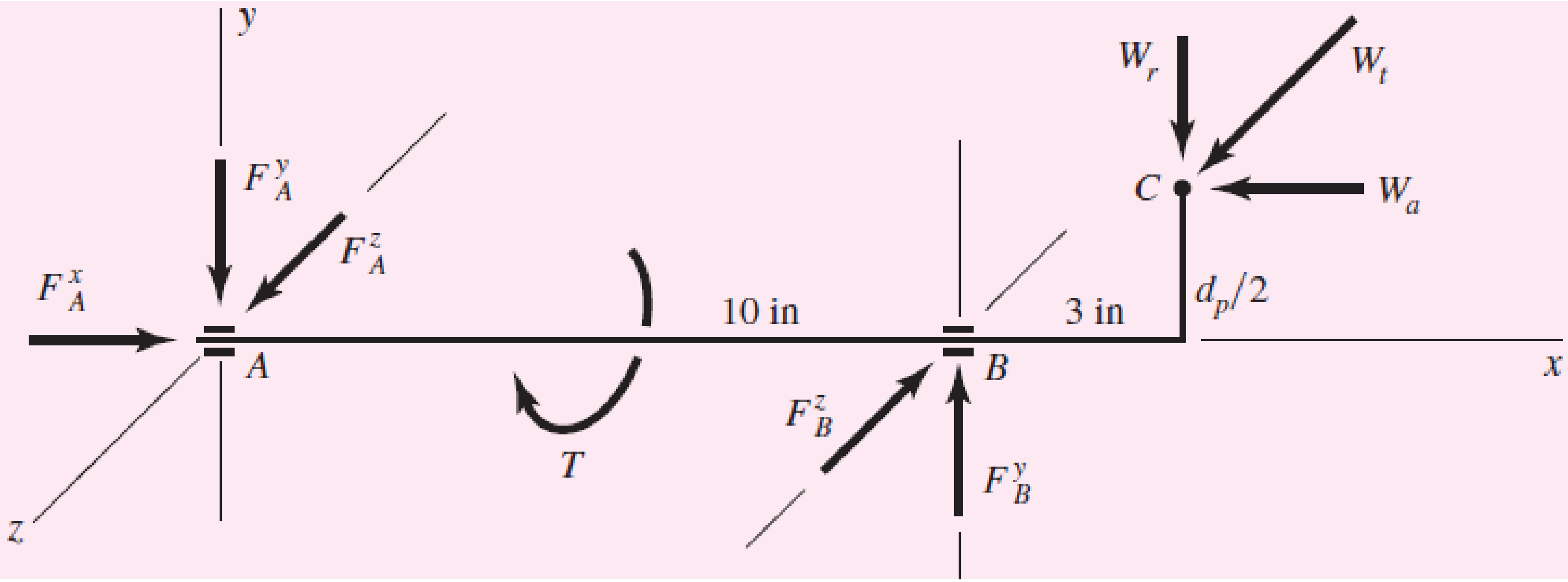

Fig. 13–39

Example 13–9

These three forces, W_r in the $-y$ direction, W_a in the $-x$ direction, and W_t in the $+z$ direction, are shown acting at point C in Fig. 13–39. We assume bearing reactions at A and B as shown. Then $F_A^x = W_a = 23.3$ lbf. Taking moments about the z axis,

$$-(17.0)(13) + (23.3)\left(\frac{1.732}{2}\right) + 10F_B^y = 0$$

or $F_B^y = 20.1$ lbf. Summing forces in the y direction then gives $F_A^y = 3.1$ lbf. Taking moments about the y axis, next

$$10F_B^z - (40.4)(13) = 0$$

or $F_B^z = 52.5$ lbf. Summing forces in the z direction and solving gives $F_A^z = 12.1$ lbf. Also, the torque is $T = W_t d_p/2 = (40.4)(1.732/2) = 35\ \text{lbf}\cdot\text{in}$.

Example 13–9

For comparison, solve the problem again using vectors. The force at C is

$$\mathbf{W} = -23.3\mathbf{i} - 17.0\mathbf{j} + 40.4\mathbf{k} \text{ lbf}$$

Position vectors to B and C from origin A are

$$\mathbf{R}_B = 10\,\mathbf{i} \qquad \mathbf{R}_C = 13\mathbf{i} + 0.866\mathbf{j}$$

Taking moments about A, we have

$$\mathbf{R}_B \times \mathbf{F}_B + \mathbf{T} + \mathbf{R}_C \times \mathbf{W} = \mathbf{0}$$

Using the directions assumed in Fig. 13–39 and substituting values gives

$$10\,\mathbf{i} \times \left(F_B^y\mathbf{j} - F_B^z\mathbf{k}\right) - T\,\mathbf{i} + (13\mathbf{i} + 0.866\mathbf{j}) \times (-23.3\mathbf{i} - 17.0\mathbf{j} + 40.4\mathbf{k}) = \mathbf{0}$$

When the cross products are formed, we get

$$\left(10F_B^y\mathbf{k} + 10F_B^z\mathbf{j}\right) - T\,\mathbf{i} + (35\mathbf{i} - 525\mathbf{j} - 201\mathbf{k}) = \mathbf{0}$$

whence $T = 35 \text{ lbf} \cdot \text{in}$, $F_B^y = 20.1 \text{ lbf}$, and $F_B^z = 52.5 \text{ lbf}$.

Next,

$$\mathbf{F}_A = -\mathbf{F}_B - \mathbf{W}, \text{ and so } \mathbf{F}_A = 23.3\mathbf{i} - 3.1\mathbf{j} + 12.1\mathbf{k} \text{ lbf}.$$

Force Analysis – Worm Gearing

$$W^x = W \cos\phi_n \sin\lambda$$
$$W^y = W \sin\phi_n \qquad (13\text{–}41)$$
$$W^z = W \cos\phi_n \cos\lambda$$

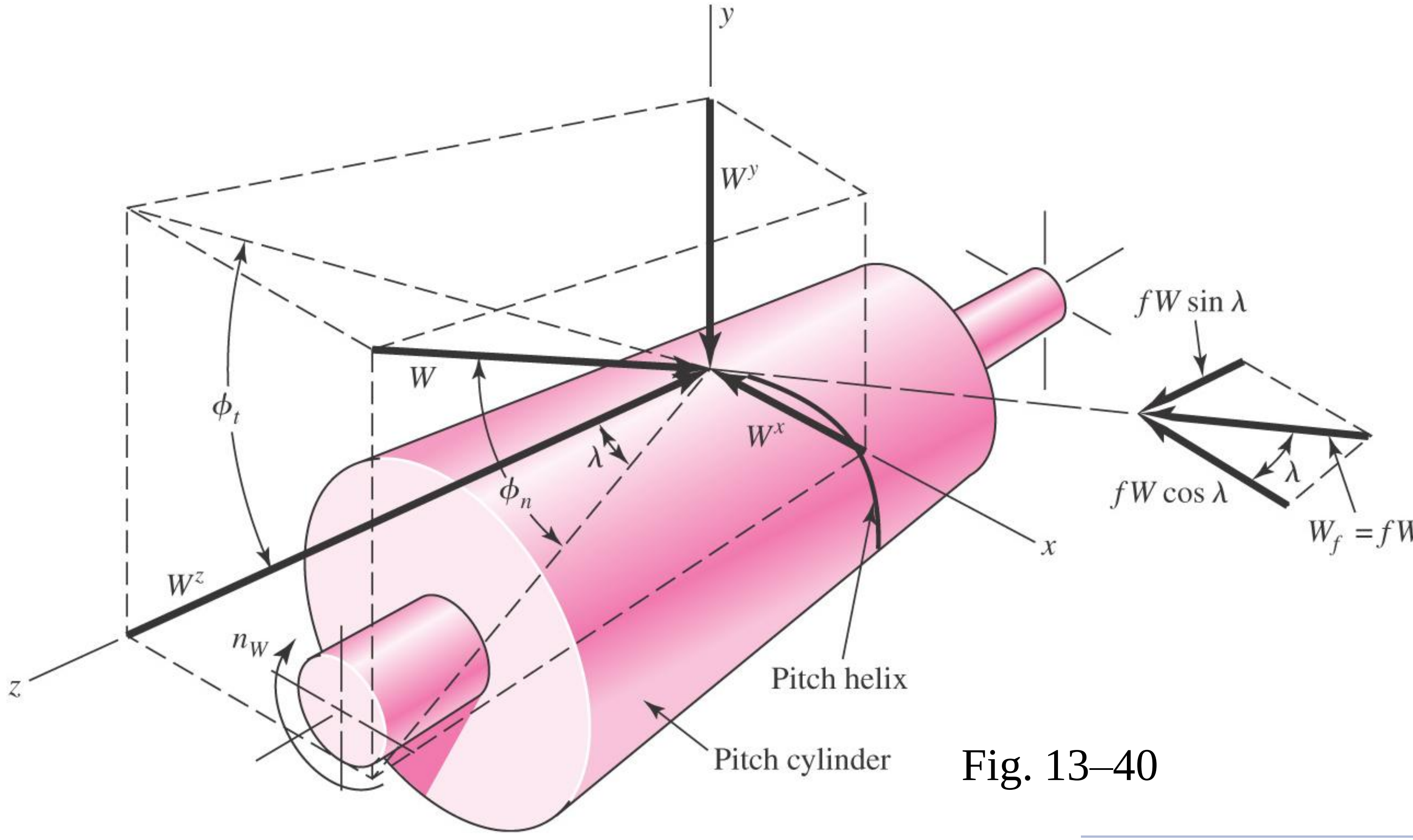

Fig. 13–40

Force Analysis – Worm Gearing

$$W_{Wt} = -W_{Ga} = W^x$$
$$W_{Wr} = -W_{Gr} = W^y \qquad (13\text{–}42)$$
$$W_{Wa} = -W_{Gt} = W^z$$

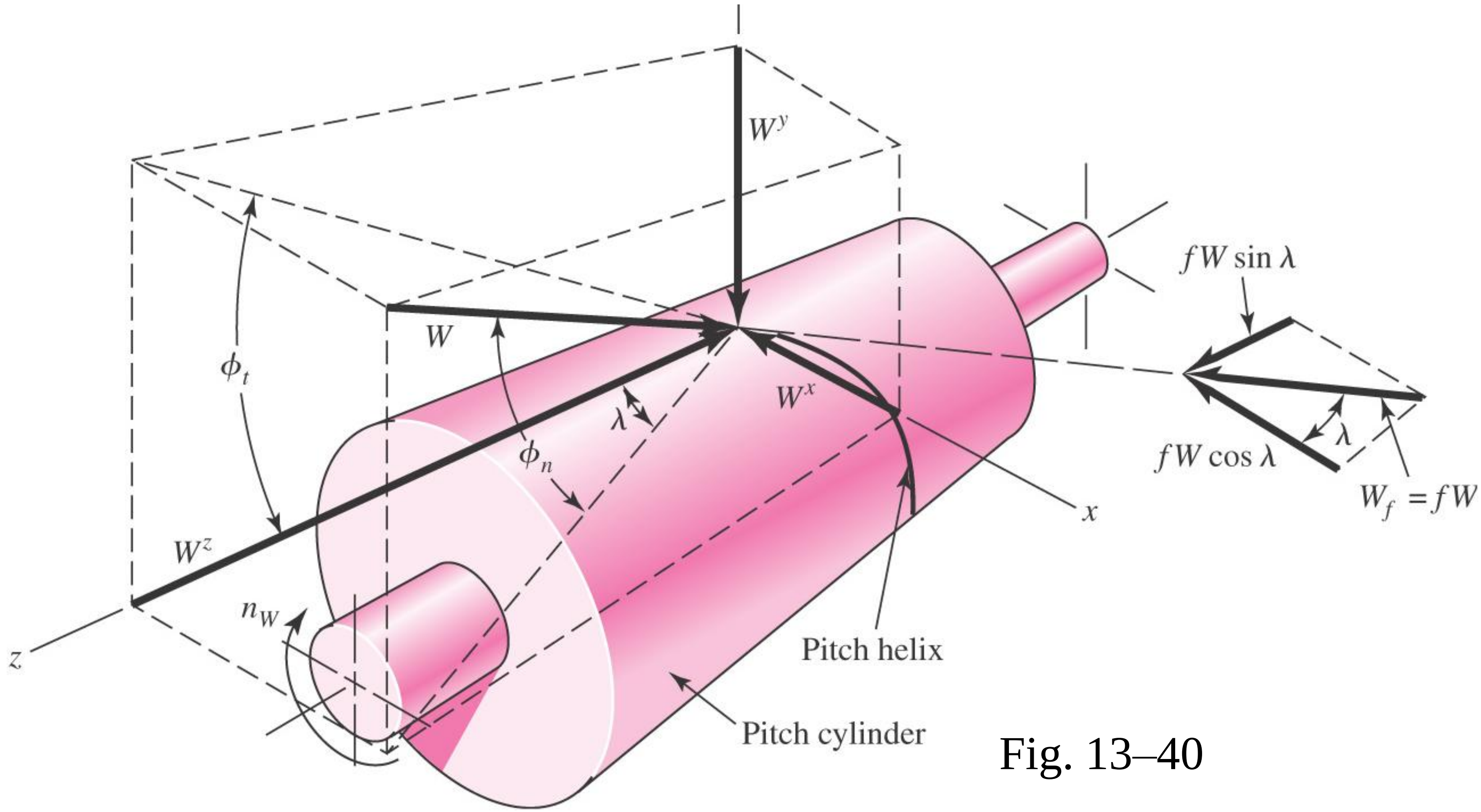

Fig. 13–40

Force Analysis – Worm Gearing

- Relative motion in worm gearing is sliding action
- Friction is much more significant than in other types of gears
- Including friction components, Eq. (13-41) can be expanded to

$$
\begin{aligned}
W^x &= W(\cos\phi_n \sin\lambda + f\cos\lambda) \\
W^y &= W\sin\phi_n \\
W^z &= W(\cos\phi_n \cos\lambda - f\sin\lambda)
\end{aligned}
\qquad (13\text{–}43)
$$

- Combining with Eqs. (13-42) and (13-43),

$$W_f = fW = \frac{fW_{Gt}}{f\sin\lambda - \cos\phi_n\cos\lambda} \qquad (13\text{–}44)$$

$$W_{Wt} = W_{Gt}\frac{\cos\phi_n\sin\lambda + f\cos\lambda}{f\sin\lambda - \cos\phi_n\cos\lambda} \qquad (13\text{–}45)$$

Worm Gearing Efficiency

- *Efficiency* is defined as

$$\eta = \frac{W_{Wt}\,(\text{without friction})}{W_{Wt}\,(\text{with friction})}$$

- From Eq. (13–45) with $f = 0$ in the numerator,

$$\eta = \frac{\cos\phi_n - f\tan\lambda}{\cos\phi_n + f\cot\lambda} \qquad (13\text{–}46)$$

Worm Gearing Efficiency

- With typical value of $f = 0.05$, and $\phi_n = 20^\circ$, efficiency as a function of helix angle is given in the table.

Helix Angle ψ, deg	Efficiency η, %
1.0	25.2
2.5	45.7
5.0	62.0
7.5	71.3
10.0	76.6
15.0	82.7
20.0	85.9
30.0	89.1

Table 13–6

Worm Gearing Efficiency

- Coefficient of friction is dependent on relative or sliding velocity V_S
- V_G is pitch line velocity of gear
- V_W is pitch line velocity of worm

$$\mathbf{V}_W = \mathbf{V}_G + \mathbf{V}_S$$

$$V_S = \frac{V_W}{\cos\lambda} \qquad (13\text{–}47)$$

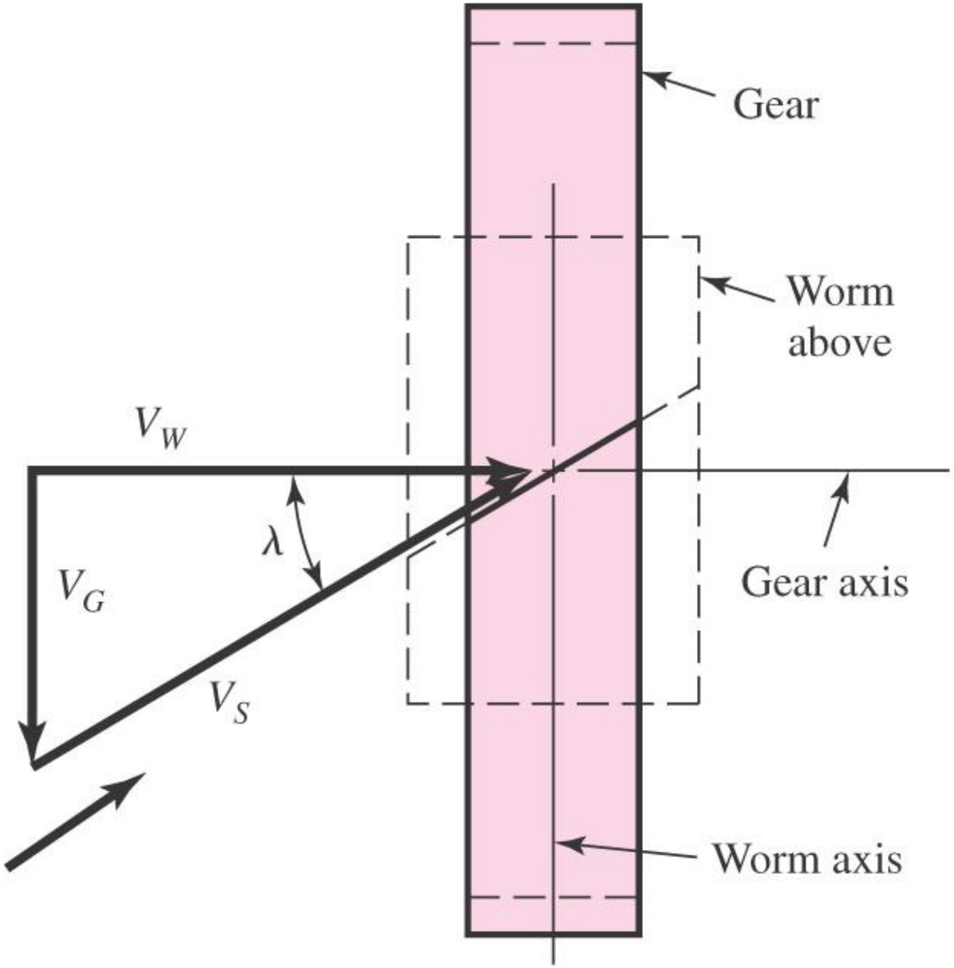

Fig. 13–41

Coefficient of Friction for Worm Gearing

- Graph shows representative values
- Curve *A* is for when more friction is expected, such as when gears are cast iron
- Curve *B* is for high-quality materials

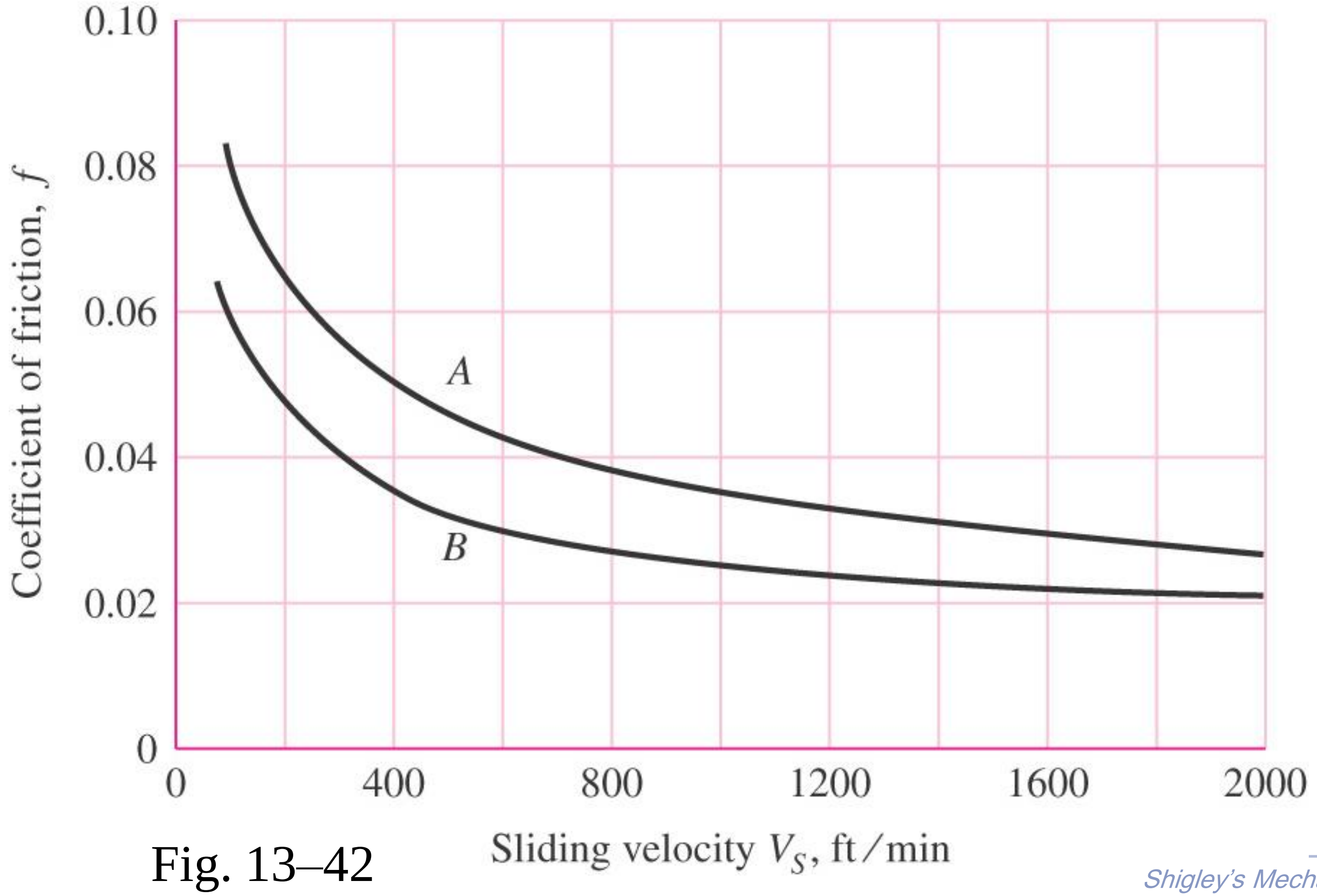

Fig. 13–42

Example 13–10

A 2-tooth right-hand worm transmits 1 hp at 1200 rev/min to a 30-tooth worm gear. The gear has a transverse diametral pitch of 6 teeth/in and a face width of 1 in. The worm has a pitch diameter of 2 in and a face width of $2\frac{1}{2}$ in. The normal pressure angle is $14\frac{1}{2}^\circ$. The materials and quality of work needed are such that curve *B* of Fig. 13–42 should be used to obtain the coefficient of friction.

(*a*) Find the axial pitch, the center distance, the lead, and the lead angle.

(*b*) Figure 13–43 is a drawing of the worm gear oriented with respect to the coordinate system described earlier in this section; the gear is supported by bearings *A* and *B*. Find the forces exerted by the bearings against the worm-gear shaft, and the output torque.

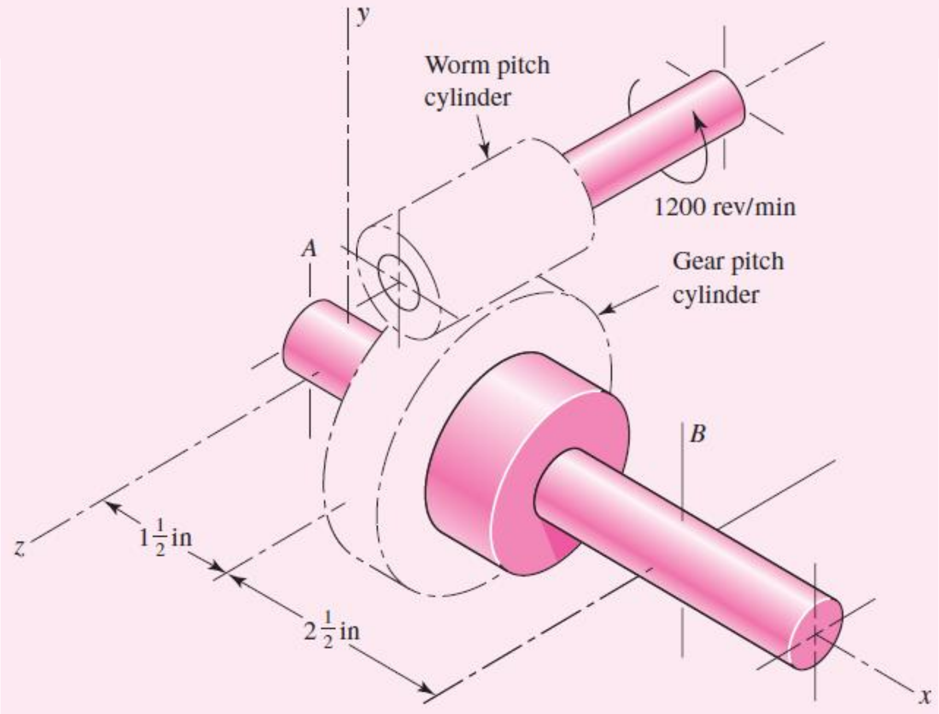

Fig. 13–43

Example 13–10

(*a*) The axial pitch is the same as the transverse circular pitch of the gear, which is

$$p_x = p_t = \frac{\pi}{P} = \frac{\pi}{6} = 0.5236 \text{ in}$$

The pitch diameter of the gear is $d_G = N_G/P = 30/6 = 5$ in. Therefore, the center distance is

$$C = \frac{d_W + d_G}{2} = \frac{2+5}{2} = 3.5 \text{ in}$$

From Eq. (13–27), the lead is

$$L = p_x N_W = (0.5236)(2) = 1.0472 \text{ in}$$

Also using Eq. (13–28), find

$$\lambda = \tan^{-1}\frac{L}{\pi d_W} = \tan^{-1}\frac{1.0472}{\pi(2)} = 9.46^\circ$$

Example 13–10

(*b*) Using the right-hand rule for the rotation of the worm, you will see that your thumb points in the positive *z* direction. Now use the bolt-and-nut analogy (the worm is right-handed, as is the screw thread of a bolt), and turn the bolt clockwise with the right hand while preventing nut rotation with the left. The nut will move axially along the bolt toward your right hand. Therefore the surface of the gear (Fig. 13–43) in contact with the worm will move in the negative *z* direction. Thus, the gear rotates clockwise about *x*, with your right thumb pointing in the negative *x* direction.

The pitch-line velocity of the worm is

$$V_W = \frac{\pi d_W n_W}{12} = \frac{\pi(2)(1200)}{12} = 628 \text{ ft/min}$$

The speed of the gear is $n_G = (\frac{2}{30})(1200) = 80$ rev/min. Therefore the pitch-line velocity of the gear is

$$V_G = \frac{\pi d_G n_G}{12} = \frac{\pi(5)(80)}{12} = 105 \text{ ft/min}$$

Example 13–10

Then, from Eq. (13–47), the sliding velocity V_S is found to be

$$V_S = \frac{V_W}{\cos\lambda} = \frac{628}{\cos 9.46^\circ} = 637 \text{ ft/min}$$

Getting to the forces now, we begin with the horsepower formula

$$W_{Wt} = \frac{33\,000 H}{V_W} = \frac{(33\,000)(1)}{628} = 52.5 \text{ lbf}$$

This force acts in the negative x direction, the same as in Fig. 13–40. Using Fig. 13–42, we find $f = 0.03$. Then, the first equation of group (13–42) and (13–43) gives

$$W = \frac{W^x}{\cos\phi_n \sin\lambda + f\cos\lambda}$$

$$= \frac{52.5}{\cos 14.5^\circ \sin 9.46^\circ + 0.03\cos 9.46^\circ} = 278 \text{ lbf}$$

Example 13–10

Also, from Eq. (13–43),

$$W^y = W \sin\phi_n = 278 \sin 14.5° = 69.6 \text{ lbf}$$

$$W^z = W(\cos\phi_n \cos\lambda - f \sin\lambda)$$

$$= 278(\cos 14.5° \cos 9.46° - 0.03 \sin 9.46°) = 264 \text{ lbf}$$

We now identify the components acting on the gear as

$$W_{Ga} = -W^x = 52.5 \text{ lbf}$$

$$W_{Gr} = -W^y = -69.6 \text{ lbf}$$

$$W_{Gt} = -W^z = -264 \text{ lbf}$$

At this point a three-dimensional line drawing should be made in order to simplify the work to follow. An isometric sketch, such as the one of Fig. 13–44, is easy to make and will help you to avoid errors.

Example 13–10

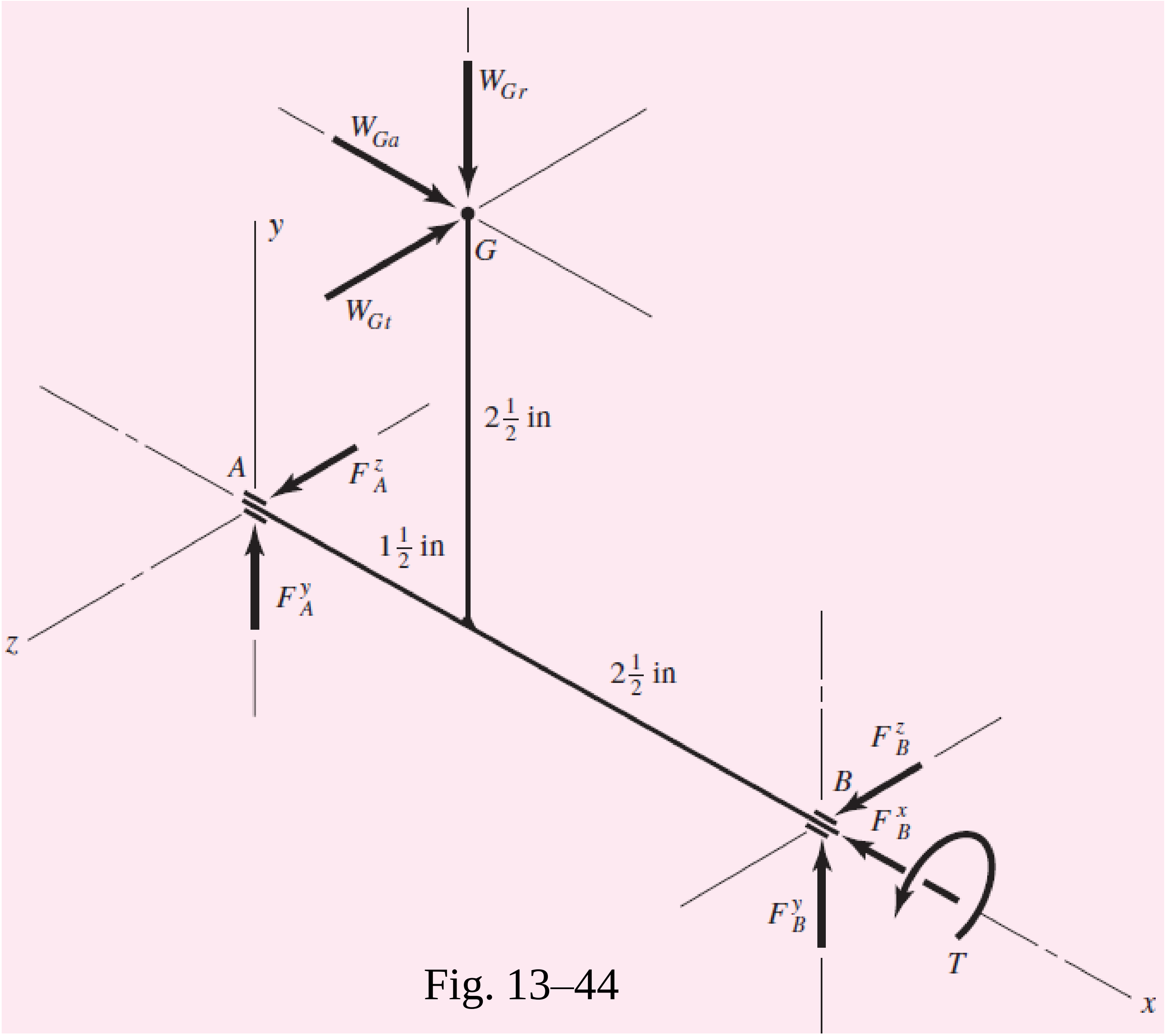

Fig. 13–44

Example 13–10

We shall make B a thrust bearing in order to place the gearshaft in compression. Thus, summing forces in the x direction gives

$$F_B^x = -52.5 \text{ lbf}$$

Taking moments about the z axis, we have

$$-(52.5)(2.5) - (69.6)(1.5) + 4F_B^y = 0 \qquad F_B^y = 58.9 \text{ lbf}$$

Taking moments about the y axis,

$$(264)(1.5) - 4F_B^z = 0 \qquad F_B^z = 99 \text{ lbf}$$

These three components are now inserted on the sketch as shown at B in Fig. 13–44.

Example 13–10

Summing forces in the y direction,

$$-69.6 + 58.9 + F_A^y = 0 \qquad F_A^y = 10.7 \text{ lbf}$$

Similarly, summing forces in the z direction,

$$-264 + 99 + F_A^z = 0 \qquad F_A^z = 165 \text{ lbf}$$

These two components can now be placed at A on the sketch. We still have one more equation to write. Summing moments about x,

$$-(264)(2.5) + T = 0 \qquad T = 660 \text{ lbf} \cdot \text{in}$$

It is because of the frictional loss that this output torque is less than the product of the gear ratio and the input torque.