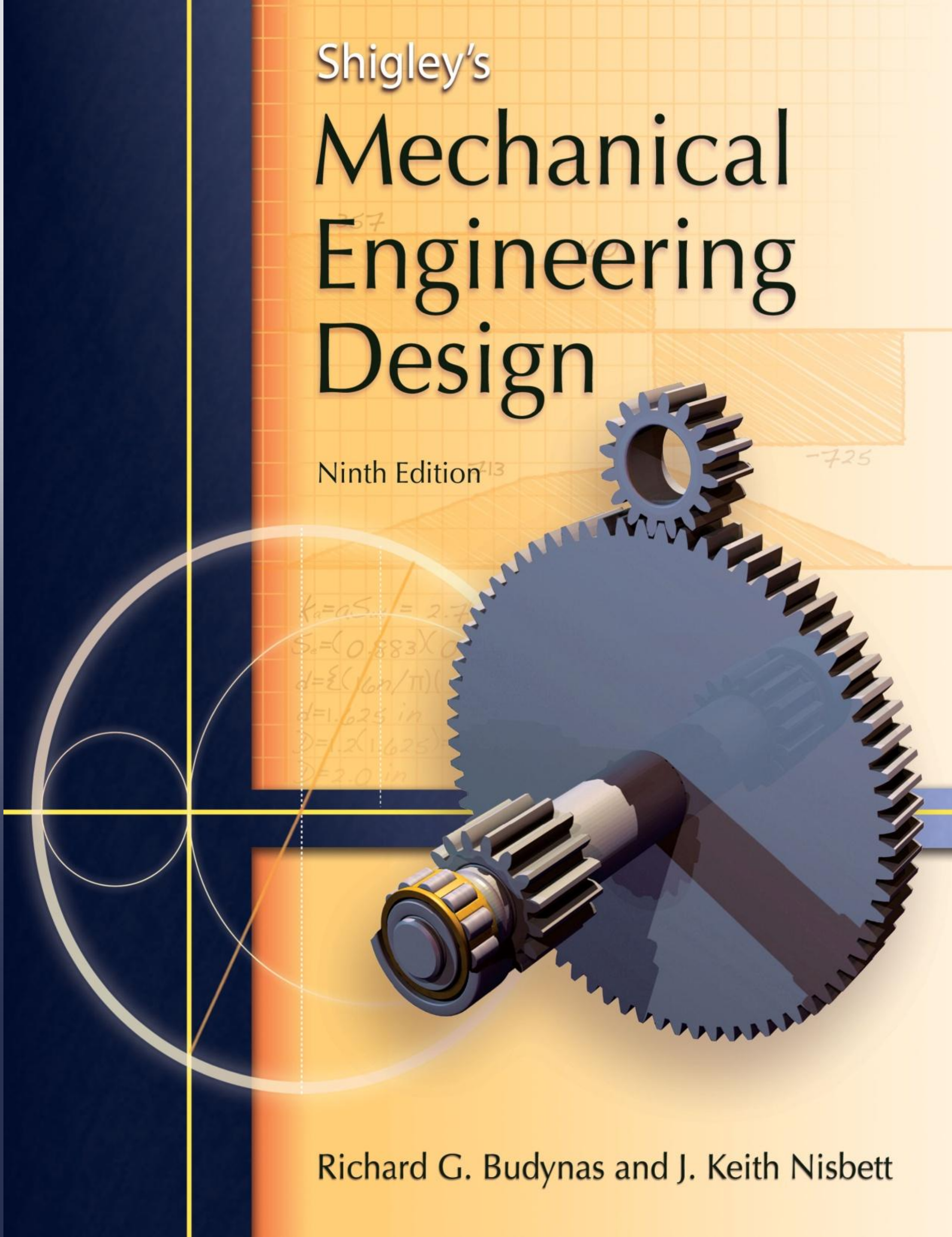
Lecture Slides

Chapter 14

Spur and Helical Gears



Chapter Outline

Cantilever Beam Model of Bending Stress in Gear Tooth

$$\sigma = \frac{M}{I/c} = \frac{6W^t l}{Ft^2}$$

$$\frac{t/2}{x} = \frac{l}{t/2} \qquad \text{or} \qquad x = \frac{t^2}{4l}$$

$$\sigma = \frac{6W^t l}{Ft^2} = \frac{W^t}{F}\frac{1}{t^2/6l} = \frac{W^t}{F}\frac{1}{t^2/4l}\frac{1}{\frac{4}{6}}$$

$$\sigma = \frac{W^t p}{F\left(\frac{2}{3}\right)xp}$$

$$y = 2x/3p$$

$$\sigma = \frac{W^t}{Fpy} \qquad (14\text{–}1)$$

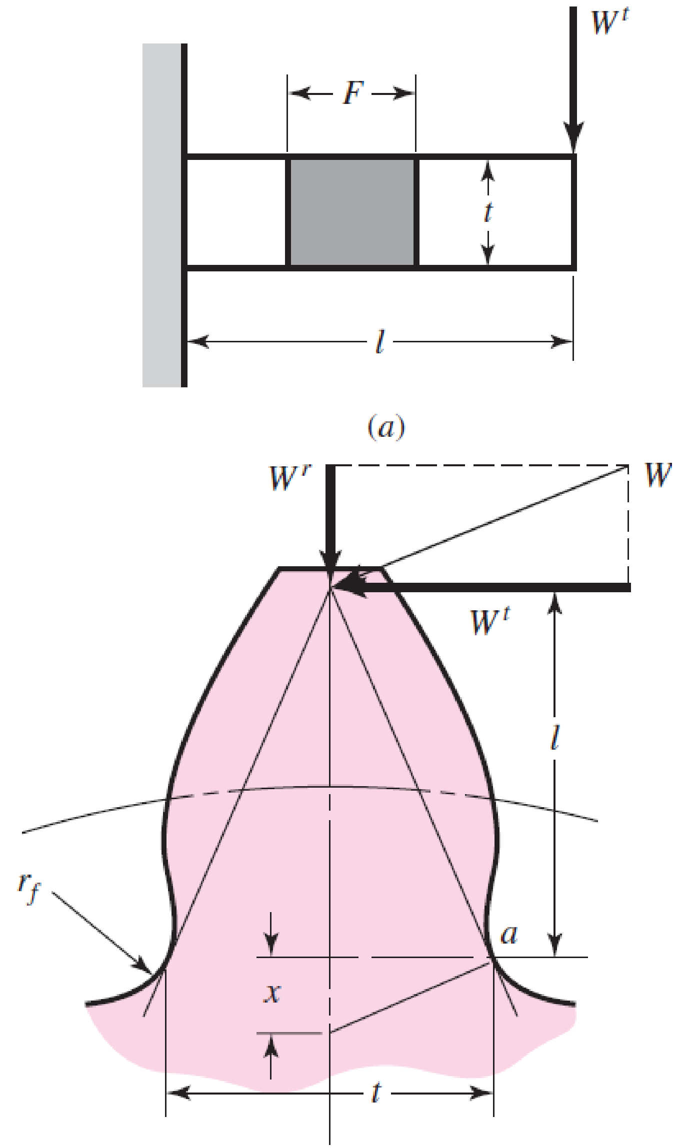


Fig. 14–1 (b)

Lewis Equation

$$\sigma = \frac{W^t}{Fpy} \tag{14–1}$$

$$P = \pi/p \qquad Y = \pi y$$

Lewis Equation $$\sigma = \frac{W^t P}{FY} \tag{14–2}$$

Lewis Form Factor $$Y = \frac{2xP}{3} \tag{14–3}$$

Values of Lewis Form Factor *Y*

Number of Teeth	Y	Number of Teeth	Y
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485

Table 14–2

Dynamic Effects

- Effective load increases as velocity increases
- *Velocity factor* K_v accounts for this
- With pitch-line velocity V in feet per minute,

$$K_v = \frac{600 + V}{600} \qquad \text{(cast iron, cast profile)} \qquad (14\text{–}4a)$$

$$K_v = \frac{1200 + V}{1200} \qquad \text{(cut or milled profile)} \qquad (14\text{–}4b)$$

$$K_v = \frac{50 + \sqrt{V}}{50} \qquad \text{(hobbed or shaped profile)} \qquad (14\text{–}5a)$$

$$K_v = \sqrt{\frac{78 + \sqrt{V}}{78}} \qquad \text{(shaved or ground profile)} \qquad (14\text{–}5b)$$

Dynamic Effects

- With pitch-line velocity V in meters per second,

$$K_v = \frac{3.05 + V}{3.05} \qquad \text{(cast iron, cast profile)} \qquad (14\text{–}6a)$$

$$K_v = \frac{6.1 + V}{6.1} \qquad \text{(cut or milled profile)} \qquad (14\text{–}6b)$$

$$K_v = \frac{3.56 + \sqrt{V}}{3.56} \qquad \text{(hobbed or shaped profile)} \qquad (14\text{–}6c)$$

$$K_v = \sqrt{\frac{5.56 + \sqrt{V}}{5.56}} \qquad \text{(shaved or ground profile)} \qquad (14\text{–}6d)$$

Lewis Equation

- The Lewis equation including velocity factor
  - U.S. Customary version

$$\sigma = \frac{K_v W^t P}{FY} \qquad (14\text{–}7)$$

  - Metric version

$$\sigma = \frac{K_v W^t}{FmY} \qquad (14\text{–}8)$$

- Acceptable for general estimation of stresses in gear teeth
- Forms basis for AGMA method, which is preferred approach

Example 14–1

A stock spur gear is available having a diametral pitch of 8 teeth/in, a $1\frac{1}{2}$-in face, 16 teeth, and a pressure angle of 20° with full-depth teeth. The material is AISI 1020 steel in as-rolled condition. Use a design factor of $n_d = 3$ to rate the horsepower output of the gear corresponding to a speed of 1200 rev/m and moderate applications.

Solution

The term *moderate applications* seems to imply that the gear can be rated by using the yield strength as a criterion of failure. From Table A–20, we find $S_{ut} = 55$ kpsi and $S_y = 30$ kpsi. A design factor of 3 means that the allowable bending stress is $30/3 = 10$ kpsi. The pitch diameter is $N/P = 16/8 = 2$ in, so the pitch-line velocity is

$$V = \frac{\pi d n}{12} = \frac{\pi(2)1200}{12} = 628 \text{ ft/min}$$

The velocity factor from Eq. (14–4*b*) is found to be

$$K_v = \frac{1200 + V}{1200} = \frac{1200 + 628}{1200} = 1.52$$

Example 14–1

Table 14–2 gives the form factor as $Y = 0.296$ for 16 teeth. We now arrange and substitute in Eq. (14–7) as follows:

$$W^t = \frac{FY\sigma_{\text{all}}}{K_v P} = \frac{1.5(0.296)10\,000}{1.52(8)} = 365 \text{ lbf}$$

The horsepower that can be transmitted is

$$hp = \frac{W^t V}{33\,000} = \frac{365(628)}{33\,000} = 6.95 \text{ hp}$$

It is important to emphasize that this is a rough estimate, and that this approach must not be used for important applications. The example is intended to help you understand some of the fundamentals that will be involved in the AGMA approach.

Example 14–2

Estimate the horsepower rating of the gear in the previous example based on obtaining an infinite life in bending.

Solution

The rotating-beam endurance limit is estimated from Eq. (6–8)

$$S'_e = 0.5S_{ut} = 0.5(55) = 27.5 \text{ kpsi}$$

To obtain the surface finish Marin factor k_a we refer to Table 6–3 for machined surface, finding $a = 2.70$ and $b = -0.265$. Then Eq. (6–19) gives the surface finish Marin factor k_a as

$$k_a = aS_{ut}^b = 2.70(55)^{-0.265} = 0.934$$

The next step is to estimate the size factor k_b. From Table 13–1, the sum of the addendum and dedendum is

$$l = \frac{1}{P} + \frac{1.25}{P} = \frac{1}{8} + \frac{1.25}{8} = 0.281 \text{ in}$$

Example 14–2

The tooth thickness t in Fig. 14–1b is given in Sec. 14–1 [Eq. (b)] as $t = (4lx)^{1/2}$ when $x = 3Y/(2P)$ from Eq. (14–3). Therefore, since from Ex. 14–1 $Y = 0.296$ and $P = 8$,

$$x = \frac{3Y}{2P} = \frac{3(0.296)}{2(8)} = 0.0555 \text{ in}$$

then

$$t = (4lx)^{1/2} = [4(0.281)0.0555]^{1/2} = 0.250 \text{ in}$$

We have recognized the tooth as a cantilever beam of rectangular cross section, so the equivalent rotating-beam diameter must be obtained from Eq. (6–25):

$$d_e = 0.808(hb)^{1/2} = 0.808(Ft)^{1/2} = 0.808[1.5(0.250)]^{1/2} = 0.495 \text{ in}$$

Then, Eq. (6–20) gives k_b as

$$k_b = \left(\frac{d_e}{0.30}\right)^{-0.107} = \left(\frac{0.495}{0.30}\right)^{-0.107} = 0.948$$

Example 14–2

The load factor k_c from Eq. (6–26) is unity. With no information given concerning temperature and reliability we will set $k_d = k_e = 1$.

In general, a gear tooth is subjected only to one-way bending. Exceptions include idler gears and gears used in reversing mechanisms. We will account for one-way bending by establishing a miscellaneous-effects Marin factor k_f.

For one-way bending the steady and alternating stress components are $\sigma_a = \sigma_m = \sigma/2$ where σ is the largest repeatedly applied bending stress as given in Eq. (14–7). If a material exhibited a Goodman failure locus,

$$\frac{S_a}{S'_e} + \frac{S_m}{S_{ut}} = 1$$

Since S_a and S_m are equal for one-way bending, we substitute S_a for S_m and solve the preceding equation for S_a, giving

$$S_a = \frac{S'_e S_{ut}}{S'_e + S_{ut}}$$

Example 14–2

Now replace S_a with $\sigma/2$, and in the denominator replace S'_e with $0.5S_{ut}$ to obtain

$$\sigma = \frac{2S'_e S_{ut}}{0.5S_{ut} + S_{ut}} = \frac{2S'_e}{0.5 + 1} = 1.33S'_e$$

Now $k_f = \sigma/S'_e = 1.33S'_e/S'_e = 1.33$. However, a Gerber fatigue locus gives mean values of

$$\frac{S_a}{S'_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

Setting $S_a = S_m$ and solving the quadratic in S_a gives

$$S_a = \frac{S_{ut}^2}{2S'_e}\left(-1 + \sqrt{1 + \frac{4S'^2_e}{S_{ut}^2}}\right)$$

Setting $S_a = \sigma/2$, $S_{ut} = S'_e/0.5$ gives

$$\sigma = \frac{S'_e}{0.5^2}\left[-1 + \sqrt{1 + 4(0.5)^2}\right] = 1.66S'_e$$

Example 14–2

and $k_f = \sigma/S'_e = 1.66$. Since a Gerber locus runs in and among fatigue data and Goodman does not, we will use $k_f = 1.66$. The Marin equation for the fully corrected endurance strength is

$$S_e = k_a k_b k_c k_d k_e k_f S'_e$$

$$= 0.934(0.948)(1)(1)(1)1.66(27.5) = 40.4 \text{ kpsi}$$

For stress, we will first determine the fatigue stress-concentration factor K_f. For a 20° full-depth tooth the radius of the root fillet is denoted r_f, where

$$r_f = \frac{0.300}{P} = \frac{0.300}{8} = 0.0375 \text{ in}$$

From Fig. A–15–6

$$\frac{r}{d} = \frac{r_f}{t} = \frac{0.0375}{0.250} = 0.15$$

Since $D/d = \infty$, we approximate with $D/d = 3$, giving $K_t = 1.68$. From Fig. 6–20, $q = 0.62$. From Eq. (6–32)

$$K_f = 1 + (0.62)(1.68 - 1) = 1.42$$

Example 14–2

For a design factor of $n_d = 3$, as used in Ex. 14–1, applied to the load or strength, the maximum bending stress is

$$\sigma_{\max} = K_f \sigma_{\text{all}} = \frac{S_e}{n_d}$$

$$\sigma_{\text{all}} = \frac{S_e}{K_f n_d} = \frac{40.4}{1.42(3)} = 9.5 \text{ kpsi}$$

The transmitted load W^t is

$$W^t = \frac{F Y \sigma_{\text{all}}}{K_v P} = \frac{1.5(0.296)9\,500}{1.52(8)} = 347 \text{ lbf}$$

and the power is, with $V = 628$ ft/min from Ex. 14–1,

$$hp = \frac{W^t V}{33\,000} = \frac{347(628)}{33\,000} = 6.6 \text{ hp}$$

Again, it should be emphasized that these results should be accepted *only* as preliminary estimates to alert you to the nature of bending in gear teeth.

Fatigue Stress-Concentration Factor

- A photoelastic investigation gives an estimate of fatigue stress-concentration factor as

$$K_f = H + \left(\frac{t}{r}\right)^L \left(\frac{t}{l}\right)^M \quad (14\text{–}9)$$

where
$$H = 0.34 - 0.458\,366\,2\phi$$
$$L = 0.316 - 0.458\,366\,2\phi$$
$$M = 0.290 + 0.458\,366\,2\phi$$
$$r = \frac{(b - r_f)^2}{(d/2) + b - r_f}$$

Surface Durability

- Another failure mode is wear due to contact stress.
- Modeling gear tooth mesh with contact stress between two cylinders, From Eq. (3–74),

$$p_{max} = \frac{2F}{\pi b l}$$

where p_{max} = largest surface pressure
F = force pressing the two cylinders together
l = length of cylinders

$$b = \left\{ \frac{2F}{\pi l} \frac{\left[\left(1-\nu_1^2\right)/E_1\right] + \left[\left(1-\nu_2^2\right)/E_2\right]}{(1/d_1) + (1/d_2)} \right\}^{1/2} \quad (14\text{–}10)$$

Surface Durability

- Converting to terms of gear tooth, the *surface compressive stress* (*Hertzian stress*) is found.

$$\sigma_C^2 = \frac{W^t}{\pi F \cos\phi} \frac{(1/r_1) + (1/r_2)}{\left[\left(1 - \nu_1^2\right)/E_1\right] + \left[\left(1 - \nu_2^2\right)/E_2\right]} \qquad (14\text{–}11)$$

- Critical location is usually at the pitch line, where

$$r_1 = \frac{d_P \sin\phi}{2} \qquad r_2 = \frac{d_G \sin\phi}{2} \qquad (14\text{–}12)$$

- Define *elastic coefficient* from denominator of Eq. (14–11),

$$C_p = \left[\frac{1}{\pi\left(\dfrac{1-\nu_P^2}{E_P} + \dfrac{1-\nu_G^2}{E_G}\right)}\right]^{1/2} \qquad (14\text{–}13)$$

Surface Durability

- Incorporating elastic coefficient and velocity factor, the contact stress equation is

$$\sigma_C = -C_p \left[\frac{K_v W^t}{F \cos\phi} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \right]^{1/2} \qquad (14\text{–}14)$$

- Again, this is useful for estimating, and as the basis for the preferred AGMA approach.

Example 14–3

The pinion of Examples 14–1 and 14–2 is to be mated with a 50-tooth gear manufactured of ASTM No. 50 cast iron. Using the tangential load of 382 lbf, estimate the factor of safety of the drive based on the possibility of a surface fatigue failure.

Solution

From Table A–5 we find the elastic constants to be $E_P = 30$ Mpsi, $\nu_P = 0.292$, $E_G = 14.5$ Mpsi, $\nu_G = 0.211$. We substitute these in Eq. (14–13) to get the elastic coefficient as

$$C_p = \left\{ \frac{1}{\pi \left[\frac{1-(0.292)^2}{30(10^6)} + \frac{1-(0.211)^2}{14.5(10^6)} \right]} \right\}^{1/2} = 1817$$

From Example 14–1, the pinion pitch diameter is $d_P = 2$ in. The value for the gear is $d_G = 50/8 = 6.25$ in. Then Eq. (14–12) is used to obtain the radii of curvature at the pitch points. Thus

$$r_1 = \frac{2 \sin 20^\circ}{2} = 0.342 \text{ in} \qquad r_2 = \frac{6.25 \sin 20^\circ}{2} = 1.069 \text{ in}$$

Example 14–3

The face width is given as $F = 1.5$ in. Use $K_v = 1.52$ from Example 14–1. Substituting all these values in Eq. (14–14) with $\phi = 20°$ gives the contact stress as

$$\sigma_C = -1817\left[\frac{1.52(380)}{1.5\cos 20°}\left(\frac{1}{0.342}+\frac{1}{1.069}\right)\right]^{1/2} = -72\ 400 \text{ psi}$$

The surface endurance strength of cast iron can be estimated from

$$S_C = 0.32 H_B \text{ kpsi}$$

for 10^8 cycles, where S_C is in kpsi. Table A–24 gives $H_B = 262$ for ASTM No. 50 cast iron. Therefore $S_C = 0.32(262) = 83.8$ kpsi. Contact stress is not linear with transmitted load [see Eq. (14–14)]. If the factor of safety is defined as the loss-of-function load divided by the imposed load, then the ratio of loads is the ratio of stresses squared. In other words,

$$n = \frac{\text{loss-of-function load}}{\text{imposed load}} = \frac{S_C^2}{\sigma_C^2} = \left(\frac{83.8}{72.4}\right)^2 = 1.34$$

AGMA Method

- The American Gear Manufacturers Association (AGMA) provides a recommended method for gear design.
- It includes bending stress and contact stress as two failure modes.
- It incorporates modifying factors to account for various situations.
- It imbeds much of the detail in tables and figures.

AGMA Bending Stress

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{bm_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases} \qquad (14\text{–}15)$$

where for U.S. customary units (SI units),

W^t is the tangential transmitted load, lbf (N)
K_o is the overload factor
K_v is the dynamic factor
K_s is the size factor
P_d is the transverse diametral pitch
F (b) is the face width of the narrower member, in (mm)
K_m (K_H) is the load-distribution factor
K_B is the rim-thickness factor
J (Y_J) is the geometry factor for bending strength (which includes root fillet stress-concentration factor K_f)
(m_t) is the transverse metric module

AGMA Contact Stress

$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F} \dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E \sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b} \dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases} \qquad (14\text{–}16)$$

where W^t, K_o, K_v, K_s, K_m, F, and b are the same terms as defined for Eq. (14–15). For U.S. customary units (SI units), the additional terms are

C_p (Z_E) is an elastic coefficient, $\sqrt{\text{lbf/in}^2}$ ($\sqrt{\text{N/mm}^2}$)
C_f (Z_R) is the surface condition factor
d_P (d_{w1}) is the pitch diameter of the *pinion*, in (mm)
I (Z_I) is the geometry factor for pitting resistance

AGMA Strengths

- AGMA uses *allowable stress numbers* rather than *strengths*.
- We will refer to them as strengths for consistency within the textbook.
- The gear strength values are only for use with the AGMA stress values, and should not be compared with other true material strengths.
- Representative values of typically available bending strengths are given in Table 14–3 for steel gears and Table 14–4 for iron and bronze gears.
- Figs. 14–2, 14–3, and 14–4 are used as indicated in the tables.
- Tables assume repeatedly applied loads at 10^7 cycles and 0.99 reliability.

Bending Strengths for Steel Gears

Table 14–3

Repeatedly Applied Bending Strength S_t at 10^7 Cycles and 0.99 Reliability for Steel Gears

Source: ANSI/AGMA 2001-D04.

Material Designation	Heat Treatment	Minimum Surface Hardness[1]	Allowable Bending Stress Number S_t,[2] psi Grade 1	Grade 2	Grade 3
Steel[3]	Through-hardened	See Fig. 14–2	See Fig. 14–2	See Fig. 14–2	—
	Flame[4] or induction hardened[4] with type A pattern[5]	See Table 8*	45 000	55 000	—
	Flame[4] or induction hardened[4] with type B pattern[5]	See Table 8*	22 000	22 000	—
	Carburized and hardened	See Table 9*	55 000	65 000 or 70 000[6]	75 000
	Nitrided[4,7] (through-hardened steels)	83.5 HR15N	See Fig. 14–3	See Fig. 14–3	—
Nitralloy 135M, Nitralloy N, and 2.5% chrome (no aluminum)	Nitrided[4,7]	87.5 HR15N	See Fig. 14–4	See Fig. 14–4	See Fig. 14–4

Bending Strengths for Iron and Bronze Gears

Table 14–4

Repeatedly Applied Bending Strength S_t for Iron and Bronze Gears at 10^7 Cycles and 0.99 Reliability

Source: ANSI/AGMA 2001-D04.

Material	Material Designation[1]	Heat Treatment	Typical Minimum Surface Hardness[2]	Allowable Bending Stress Number, S_t,[3] psi
ASTM A48 gray cast iron	Class 20	As cast	—	5000
	Class 30	As cast	174 HB	8500
	Class 40	As cast	201 HB	13 000
ASTM A536 ductile (nodular) Iron	Grade 60–40–18	Annealed	140 HB	22 000–33 000
	Grade 80–55–06	Quenched and tempered	179 HB	22 000–33 000
	Grade 100–70–03	Quenched and tempered	229 HB	27 000–40 000
	Grade 120–90–02	Quenched and tempered	269 HB	31 000–44 000
Bronze		Sand cast	Minimum tensile strength 40 000 psi	5700
	ASTM B–148 Alloy 954	Heat treated	Minimum tensile strength 90 000 psi	23 600

Bending Strengths for Through-hardened Steel Gears

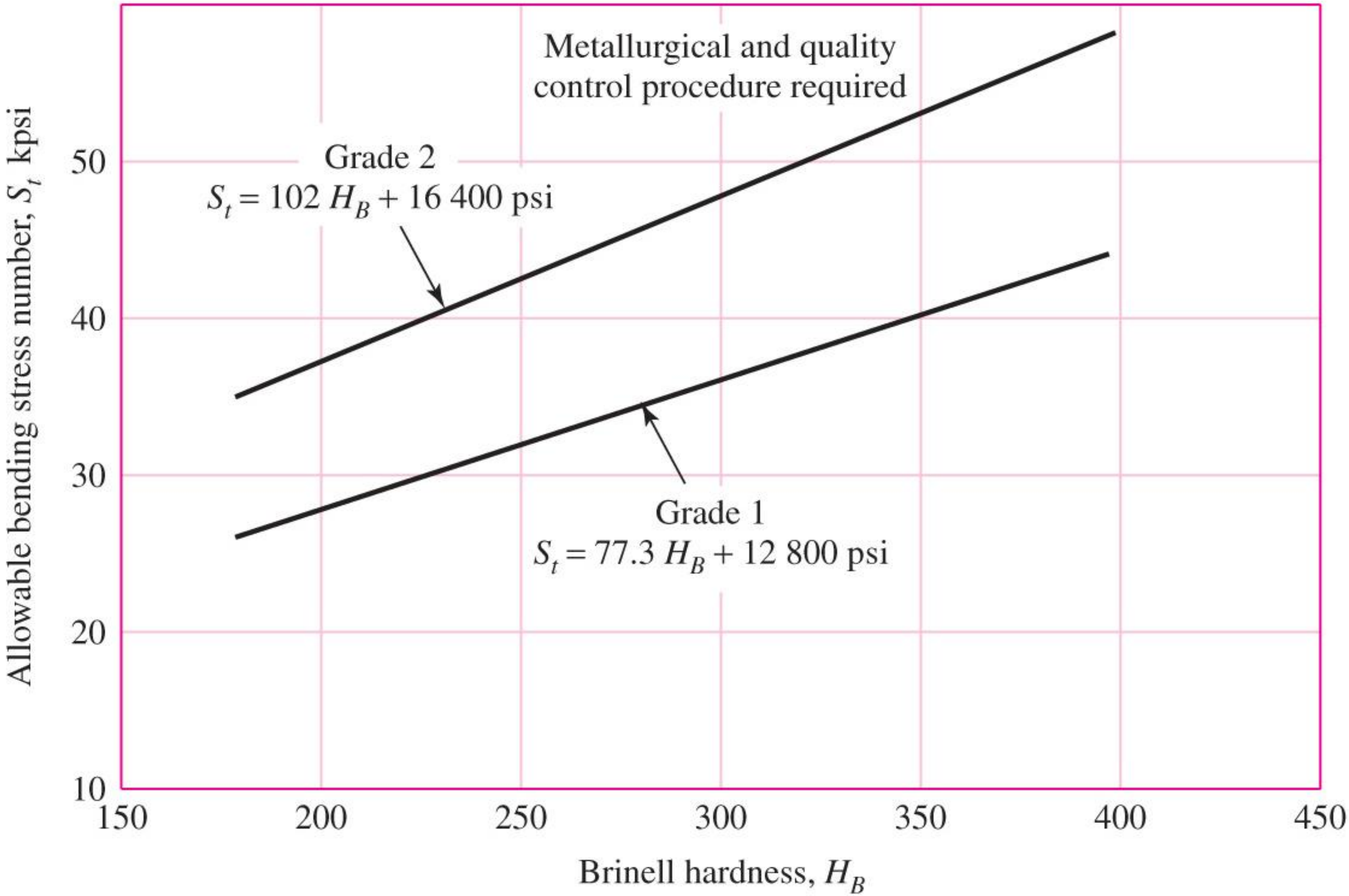


Fig. 14–2

Bending Strengths for Nitrided Through-hardened Steel Gears

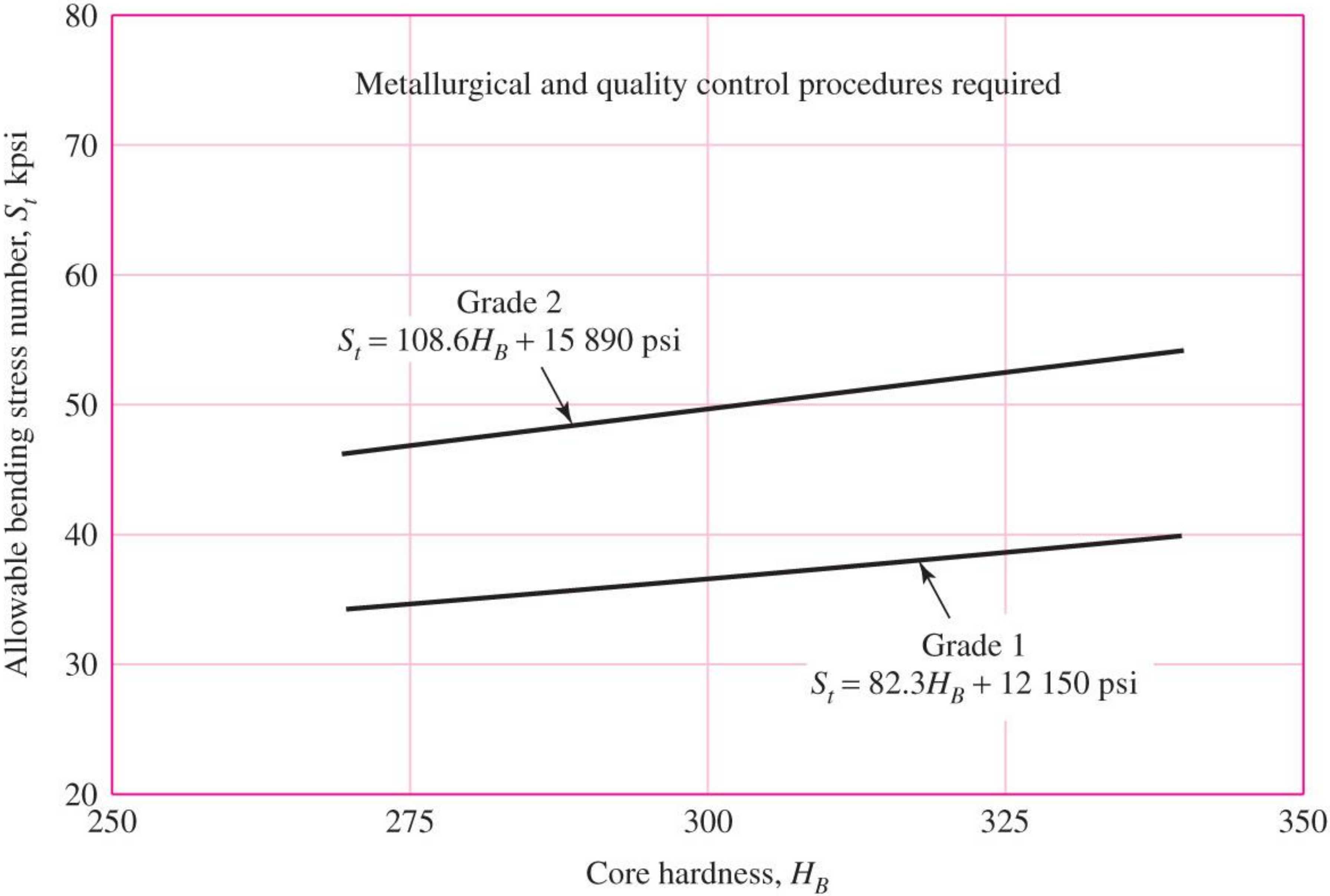


Fig. 14–3

Bending Strengths for Nitriding Steel Gears

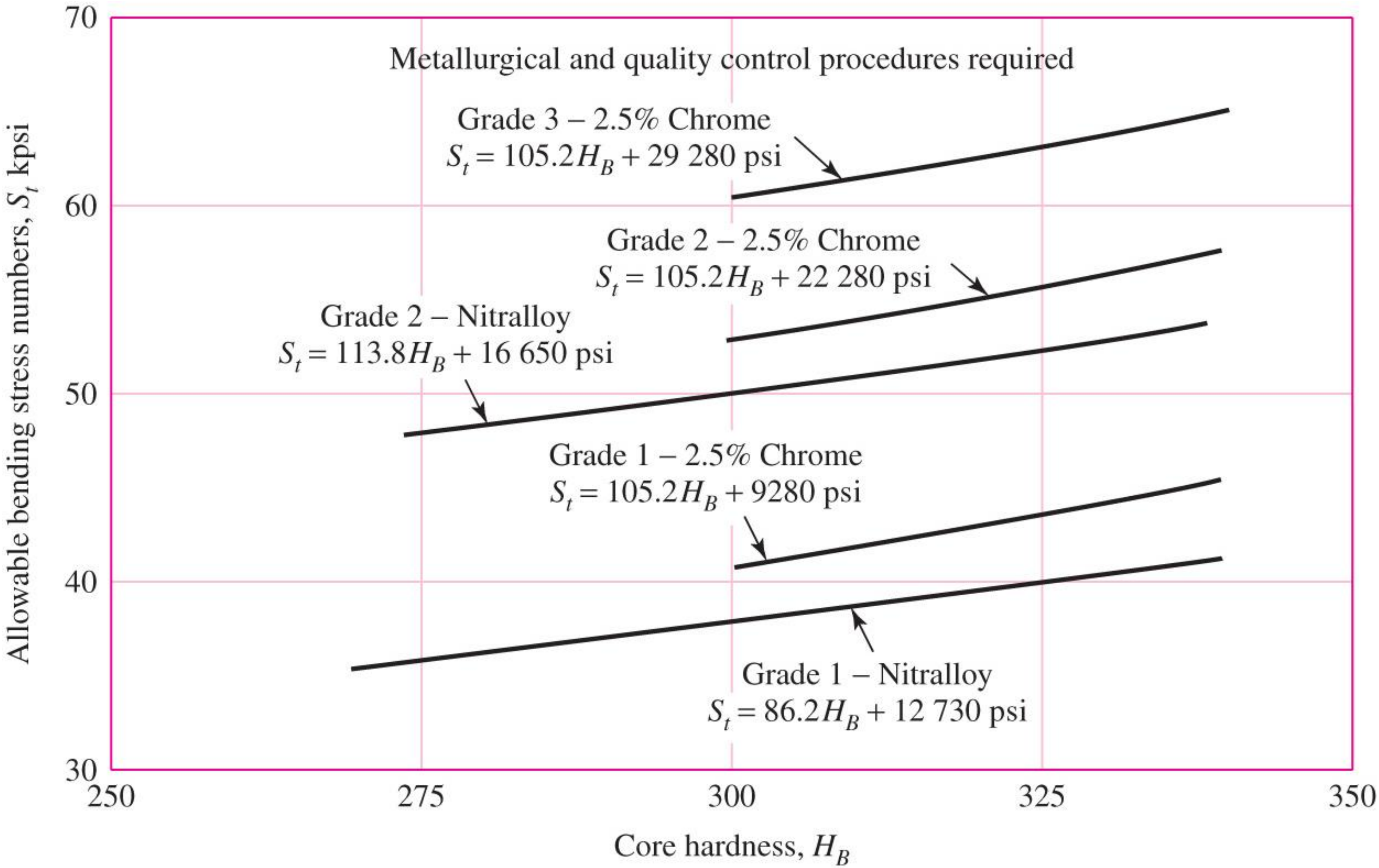


Fig. 14–4

Allowable Bending Stress

$$\sigma_{all} = \begin{cases} \dfrac{S_t}{S_F}\dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F}\dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases} \tag{14–17}$$

where for U.S. customary units (SI units),

S_t is the allowable bending stress, lbf/in^2 (N/mm^2)
Y_N is the stress cycle factor for bending stress
K_T (Y_θ) are the temperature factors
K_R (Y_Z) are the reliability factors
S_F is the AGMA factor of safety, a stress ratio

Allowable Contact Stress

$$\sigma_{c,\text{all}} = \begin{cases} \dfrac{S_c}{S_H}\dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_c}{S_H}\dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases} \qquad (14\text{–}18)$$

S_c is the allowable contact stress, lbf/in^2 (N/mm^2)

Z_N is the stress cycle life factor

C_H (Z_W) are the hardness ratio factors for pitting resistance

K_T (Y_θ) are the temperature factors

K_R (Y_Z) are the reliability factors

S_H is the AGMA factor of safety, a stress ratio

Nominal Temperature Used in Nitriding and Hardness Obtained

Steel	Temperature Before Nitriding, °F	Nitriding, °F	Hardness, Rockwell *C* Scale Case	Hardness, Rockwell *C* Scale Core
Nitralloy 135*	1150	975	62–65	30–35
Nitralloy 135M	1150	975	62–65	32–36
Nitralloy N	1000	975	62–65	40–44
AISI 4340	1100	975	48–53	27–35
AISI 4140	1100	975	49–54	27–35
31 Cr Mo V 9	1100	975	58–62	27–33

*Nitralloy is a trademark of the Nitralloy Corp., New York.

Table 14–5

Contact Strength for Steel Gears

Table 14–6

Repeatedly Applied Contact Strength S_c at 10^7 Cycles and 0.99 Reliability for Steel Gears

Source: ANSI/AGMA 2001-D04.

Material Designation	Heat Treatment	Minimum Surface Hardness[1]	Allowable Contact Stress Number,[2] S_c, psi Grade 1	Grade 2	Grade 3
Steel[3]	Through hardened[4]	See Fig. 14–5	See Fig. 14–5	See Fig. 14–5	—
	Flame[5] or induction hardened[5]	50 HRC	170 000	190 000	—
		54 HRC	175 000	195 000	—
	Carburized and hardened[5]	See Table 9*	180 000	225 000	275 000
	Nitrided[5] (through hardened steels)	83.5 HR15N	150 000	163 000	175 000
		84.5 HR15N	155 000	168 000	180 000
2.5% chrome (no aluminum)	Nitrided[5]	87.5 HR15N	155 000	172 000	189 000
Nitralloy 135M	Nitrided[5]	90.0 HR15N	170 000	183 000	195 000
Nitralloy N	Nitrided[5]	90.0 HR15N	172 000	188 000	205 000
2.5% chrome (no aluminum)	Nitrided[5]	90.0 HR15N	176 000	196 000	216 000

Contact Strength for Iron and Bronze Gears

Table 14–7

Repeatedly Applied Contact Strength S_c 10^7 Cycles and 0.99 Reliability for Iron and Bronze Gears

Source: ANSI/AGMA 2001-D04.

Material	Material Designation[1]	Heat Treatment	Typical Minimum Surface Hardness[2]	Allowable Contact Stress Number,[3] S_c, psi
ASTM A48 gray cast iron	Class 20	As cast	—	50 000–60 000
	Class 30	As cast	174 HB	65 000–75 000
	Class 40	As cast	201 HB	75 000–85 000
ASTM A536 ductile (nodular) iron	Grade 60–40–18	Annealed	140 HB	77 000–92 000
	Grade 80–55–06	Quenched and tempered	179 HB	77 000–92 000
	Grade 100–70–03	Quenched and tempered	229 HB	92 000–112 000
	Grade 120–90–02	Quenched and tempered	269 HB	103 000–126 000
Bronze	—	Sand cast	Minimum tensile strength 40 000 psi	30 000
	ASTM B-148 Alloy 954	Heat treated	Minimum tensile strength 90 000 psi	65 000

Contact Strength for Through-hardened Steel Gears

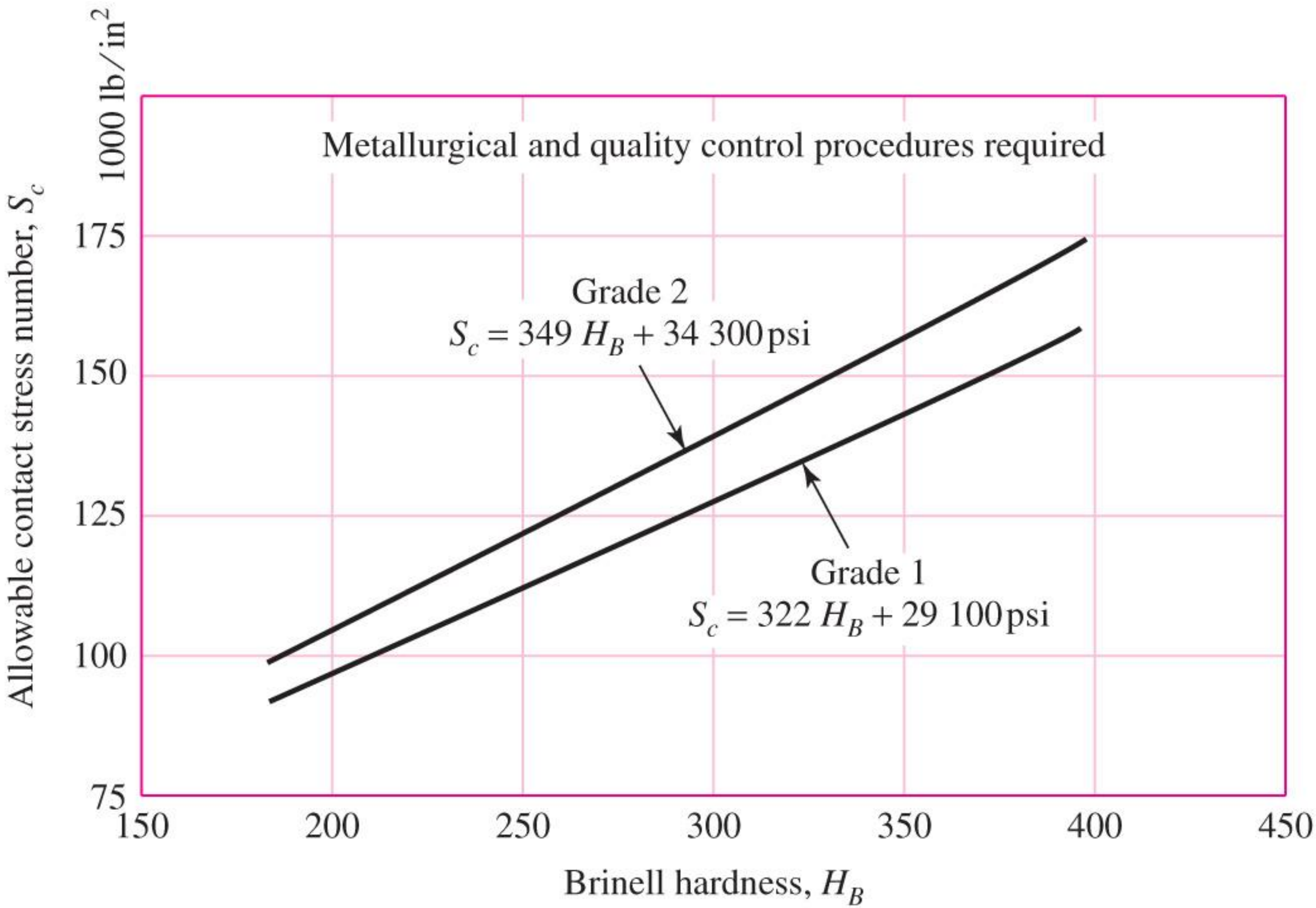


Fig. 14–5

Geometry Factor J (Y_J in metric)

- Accounts for shape of tooth in bending stress equation
- Includes
  - A modification of the Lewis form factor Y
  - Fatigue stress-concentration factor K_f
  - Tooth *load-sharing ratio* m_N
- AGMA equation for geometry factor is

$$J = \frac{Y}{K_f m_N} \qquad (14\text{–}20)$$

$$m_N = \frac{p_N}{0.95Z} \qquad (14\text{–}21)$$

- Values for Y and Z are found in the AGMA standards.
- For most common case of spur gear with 20º pressure angle, J can be read directly from Fig. 14–6.
- For helical gears with 20º normal pressure angle, use Figs. 14–7 and 14–8.

Spur-Gear Geometry Factor *J*

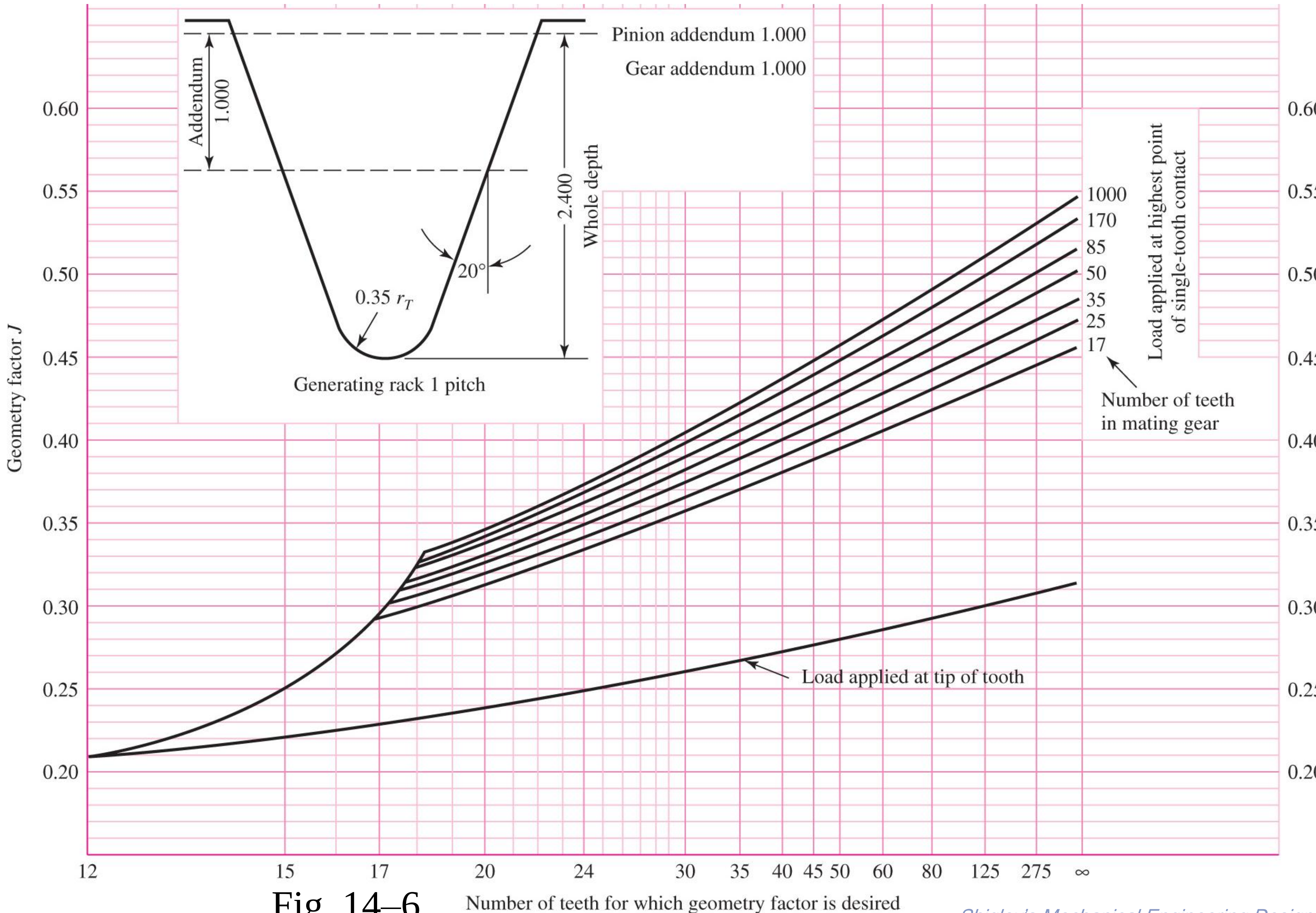


Fig. 14–6

Helical-Gear Geometry Factor *J*

- Get *J′* from Fig. 14–7, which assumes the mating gear has 75 teeth
- Get multiplier from Fig. 14–8 for mating gear with other than 75 teeth
- Obtain *J* by applying multiplier to *J′*

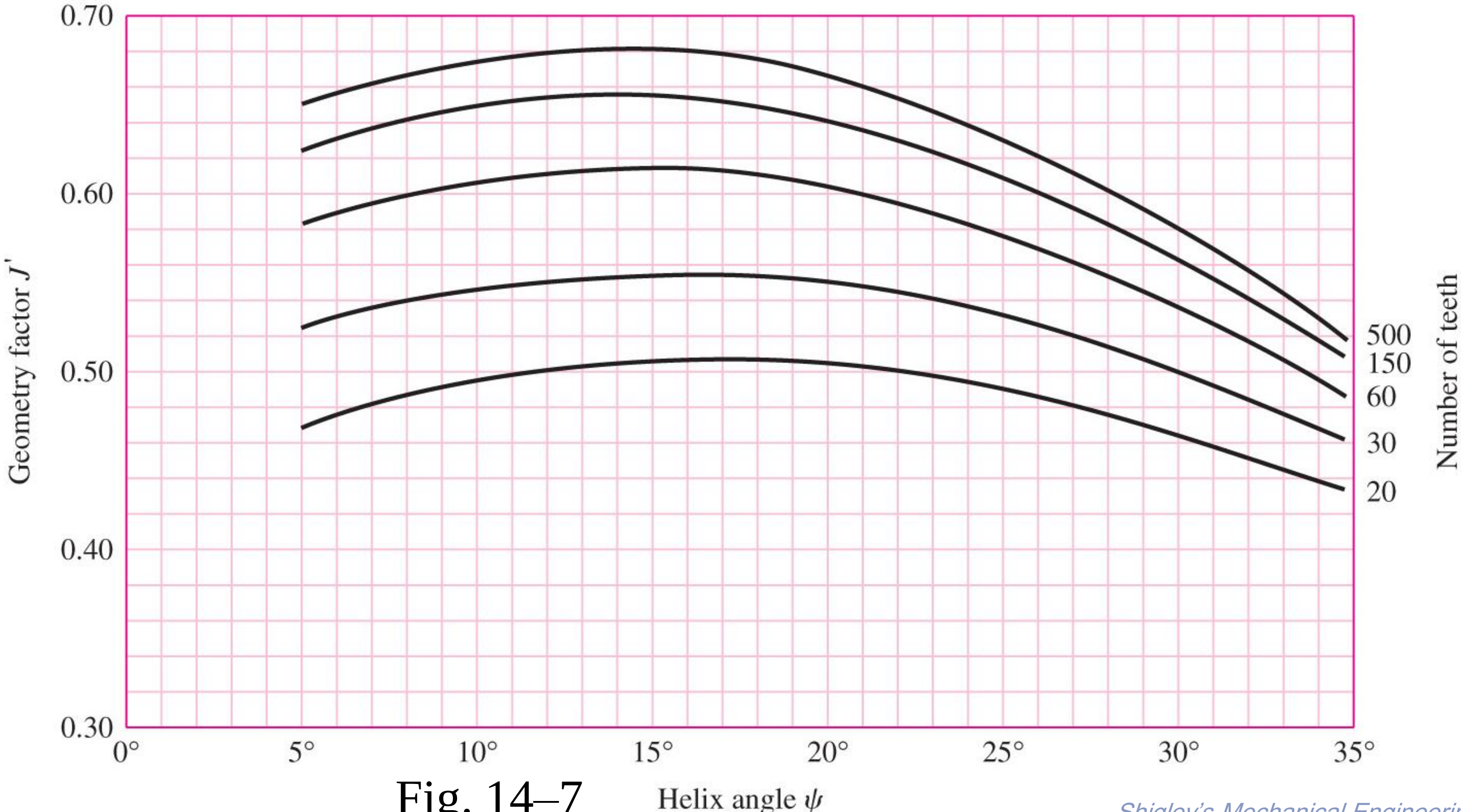


Fig. 14–7

Modifying Factor for *J*

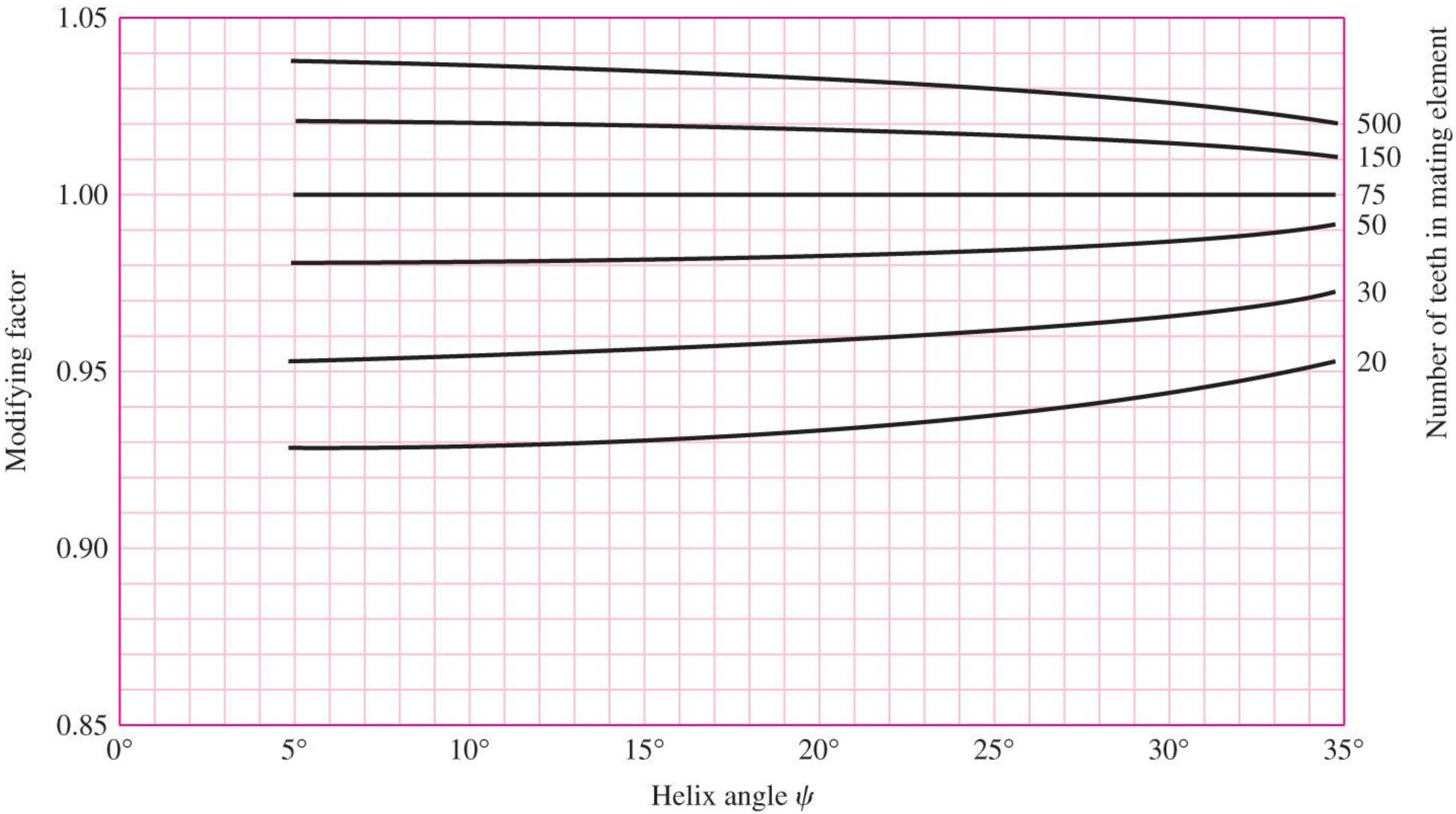


Fig. 14–8

Surface Strength Geometry Factor I (Z_I in metric)

- Called *pitting resistance geometry factor* by AGMA

$$I = \begin{cases} \dfrac{\cos\phi_t \sin\phi_t}{2m_N} \dfrac{m_G}{m_G + 1} & \text{external gears} \\ \dfrac{\cos\phi_t \sin\phi_t}{2m_N} \dfrac{m_G}{m_G - 1} & \text{internal gears} \end{cases} \qquad (14\text{–}23)$$

$$m_G = \frac{N_G}{N_P} = \frac{d_G}{d_P} \qquad (14\text{–}22)$$

$$m_N = \frac{p_N}{0.95Z} \qquad (14\text{–}21)$$

$$p_N = p_n \cos\phi_n \qquad (14\text{–}24)$$

$$Z = \left[(r_P + a)^2 - r_{bP}^2\right]^{1/2} + \left[(r_G + a)^2 - r_{bG}^2\right]^{1/2} - (r_P + r_G)\sin\phi_t \qquad (14\text{–}25)$$

$$r_b = r\cos\phi_t \qquad (14\text{–}26)$$

Elastic Coefficient $C_P (Z_E)$

- Obtained from Eq. (14–13) or from Table 14–8.

$$C_p = \left[\frac{1}{\pi \left(\frac{1 - \nu_P^2}{E_P} + \frac{1 - \nu_G^2}{E_G} \right)} \right]^{1/2} \tag{14–13}$$

Elastic Coefficient

Table 14–8

Elastic Coefficient C_p (Z_E), $\sqrt{\text{psi}}$ ($\sqrt{\text{MPa}}$) *Source: AGMA 218.01*

		Gear Material and Modulus of Elasticity E_G, lbf/in^2 (MPa)*					
Pinion Material	**Pinion Modulus of Elasticity E_p psi (MPa)***	**Steel 30×10^6 (2×10^5)**	**Malleable Iron 25×10^6 (1.7×10^5)**	**Nodular Iron 24×10^6 (1.7×10^5)**	**Cast Iron 22×10^6 (1.5×10^5)**	**Aluminum Bronze 17.5×10^6 (1.2×10^5)**	**Tin Bronze 16×10^6 (1.1×10^5)**
Steel	30×10^6 (2×10^5)	2300 (191)	2180 (181)	2160 (179)	2100 (174)	1950 (162)	1900 (158)
Malleable iron	25×10^6 (1.7×10^5)	2180 (181)	2090 (174)	2070 (172)	2020 (168)	1900 (158)	1850 (154)
Nodular iron	24×10^6 (1.7×10^5)	2160 (179)	2070 (172)	2050 (170)	2000 (166)	1880 (156)	1830 (152)
Cast iron	22×10^6 (1.5×10^5)	2100 (174)	2020 (168)	2000 (166)	1960 (163)	1850 (154)	1800 (149)
Aluminum bronze	17.5×10^6 (1.2×10^5)	1950 (162)	1900 (158)	1880 (156)	1850 (154)	1750 (145)	1700 (141)
Tin bronze	16×10^6 (1.1×10^5)	1900 (158)	1850 (154)	1830 (152)	1800 (149)	1700 (141)	1650 (137)

Dynamic Factor K_v

- Accounts for increased forces with increased speed
- Affected by manufacturing quality of gears
- A set of *quality numbers* define tolerances for gears manufactured to a specified accuracy.
- Quality numbers 3 to 7 include most commercial-quality gears.
- Quality numbers 8 to 12 are of precision quality.
- The AGMA *transmission accuracy-level number* Q_v is basically the same as the quality number.

Dynamic Factor K_v

- Dynamic Factor equation

$$K_v = \begin{cases} \left(\dfrac{A + \sqrt{V}}{A}\right)^B & V \text{ in ft/min} \\ \left(\dfrac{A + \sqrt{200V}}{A}\right)^B & V \text{ in m/s} \end{cases} \quad (14\text{–}27)$$

$$A = 50 + 56(1 - B)$$
$$B = 0.25(12 - Q_v)^{2/3} \quad (14\text{–}28)$$

- Or can obtain value directly from Fig. 14–9
- Maximum recommended velocity for a given quality number,

$$(V_t)_{\max} = \begin{cases} [A + (Q_v - 3)]^2 & \text{ft/min} \\ \dfrac{[A + (Q_v - 3)]^2}{200} & \text{m/s} \end{cases} \quad (14\text{–}29)$$

Dynamic Factor K_v

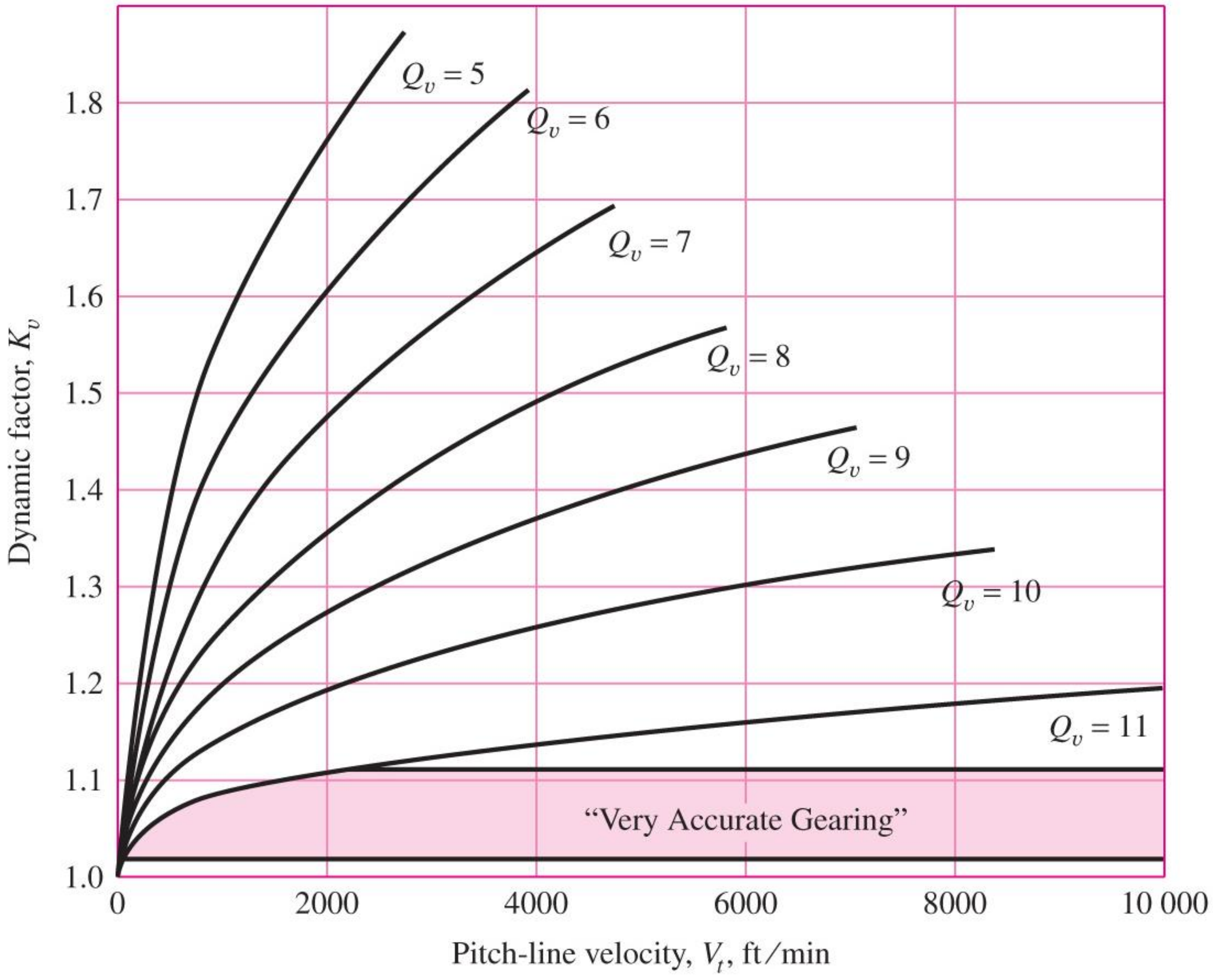


Fig. 14–9

Overload Factor K_o

- To account for likelihood of increase in nominal tangential load due to particular application.
- Recommended values,

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Surface Condition Factor $C_f\,(Z_R)$

- To account for detrimental surface finish
- No values currently given by AGMA
- Use value of 1 for normal commercial gears

Size Factor K_s

- Accounts for fatigue size effect, and non-uniformity of material properties for large sizes
- AGMA has not established size factors
- Use 1 for normal gear sizes
- Could apply fatigue size factor method from Ch. 6, where this size factor is the reciprocal of the Marin size factor k_b. Applying known geometry information for the gear tooth,

$$K_s = \frac{1}{k_b} = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535}$$

Load-Distribution Factor K_m (K_H)

- Accounts for non-uniform distribution of load across the line of contact
- Depends on mounting and face width
- Load-distribution factor is currently only defined for
  - Face width to pinion pitch diameter ratio $F/d \leq 2$
  - Gears mounted between bearings
  - Face widths up to 40 in
  - Contact across the full width of the narrowest member

Load-Distribution Factor K_m (K_H)

- Face load-distribution factor

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) \qquad (14\text{–}30)$$

$$C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases} \qquad (14\text{–}31)$$

$$C_{pf} = \begin{cases} \dfrac{F}{10d} - 0.025 & F \le 1 \text{ in} \\ \dfrac{F}{10d} - 0.0375 + 0.0125F & 1 < F \le 17 \text{ in} \\ \dfrac{F}{10d} - 0.1109 + 0.0207F - 0.000\,228F^2 & 17 < F \le 40 \text{ in} \end{cases} \qquad (14\text{–}32)$$

$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or compatibility} \\ & \text{is improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases} \qquad (14\text{–}35)$$

Load-Distribution Factor K_m (K_H)

$$C_{pm} = \begin{cases} 1 & \text{for straddle-mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle-mounted pinion with } S_1/S \geq 0.175 \end{cases} \qquad (14\text{–}33)$$

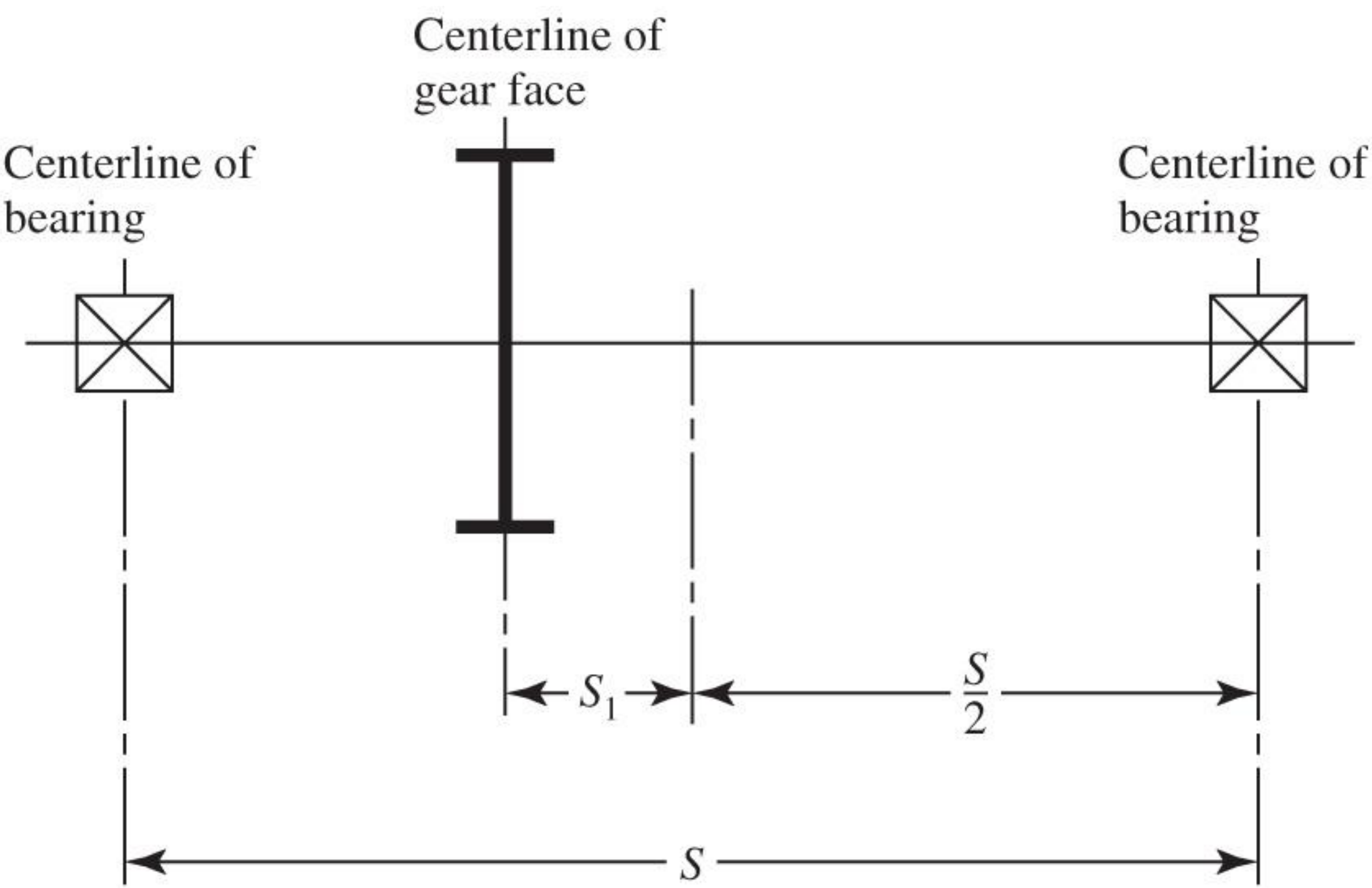


Fig. 14–10

Load-Distribution Factor K_m (K_H)

- C_{ma} can be obtained from Eq. (14–34) with Table 14–9

$$C_{ma} = A + BF + CF^2 \qquad \text{(see Table 14–9 for values of } A, B, \text{and } C) \qquad (14\text{–}34)$$

Table 14–9

Empirical Constants A, B, and C for Eq. (14–34), Face Width F in Inches*

Source: ANSI/AGMA 2001-D04.

Condition	A	B	C
Open gearing	0.247	0.0167	$-0.765(10^{-4})$
Commercial, enclosed units	0.127	0.0158	$-0.930(10^{-4})$
Precision, enclosed units	0.0675	0.0128	$-0.926(10^{-4})$
Extraprecision enclosed gear units	0.00360	0.0102	$-0.822(10^{-4})$

*See ANSI/AGMA 2101-D04, pp. 20–22, for SI formulation.

- Or can read C_{ma} directly from Fig. 14–11

Load-Distribution Factor K_m (K_H)

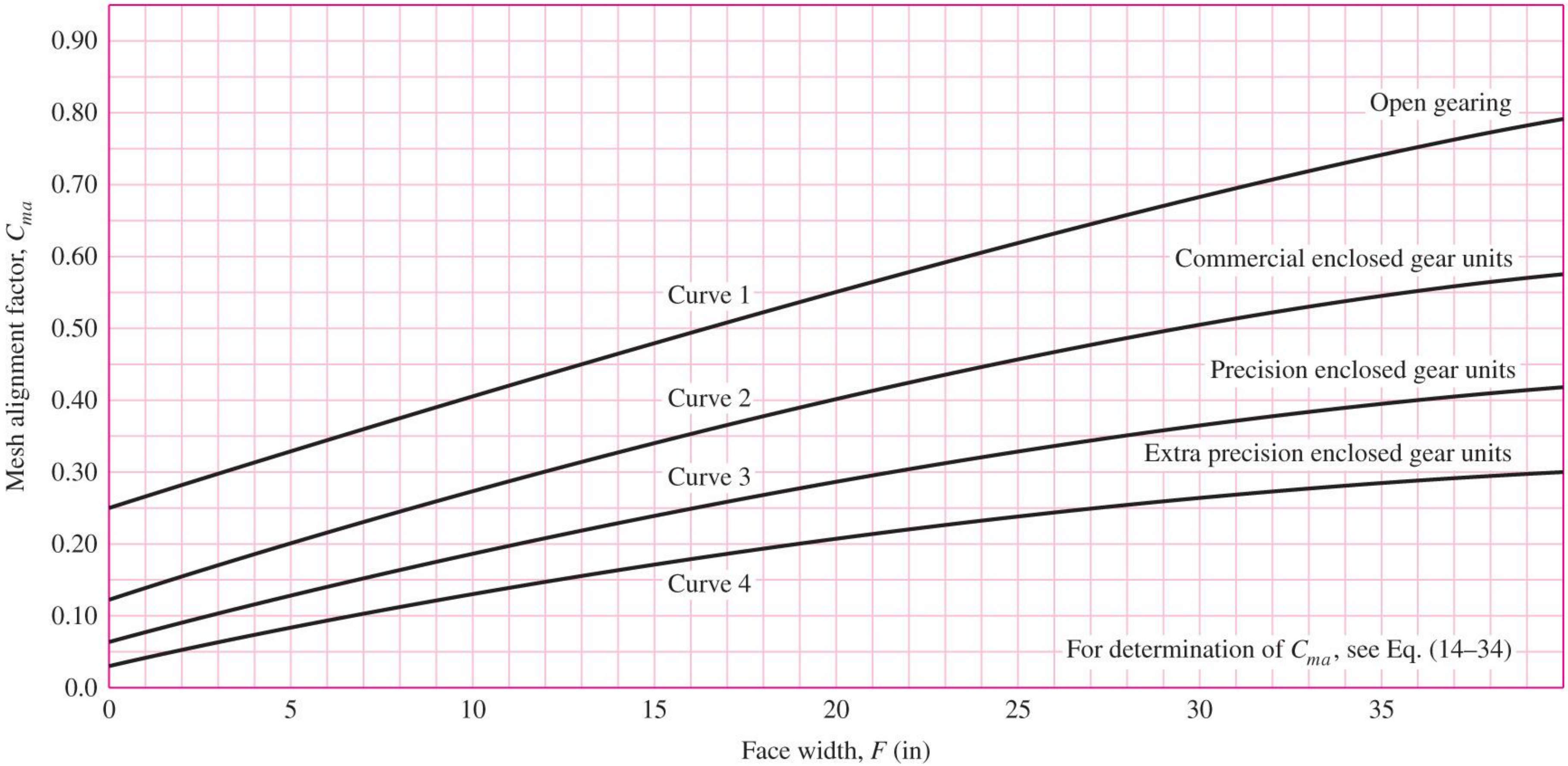


Fig. 14–11

Hardness-Ratio Factor C_H (Z_W)

- Since the pinion is subjected to more cycles than the gear, it is often hardened more than the gear.
- The hardness-ratio factor accounts for the difference in hardness of the pinion and gear.
- C_H is only applied to the gear. That is, $C_H = 1$ for the pinion.
- For the gear,

$$C_H = 1.0 + A'(m_G - 1.0) \qquad (14\text{–}36)$$

$$A' = 8.98(10^{-3})\left(\frac{H_{BP}}{H_{BG}}\right) - 8.29(10^{-3})\quad 1.2 \le \frac{H_{BP}}{H_{BG}} \le 1.7$$

- Eq. (14–36) in graph form is given in Fig. 14–12.

Hardness-Ratio Factor C_H

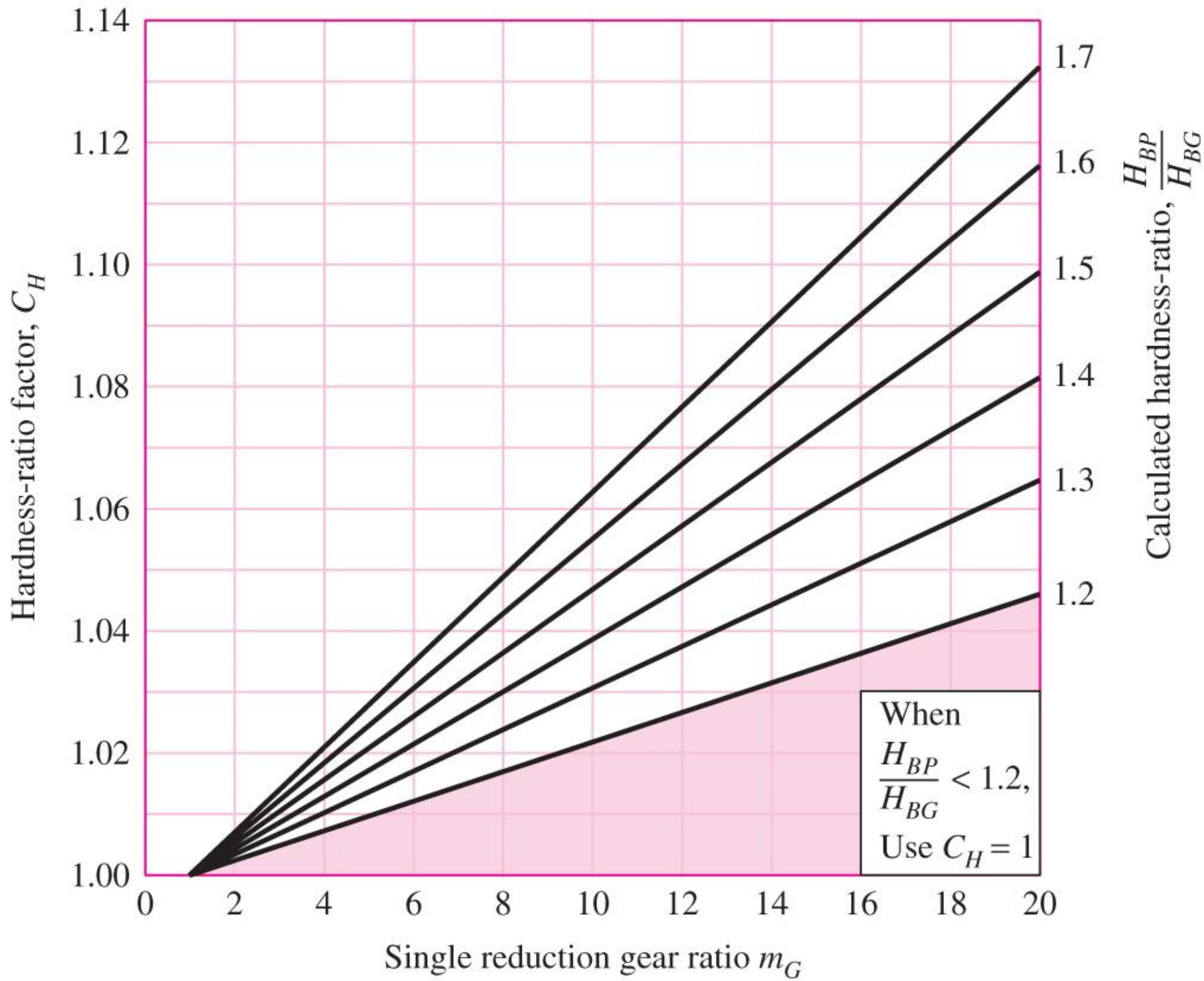


Fig. 14–12

Hardness-Ratio Factor

- If the pinion is surface-hardened to 48 Rockwell C or greater, the softer gear can experience work-hardening during operation. In this case,

$$C_H = 1 + B'(450 - H_{BG}) \qquad (14\text{–}37)$$

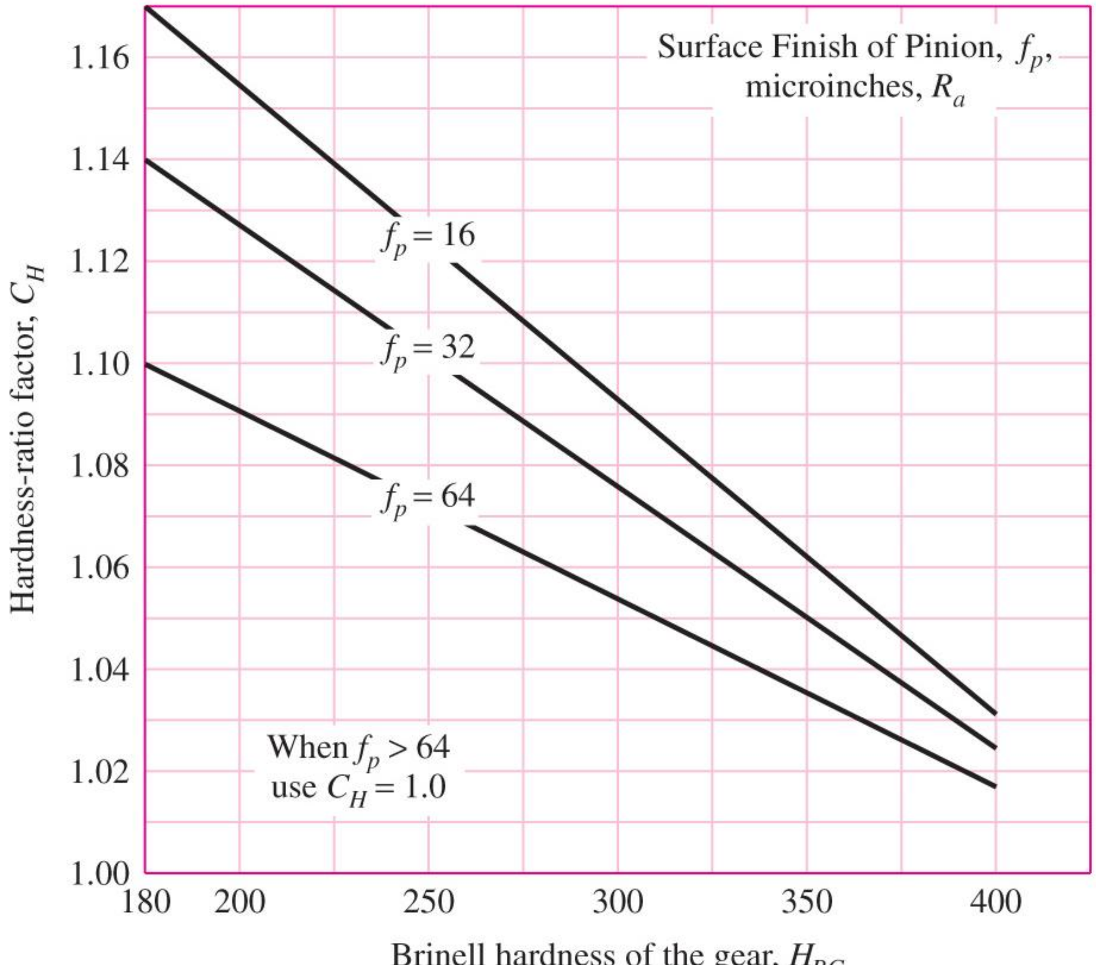


Fig. 14–13

Stress-Cycle Factors Y_N and Z_N

- AGMA strengths are for 10^7 cycles
- Stress-cycle factors account for other design cycles
- Fig. 14–14 gives Y_N for bending
- Fig. 14–15 gives Z_N for contact stress

Stress-Cycle Factor Y_N

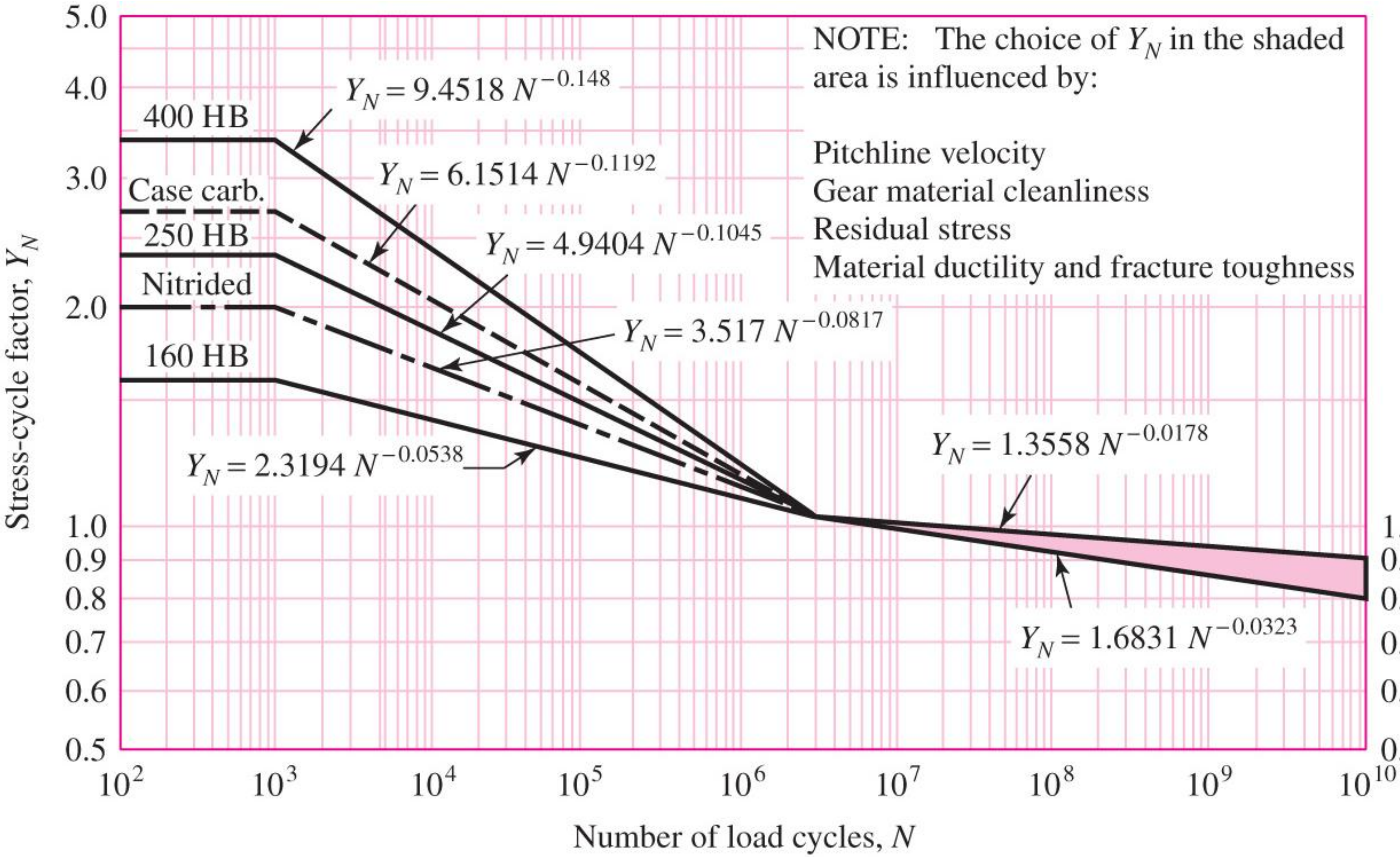


Fig. 14–14

Stress-Cycle Factor Z_N

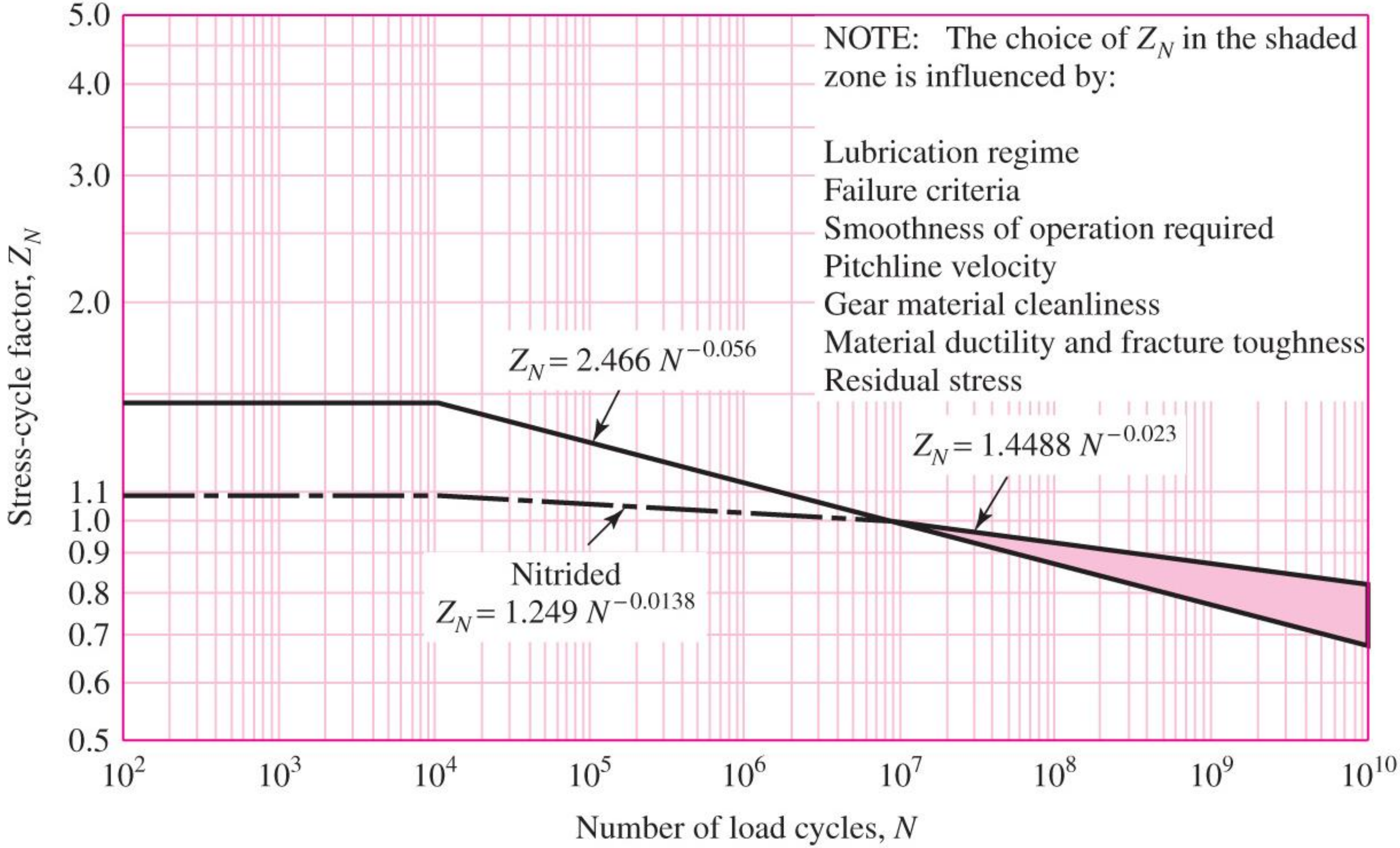


Fig. 14–15

Reliability Factor K_R (Y_Z)

- Accounts for statistical distributions of material fatigue failures
- Does not account for load variation
- Use Table 14–10
- Since reliability is highly nonlinear, if interpolation between table values is needed, use the least-squares regression fit,

$$K_R = \begin{cases} 0.658 - 0.0759\ln(1-R) & 0.5 < R < 0.99 \\ 0.50 - 0.109\ln(1-R) & 0.99 \le R \le 0.9999 \end{cases} \qquad (14\text{–}38)$$

Reliability	K_R (Y_Z)
0.9999	1.50
0.999	1.25
0.99	1.00
0.90	0.85
0.50	0.70

Table 14–10

Temperature Factor K_T (Y_θ)

- AGMA has not established values for this factor.
- For temperatures up to 250ºF (120ºC), $K_T = 1$ is acceptable.

Rim-Thickness Factor K_B

- Accounts for bending of rim on a gear that is not solid

$$K_B = \begin{cases} 1.6 \ln \dfrac{2.242}{m_B} & m_B < 1.2 \\ 1 & m_B \geq 1.2 \end{cases} \qquad (14\text{–}40)$$

$$m_B = \frac{t_R}{h_t} \qquad (14\text{–}39)$$

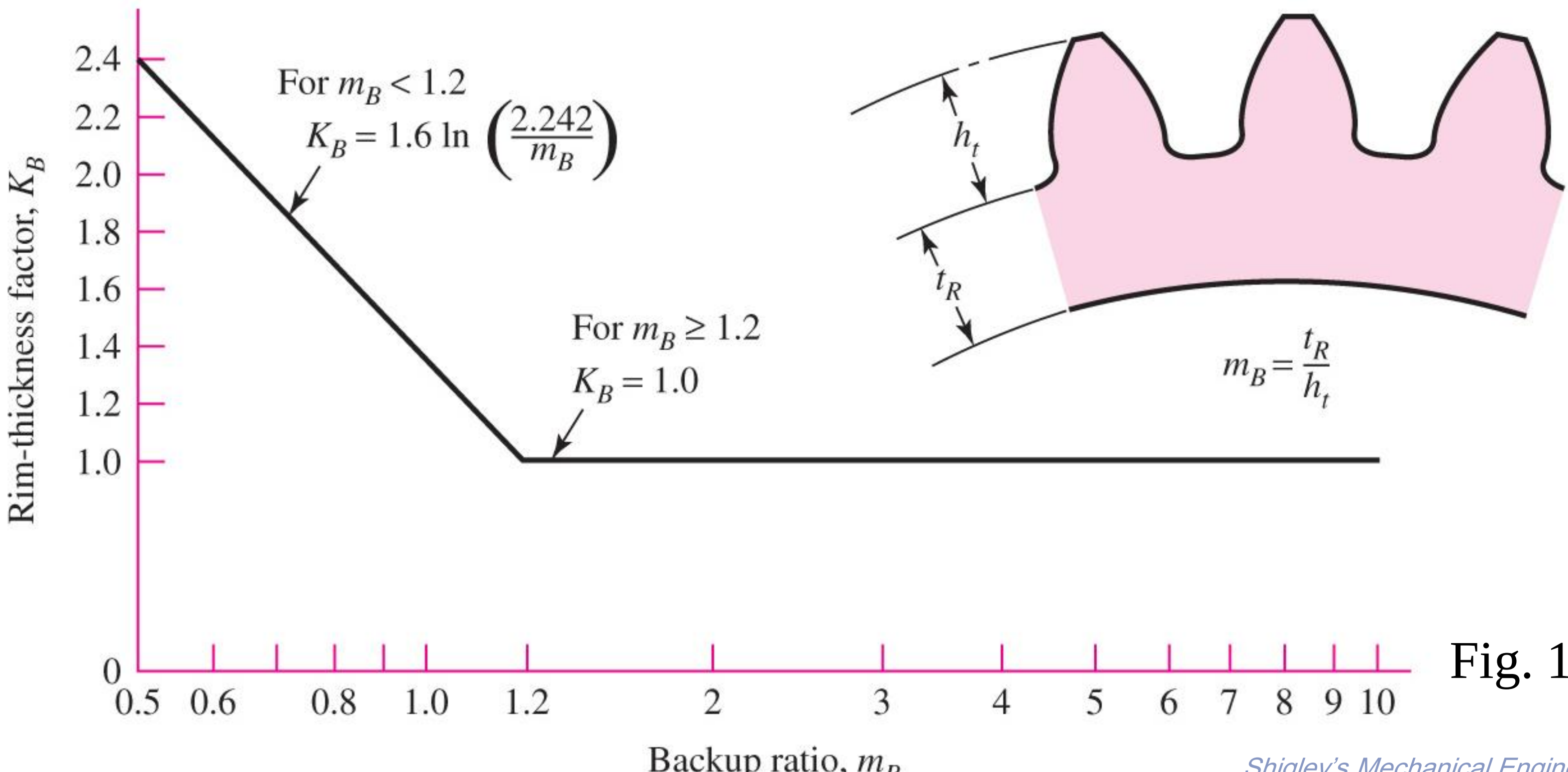


Fig. 14–16

Safety Factors S_F and S_H

- Included as design factors in the strength equations
- Can be solved for and used as factor of safety

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \quad (14\text{–}41)$$

$$S_H = \frac{S_c Z_N C_H/(K_T K_R)}{\sigma_c} = \frac{\text{fully corrected contact strength}}{\text{contact stress}} \quad (14\text{–}42)$$

- Or, can set equal to unity, and solve for traditional factor of safety as $n = \sigma_{\text{all}}/\sigma$

Comparison of Factors of Safety

- Bending stress is linear with transmitted load.
- Contact stress is not linear with transmitted load
- To compare the factors of safety between the different failure modes, to determine which is critical,
  - Compare S_F with S_H^2 for linear or helical contact
  - Compare S_F with S_H^3 for spherical contact

Summary for Bending of Gear Teeth

$$d_P = \frac{N_P}{P_d}$$

$$V = \frac{\pi d n}{12}$$

$$W^t = \frac{33\,000\ H}{V}$$

Gear bending stress equation Eq. (14–15)

$$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

- K_o: Table below
- K_v: Eq. (14–27); p. 756
- K_s: 1 [or Eq. (a), Sec. 14–10]; p. 759
- K_m: Eq. (14–30); p. 759
- K_B: Eq. (14–40); p. 764
- J: Fig. 14–6; p. 753

Gear bending endurance strength equation Eq. (14–17)

$$\sigma_{all} = \frac{S_t}{S_F} \frac{Y_N}{K_T K_R}$$

- S_t: ${}_{0.99}(S_t)_{10^7}$ Tables 14–3, 14–4; pp. 748, 749
- Y_N: Fig. 14–14; p. 763
- K_T: 1 if $T < 250°$F
- K_R: Table 14–10, Eq. (14–38); pp. 763, 764

Bending factor of safety Eq. (14–41)

$$S_F = \frac{S_t Y_N / (K_T K_R)}{\sigma}$$

Fig. 14–17

Summary for Surface Wear of Gear Teeth

$$d_P = \frac{N_P}{P_d}$$

$$V = \frac{\pi d n}{12}$$

$$W^t = \frac{33\,000\ H}{V}$$

Gear contact stress equation Eq. (14–16)

$$\sigma_c = C_p \left(W^t K_o K_v K_s \frac{K_m}{d_P F} \frac{C_f}{I} \right)^{1/2}$$

- C_p: Eq. (14–13), Table 14–8; pp. 744, 757
- K_o: Table below
- K_v: Eq. (14–27); p. 756
- K_s: 1 [or Eq. (a), Sec. 14–10]; p. 759
- K_m: Eq. (14–30); p. 759
- C_f: 1
- I: Eq. (14–23); p. 755

Gear contact endurance strength Eq. (14–18)

$$\sigma_{c,\text{all}} = \frac{S_c Z_N C_H}{S_H K_T K_R}$$

- S_c: ${}_{0.99}(S_c)_{10^7}$ Tables 14–6, 14–7; pp. 751, 752
- Z_N: Fig. 14–15; p. 763
- C_H: Section 14–12, gear only; pp. 761, 762
- K_T: 1 if $T < 250°F$
- K_R: Table 14–10, Eq. (14–38); pp. 763, 764

Wear factor of safety Eq. (14–42)

$$S_H = \frac{S_c Z_N C_H / (K_T K_R)}{\sigma_c}$$

- C_H: Gear only

Fig. 14–18

Example 14–4

A 17-tooth 20° pressure angle spur pinion rotates at 1800 rev/min and transmits 4 hp to a 52-tooth disk gear. The diametral pitch is 10 teeth/in, the face width 1.5 in, and the quality standard is No. 6. The gears are straddle-mounted with bearings immediately adjacent. The pinion is a grade 1 steel with a hardness of 240 Brinell tooth surface and through-hardened core. The gear is steel, through-hardened also, grade 1 material, with a Brinell hardness of 200, tooth surface and core. Poisson's ratio is 0.30, $J_P = 0.30$, $J_G = 0.40$, and Young's modulus is $30(10^6)$ psi. The loading is smooth because of motor and load. Assume a pinion life of 10^8 cycles and a reliability of 0.90, and use $Y_N = 1.3558N^{-0.0178}$, $Z_N = 1.4488N^{-0.023}$. The tooth profile is uncrowned. This is a commercial enclosed gear unit.

(*a*) Find the factor of safety of the gears in bending.

(*b*) Find the factor of safety of the gears in wear.

(*c*) By examining the factors of safety, identify the threat to each gear and to the mesh.

Example 14–4

There will be many terms to obtain so use Figs. 14–17 and 14–18 as guides to what is needed.

$$d_P = N_P/P_d = 17/10 = 1.7 \text{ in} \qquad d_G = 52/10 = 5.2 \text{ in}$$

$$V = \frac{\pi d_P n_P}{12} = \frac{\pi(1.7)1800}{12} = 801.1 \text{ ft/min}$$

$$W^t = \frac{33\,000\ H}{V} = \frac{33\ 000(4)}{801.1} = 164.8 \text{ lbf}$$

Example 14–4

Assuming uniform loading, $K_o = 1$. To evaluate K_v, from Eq. (14–28) with a quality number $Q_v = 6$,

$$B = 0.25(12 - 6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

Then from Eq. (14–27) the dynamic factor is

$$K_v = \left(\frac{59.77 + \sqrt{801.1}}{59.77}\right)^{0.8255} = 1.377$$

Example 14–4

To determine the size factor, K_s, the Lewis form factor is needed. From Table 14–2, with $N_P = 17$ teeth, $Y_P = 0.303$. Interpolation for the gear with $N_G = 52$ teeth yields $Y_G = 0.412$. Thus from Eq. (*a*) of Sec. 14–10, with $F = 1.5$ in,

$$(K_s)_P = 1.192\left(\frac{1.5\sqrt{0.303}}{10}\right)^{0.0535} = 1.043$$

$$(K_s)_G = 1.192\left(\frac{1.5\sqrt{0.412}}{10}\right)^{0.0535} = 1.052$$

Example 14–4

The load distribution factor K_m is determined from Eq. (14–30), where five terms are needed. They are, where $F = 1.5$ in when needed:

Uncrowned, Eq. (14–30): $C_{mc} = 1$,
Eq. (14–32): $C_{pf} = 1.5/[10(1.7)] - 0.0375 + 0.0125(1.5) = 0.0695$
Bearings immediately adjacent, Eq. (14–33): $C_{pm} = 1$
Commercial enclosed gear units (Fig. 14–11): $C_{ma} = 0.15$
Eq. (14–35): $C_e = 1$

Thus,

$$K_m = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) = 1 + (1)[0.0695(1) + 0.15(1)] = 1.22$$

Assuming constant thickness gears, the rim-thickness factor $K_B = 1$. The speed ratio is $m_G = N_G/N_P = 52/17 = 3.059$. The load cycle factors given in the problem statement, with $N(\text{pinion}) = 10^8$ cycles and $N(\text{gear}) = 10^8/m_G = 10^8/3.059$ cycles, are

$$(Y_N)_P = 1.3558(10^8)^{-0.0178} = 0.977$$

$$(Y_N)_G = 1.3558(10^8/3.059)^{-0.0178} = 0.996$$

Example 14–4

Assuming constant thickness gears, the rim-thickness factor $K_B = 1$. The speed ratio is $m_G = N_G/N_P = 52/17 = 3.059$. The load cycle factors given in the problem statement, with $N(\text{pinion}) = 10^8$ cycles and $N(\text{gear}) = 10^8/m_G = 10^8/3.059$ cycles, are

$$(Y_N)_P = 1.3558(10^8)^{-0.0178} = 0.977$$

$$(Y_N)_G = 1.3558(10^8/3.059)^{-0.0178} = 0.996$$

From Table 14.10, with a reliability of 0.9, $K_R = 0.85$. From Fig. 14–18, the temperature and surface condition factors are $K_T = 1$ and $C_f = 1$. From Eq. (14–23), with $m_N = 1$ for spur gears,

$$I = \frac{\cos 20^\circ \sin 20^\circ}{2} \frac{3.059}{3.059 + 1} = 0.121$$

From Table 14–8, $C_p = 2300\sqrt{\text{psi}}$.

Example 14–4

Next, we need the terms for the gear endurance strength equations. From Table 14–3, for grade 1 steel with $H_{BP} = 240$ and $H_{BG} = 200$, we use Fig. 14–2, which gives

$$(S_t)_P = 77.3(240) + 12\,800 = 31\,350 \text{ psi}$$

$$(S_t)_G = 77.3(200) + 12\,800 = 28\,260 \text{ psi}$$

Similarly, from Table 14–6, we use Fig. 14–5, which gives

$$(S_c)_P = 322(240) + 29\,100 = 106\,400 \text{ psi}$$

$$(S_c)_G = 322(200) + 29\,100 = 93\,500 \text{ psi}$$

From Fig. 14–15,

$$(Z_N)_P = 1.4488(10^8)^{-0.023} = 0.948$$

$$(Z_N)_G = 1.4488(10^8/3.059)^{-0.023} = 0.973$$

Example 14–4

For the hardness ratio factor C_H, the hardness ratio is $H_{BP}/H_{BG} = 240/200 = 1.2$. Then, from Sec. 14–12,

$$A' = 8.98(10^{-3})(H_{BP}/H_{BG}) - 8.29(10^{-3})$$

$$= 8.98(10^{-3})(1.2) - 8.29(10^{-3}) = 0.002\,49$$

Thus, from Eq. (14–36),

$$C_H = 1 + 0.002\,49(3.059 - 1) = 1.005$$

Example 14–4

(*a*) **Pinion tooth bending.** Substituting the appropriate terms for the pinion into Eq. (14–15) gives

$$(\sigma)_P = \left(W^t K_o K_v K_s \frac{P_d}{F}\frac{K_m K_B}{J}\right)_P = 164.8(1)1.377(1.043)\frac{10}{1.5}\frac{1.22\,(1)}{0.30}$$

$$= 6417 \text{ psi}$$

Substituting the appropriate terms for the pinion into Eq. (14–41) gives

$$(S_F)_P = \left(\frac{S_t Y_N/(K_T K_R)}{\sigma}\right)_P = \frac{31\,350(0.977)/[1(0.85)]}{6417} = 5.62$$

Example 14–4

Gear tooth bending. Substituting the appropriate terms for the gear into Eq. (14–15) gives

$$(\sigma)_G = 164.8(1)1.377(1.052)\frac{10}{1.5}\frac{1.22(1)}{0.40} = 4854 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–41) gives

$$(S_F)_G = \frac{28\ 260(0.996)/[1(0.85)]}{4854} = 6.82$$

Example 14–4

(*b*) **Pinion tooth wear.** Substituting the appropriate terms for the pinion into Eq. (14–16) gives

$$(\sigma_c)_P = C_p\left(W^t K_o K_v K_s \frac{K_m}{d_P F}\frac{C_f}{I}\right)_P^{1/2}$$

$$= 2300\left[164.8(1)1.377(1.043)\frac{1.22}{1.7(1.5)}\frac{1}{0.121}\right]^{1/2} = 70\,360 \text{ psi}$$

Substituting the appropriate terms for the pinion into Eq. (14–42) gives

$$(S_H)_P = \left[\frac{S_c Z_N/(K_T K_R)}{\sigma_c}\right]_P = \frac{106\,400(0.948)/[1(0.85)]}{70\,360} = 1.69$$

Example 14–4

Gear tooth wear. The only term in Eq. (14–16) that changes for the gear is K_s. Thus,

$$(\sigma_c)_G = \left[\frac{(K_s)_G}{(K_s)_P}\right]^{1/2} (\sigma_c)_P = \left(\frac{1.052}{1.043}\right)^{1/2} 70\,360 = 70\,660 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–42) with $C_H = 1.005$ gives

$$(S_H)_G = \frac{93\,500(0.973)1.005/[1(0.85)]}{70\,660} = 1.52$$

(*c*) For the pinion, we compare $(S_F)_P$ with $(S_H)_P^2$, or 5.73 with $1.69^2 = 2.86$, so the threat in the pinion is from wear. For the gear, we compare $(S_F)_G$ with $(S_H)_G^2$, or 6.96 with $1.52^2 = 2.31$, so the threat in the gear is also from wear.

Example 14–5

A 17-tooth 20° normal pitch-angle helical pinion with a right-hand helix angle of 30° rotates at 1800 rev/min when transmitting 4 hp to a 52-tooth helical gear. The normal diametral pitch is 10 teeth/in, the face width is 1.5 in, and the set has a quality number of 6. The gears are straddle-mounted with bearings immediately adjacent. The pinion and gear are made from a through-hardened steel with surface and core hardnesses of 240 Brinell on the pinion and surface and core hardnesses of 200 Brinell on the gear. The transmission is smooth, connecting an electric motor and a centrifugal pump. Assume a pinion life of 10^8 cycles and a reliability of 0.9 and use the upper curves in Figs. 14–14 and 14–15.

(*a*) Find the factors of safety of the gears in bending.

(*b*) Find the factors of safety of the gears in wear.

(*c*) By examining the factors of safety identify the threat to each gear and to the mesh.

Example 14–5

All of the parameters in this example are the same as in Ex. 14–4 with the exception that we are using helical gears. Thus, several terms will be the same as Ex. 14–4. The reader should verify that the following terms remain unchanged: $K_o = 1$, $Y_P = 0.303$, $Y_G = 0.412$, $m_G = 3.059$, $(K_s)_P = 1.043$, $(K_s)_G = 1.052$, $(Y_N)_P = 0.977$, $(Y_N)_G = 0.996$, $K_R = 0.85$, $K_T = 1$, $C_f = 1$, $C_p = 2300\sqrt{\text{psi}}$, $(S_t)_P = 31\,350$ psi, $(S_t)_G = 28\,260$ psi, $(S_c)_P = 106\,380$ psi, $(S_c)_G = 93\,500$ psi, $(Z_N)_P = 0.948$, $(Z_N)_G = 0.973$, and $C_H = 1.005$.

For helical gears, the transverse diametral pitch, given by Eq. (13–18), is

$$P_t = P_n \cos\psi = 10 \cos 30° = 8.660 \text{ teeth/in}$$

Thus, the pitch diameters are $d_P = N_P/P_t = 17/8.660 = 1.963$ in and $d_G = 52/8.660 = 6.005$ in. The pitch-line velocity and transmitted force are

$$V = \frac{\pi d_P n_P}{12} = \frac{\pi(1.963)1800}{12} = 925 \text{ ft/min}$$

$$W^t = \frac{33\,000H}{V} = \frac{33\,000(4)}{925} = 142.7 \text{ lbf}$$

Example 14–5

As in Ex. 14–4, for the dynamic factor, $B = 0.8255$ and $A = 59.77$. Thus, Eq. (14–27) gives

$$K_v = \left(\frac{59.77 + \sqrt{925}}{59.77}\right)^{0.8255} = 1.404$$

The geometry factor I for helical gears requires a little work. First, the transverse pressure angle is given by Eq. (13–19)

$$\phi_t = \tan^{-1}\left(\frac{\tan \phi_n}{\cos \psi}\right) = \tan^{-1}\left(\frac{\tan 20^\circ}{\cos 30^\circ}\right) = 22.80^\circ$$

The radii of the pinion and gear are $r_P = 1.963/2 = 0.9815$ in and $r_G = 6.004/2 = 3.002$ in, respectively. The addendum is $a = 1/P_n = 1/10 = 0.1$, and the base-circle radii of the pinion and gear are given by Eq. (13–6) with $\phi = \phi_t$:

$$(r_b)_P = r_P \cos \phi_t = 0.9815 \cos 22.80^\circ = 0.9048 \text{ in}$$

$$(r_b)_G = 3.002 \cos 22.80^\circ = 2.767 \text{ in}$$

Example 14–5

From Eq. (14–25), the surface strength geometry factor

$$Z = \sqrt{(0.9815 + 0.1)^2 - 0.9048^2} + \sqrt{(3.004 + 0.1)^2 - 2.769^2}$$

$$- (0.9815 + 3.004) \sin 22.80°$$

$$= 0.5924 + 1.4027 - 1.544\,4 = 0.4507 \text{ in}$$

Since the first two terms are less than 1.544 4, the equation for Z stands. From Eq. (14–24) the normal circular pitch p_N is

$$p_N = p_n \cos \phi_n = \frac{\pi}{P_n} \cos 20° = \frac{\pi}{10} \cos 20° = 0.2952 \text{ in}$$

From Eq. (14–21), the load sharing ratio

$$m_N = \frac{p_N}{0.95Z} = \frac{0.2952}{0.95(0.4507)} = 0.6895$$

Substituting in Eq. (14–23), the geometry factor I is

$$I = \frac{\sin 22.80° \cos 22.80°}{2(0.6895)} \frac{3.06}{3.06 + 1} = 0.195$$

Example 14–5

From Fig. 14–7, geometry factors $J'_P = 0.45$ and $J'_G = 0.54$. Also from Fig. 14–8 the J-factor multipliers are 0.94 and 0.98, correcting J'_P and J'_G to

$$J_P = 0.45(0.94) = 0.423$$

$$J_G = 0.54(0.98) = 0.529$$

The load-distribution factor K_m is estimated from Eq. (14–32):

$$C_{pf} = \frac{1.5}{10(1.963)} - 0.0375 + 0.0125(1.5) = 0.0577$$

with $C_{mc} = 1$, $C_{pm} = 1$, $C_{ma} = 0.15$ from Fig. 14–11, and $C_e = 1$. Therefore, from Eq. (14–30),

$$K_m = 1 + (1)[0.0577(1) + 0.15(1)] = 1.208$$

Example 14–5

(*a*) **Pinion tooth bending.** Substituting the appropriate terms into Eq. (14–15) using P_t gives

$$(\sigma)_P = \left(W^t K_o K_v K_s \frac{P_t}{F} \frac{K_m K_B}{J}\right)_P = 142.7(1)1.404(1.043)\frac{8.66}{1.5}\frac{1.208(1)}{0.423}$$

$$= 3445 \text{ psi}$$

Substituting the appropriate terms for the pinion into Eq. (14–41) gives

$$(S_F)_P = \left(\frac{S_t Y_N/(K_T K_R)}{\sigma}\right)_P = \frac{31\,350(0.977)/[1(0.85)]}{3445} = 10.5$$

Gear tooth bending. Substituting the appropriate terms for the gear into Eq. (14–15) gives

$$(\sigma)_G = 142.7(1)1.404(1.052)\frac{8.66}{1.5}\frac{1.208(1)}{0.529} = 2779 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–41) gives

$$(S_F)_G = \frac{28\,260(0.996)/[1(0.85)]}{2779} = 11.9$$

Example 14–5

(*b*) **Pinion tooth wear.** Substituting the appropriate terms for the pinion into Eq. (14–16) gives

$$(\sigma_c)_P = C_p \left(W^t K_o K_v K_s \frac{K_m}{d_P F} \frac{C_f}{I} \right)_P^{1/2}$$

$$= 2300 \left[142.7(1)1.404(1.043) \frac{1.208}{1.963(1.5)} \frac{1}{0.195} \right]^{1/2} = 48\,230 \text{ psi}$$

Substituting the appropriate terms for the pinion into Eq. (14–42) gives

$$(S_H)_P = \left(\frac{S_c Z_N/(K_T K_R)}{\sigma_c} \right)_P = \frac{106\,400(0.948)/[1(0.85)]}{48\,230} = 2.46$$

Example 14–5

Gear tooth wear. The only term in Eq. (14–16) that changes for the gear is K_s. Thus,

$$(\sigma_c)_G = \left[\frac{(K_s)_G}{(K_s)_P}\right]^{1/2} (\sigma_c)_P = \left(\frac{1.052}{1.043}\right)^{1/2} 48\,230 = 48\,440 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–42) with $C_H = 1.005$ gives

$$(S_H)_G = \frac{93\,500(0.973)1.005/[1(0.85)]}{48\,440} = 2.22$$

(*c*) For the pinion we compare S_F with S_H^2, or 10.5 with $2.46^2 = 6.05$, so the threat in the pinion is from wear. For the gear we compare S_F with S_H^2, or 11.9 with $2.22^2 = 4.93$, so the threat is also from wear in the gear. For the meshing gearset wear controls.

Comparing Pinion with Gear

- Comparing the pinion with the gear can provide insight.
- Equating factors of safety from bending equations for pinion and gear, and cancelling all terms that are equivalent for the two, and solving for the gear strength, we get

$$(S_t)_G = (S_t)_P \frac{(Y_N)_P}{(Y_N)_G} \frac{J_P}{J_G}$$

- Substituting in equations for the stress-cycle factor Y_N,

$$(S_t)_G = (S_t)_P m_G^{\beta} \frac{J_P}{J_G} \qquad (14\text{–}44)$$

- Normally, $m_G > 1$, and $J_G > J_P$, so Eq. (14–44) indicates the gear can be less strong than the pinion for the same safety factor.

Comparing Pinion and Gear

- Repeating the same process for contact stress equations,

$$(S_c)_G = (S_c)_P \frac{(Z_N)_P}{(Z_N)_G}\left(\frac{1}{C_H}\right)_G = (S_C)_P m_G^{\beta}\left(\frac{1}{C_H}\right)_G$$

- Neglecting C_H which is near unity,

$$(S_c)_G = (S_c)_P m_G^{\beta} \qquad (14\text{–}45)$$

Example 14–6

In a set of spur gears, a 300-Brinell 18-tooth 16-pitch 20° full-depth pinion meshes with a 64-tooth gear. Both gear and pinion are of grade 1 through-hardened steel. Using $\beta = -0.023$, what hardness can the gear have for the same factor of safety?

Solution

For through-hardened grade 1 steel the pinion strength $(S_t)_P$ is given in Fig. 14–2:

$$(S_t)_P = 77.3(300) + 12\,800 = 35\,990 \text{ psi}$$

From Fig. 14–6 the form factors are $J_P = 0.32$ and $J_G = 0.41$. Equation (14–44) gives

$$(S_t)_G = 35\,990 \left(\frac{64}{18}\right)^{-0.023} \frac{0.32}{0.41} = 27\,280 \text{ psi}$$

Use the equation in Fig. 14–2 again.

$$(H_B)_G = \frac{27\,280 - 12\,800}{77.3} = 187 \text{ Brinell}$$

Example 14–7

For $\beta = -0.056$ for a through-hardened steel, grade 1, continue Ex. 14–6 for wear.

Solution

From Fig. 14–5,

$$(S_c)_P = 322(300) + 29\,100 = 125\,700 \text{ psi}$$

From Eq. (14–45),

$$(S_c)_G = (S_c)_P \left(\frac{64}{18}\right)^{-0.056} = 125\,700 \left(\frac{64}{18}\right)^{-0.056} = 117\,100 \text{ psi}$$

$$(H_B)_G = \frac{117\,100 - 29\,200}{322} = 273 \text{ Brinell}$$

which is slightly less than the pinion hardness of 300 Brinell.