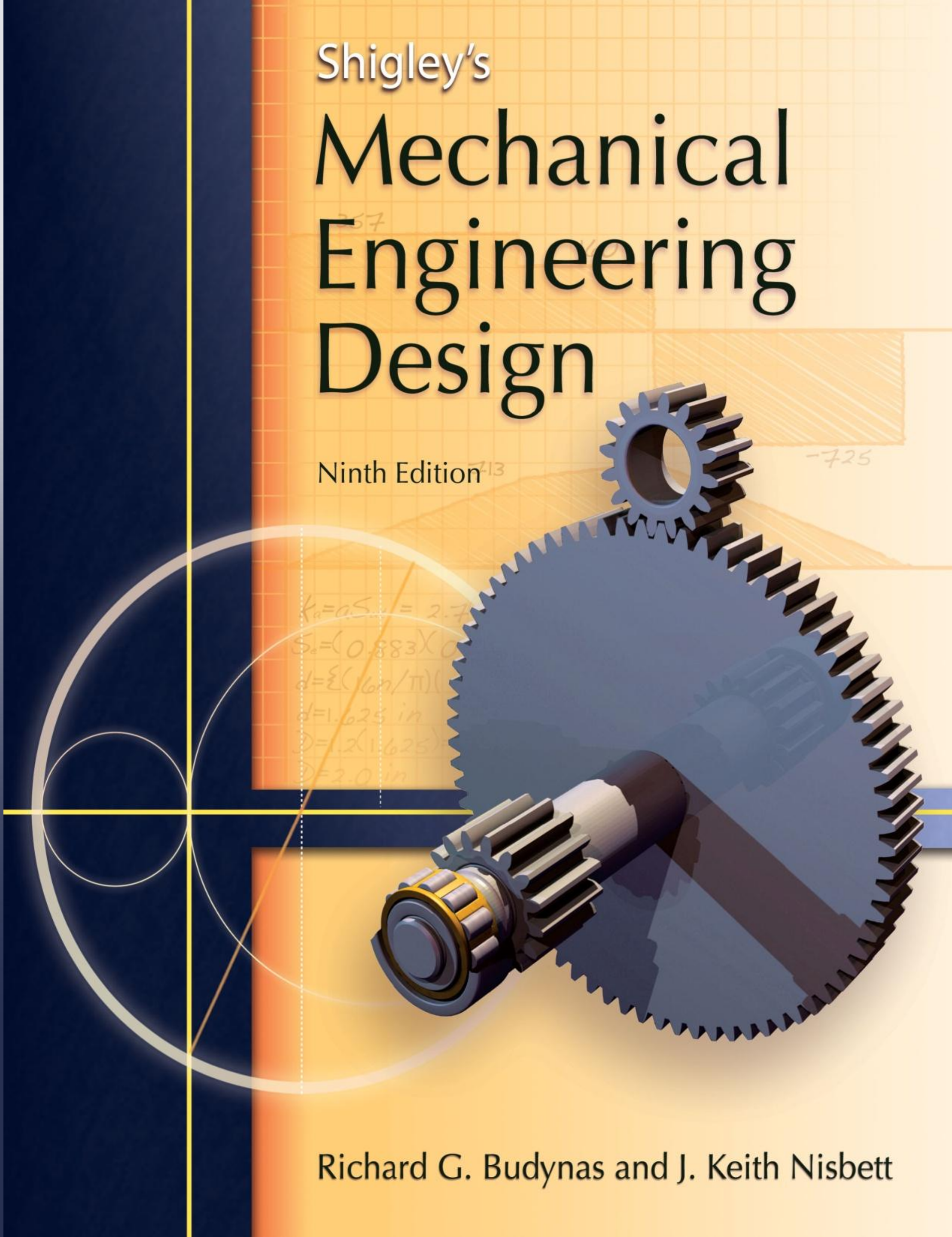
Lecture Slides

Chapter 15

Bevel and Worm Gears



Chapter Outline

Bevel Gearing - General

- Bevel gear classifications
  - Straight bevel gears
  - Spiral bevel gears
  - Zerol bevel gears
  - Hypoid gears
  - Spiroid gears

Straight Bevel Gear

- Perpendicular shafts lying in a plane
- Usually used for pitch line velocities up to 1000 ft/min (5 m/s)

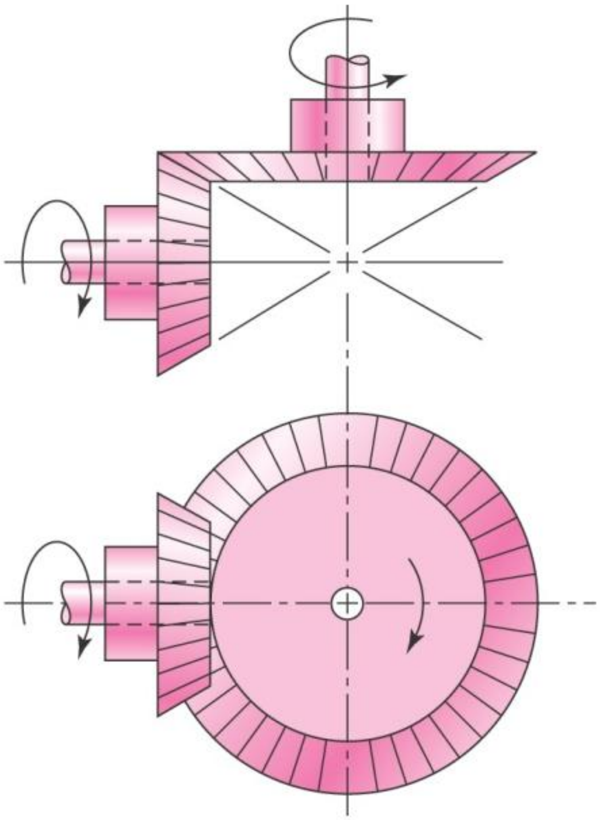


Fig. 13–3

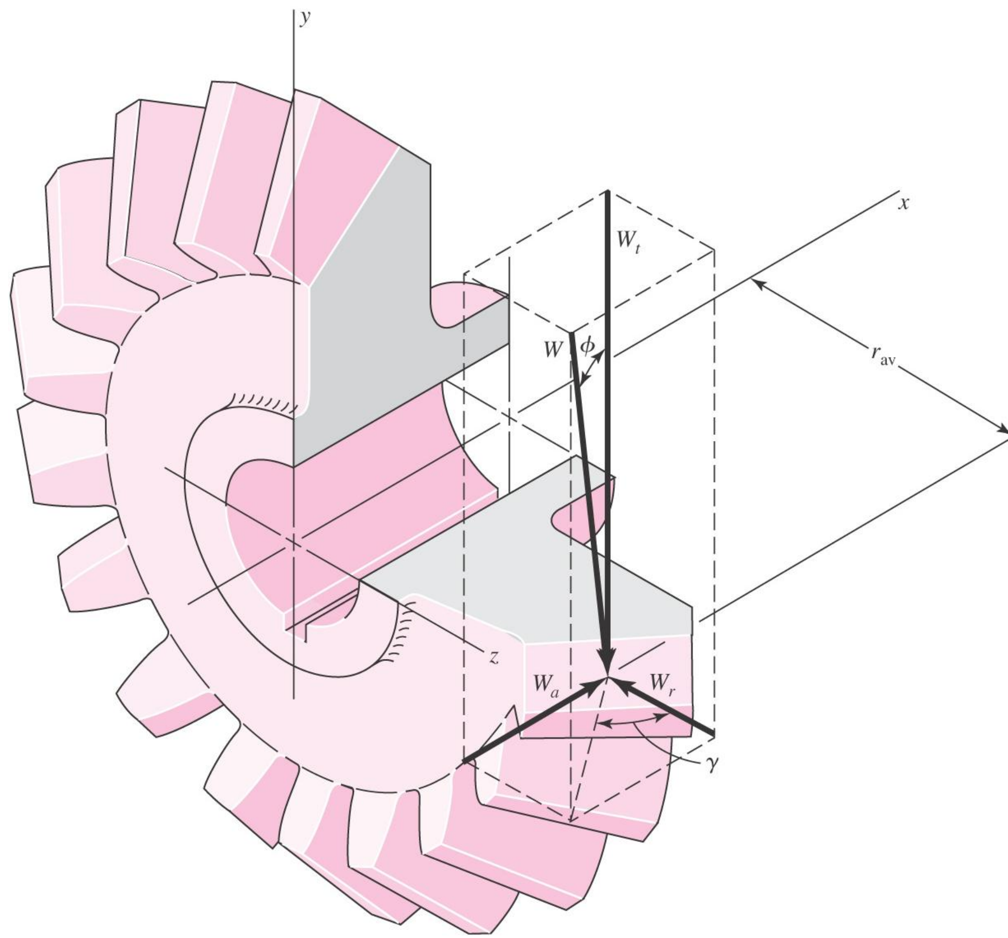


Fig. 13–35

Spiral Bevel Gear

- Recommended for higher speeds
- Recommended for lower noise levels
- The bevel counterpart of the helical gear

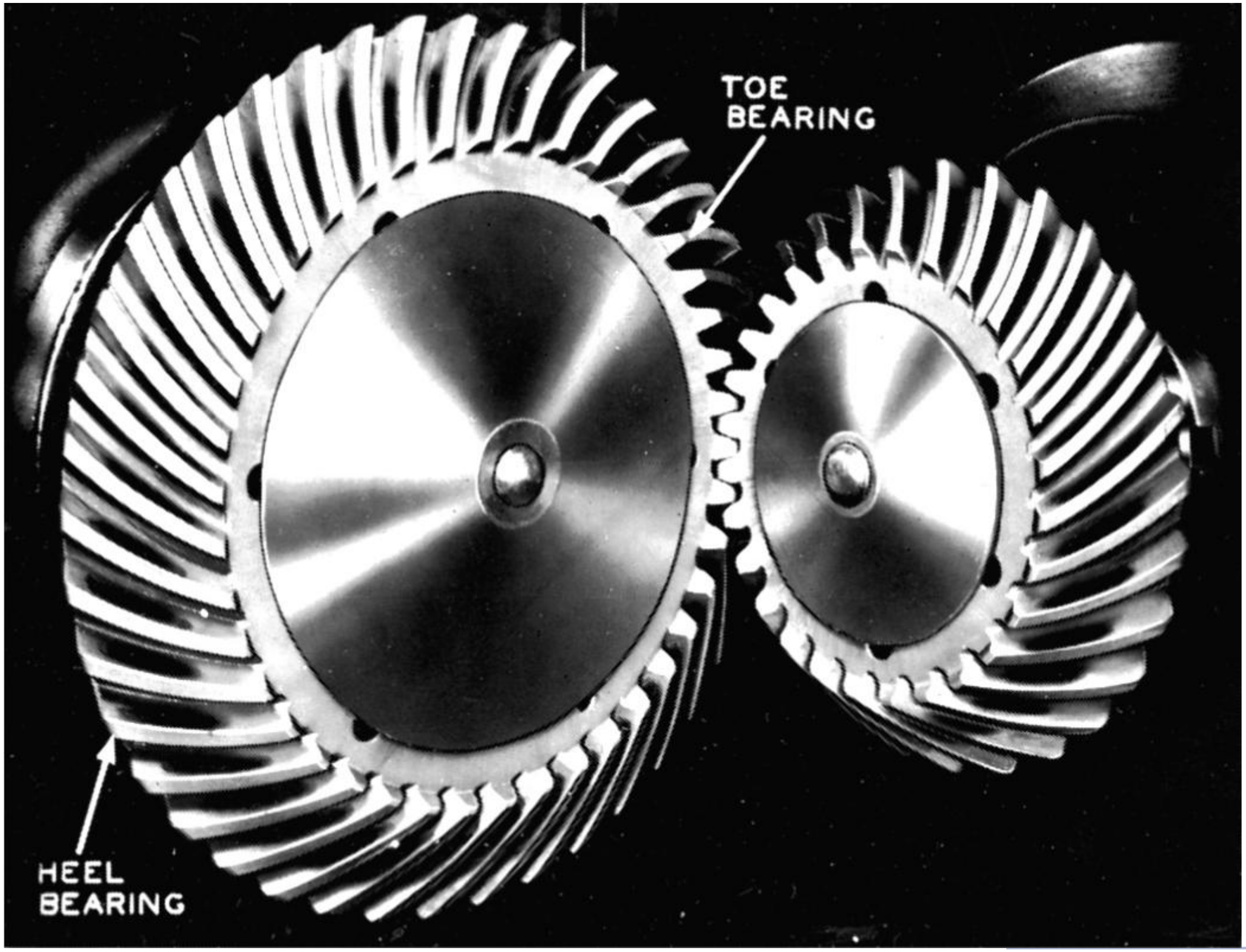


Fig. 15–1

Spiral Bevel Gear

- Cutting spiral-gear teeth

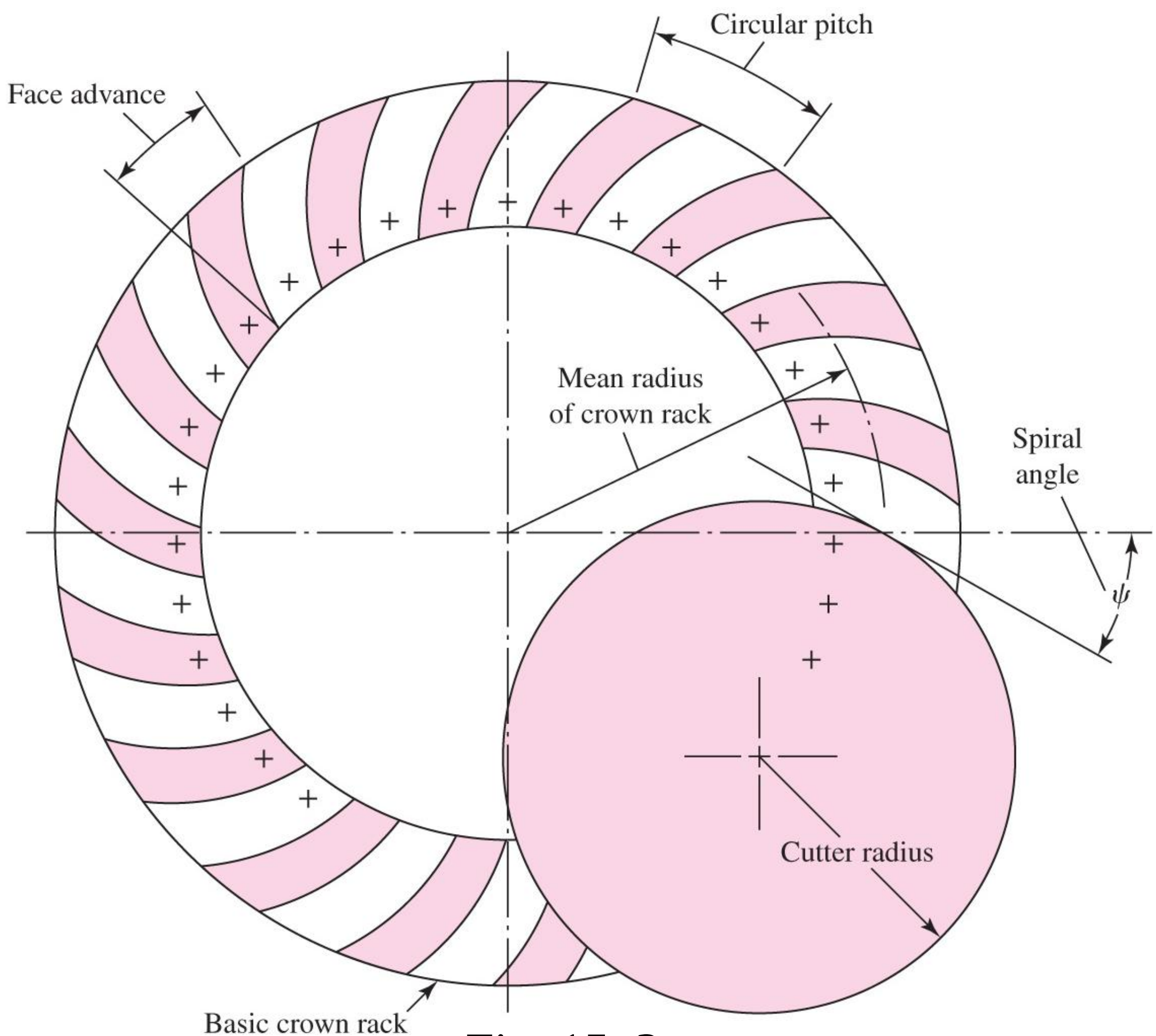


Fig. 15–2

Zerol Bevel Gear

- Patented gear with curved teeth but with a zero spiral angle
- Axial thrust loads are less than spiral bevel gear
- Often used instead of straight bevel gears

Hypoid Gears

- Allows for offset in shaft center-lines
- Pitch surfaces are hyperboloids of revolution

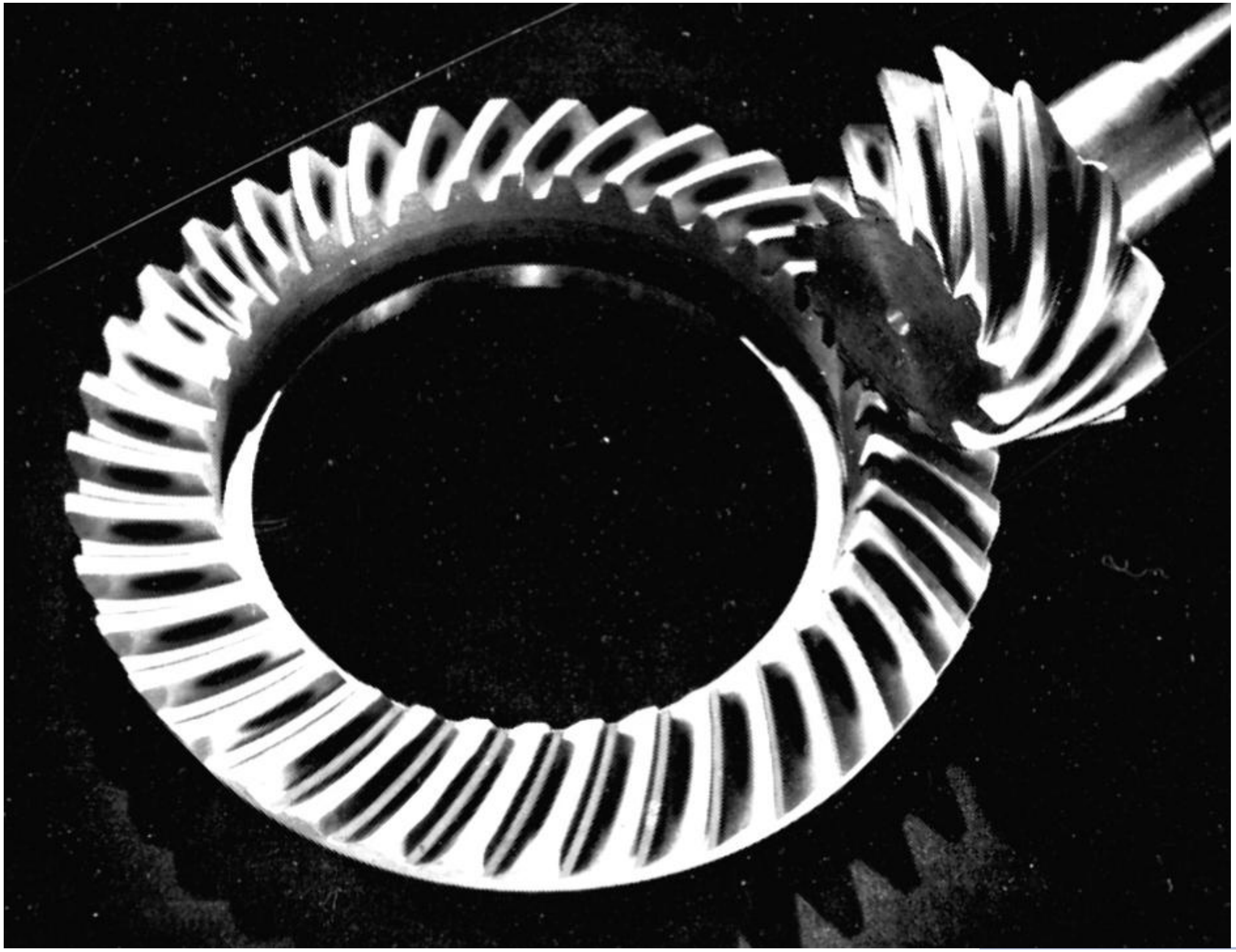


Fig. 15–3

Spiroid Gears

- Greater offset of center-lines than hypoid gears
- Hypoid and Spiroid gears are progressions from spiral gear to worm gear

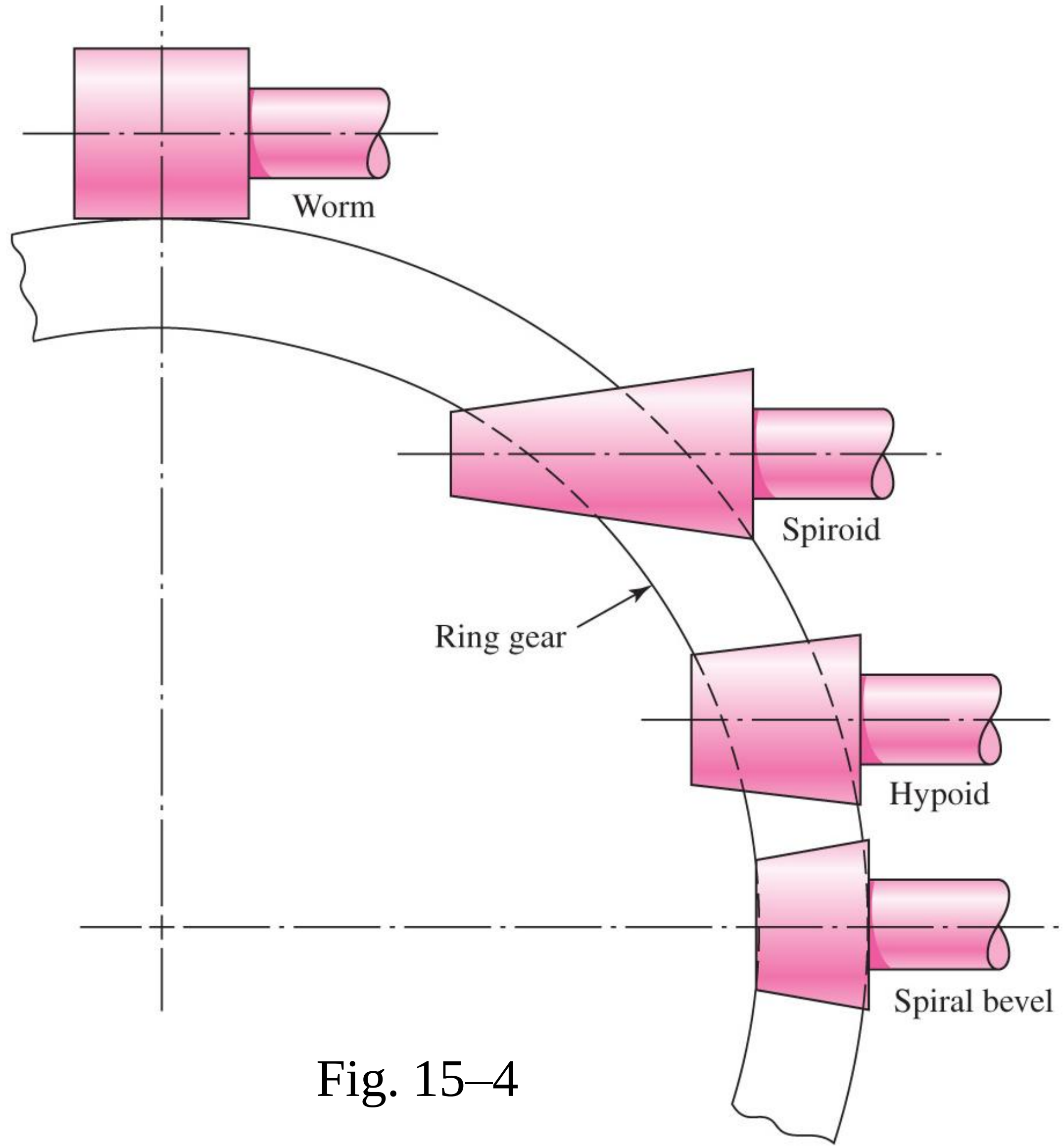


Fig. 15–4

AGMA Straight-Bevel Gear Equations

Fundamental Contact Stress Equation

$$s_c = \sigma_c = C_p \left(\frac{W^t}{F d_P I} K_o K_v K_m C_s C_{xc} \right)^{1/2} \qquad \text{(U.S. customary units)}$$

(15–1)

$$\sigma_H = Z_E \left(\frac{1000 W^t}{b d Z_1} K_A K_v K_{H\beta} Z_x Z_{xc} \right)^{1/2} \qquad \text{(SI units)}$$

Permissible Contact Stress Number (Strength) Equation

$$s_{wc} = (\sigma_c)_{\text{all}} = \frac{s_{ac} C_L C_H}{S_H K_T C_R} \qquad \text{(U.S. customary units)}$$

(15–2)

$$\sigma_{HP} = \frac{\sigma_{H\,\text{lim}} Z_{NT} Z_W}{S_H K_\theta Z_Z} \qquad \text{(SI units)}$$

AGMA Straight-Bevel Gear Equations

Bending Stress

$$s_t = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J} \qquad \text{(U.S. customary units)}$$

(15–3)

$$\sigma_F = \frac{1000 W^t}{b} \frac{K_A K_v}{m_{et}} \frac{Y_x K_{H\beta}}{Y_\beta Y_J} \qquad \text{(SI units)}$$

Permissible Bending Stress Equation

$$s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R} \qquad \text{(U.S. customary units)}$$

(15–4)

$$\sigma_{FP} = \frac{\sigma_{F\,\lim} Y_{NT}}{S_F K_\theta Y_z} \qquad \text{(SI units)}$$

Overload Factor K_O (K_A)

Character of Prime Mover	Character of Load on Driven Machine: Uniform	Light Shock	Medium Shock	Heavy Shock
Uniform	1.00	1.25	1.50	1.75 or higher
Light shock	1.10	1.35	1.60	1.85 or higher
Medium shock	1.25	1.50	1.75	2.00 or higher
Heavy shock	1.50	1.75	2.00	2.25 or higher

Note: This table is for speed-decreasing drives. For speed-increasing drives, add $0.01(N/n)^2$ or $0.01(z_2/z_1)^2$ to the above factors.

Table 15–2

Dynamic Factor K_v

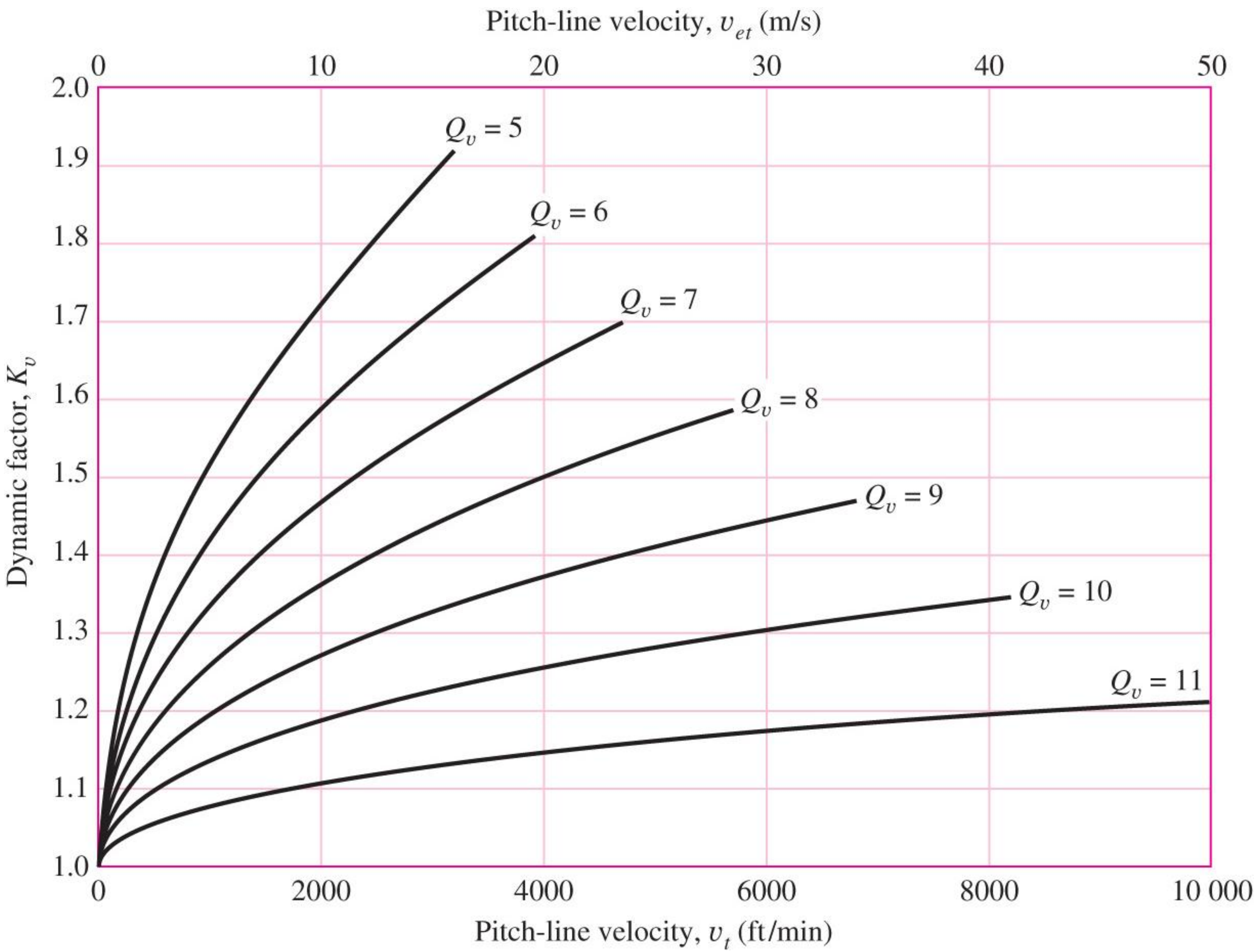


Fig. 15–5

Dynamic Factor K_v

$$K_v = \left(\frac{A + \sqrt{v_t}}{A} \right)^B \qquad \text{(U.S. customary units)}$$

$$K_v = \left(\frac{A + \sqrt{200 v_{et}}}{A} \right)^B \qquad \text{(SI units)} \tag{15–5}$$

$$A = 50 + 56(1 - B)$$

$$B = 0.25(12 - Q_v)^{2/3} \tag{15–6}$$

$$v_t = \pi d_P n_P / 12 \qquad \text{(U.S. customary units)}$$

$$v_{et} = 5.236(10^{-5}) d_1 n_1 \qquad \text{(SI units)} \tag{15–7}$$

$$v_{t\,\max} = [A + (Q_v - 3)]^2 \qquad \text{(U.S. customary units)}$$

$$v_{et\,\max} = \frac{[A + (Q_v - 3)]^2}{200} \qquad \text{(SI units)} \tag{15–8}$$

Size Factor for Pitting Resistance C_s (Z_x)

$$C_s = \begin{cases} 0.5 & F < 0.5 \text{ in} \\ 0.125F + 0.4375 & 0.5 \le F \le 4.5 \text{ in} \\ 1 & F > 4.5 \text{ in} \end{cases} \qquad \text{(U.S. customary units)}$$

$$Z_x = \begin{cases} 0.5 & b < 12.7 \text{ mm} \\ 0.00492b + 0.4375 & 12.7 \le b \le 114.3 \text{ mm} \\ 1 & b > 114.3 \text{ mm} \end{cases} \qquad \text{(SI units)} \tag{15–9}$$

Size Factor for Bending K_s (Y_x)

$$K_s = \begin{cases} 0.4867 + 0.2132/P_d & 0.5 \le P_d \le 16 \text{ teeth/in} \\ 0.5 & P_d > 16 \text{ teeth/in} \end{cases} \quad \text{(U.S. customary units)}$$

$$Y_x = \begin{cases} 0.5 & m_{et} < 1.6 \text{ mm} \\ 0.4867 + 0.008\,339 m_{et} & 1.6 \le m_{et} \le 50 \text{ mm} \end{cases} \quad \text{(SI units)} \tag{15–10}$$

Load-Distribution Factor K_m ($K_{H\beta}$)

$$K_m = K_{mb} + 0.0036F^2 \qquad \text{(U.S. customary units)}$$

$$K_{H\beta} = K_{mb} + 5.6(10^{-6})b^2 \qquad \text{(SI units)} \tag{15–11}$$

where

$$K_{mb} = \begin{cases} 1.00 & \text{both members straddle-mounted} \\ 1.10 & \text{one member straddle-mounted} \\ 1.25 & \text{neither member straddle-mounted} \end{cases}$$

Crowning Factor for Pitting C_{xc} (Z_{xc})

$$C_{xc} = Z_{xc} = \begin{cases} 1.5 & \text{properly crowned teeth} \\ 2.0 & \text{or larger uncrowned teeth} \end{cases} \qquad (15\text{–}12)$$

Lengthwise Curvature Factor for Bending Strength K_x (Y_β)

$$K_x = Y_\beta = 1 \qquad (15\text{–}13)$$

Pitting Resistance Geometry Factor $I\,(Z_I)$

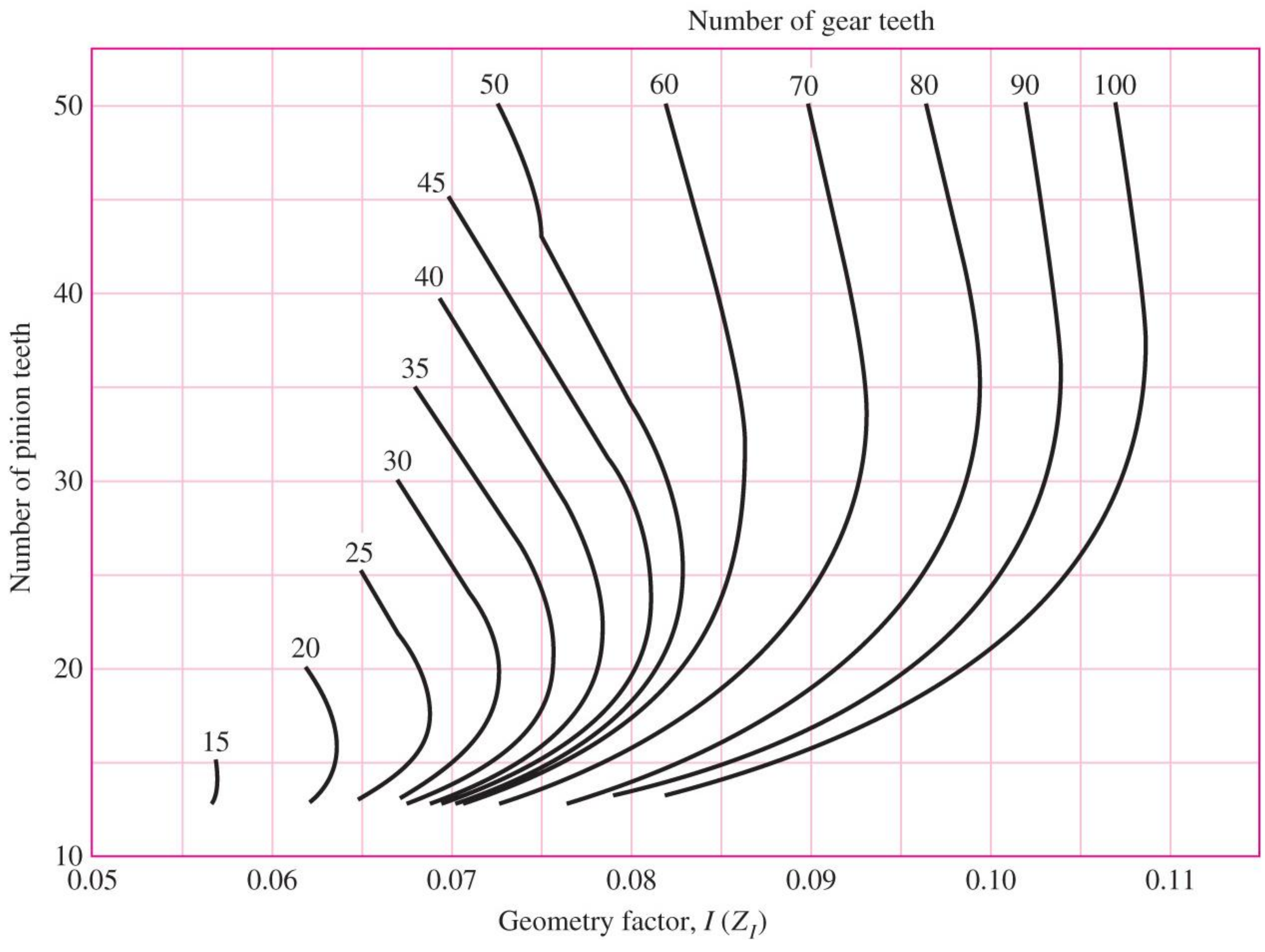


Fig. 15–6

Bending Strength Geometry Factor $J\,(Y_J)$

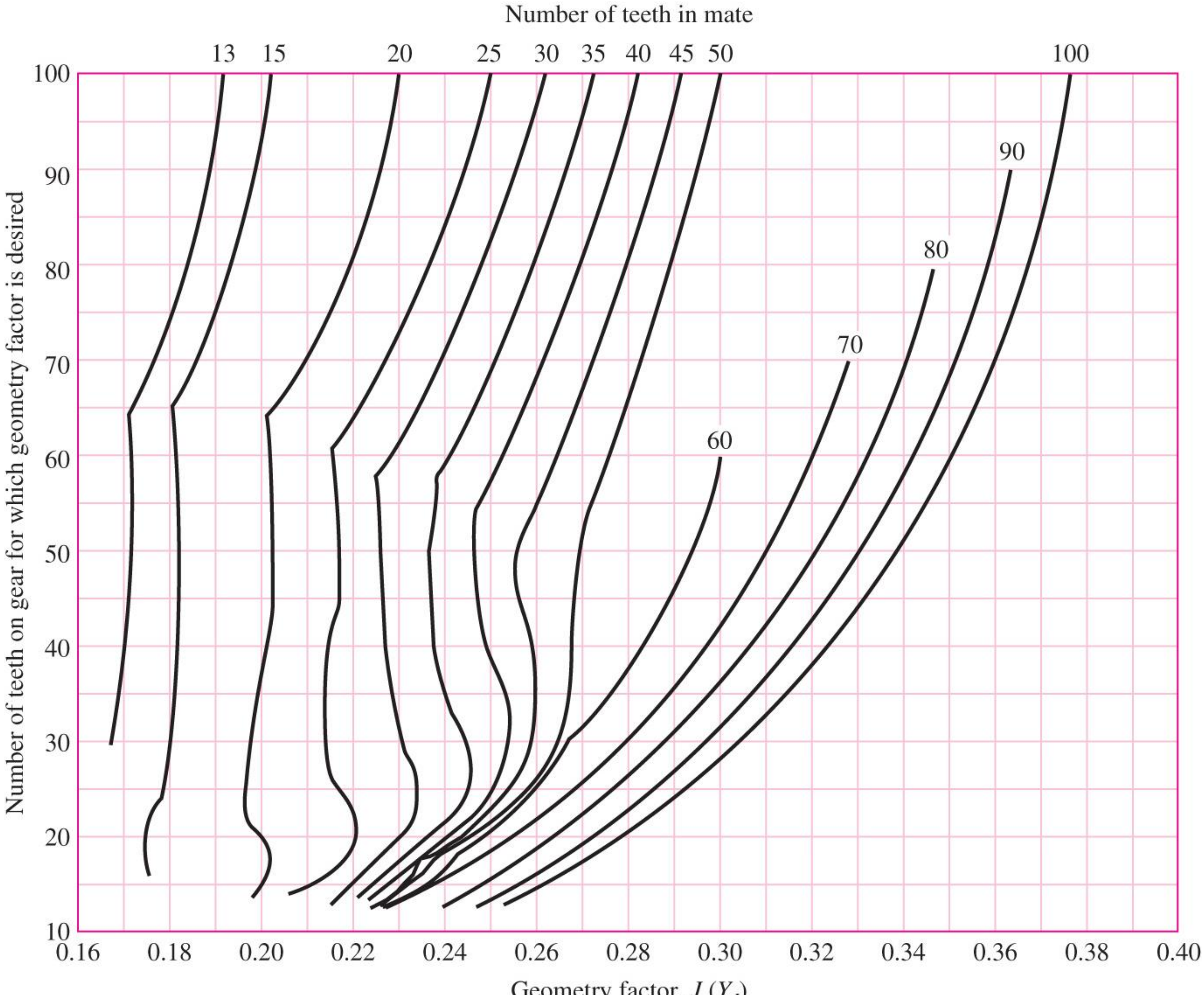


Fig. 15–7

Stress-Cycle Factor for Pitting Resistance $C_L\,(Z_{NT})$

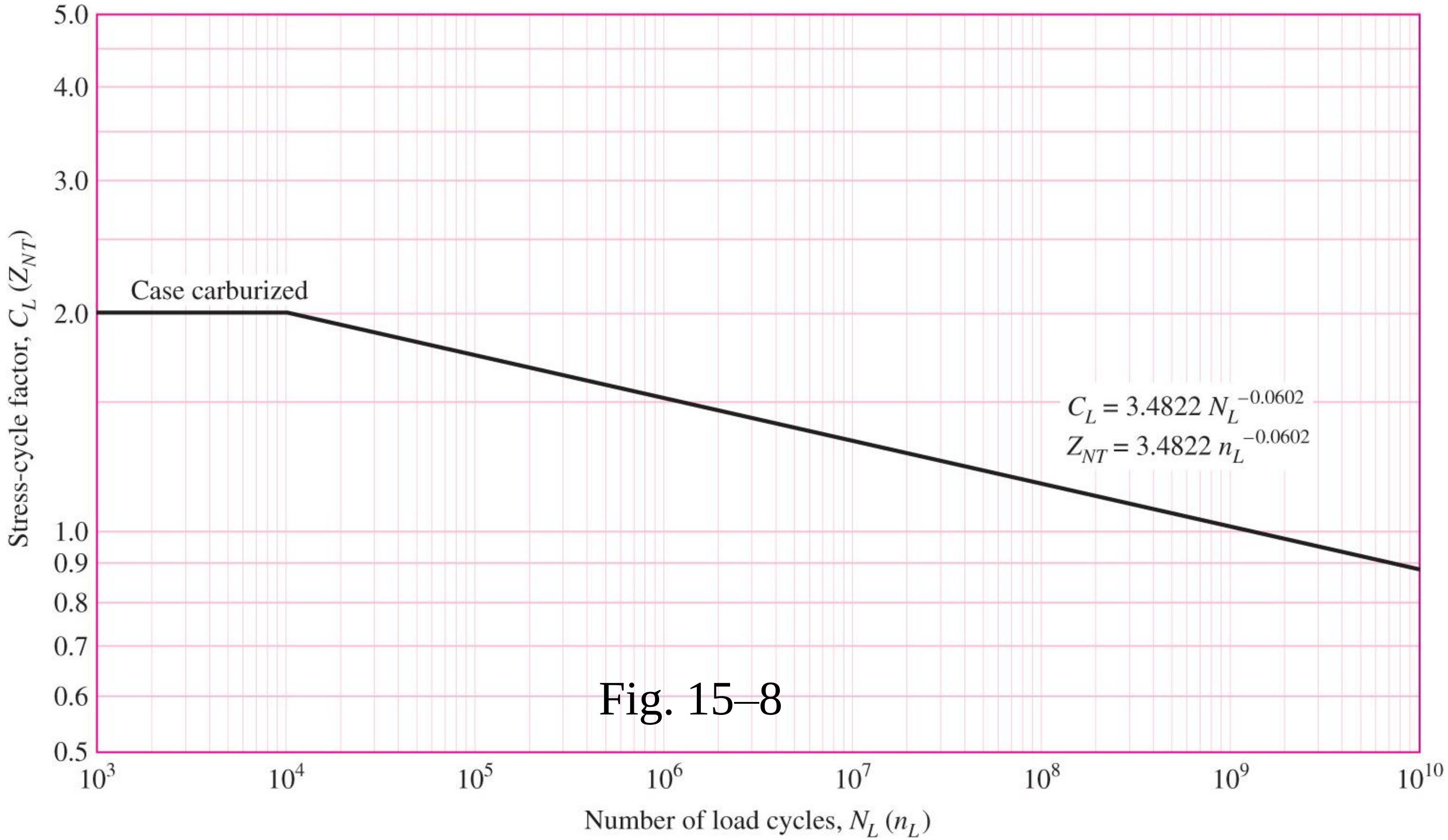


Fig. 15–8

$$C_L = \begin{cases} 2 & 10^3 \le N_L < 10^4 \\ 3.4822 N_L^{-0.0602} & 10^4 \le N_L \le 10^{10} \end{cases}$$

$$Z_{NT} = \begin{cases} 2 & 10^3 \le n_L < 10^4 \\ 3.4822 n_L^{-0.0602} & 10^4 \le n_L \le 10^{10} \end{cases}$$

(15–14)

Stress-Cycle Factor for Bending Strength K_L (Y_{NT})

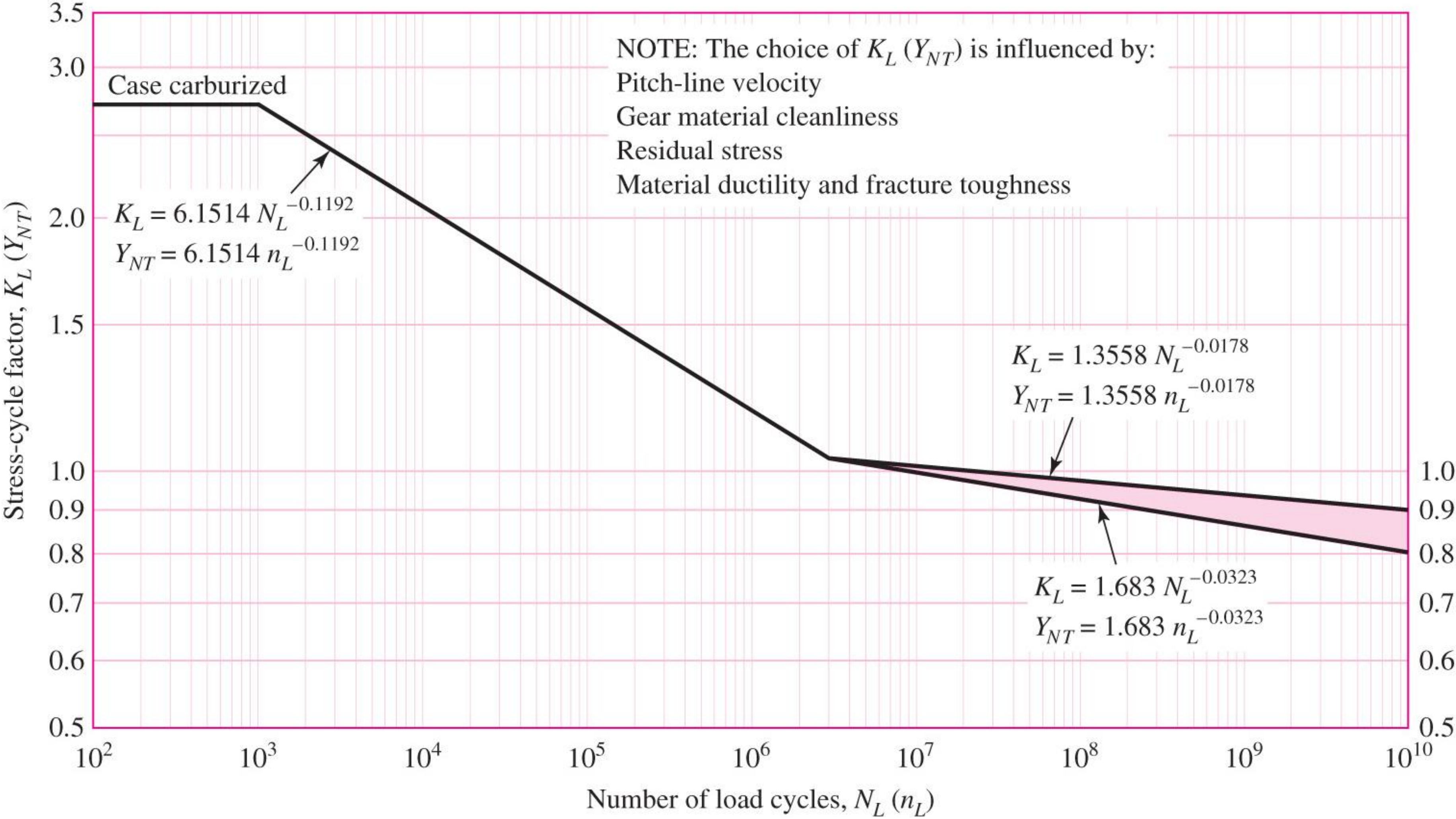


Fig. 15–9

Stress-Cycle Factor for Bending Strength K_L (Y_{NT})

$$K_L = \begin{cases} 2.7 & 10^2 \le N_L < 10^3 \\ 6.1514 N_L^{-0.1182} & 10^3 \le N_L < 3(10^6) \\ 1.6831 N_L^{-0.0323} & 3(10^6) \le N_L \le 10^{10} \quad \text{general} \\ 1.3558 N_L^{-0.0178} & 3(10^6) \le N_L \le 10^{10} \quad \text{critical} \end{cases}$$

(15–15)

$$Y_{NT} = \begin{cases} 2.7 & 10^2 \le n_L < 10^3 \\ 6.1514 n_L^{-0.1182} & 10^3 \le n_L < 3(10^6) \\ 1.6831 n_L^{-0.0323} & 3(10^6) \le n_L \le 10^{10} \quad \text{general} \\ 1.3558 n_L^{-0.0323} & 3(10^6) \le n_L \le 10^{10} \quad \text{critical} \end{cases}$$

Hardness-Ratio Factor $C_H\,(Z_W)$

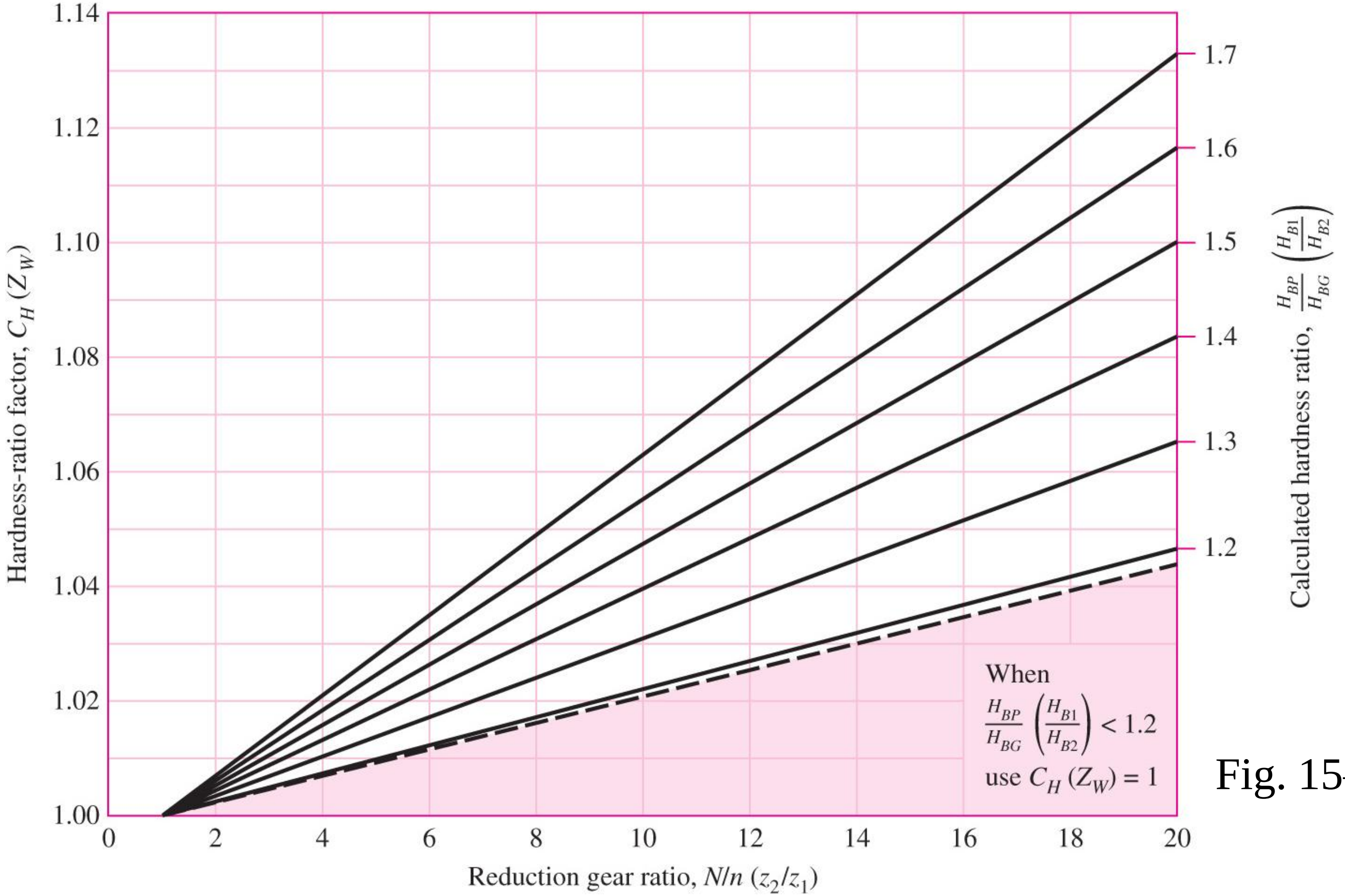


Fig. 15–10

$$C_H = 1 + B_1(N/n - 1) \qquad B_1 = 0.008\,98(H_{BP}/H_{BG}) - 0.008\,29$$

(15–16)

$$Z_W = 1 + B_1(z_1/z_2 - 1) \qquad B_1 = 0.008\,98(H_{B1}/H_{B2}) - 0.008\,29$$

Hardness-Ratio Factor $C_H\,(Z_W)$ for Work-Hardened Gear

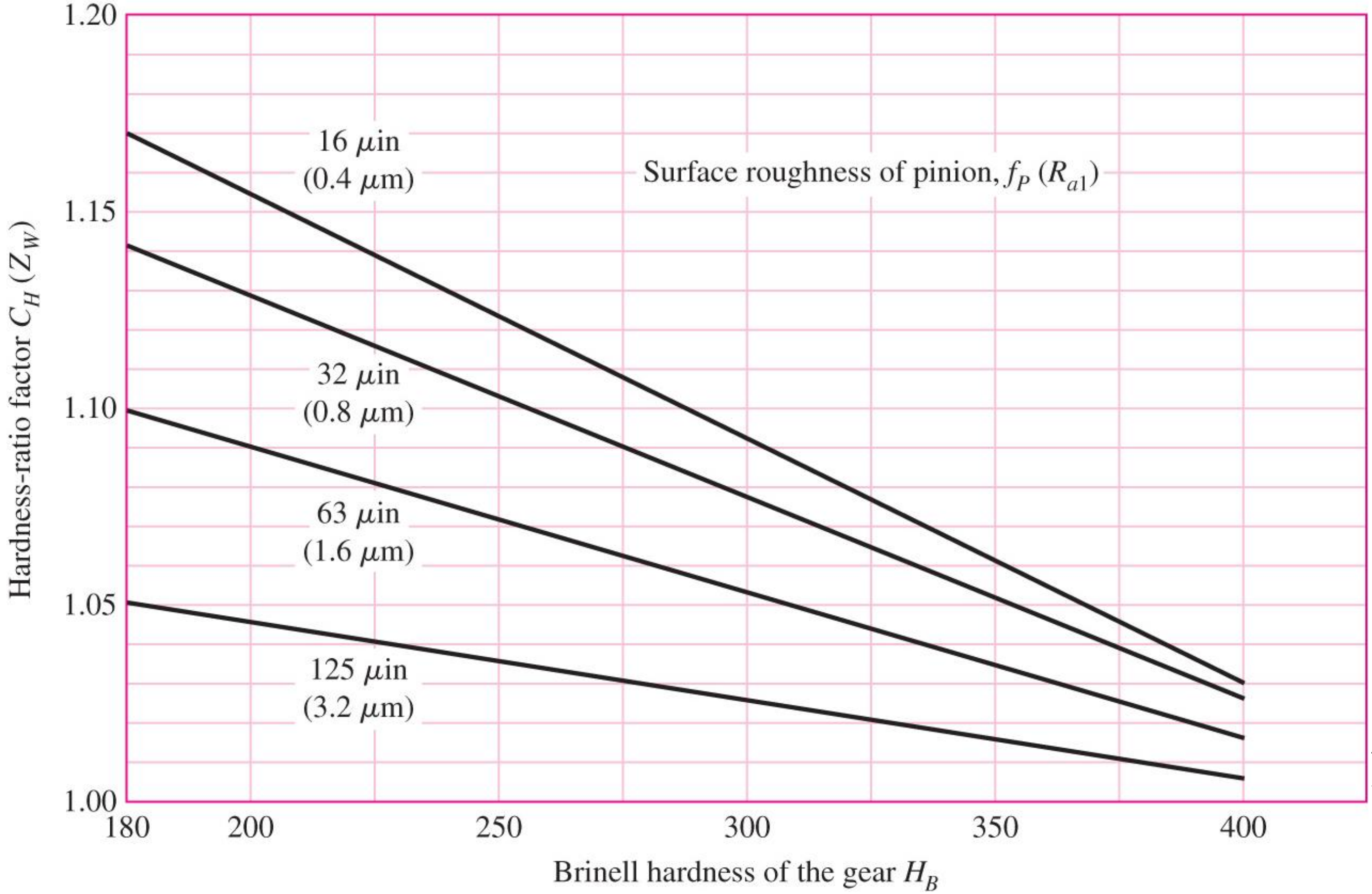


Fig. 15–11

$$C_H = 1 + B_2(450 - H_{BG}) \qquad B_2 = 0.000\,75\exp(-0.0122 f_P)$$

$$Z_W = 1 + B_2(450 - H_{B2}) \qquad B_2 = 0.000\,75\exp(-0.52 f_P) \tag{15–17}$$

where $\quad f_P(R_{a1})$ = pinion surface hardness μin (μm)

$H_{BG}(H_{B2})$ = minimum Brinell hardness

Temperature Factor K_T (K_θ)

$$K_T = \begin{cases} 1 & 32°\text{F} \le t \le 250°\text{F} \\ (460 + t)/710 & t > 250°\text{F} \end{cases} \tag{15–18}$$

$$K_\theta = \begin{cases} 1 & 0°\text{C} \le \theta \le 120°\text{C} \\ (273 + \theta)/393 & \theta > 120°\text{C} \end{cases}$$

Reliability Factors $C_R\,(Z_Z)$ and $K_R\,(Y_Z)$

Requirements of Application	Reliability Factors for Steel* $C_R\,(Z_Z)$	$K_R\,(Y_Z)$†
Fewer than one failure in 10 000	1.22	1.50
Fewer than one failure in 1000	1.12	1.25
Fewer than one failure in 100	1.00	1.00
Fewer than one failure in 10	0.92	0.85‡
Fewer than one failure in 2	0.84	0.70§

Table 15–3

$$Y_Z = K_R = \begin{cases} 0.50 - 0.25\ \log(1 - R) & 0.99 \le R \le 0.999 \quad (15\text{–}19) \\ 0.70 - 0.15\ \log(1 - R) & 0.90 \le R < 0.99 \quad (15\text{–}20) \end{cases}$$

Elastic Coefficient for Pitting Resistance C_p (Z_E)

$$C_p = \sqrt{\frac{1}{\pi\left[\left(1-\nu_P^2\right)/E_P + \left(1-\nu_G^2\right)/E_G\right]}}$$

$$Z_E = \sqrt{\frac{1}{\pi\left[\left(1-\nu_1^2\right)/E_1 + \left(1-\nu_2^2\right)/E_2\right]}} \tag{15–21}$$

where C_p = elastic coefficient, 2290 $\sqrt{\text{psi}}$ for steel

Z_E = elastic coefficient, 190 $\sqrt{\text{N/mm}^2}$ for steel

E_P and E_G = Young's moduli for pinion and gear respectively, psi

E_1 and E_2 = Young's moduli for pinion and gear respectively, N/mm^2

Allowable Contact Stress Number for Steel Gears

Table 15–4

Allowable Contact Stress Number for Steel Gears, s_{ac} ($\sigma_{H\,\mathrm{lim}}$) *Source: ANSI/AGMA 2003-B97.*

Material Designation	Heat Treatment	Minimum Surface* Hardness	Allowable Contact Stress Number, s_{ac} ($\sigma_{H\,\mathrm{lim}}$) lbf/in² (N/mm²) Grade 1†	Grade 2†	Grade 3†
Steel	Through-hardened‡	Fig. 15–12	Fig. 15–12	Fig. 15–12	
	Flame or induction hardened§	50 HRC	175 000 (1210)	190 000 (1310)	
	Carburized and case hardened§	2003-B97 Table 8	200 000 (1380)	225 000 (1550)	250 000 (1720)
AISI 4140	Nitrided§	84.5 HR15N		145 000 (1000)	
Nitralloy 135M	Nitrided§	90.0 HR15N		160 000 (1100)	

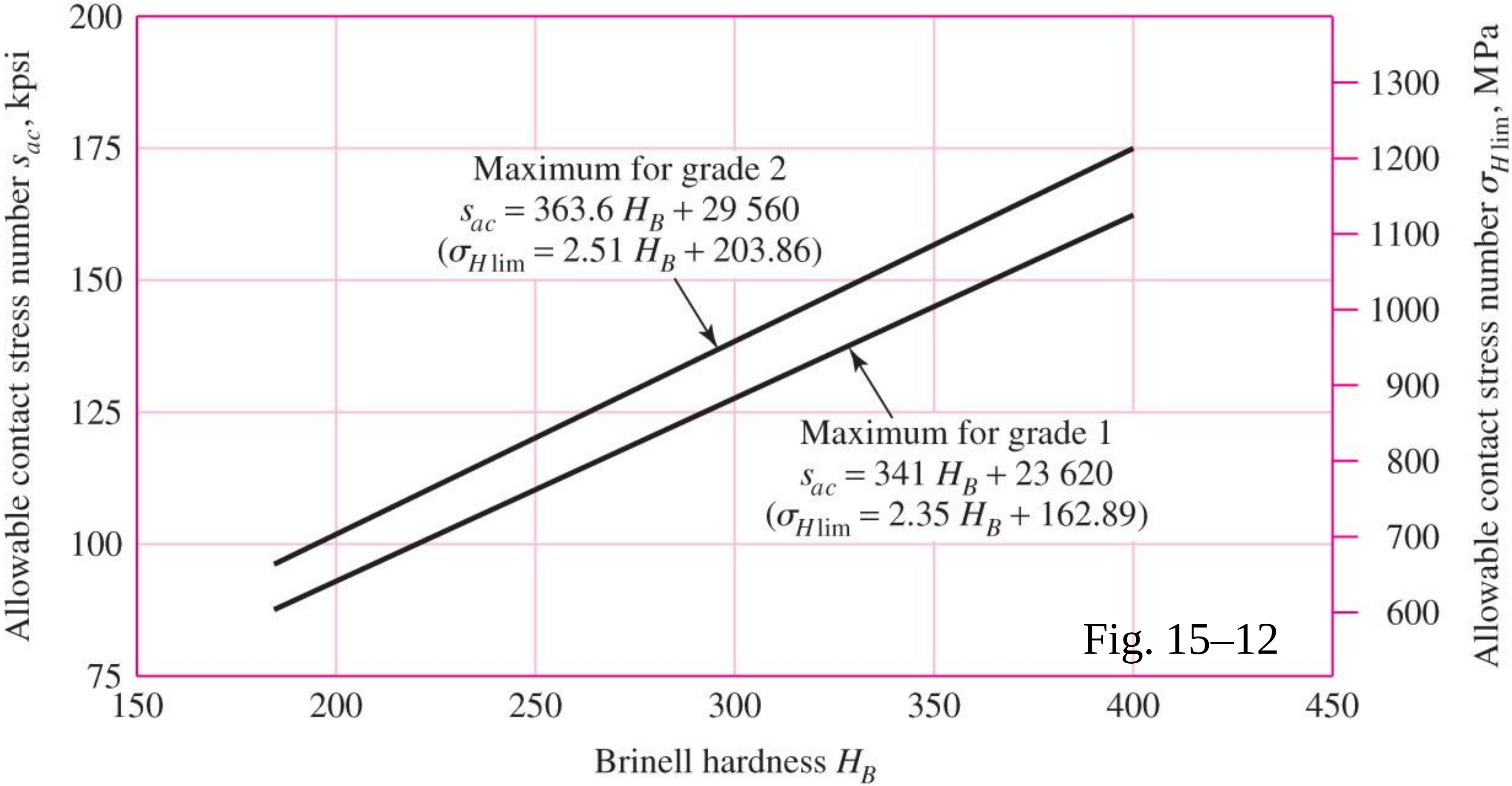
*Hardness to be equivalent to that at the tooth middepth in the center of the face width.

†See ANSI/AGMA 2003-B97, Tables 8 through 11, for metallurgical factors for each stress grade of steel gears.

‡These materials must be annealed or normalized as a minumum.

§The allowable stress numbers indicated may be used with the case depths prescribed in 21.1, ANSI/AGMA 2003-B97.

Allowable Contact Stress Number for Through-Hardened Steel Gears



$$
\begin{aligned}
s_{ac} &= 341 H_B + 23\ 620 \text{ psi} && \text{grade 1} \\
\sigma_{H\,\mathrm{lim}} &= 2.35 H_B + 162.89 \text{ MPa} && \text{grade 1} \\
s_{ac} &= 363.6 H_B + 29\ 560 \text{ psi} && \text{grade 2} \\
\sigma_{H\,\mathrm{lim}} &= 2.51 H_B + 203.86 \text{ MPa} && \text{grade 2}
\end{aligned} \qquad (15\text{–}22)
$$

Allowable Contact Stress Number for Iron Gears

Table 15–5

Allowable Contact Stress Number for Iron Gears, s_{ac} ($\sigma_{H\,\mathrm{lim}}$) *Source: ANSI/AGMA 2003-B97.*

Material	Material Designation ASTM	ISO	Heat Treatment	Typical Minimum Surface Hardness	Allowable Contact Stress Number, s_{ac} ($\sigma_{H\,\mathrm{lim}}$) lbf/in^2 (N/mm^2)
Cast iron	ASTM A48	ISO/DR 185			
	Class 30	Grade 200	As cast	175 HB	50 000 (345)
	Class 40	Grade 300	As cast	200 HB	65 000 (450)
Ductile	ASTM A536	ISO/DIS 1083			
(nodular)	Grade 80-55-06	Grade 600-370-03	Quenched	180 HB	94 000 (650)
iron	Grade 120-90-02	Grade 800-480-02	and tempered	300 HB	135 000 (930)

Allowable Bending Stress Number for Steel Gears

Table 15–6

Allowable Bending Stress Numbers for Steel Gears, s_{at} ($\sigma_{F\,\lim}$) *Source: ANSI/AGMA 2003-B97.*

Material Designation	Heat Treatment	Minimum Surface Hardness	Bending Stress Number (Allowable), s_{at} ($\sigma_{F\,\lim}$) lbf/in^2 (N/mm^2) Grade 1*	Grade 2*	Grade 3*
Steel	Through-hardened	Fig. 15–13	Fig. 15–13	Fig. 15–13	
	Flame or induction hardened				
	Unhardened roots	50 HRC	15 000 (85)	13 500 (95)	
	Hardened roots		22 500 (154)		
	Carburized and case hardened†	2003-B97 Table 8	30 000 (205)	35 000 (240)	40 000 (275)
AISI 4140	Nitrided†,‡	84.5 HR15N		22 000 (150)	
Nitralloy 135M	Nitrided†,‡	90.0 HR15N		24 000 (165)	

*See ANSI/AGMA 2003-B97, Tables 8–11, for metallurgical factors for each stress grade of steel gears.

†The allowable stress numbers indicated may be used with the case depths prescribed in 21.1, ANSI/AGMA 2003-B97.

‡The overload capacity of nitrided gears is low. Since the shape of the effective S-N curve is flat, the sensitivity to shock should be investigated before proceeding with the design.

Allowable Bending Stress Number for Through-Hardened Steel Gears

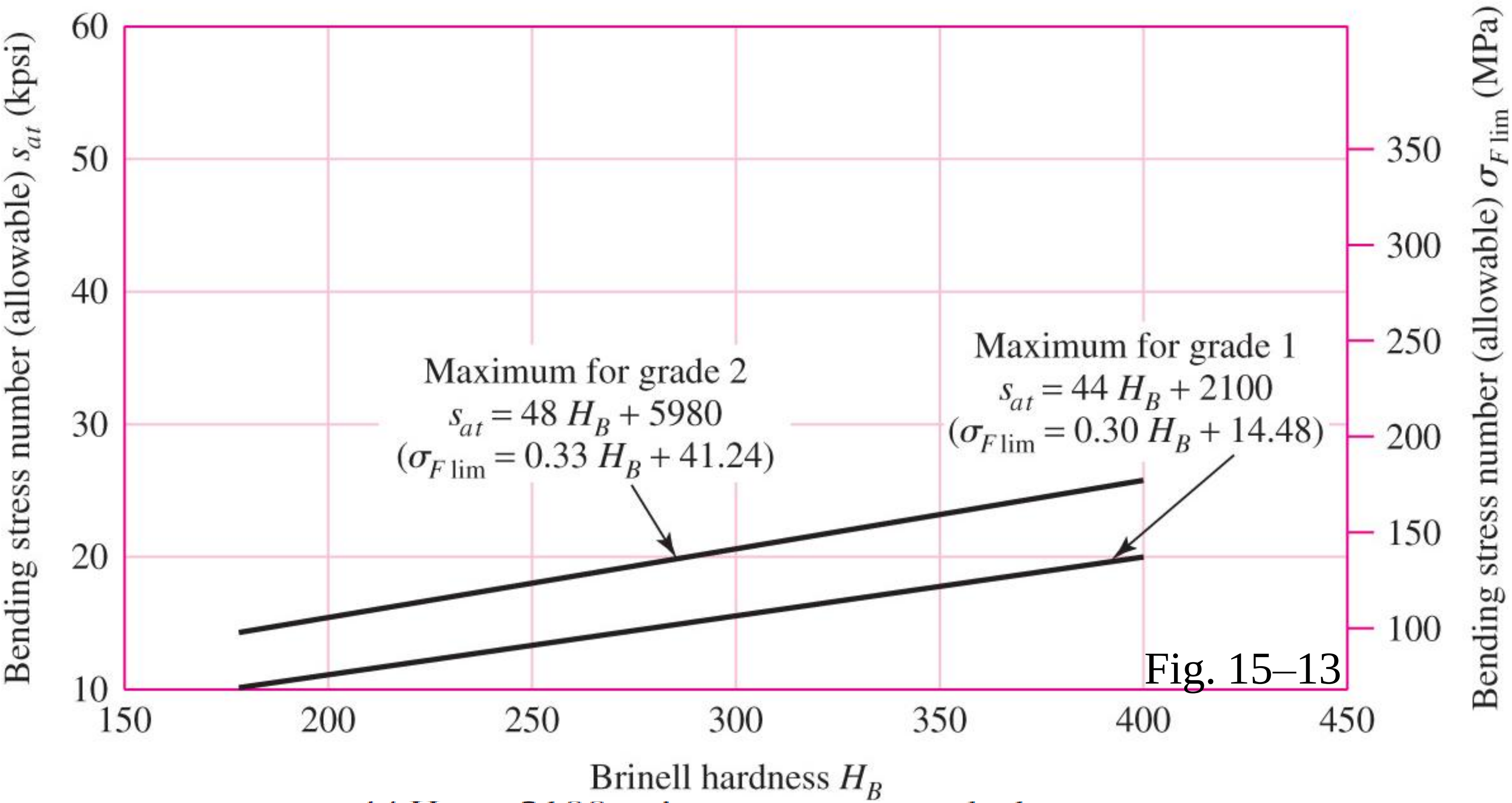


Fig. 15–13

$$
\begin{aligned}
s_{at} &= 44H_B + 2100 \text{ psi} && \text{grade 1} \\
\sigma_{F\,\lim} &= 0.30H_B + 14.48 \text{ MPa} && \text{grade 1} \\
s_{at} &= 48H_B + 5980 \text{ psi} && \text{grade 2} \\
\sigma_{H\,\lim} &= 0.33H_B + 41.24 \text{ MPa} && \text{grade 2}
\end{aligned}
\qquad (15\text{–}23)
$$

Allowable Bending Stress Number for Iron Gears

Table 15–7

Allowable Bending Stress Number for Iron Gears, s_{at} ($\sigma_{F\,\mathrm{lim}}$) *Source: ANSI/AGMA 2003-B97.*

Material	Material Designation ASTM	ISO	Heat Treatment	Typical Minimum Surface Hardness	Bending Stress Number (Allowable), s_{at} ($\sigma_{F\,\mathrm{lim}}$) lbf/in^2 (N/mm^2)
Cast iron	ASTM A48	ISO/DR 185			
	Class 30	Grade 200	As cast	175 HB	4500 (30)
	Class 40	Grade 300	As cast	200 HB	6500 (45)
Ductile	ASTM A536	ISO/DIS 1083			
(nodular)	Grade 80-55-06	Grade 600-370-03	Quenched	180 HB	10 000 (70)
iron	Grade 120-90-02	Grade 800-480-02	and tempered	300 HB	13 500 (95)

Summary for Straight-Bevel Gear Wear

Geometry	Force Analysis	Strength Analysis
$d_p = \dfrac{N_P}{P_d}$	$W^t = \dfrac{2T}{d_{av}}$	$W^t = \dfrac{2T}{d_p}$
$\gamma = \tan^{-1}\dfrac{N_P}{N_G}$	$W^r = W^t \tan\phi \cos\gamma$	$W^r = W^t \tan\phi \cos\gamma$
$\Gamma = \tan^{-1}\dfrac{N_G}{N_P}$	$W^a = W^t \tan\phi \sin\gamma$	$W^a = W^t \tan\phi \sin\gamma$
$d_{av} = d_p - F\cos\Gamma$		

Gear contact stress

$$S_c = \sigma_c = C_p\left(\frac{W^t}{Fd_P I} K_o K_v K_m C_s C_{xc}\right)^{1/2}$$

At large end of tooth

Table 15-2, p. 791

Eqs. (15-5) to (15-8), p. 792

Eq. (15-11), p. 793

Eq. (15-12), p. 793

Eq. (15-9), p. 793

Fig. 15-6, p. 794

Eq. (15-21), p. 798

Gear wear strength

$$S_{wc} = (\sigma_c)_{all} = \frac{s_{ac} C_L C_H}{S_H K_T C_R}$$

Tables 15-4, 15-5, Fig. 15-12, Eq. (15-22), pp. 798–800

Fig. 15-8, Eq. (15-14), p. 795

Eqs. (15-16), (15-17), gear only, p. 796

Eqs. (15-19), (15-20), Table 15-3, pp. 797, 798

Eq. (15-18), p. 796

Wear factor of safety

$S_H = \dfrac{(\sigma_c)_{all}}{\sigma_c}$, based on strength

$n_w = \left(\dfrac{(\sigma_c)_{all}}{\sigma_c}\right)^2$, based on W^t; can be compared directly with S_F

Fig. 15–14

Summary for Straight-Bevel Gear Bending

Geometry	Force Analysis	Strength Analysis
$d_p = \dfrac{N_P}{P}$	$W^t = \dfrac{2T}{d_{av}}$	$W^t = \dfrac{2T}{d_p}$
$\gamma = \tan^{-1}\dfrac{N_P}{N_G}$	$W^r = W^t \tan\phi \cos\gamma$	$W^r = W^t \tan\phi \cos\gamma$
$\Gamma = \tan^{-1}\dfrac{N_G}{N_P}$	$W^a = W^t \tan\phi \sin\gamma$	$W^a = W^t \tan\phi \sin\gamma$
$d_{av} = d_p - F\cos\Gamma$		

Gear bending stress

$$S_t = \sigma = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J}$$

At large end of tooth

Table 15-2, p. 791

Eqs. (15-5) to (15-8), p. 792

Eq. (15-10), p. 793

Eq. (15-11), p. 793

Fig. 15-7, p. 794

Eq. (15-13), p. 793

Gear bending strength

$$S_{wt} = \sigma_{all} = \frac{s_{at} K_L}{S_F K_T K_R}$$

Table 15-6 or 15-7, pp. 799, 800

Fig. 15-9, Eq. (15-15), pp. 795, 796

Eqs. (15-19), (15-20), Table 15-3, pp. 797, 798

Eq. (15-18), p. 796

Bending factor of safety

$S_F = \dfrac{\sigma_{all}}{\sigma}$, based on strength

$n_B = \dfrac{\sigma_{all}}{\sigma}$, based on W^t, same as S_F

Fig. 15–15

Example 15–1

A pair of identical straight-tooth miter gears listed in a catalog has a diametral pitch of 5 at the large end, 25 teeth, a 1.10-in face width, and a 20° normal pressure angle; the gears are grade 1 steel through-hardened with a core and case hardness of 180 Brinell. The gears are uncrowned and intended for general industrial use. They have a quality number of $Q_v = 7$. It is likely that the application intended will require outboard mounting of the gears. Use a safety factor of 1, a 10^7 cycle life, and a 0.99 reliability.

(*a*) For a speed of 600 rev/min find the power rating of this gearset based on AGMA bending strength.

(*b*) For the same conditions as in part (*a*) find the power rating of this gearset based on AGMA wear strength.

(*c*) For a reliability of 0.995, a gear life of 10^9 revolutions, and a safety factor of $S_F = S_H = 1.5$, find the power rating for this gearset using AGMA strengths.

Example 15–1

From Figs. 15–14 and 15–15,

$$d_P = N_P/P = 25/5 = 5.000 \text{ in}$$

$$v_t = \pi d_P n_P/12 = \pi(5)600/12 = 785.4 \text{ ft/min}$$

Overload factor: uniform-uniform loading, Table 15–2, $K_o = 1.00$.
Safety factor: $S_F = 1$, $S_H = 1$.
Dynamic factor K_v: from Eq. (15–6),

$$B = 0.25(12-7)^{2/3} = 0.731$$

$$A = 50 + 56(1 - 0.731) = 65.06$$

$$K_v = \left(\frac{65.06 + \sqrt{785.4}}{65.06}\right)^{0.731} = 1.299$$

From Eq. (15–8),

$$v_{t\,\max} = [65.06 + (7-3)]^2 = 4769 \text{ ft/min}$$

$v_t < v_{t\,\max}$, that is, $785.4 < 4769$ ft/min, therefore K_v is valid. From Eq. (15–10),

$$K_s = 0.4867 + 0.2132/5 = 0.529$$

Example 15–1

From Eq. (15–11),

$$K_{mb} = 1.25 \quad \text{and} \quad K_m = 1.25 + 0.0036(1.10)^2 = 1.254$$

From Eq. (15–13), $K_x = 1$. From Fig. 15–6, $I = 0.065$; from Fig. 15–7, $J_P = 0.216$, $J_G = 0.216$. From Eq. (15–15),

$$K_L = 1.683(10^7)^{-0.0323} = 0.999\,96 \doteq 1$$

From Eq. (15–14),

$$C_L = 3.4822(10^7)^{-0.0602} = 1.32$$

Since $H_{BP}/H_{BG} = 1$, then from Fig. 15–10, $C_H = 1$. From Eqs. (15–13) and (15–18), $K_x = 1$ and $K_T = 1$, respectively. From Eq. (15–20),

$$K_R = 0.70 - 0.15 \log(1 - 0.99) = 1, \qquad C_R = \sqrt{K_R} = \sqrt{1} = 1$$

Example 15–1

(*a*) *Bending:* From Eq. (15–23),

$$s_{at} = 44(180) + 2100 = 10\,020 \text{ psi}$$

From Eq. (15–3),

$$s_t = \sigma = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J} = \frac{W^t}{1.10}(5)(1)1.299\frac{0.529(1.254)}{(1)0.216}$$
$$= 18.13W^t$$

From Eq. (15–4),

$$s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R} = \frac{10\,020(1)}{(1)(1)(1)} = 10\,020 \text{ psi}$$

Equating s_t and s_{wt},

$$18.13W^t = 10\,020 \qquad W^t = 552.6 \text{ lbf}$$

$$H = \frac{W^t v_t}{33\,000} = \frac{552.6(785.4)}{33\,000} = 13.2 \text{ hp}$$

Example 15–1

(*b*) *Wear:* From Fig. 15–12,

$$s_{ac} = 341(180) + 23\,620 = 85\,000 \text{ psi}$$

From Eq. (15–2),

$$\sigma_{c,\text{all}} = \frac{s_{ac} C_L C_H}{S_H K_T C_R} = \frac{85\,000(1.32)(1)}{(1)(1)(1)} = 112\,200 \text{ psi}$$

Now $C_p = 2290\sqrt{\text{psi}}$ from definitions following Eq. (15–21). From Eq. (15–9),

$$C_s = 0.125(1.1) + 0.4375 = 0.575$$

From Eq. (15–12), $C_{xc} = 2$. Substituting in Eq. (15–1) gives

$$\sigma_c = C_p \left(\frac{W^t}{F d_P I} K_o K_v K_m C_s C_{xc} \right)^{1/2}$$

$$= 2290 \left[\frac{W^t}{1.10(5)0.065} (1)1.299(1.254)0.575(2) \right]^{1/2} = 5242\sqrt{W^t}$$

Example 15–1

Equating σ_c and $\sigma_{c,\text{all}}$ gives

$$5242\sqrt{W^t} = 112\,200, \qquad W^t = 458.1 \text{ lbf}$$

$$H = \frac{458.1(785.4)}{33\,000} = 10.9 \text{ hp}$$

Rated power for the gearset is

$$H = \min(12.9, 10.9) = 10.9 \text{ hp}$$

Example 15–1

(*c*) Life goal 10^9 cycles, $R = 0.995$, $S_F = S_H = 1.5$, and from Eq. (15–15),

$$K_L = 1.683(10^9)^{-0.0323} = 0.8618$$

From Eq. (15–19),

$$K_R = 0.50 - 0.25\log(1 - 0.995) = 1.075, \qquad C_R = \sqrt{K_R} = \sqrt{1.075} = 1.037$$

From Eq. (15–14),

$$C_L = 3.4822(10^9)^{-0.0602} = 1$$

Example 15–1

Bending: From Eq. (15–23) and part (*a*), $s_{at} = 10\,020$ psi. From Eq. (15–3),

$$s_t = \sigma = \frac{W^t}{1.10} 5(1)1.299 \frac{0.529(1.254)}{(1)0.216} = 18.13W^t$$

From Eq. (15–4),

$$s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R} = \frac{10\,020(0.8618)}{1.5(1)1.075} = 5355 \text{ psi}$$

Equating s_t to s_{wt} gives

$$18.13W^t = 5355 \qquad W^t = 295.4 \text{ lbf}$$

$$H = \frac{295.4(785.4)}{33\,000} = 7.0 \text{ hp}$$

Example 15–1

Wear: From Eq. (15–22), and part (*b*), $s_{ac} = 85\,000$ psi.

Substituting into Eq. (15–2) gives

$$\sigma_{c,\text{all}} = \frac{s_{ac} C_L C_H}{S_H K_T C_R} = \frac{85\,000(1)(1)}{1.5(1)1.037} = 54\,640 \text{ psi}$$

Substituting into Eq. (15–1) gives, from part (*b*), $\sigma_c = 5242\sqrt{W^t}$.

Equating σ_c to $\sigma_{c,\text{all}}$ gives

$$\sigma_c = \sigma_{c,\text{all}} = 54\,640 = 5242\sqrt{W^t} \qquad W^t = 108.6 \text{ lbf}$$

The wear power is

$$H = \frac{108.6(785.4)}{33\,000} = 2.58 \text{ hp}$$

The mesh rated power is $H = \min(7.0, 2.58) = 2.6$ hp.

Design of Straight-Bevel Gear Mesh

A useful decision set for straight-bevel gear design is

- Function
- Design factor
- Tooth system
- Tooth count

A priori decisions

- Pitch and face width
- Quality number
- Gear material, core and case hardness
- Pinion material, core and case hardness

Design variables

Recommended Face Width

- Bending strength is not linear with face width
- Added material is placed at the small end of the teeth
- Recommended face width,

$$F = \min(0.3A_0,\ 10/P_d) \tag{15–24}$$

$$A_0 = \frac{d_P}{2\sin\gamma} = \frac{d_G}{2\sin\Gamma} \tag{15–25}$$

Example 15–2

Design a straight-bevel gear mesh for shaft centerlines that intersect perpendicularly, to deliver 6.85 hp at 900 rev/min with a gear ratio of 3:1, temperature of 300°F, normal pressure angle of 20°, using a design factor of 2. The load is uniform-uniform. Although the minimum number of teeth on the pinion is 13, which will mesh with 31 or more teeth without interference, use a pinion of 20 teeth. The material is to be AGMA grade 1 and the teeth are to be crowned. The reliability goal is 0.995 with a pinion life of 10^9 revolutions.

Example 15–2

First we list the a priori decisions and their immediate consequences.

Function: 6.85 hp at 900 rev/min, gear ratio $m_G = 3$, 300°F environment, neither gear straddle-mounted, $K_{mb} = 1.25$ [Eq. (15–11)], $R = 0.995$ at 10^9 revolutions of the pinion,

Eq. (15–14): $$(C_L)_G = 3.4822(10^9/3)^{-0.0602} = 1.068$$

$$(C_L)_P = 3.4822(10^9)^{-0.0602} = 1$$

Eq. (15–15): $$(K_L)_G = 1.683(10^9/3)^{-0.0323} = 0.8929$$

$$(K_L)_P = 1.683(10^9)^{-0.0323} = 0.8618$$

Eq. (15–19): $$K_R = 0.50 - 0.25\log(1 - 0.995) = 1.075$$

$$C_R = \sqrt{K_R} = \sqrt{1.075} = 1.037$$

Eq. (15–18): $$K_T = C_T = (460 + 300)/710 = 1.070$$

Example 15–2

Design factor: $n_d = 2$, $S_F = 2$, $S_H = \sqrt{2} = 1.414$.

Tooth system: crowned, straight-bevel gears, normal pressure angle 20°,

Eq. (15–13): $$K_x = 1$$

Eq. (15–12): $$C_{xc} = 1.5.$$

With $N_P = 20$ teeth, $N_G = (3)20 = 60$ teeth and from Fig. 15–14,

$$\gamma = \tan^{-1}(N_P/N_G) = \tan^{-1}(20/60) = 18.43^\circ \qquad \Gamma = \tan^{-1}(60/20) = 71.57^\circ$$

From Figs. 15–6 and 15–7, $I = 0.0825$, $J_P = 0.248$, and $J_G = 0.202$. Note that $J_P > J_G$.

Example 15–2

Decision 1: Trial diametral pitch, $P_d = 8$ teeth/in.

Eq. (15–10): $K_s = 0.4867 + 0.2132/8 = 0.5134$

$$d_P = N_P/P_d = 20/8 = 2.5 \text{ in}$$

$$d_G = 2.5(3) = 7.5 \text{ in}$$

$$v_t = \pi d_P n_P/12 = \pi(2.5)900/12 = 589.0 \text{ ft/min}$$

$$W^t = 33\,000 \text{ hp}/v_t = 33\,000(6.85)/589.0 = 383.8 \text{ lbf}$$

Eq. (15–25): $A_0 = d_P/(2\sin\gamma) = 2.5/(2\sin 18.43°) = 3.954$ in

Eq. (15–24):

$$F = \min(0.3A_0, 10/P_d) = \min[0.3(3.954), 10/8] = \min(1.186, 1.25) = 1.186 \text{ in}$$

Example 15–2

Decision 2: Let $F = 1.25$ in. Then,

Eq. (15–9): $$C_s = 0.125(1.25) + 0.4375 = 0.5937$$

Eq. (15–11): $$K_m = 1.25 + 0.0036(1.25)^2 = 1.256$$

Decision 3: Let the transmission accuracy number be 6. Then, from Eq. (15–6),

$$B = 0.25(12 - 6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

Eq. (15–5): $$K_v = \left(\frac{59.77 + \sqrt{589.0}}{59.77}\right)^{0.8255} = 1.325$$

Decision 4: Pinion and gear material and treatment. Carburize and case-harden grade ASTM 1320 to

Core 21 HRC (H_B is 229 Brinell)
Case 55-64 HRC (H_B is 515 Brinell)

From Table 15–4, $s_{ac} = 200\,000$ psi and from Table 15–6, $s_{at} = 30\,000$ psi.

Example 15–2

Gear bending: From Eq. (15–3), the bending stress is

$$(s_t)_G = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J_G} = \frac{383.8}{1.25} 8(1)1.325 \frac{0.5134(1.256)}{(1)0.202}$$

$$= 10\,390 \text{ psi}$$

The bending strength, from Eq. (15–4), is given by

$$(s_{wt})_G = \left(\frac{s_{at} K_L}{S_F K_T K_R}\right)_G = \frac{30\,000(0.8929)}{2(1.070)1.075} = 11\,640 \text{ psi}$$

The strength exceeds the stress by a factor of $11640/10390 = 1.12$, giving an actual factor of safety of $(S_F)_G = 2(1.12) = 2.24$.

Example 15–2

Pinion bending: The bending stress can be found from

$$(s_t)_P = (s_t)_G \frac{J_G}{J_P} = 10\,390 \frac{0.202}{0.248} = 8463 \text{ psi}$$

The bending strength, again from Eq. (15–4), is given by

$$(s_{wt})_P = \left(\frac{s_{at} K_L}{S_F K_T K_R}\right)_P = \frac{30\,000\,(0.8618)}{2(1.070)1.075} = 11\,240 \text{ psi}$$

The strength exceeds the stress by a factor of $11\,240/8463 = 1.33$, giving an actual factor of safety of $(S_F)_P = 2(1.33) = 2.66$.

Example 15–2

Gear wear: The load-induced contact stress for the pinion and gear, from Eq. (15–1), is

$$s_c = C_p\left(\frac{W^t}{Fd_P I}K_o K_v K_m C_s C_{xc}\right)^{1/2}$$

$$= 2290\left[\frac{383.8}{1.25(2.5)0.0825}(1)1.325(1.256)0.5937(1.5)\right]^{1/2}$$

$$= 107\ 560 \text{ psi}$$

From Eq. (15–2) the contact strength of the gear is

$$(s_{wc})_G = \left(\frac{s_{ac}C_L C_H}{S_H K_T C_R}\right)_G = \frac{200\ 000(1.068)(1)}{\sqrt{2}(1.070)1.037} = 136\ 120 \text{ psi}$$

The strength exceeds the stress by a factor of $136\ 120/107\ 560 = 1.266$, giving an actual factor of safety of $(S_H)_G^2 = 1.266^2(2) = 3.21$.

Example 15–2

Pinion wear: From Eq. (15–2) the contact strength of the pinion is

$$(s_{wc})_P = \left(\frac{s_{ac} C_L C_H}{S_H K_T C_R}\right)_P = \frac{200\ 000(1)(1)}{\sqrt{2}(1.070)1.037} = 127\ 450 \text{ psi}$$

The strength exceeds the stress by a factor of 136 120/127 450 = 1.068, giving an actual factor of safety of $(S_H)_P^2 = 1.068^2(2) = 2.28$.

The actual factors of safety are 2.24, 2.66, 3.21, and 2.28. Making a direct comparison of the factors, we note that the threat from gear bending and pinion wear are practically equal. We also note that three of the ratios are comparable. Our goal would be to make changes in the design decisions that drive the factors closer to 2. The next step would be to adjust the design variables. It is obvious that an iterative process is involved. We need a figure of merit to order the designs. A computer program clearly is desirable.

Worm Gearing

- Used to transmit rotary motion between non-parallel and non-intersecting shafts
- Usually perpendicular
- Relation between shaft angle and helix angles is

$$\sum = \psi_P \pm \psi_G \qquad (15\text{–}26)$$

- Crossed helical gears can be considered as non-enveloping worm gears

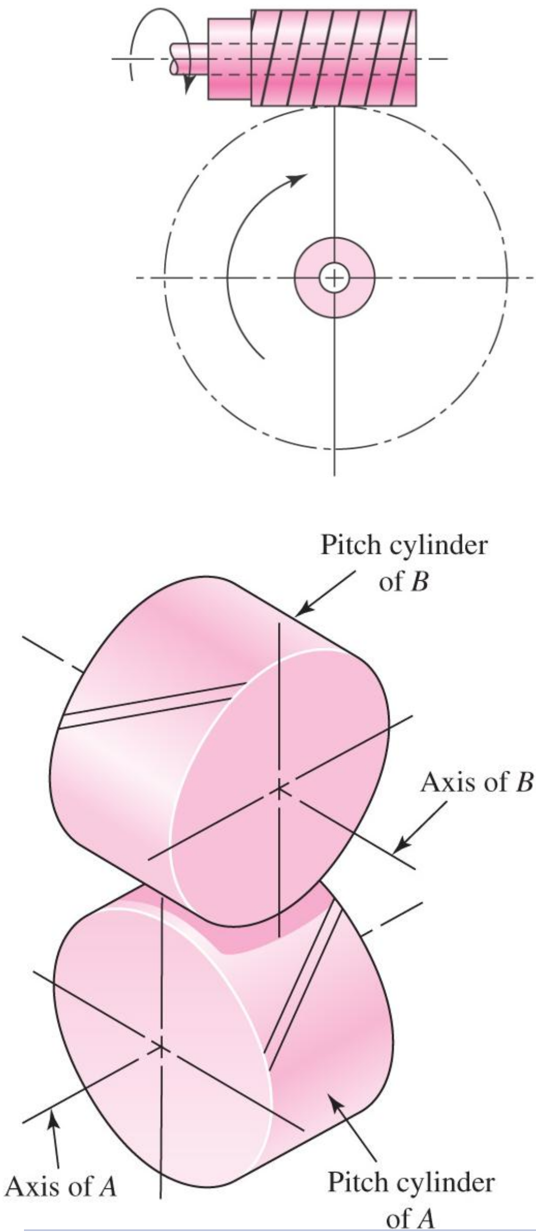


Fig. 15–16

Worm Gear Dimensions

- With center-to-center distance *C*, good proportions indicate the pitch worm diameter *d* should be in the range

$$\frac{C^{0.875}}{3} \leq d \leq \frac{C^{0.875}}{1.6} \qquad (15\text{–}27)$$

- Cylindrical worm dimensions common to both worm and gear,

Quantity	Symbol	ϕ_n 14.5° $N_W \leq 2$	20° $N_W \leq 2$	25° $N_W > 2$
Addendum	a	$0.3183p_x$	$0.3183p_x$	$0.286p_x$
Dedendum	b	$0.3683p_x$	$0.3683p_x$	$0.349p_x$
Whole depth	h_t	$0.6866p_x$	$0.6866p_x$	$0.635p_x$

*The table entries are for a tangential diametral pitch of the gear of $P_t = 1$.

Table 15–8

Friction Force

$$W_f = \frac{f W^t}{\cos\lambda \cos\phi_n} \tag{15–29}$$

where f = coefficient of friction

λ = lead angle at mean worm diameter

ϕ_n = normal pressure angle

Sliding Velocity and Torque

$$V_s = \frac{\pi n_W d_m}{12 \cos \lambda} \qquad (15\text{–}30)$$

$$T_G = \frac{W^t D_m}{2} \qquad (15\text{–}31)$$

Worm Gearing Equations for Allowable Tangential Force

$$(W^t)_{\text{all}} = C_s D_m^{0.8} F_e C_m C_v \qquad (15\text{–}28)$$

where C_s = materials factor

D_m = mean gear diameter, in (mm)

F_e = effective face width of the gear (actual face width, but not to exceed $0.67d_m$, the mean worm diameter), in (mm)

C_m = ratio correction factor

C_v = velocity factor

Worm Gearing Equations for Allowable Tangential Force

$$C_s = 270 + 10.37C^3 \qquad C \le 3 \text{ in} \tag{15–32}$$

For sand-cast gears,

$$C_s = \begin{cases} 1000 & C > 3 \quad d_G \le 2.5 \text{ in} \\ 1190 - 477 \log d_G & C > 3 \quad d_G > 2.5 \text{ in} \end{cases} \tag{15–33}$$

For chilled-cast gears,

$$C_s = \begin{cases} 1000 & C > 3 \quad d_G \le 8 \text{ in} \\ 1412 - 456 \log d_G & C > 3 \quad d_G > 8 \text{ in} \end{cases} \tag{15–34}$$

For centrifugally cast gears,

$$C_s = \begin{cases} 1000 & C > 3 \quad d_G \le 25 \text{ in} \\ 1251 - 180 \log d_G & C > 3 \quad d_G > 25 \text{ in} \end{cases} \tag{15–35}$$

Worm Gearing Equations for Allowable Tangential Force

The ratio correction factor C_m is given by

$$C_m = \begin{cases} 0.02\sqrt{-m_G^2 + 40m_G - 76} + 0.46 & 3 < m_G \leq 20 \\ 0.0107\sqrt{-m_G^2 + 56m_G + 5145} & 20 < m_G \leq 76 \\ 1.1483 - 0.006\,58m_G & m_G > 76 \end{cases} \qquad (15\text{–}36)$$

The velocity factor C_v is given by

$$C_v = \begin{cases} 0.659\exp(-0.0011V_s) & V_s < 700 \text{ ft/min} \\ 13.31V_s^{-0.571} & 700 \leq V_s < 3000 \text{ ft/min} \\ 65.52V_s^{-0.774} & V_s > 3000 \text{ ft/min} \end{cases} \qquad (15\text{–}37)$$

Coefficient of Friction *f*

$$f = \begin{cases} 0.15 & V_s = 0 \\ 0.124\ \exp\left(-0.074 V_s^{0.645}\right) & 0 < V_s \le 10\ \text{ft/min} \\ 0.103\ \exp\left(-0.110 V_s^{0.450}\right) + 0.012 & V_s > 10\ \text{ft/min} \end{cases} \qquad (15\text{–}38)$$

Worm-Gear Geometry

$$a = \frac{p_x}{\pi} = 0.3183 p_x \tag{15–39}$$

$$b = \frac{1.157 p_x}{\pi} = 0.3683 p_x \tag{15–40}$$

$$h_t = \begin{cases} \dfrac{2.157 p_x}{\pi} = 0.6866 p_x & p_x \geq 0.16 \text{ in} \\ \dfrac{2.200 p_x}{\pi} + 0.002 = 0.7003 p_x + 0.002 & p_x < 0.16 \text{ in} \end{cases} \tag{15–41}$$

$$d_0 = d + 2a \tag{15–42}$$

$$d_r = d - 2b \tag{15–43}$$

$$D_t = D + 2a \tag{15–44}$$

$$D_r = D - 2b \tag{15–45}$$

$$c = b - a \tag{15–46}$$

Face Width

$$(F_W)_{\max} = 2\sqrt{\left(\frac{D_t}{2}\right)^2 - \left(\frac{D}{2} - a\right)^2} = 2\sqrt{2Da} \qquad (15\text{–}47)$$

$$F_G = \begin{cases} 2d_m/3 & p_x > 0.16 \text{ in} \\ 1.125\sqrt{(d_0 + 2c)^2 - (d_0 - 4a)^2} & p_x \le 0.16 \text{ in} \end{cases} \qquad (15\text{–}48)$$

Heat Loss Rate From Worm-Gear Case

$$H_{\text{loss}} = 33\,000(1 - e)H_{\text{in}} \tag{15–49}$$

$$\hbar_{CR} = \begin{cases} \dfrac{n_W}{6494} + 0.13 & \text{no fan on worm shaft} \\[2ex] \dfrac{n_W}{3939} + 0.13 & \text{fan on worm shaft} \end{cases} \tag{15–50}$$

Energy Issues

- Heat loss rate from worm-gear case in ft·lbf/min,

$$H_{\text{loss}} = 33\,000(1 - e)H_{\text{in}} \qquad (15\text{–}49)$$

- Overall coefficient for combined convective and radiative heat transfer from the worm-gear case,

$$\hbar_{CR} = \begin{cases} \dfrac{n_W}{6494} + 0.13 & \text{no fan on worm shaft} \\[2ex] \dfrac{n_W}{3939} + 0.13 & \text{fan on worm shaft} \end{cases} \qquad (15\text{–}50)$$

- With case lateral area A, the oil sump temperature,

$$t_s = t_a + \frac{H_{\text{loss}}}{\hbar_{CR} A} = \frac{33\,000(1 - e)(H)_{\text{in}}}{\hbar_{CR} A} + t_a \qquad (15\text{–}51)$$

- AGMA recommended minimum lateral area in in^2

$$A_{\min} = 43.20C^{1.7} \qquad (15\text{–}52)$$

Buckingham Stress Equation

- Worm teeth are inherently much stronger than worm-gear teeth
- Worm-gear teeth are short and thick on the edges of the face
- Midplane they are thinner as well as curved
- Buckingham adapted the Lewis equation for this case,

$$\sigma_a = \frac{W_G^t}{p_n F_e y} \tag{15–53}$$

- y is the Lewis form factor

Worm-Gear Analysis

- Mechanical efficiency with worm driving,

$$e_W = \frac{\cos\phi_n - f\tan\lambda}{\cos\phi_n + f\cot\lambda} \qquad (15\text{–}54)$$

- Mechanical efficiency with gear driving,

$$e_G = \frac{\cos\phi_n - f\cot\lambda}{\cos\phi_n + f\tan\lambda} \qquad (15\text{–}55)$$

- To ensure worm gear will drive the worm,

$$f_{\text{stat}} < \cos\phi_n \tan\lambda \qquad (15\text{–}56)$$

Worm-Gear Analysis

- Relation of tangential worm force and tangential gear force,

$$W_W^t = W_G^t \frac{\cos\phi_n \sin\lambda + f\cos\lambda}{\cos\phi_n \cos\lambda - f\sin\lambda} \tag{15–57}$$

- Due to low efficiency of worm gearing, output power is not considered equivalent to input power
- Relating tangential gear force to output power and efficiency,

$$W_G^t = \frac{33\,000 n_d H_0 K_a}{V_G e} \tag{15–58}$$

- Power for worm and gear,

$$H_W = \frac{W_W^t V_W}{33\,000} = \frac{\pi d_W n_W W_W^t}{12(33\,000)} \text{ hp} \tag{15–59}$$

$$H_G = \frac{W_G^t V_G}{33\,000} = \frac{\pi d_G n_G W_G^t}{12(33\,000)} \text{ hp} \tag{15–60}$$

Worm-Gear Analysis

- Friction force,

$$W_f = \frac{f W_G^t}{f \sin\lambda - \cos\phi_n \cos\lambda} \tag{15–61}$$

- Sliding velocity of worm at pitch cylinder,

$$V_s = \frac{\pi d n_W}{12 \cos\lambda} \tag{15–62}$$

- Friction power,

$$H_f = \frac{|W_f| V_s}{33\,000} \text{ hp} \tag{15–63}$$

Maximum Lead Angle for Worm Gearing

Table 15–9

Largest Lead Angle Associated with a Normal Pressure Angle ϕ_n for Worm Gearing

ϕ_n	Maximum Lead Angle λ_{max}
14.5°	16°
20°	25°
25°	35°
30°	45°

Example 15–3

A single-thread steel worm rotates at 1800 rev/min, meshing with a 24-tooth worm gear transmitting 3 hp to the output shaft. The worm pitch diameter is 3 in and the tangential diametral pitch of the gear is 4 teeth/in. The normal pressure angle is 14.5°. The ambient temperature is 70°F. The application factor is 1.25 and the design factor is 1; gear face width is 2 in, lateral case area 600 in^2, and the gear is chill-cast bronze.

(*a*) Find the gear geometry.

(*b*) Find the transmitted gear forces and the mesh efficiency.

(*c*) Is the mesh sufficient to handle the loading?

(*d*) Estimate the lubricant sump temperature.

Example 15–3

(*a*) $m_G = N_G/N_W = 24/1 = 24$, gear: $D = N_G/P_t = 24/4 = 6.000$ in, worm: $d = 3.000$ in. The axial circular pitch p_x is $p_x = \pi/P_t = \pi/4 = 0.7854$ in. $C = (3+6)/2 = 4.5$ in.

Eq. (15–39): $a = p_x/\pi = 0.7854/\pi = 0.250$ in

Eq. (15–40): $b = 0.3683 p_x = 0.3683(0.7854) = 0.289$ in

Eq. (15–41): $h_t = 0.6866 p_x = 0.6866(0.7854) = 0.539$ in

Eq. (15–42): $d_0 = 3 + 2(0.250) = 3.500$ in

Eq. (15–43): $d_r = 3 - 2(0.289) = 2.422$ in

Eq. (15–44): $D_t = 6 + 2(0.250) = 6.500$ in

Eq. (15–45): $D_r = 6 - 2(0.289) = 5.422$ in

Eq. (15–46): $c = 0.289 - 0.250 = 0.039$ in

Eq. (15–47): $(F_W)_{\max} = 2\sqrt{2(6)0.250} = 3.464$ in

Example 15–3

The tangential speeds of the worm, V_W, and gear, V_G, are, respectively,

$$V_W = \pi(3)1800/12 = 1414 \text{ ft/min} \qquad V_G = \frac{\pi(6)1800/24}{12} = 117.8 \text{ ft/min}$$

The lead of the worm, from Eq. (13–27), is $L = p_x N_W = 0.7854(1) = 0.7854$ in. The lead angle λ, from Eq. (13–28), is

$$\lambda = \tan^{-1}\frac{L}{\pi d} = \tan^{-1}\frac{0.7854}{\pi(3)} = 4.764^\circ$$

The normal diametral pitch for a worm gear is the same as for a helical gear, which from Eq. (13–18) with $\psi = \lambda$ is

$$P_n = \frac{P_t}{\cos\lambda} = \frac{4}{\cos 4.764^\circ} = 4.014$$

$$p_n = \frac{\pi}{P_n} = \frac{\pi}{4.014} = 0.7827 \text{ in}$$

The sliding velocity, from Eq. (15–62), is

$$V_s = \frac{\pi d n_W}{12\cos\lambda} = \frac{\pi(3)1800}{12\cos 4.764^\circ} = 1419 \text{ ft/min}$$

Example 15–3

(*b*) The coefficient of friction, from Eq. (15–38), is

$$f = 0.103\exp[-0.110(1419)^{0.450}] + 0.012 = 0.0178$$

The efficiency e, from Eq. (13–46), is

$$e = \frac{\cos\phi_n - f\tan\lambda}{\cos\phi_n + f\cot\lambda} = \frac{\cos 14.5^\circ - 0.0178\tan 4.764^\circ}{\cos 14.5^\circ + 0.0178\cot 4.764^\circ} = 0.818$$

The designer used $n_d = 1$, $K_a = 1.25$ and an output horsepower of $H_0 = 3$ hp. The gear tangential force component W_G^t, from Eq. (15–58), is

$$W_G^t = \frac{33\,000 n_d H_0 K_a}{V_G e} = \frac{33\,000(1)3(1.25)}{117.8(0.818)} = 1284 \text{ lbf}$$

The tangential force on the worm is given by Eq. (15–57):

$$W_W^t = W_G^t \frac{\cos\phi_n \sin\lambda + f\cos\lambda}{\cos\phi_n\cos\lambda - f\sin\lambda}$$

$$= 1284\frac{\cos 14.5^\circ \sin 4.764^\circ + 0.0178\cos 4.764^\circ}{\cos 14.5^\circ \cos 4.764^\circ - 0.0178\sin 4.764^\circ} = 131 \text{ lbf}$$

Example 15–3

(*c*)

Eq. (15–34): $C_s = 1000$

Eq. (15–36): $C_m = 0.0107\sqrt{-24^2 + 56(24) + 5145} = 0.823$

Eq. (15–37): $C_v = 13.31(1419)^{-0.571} = 0.211^4$

Eq. (15–28): $(W^t)_{\text{all}} = C_s D^{0.8}(F_e)_G C_m C_v$

$$= 1000(6)^{0.8}(2)0.823(0.211) = 1456 \text{ lbf}$$

Since $W^t_G < (W^t)_{\text{all}}$, the mesh will survive at least 25 000 h. The friction force W_f is given by Eq. (15–61):

$$W_f = \frac{f W^t_G}{f \sin\lambda - \cos\phi_n \cos\lambda} = \frac{0.0178(1284)}{0.0178 \sin 4.764^\circ - \cos 14.5^\circ \cos 4.764^\circ}$$

$$= -23.7 \text{ lbf}$$

Example 15–3

The power dissipated in frictional work H_f is given by Eq. (15–63):

$$H_f = \frac{|W_f|V_s}{33\,000} = \frac{|-23.7|1419}{33\,000} = 1.02 \text{ hp}$$

The worm and gear powers, H_W and H_G, are given by

$$H_W = \frac{W_W^t V_W}{33\,000} = \frac{131(1414)}{33\,000} = 5.61 \text{ hp} \qquad H_G = \frac{W_G^t V_G}{33\,000} = \frac{1284(117.8)}{33\,000} = 4.58 \text{ hp}$$

Gear power is satisfactory. Now,

$$P_n = P_t/\cos\lambda = 4/\cos 4.764° = 4.014$$

$$p_n = \pi/P_n = \pi/4.014 = 0.7827 \text{ in}$$

Example 15–3

The bending stress in a gear tooth is given by Buckingham's adaptation of the Lewis equation, Eq. (15–53), as

$$(\sigma)_G = \frac{W_G^t}{p_n F_G y} = \frac{1284}{0.7827(2)(0.1)} = 8200 \text{ psi}$$

Stress in gear satisfactory.

Example 15–3

(*d*)

Eq. (15–52): $$A_{\text{min}} = 43.2C^{1.7} = 43.2(4.5)^{1.7} = 557 \text{ in}^2$$

The gear case has a lateral area of 600 in^2.

Eq. (15–49): $$H_{\text{loss}} = 33\,000(1 - e)H_{\text{in}} = 33\,000(1 - 0.818)5.61$$
$$= 33\,690 \text{ ft} \cdot \text{lbf/min}$$

Eq. (15–50): $$\hbar_{\text{CR}} = \frac{n_W}{3939} + 0.13 = \frac{1800}{3939} + 0.13 = 0.587 \text{ ft} \cdot \text{lbf/(min} \cdot \text{in}^2 \cdot {}^\circ\text{F)}$$

Eq. (15–51): $$t_s = t_a + \frac{H_{\text{loss}}}{\hbar_{\text{CR}} A} = 70 + \frac{33\,690}{0.587(600)} = 166^\circ\text{F}$$

Recommended Minimum Number of Worm-Gear Teeth

Table 15–10

Minimum Number of Gear Teeth for Normal Pressure Angle ϕ_n

ϕ_n	$(N_G)_{min}$
14.5	40
17.5	27
20	21
22.5	17
25	14
27.5	12
30	10

Example 15–4

Design a 10-hp 11:1 worm-gear speed-reducer mesh for a lumber mill planer feed drive for 3- to 10-h daily use. A 1720-rev/min squirrel-cage induction motor drives the planer feed ($K_a = 1.25$), and the ambient temperature is 70°F.

Solution

Function: $H_0 = 10$ hp, $m_G = 11$, $n_W = 1720$ rev/min.

Design factor: $n_d = 1.2$.

Materials and processes: case-hardened alloy steel worm, sand-cast bronze gear.

Worm threads: double, $N_W = 2$, $N_G = m_G N_W = 11(2) = 22$ gear teeth acceptable for $\phi_n = 20°$, according to Table 15–10.

Decision 1: Choose an axial pitch of worm $p_x = 1.5$ in. Then,

$$P_t = \pi/p_x = \pi/1.5 = 2.0944$$

$$D = N_G/P_t = 22/2.0944 = 10.504 \text{ in}$$

Eq. (15–39): $a = 0.3183 p_x = 0.3183(1.5) = 0.4775$ in (addendum)

Eq. (15–40): $b = 0.3683(1.5) = 0.5525$ in (dedendum)

Eq. (15–41): $h_t = 0.6866(1.5) = 1.030$ in

Example 15–4

Decision 2: Choose a mean worm diameter $d = 2.000$ in. Then

$$C = (d + D)/2 = (2.000 + 10.504)/2 = 6.252 \text{ in}$$

$$(d)_{\mathrm{lo}} = 6.252^{0.875}/3 = 1.657 \text{ in}$$

$$(d)_{\mathrm{hi}} = 6.252^{0.875}/1.6 = 3.107 \text{ in}$$

The range, given by Eq. (15–27), is $1.657 \le d \le 3.107$ in, which is satisfactory. Try $d = 2.500$ in. Recompute C:

$$C = (2.5 + 10.504)/2 = 6.502 \text{ in}$$

The range is now $1.715 \le d \le 3.216$ in, which is still satisfactory. Decision: $d = 2.500$ in. Then

Eq. (13–27): $$L = p_x N_W = 1.5(2) = 3.000 \text{ in}$$

Eq. (13–28):

$$\lambda = \tan^{-1}[L/(\pi d)] = \tan^{-1}[3/(\pi 2.5)] = 20.905^\circ \quad \text{(from Table 15–9 lead angle OK)}$$

Example 15–4

Eq. (15–62): $$V_s = \frac{\pi d n_W}{12 \cos \lambda} = \frac{\pi (2.5)1720}{12 \cos 20.905^\circ} = 1205.1 \text{ ft/min}$$

$$V_W = \frac{\pi d n_W}{12} = \frac{\pi (2.5)1720}{12} = 1125.7 \text{ ft/min}$$

$$V_G = \frac{\pi D n_G}{12} = \frac{\pi (10.504)1720/11}{12} = 430.0 \text{ ft/min}$$

Eq. (15–33): $$C_s = 1190 - 477 \log 10.504 = 702.8$$

Eq. (15–36): $$C_m = 0.02\sqrt{-11^2 + 40(11) - 76} + 0.46 = 0.772$$

Eq. (15–37): $$C_v = 13.31(1205.1)^{-0.571} = 0.232$$

Eq. (15–38): $$f = 0.103 \exp[-0.11(1205.1)^{0.45}] + 0.012 = 0.0191^5$$

Eq. (15–54): $$e_W = \frac{\cos 20^\circ - 0.0191 \tan 20.905^\circ}{\cos 20^\circ + 0.0191 \cot 20.905^\circ} = 0.942$$

Example 15–4

(If the worm gear drives, $e_G = 0.939$.) To ensure nominal 10-hp output, with adjustments for K_a, n_d, and e,

Eq. (15–57): $$W_W^t = 1222 \frac{\cos 20^\circ \sin 20.905^\circ + 0.0191 \cos 20.905^\circ}{\cos 20^\circ \cos 20.905^\circ - 0.0191 \sin 20.905^\circ} = 495.4 \text{ lbf}$$

Eq. (15–58): $$W_G^t = \frac{33\,000(1.2)10(1.25)}{430(0.942)} = 1222 \text{ lbf}$$

Eq. (15–59): $$H_W = \frac{\pi(2.5)1720(495.4)}{12(33\,000)} = 16.9 \text{ hp}$$

Eq. (15–60): $$H_G = \frac{\pi(10.504)1720/11(1222)}{12(33\,000)} = 15.92 \text{ hp}$$

Eq. (15–61): $$W_f = \frac{0.0191(1222)}{0.0191 \sin 20.905^\circ - \cos 20^\circ \cos 20.905^\circ} = -26.8 \text{ lbf}$$

Eq. (15–63): $$H_f = \frac{|-26.8|1205.1}{33\,000} = 0.979 \text{ hp}$$

Example 15–4

With $C_s = 702.8$, $C_m = 0.772$, and $C_v = 0.232$,

$$(F_e)_{\text{req}} = \frac{W_G^t}{C_s D^{0.8} C_m C_v} = \frac{1222}{702.8(10.504)^{0.8}0.772(0.232)} = 1.479 \text{ in}$$

Decision 3: The available range of $(F_e)_G$ is $1.479 \le (F_e)_G \le 2d/3$ or $1.479 \le (F_e)_G \le 1.667$ in. Set $(F_e)_G = 1.5$ in.

Eq. (15–28): $\quad W_{\text{all}}^t = 702.8(10.504)^{0.8}1.5(0.772)0.232 = 1239 \text{ lbf}$

This is greater than 1222 lbf. There is a little excess capacity. The force analysis stands.

Example 15–4

Decision 4:

Eq. (15–50): $\hbar_{CR} = \dfrac{n_W}{6494} + 0.13 = \dfrac{1720}{6494} + 0.13 = 0.395\ \text{ft} \cdot \text{lbf/(min} \cdot \text{in}^2 \cdot {}^\circ\text{F)}$

Eq. (15–49): $H_{\text{loss}} = 33\,000(1 - e)H_W = 33\,000(1 - 0.942)16.9 = 32\,347\ \text{ft} \cdot \text{lbf/min}$

The AGMA area, from Eq. (15–52), is $A_{\min} = 43.2C^{1.7} = 43.2(6.502)^{1.7} = 1041.5\ \text{in}^2$. A rough estimate of the lateral area for 6-in clearances:

$$
\begin{array}{ll}
\text{Vertical:} & d + D + 6 = 2.5 + 10.5 + 6 = 19\ \text{in} \\
\text{Width:} & D + 6 = 10.5 + 6 = 16.5\ \text{in} \\
\text{Thickness:} & d + 6 = 2.5 + 6 = 8.5\ \text{in} \\
\text{Area:} & 2(19)16.5 + 2(8.5)19 + 16.5(8.5) \doteq 1090\ \text{in}^2
\end{array}
$$

Expect an area of 1100 in^2. Choose: Air-cooled, no fan on worm, with an ambient temperature of 70°F.

$$t_s = t_a + \frac{H_{\text{loss}}}{\hbar_{CR} A} = 70 + \frac{32\,350}{0.395(1100)} = 70 + 74.5 = 144.5^\circ\text{F}$$

Lubricant is safe with some margin for smaller area.

Example 15–4

Eq. (13–18): $$P_n = \frac{P_t}{\cos\lambda} = \frac{2.094}{\cos 20.905^\circ} = 2.242$$

$$p_n = \frac{\pi}{P_n} = \frac{\pi}{2.242} = 1.401 \text{ in}$$

Gear bending stress, for reference, is

Eq. (15–53): $$\sigma = \frac{W_G^t}{p_n F_e y} = \frac{1222}{1.401(1.5)0.125} = 4652 \text{ psi}$$

The risk is from wear, which is addressed by the AGMA method that provides $(W_G^t)_{\text{all}}$.

Buckingham Wear Load

- Buckingham showed that the allowable gear-tooth loading for wear can be estimated from

$$(W_G^t)_{\text{all}} = K_w d_G F_e \qquad (15\text{–}64)$$

where K_w = worm-gear load factor

d_G = gear-pitch diameter

F_e = worm-gear effective face width

Wear Factor K_w for Worm Gearing

Material		Thread Angle ϕ_n			
Worm	**Gear**	**$14\frac{1}{2}°$**	**20°**	**25°**	**30°**
Hardened steel*	Chilled bronze	90	125	150	180
Hardened steel*	Bronze	60	80	100	120
Steel, 250 BHN (min.)	Bronze	36	50	60	72
High-test cast iron	Bronze	80	115	140	165
Gray iron†	Aluminum	10	12	15	18
High-test cast iron	Gray iron	90	125	150	180
High-test cast iron	Cast steel	22	31	37	45
High-test cast iron	High-test cast iron	135	185	225	270
Steel 250 BHN (min.)	Laminated phenolic	47	64	80	95
Gray iron	Laminated phenolic	70	96	120	140

*Over 500 BHN surface.

†For steel worms, multiply given values by 0.6.

Table 15–11

Example 15–5

Estimate the allowable gear wear load $(W_G^t)_{\text{all}}$ for the gearset of Ex. 15–4 using Buckingham's wear equation.

Solution

From Table 15–11 for a hardened steel worm and a bronze bear, K_w is given as 80 for $\phi_n = 20°$. Equation (15–64) gives

$$\left(W_G^t\right)_{\text{all}} = 80(10.504)1.5 = 1260 \text{ lbf}$$

which is larger than the 1239 lbf of the AGMA method. The method of Buckingham does not have refinements of the AGMA method. [Is $(W_G^t)_{\text{all}}$ linear with gear diameter?]