9. DC-DC Converters (DC –Choppers)

A dc-to-dc converter, also known as dc chopper, is a static device which is used to obtain a variable dc voltage from a constant dc voltage source. Choppers are widely used in trolley cars, battery operated vehicles, traction motor control, control of large number of dc motors, etc….. They are also used as dc voltage regulators.

Choppers are of two types: (1) Step-down choppers, and (2) Step-up choppers. In step-down choppers, the output voltage will be less than the input voltage, whereas in step-up choppers output voltage will be more than the input voltage.

9.1 PRINCIPLE OF STEP-DOWN CHOPPER

Figure 9.1 shows a step-down chopper with resistive load. The thyristor in the circuit acts as a switch. When thyristor is ON, supply voltage appears across the load and when thyristor is OFF, the voltage across the load will be zero. The output voltage waveform is as shown in Fig. 9.2.

![Chopper circuit](image)

![Chopper output voltage waveform](image)
Methods of Control
The output dc voltage can be varied by the following methods.
- Pulse width modulation control or constant frequency operation.
- Variable frequency control.

Pulse Width Modulation
- $t_{ON}$ is varied keeping chopping frequency 'f' & chopping period 'T' constant.
- Output voltage is varied by varying the ON time $t_{ON}$

9.2 ANALYSIS OF A STEP-DOWN CHOPPER WITH R-LOAD

Referring to Fig.9.2, the average output voltage $v_o$ can be found as

Let $T = control\ period = t_{on} + t_{off}$

$$v_o = V_{av} = \frac{1}{T} \int_0^{t_{on}} V_d \, dt$$

$$V_o = V_d \frac{t_{on}}{T} = V_d (\gamma)$$

where, $\gamma = \frac{t_{on}}{T} = Duty\ cycle$

- Maximum value of $\gamma = 1$ when $t_{on} = T$ ($t_{off} = 0$)
- Minimum value of $\gamma = 0$ when $t_{on} = 0$ ($t_{off} = 0$)

The output voltage is stepped down by the factor $\gamma$ ($0 \leq V_o \leq V_d$). Therefore this form of chopper is a step down chopper.

The R.M.S. value of the output voltage $v_{o,rms} = \sqrt{\frac{1}{T} \int_0^{t_{on}} v_0^2 \, dt} = \gamma \sqrt{V_d}$

The Output power $= \frac{v_{o,rms}^2}{R} = \gamma \frac{V_d^2}{R}$
Input current (Assume 100% efficiency) \[ I_a = \frac{P}{V} = \frac{\gamma v_d^2}{R} \cdot \frac{1}{v_d} = \frac{\gamma v_d}{R} \]

f= chopping frequency \( f = \frac{1}{\text{chopping period}(T)} \) = \( \frac{1}{T} \)

The ripple factor, \( RF \)

It is a measure of the ripple content.

\[ RF = \sqrt{\left(\frac{V_{o_{rms}}}{V_o}\right)^2 - 1} = \sqrt{\left(\frac{\gamma v_d^2}{\gamma^2 V_o^2}\right) - 1} = \sqrt{\frac{1}{\gamma} - 1} = \sqrt{\frac{1 - \gamma}{\gamma}} \]

Note1: In this type of chopper both the voltage and current are always positive, hence this chopper is called a single-quadrant Buck converter or class – A chopper.

Note2: The chopper switch can also be implemented by using a power BJT, power MOSFET, GTO, and IGBT transistor. The practical devices have a finite voltage drop ranging from 0.5V to 2V, and for the sake of simplicity, the voltage drop of their power semiconductor devices are neglected.
Example 1: A transistor dc chopper circuit (Buck converter) is supplied with power form an ideal battery of 100 V. The load voltage waveform consists of rectangular pulses of duration 1 ms in an overall cycle time of 2.5 ms. Calculate, for resistive load of 10 Ω.

(a) The duty cycle $\gamma$.
(b) The average value of the output voltage $V_o$.
(c) The rms value of the output voltage $V_{rms}$.
(d) The ripple factor $RF$.
(e) The output dc power.

Solution:

(a) $ton = 1$ ms, $T = 2.5$ ms

$$\gamma = \frac{ton}{T} = \frac{1\text{ ms}}{2.5\text{ ms}} = 0.4$$

(b) $V_{av} = V_o = \gamma \ V_d = 0.4 \times 100 = 40$ V.

(c) $V_{rms} = \sqrt{\gamma} \ V_i = \sqrt{0.4} \times 100 = 63.2$ V.

(d) $RF = \sqrt{\frac{1 - \gamma}{\gamma}} = \sqrt{\frac{1 - 0.4}{0.4}} = 1.225$

(e) $I_a = \frac{V_o}{R} = \frac{40}{10} = 4$ A

$$P_{av} = I_a \ V_o = 4 \times 40 = 160 \text{ W}$$
9.3 STEP-DOWN CHOPPER WITH R-L LOAD

Consider a class-A chopper circuit with R-L load as shown in Fig.9.4. This is a step down chopper with one quadrant operation.

If we use the simplified linear analysis by considering that $T \ll \tau$, where $(T = t_{on} + t_{off})$. In this case the current is continuous as shown in Fig.9.5.
Referring to Fig. 9.5:

- The current variation is almost linear and the current waveform becomes triangular.
- During the ON period, the equation govern the circuit is

\[ V_d = Ri + L \frac{di}{dt} \]

Since \( \frac{di}{dt} \) = constant, hence during ON period:

\[ \frac{di}{dt} = \frac{l_2 - l_1}{t_{on}} = \frac{\Delta I}{t_{on}} \]

Where \( \Delta I \) is the peak – to – peak of the load current. Thus the equation of the current is given by:

\[ i_1 = I_1 + \frac{\Delta I}{\gamma T} t \quad 0 \leq t \leq t_{on} \]

Where \( \gamma = \frac{t_{on}}{T} \)

During the off period:

\[ \frac{di}{dt} = \frac{l_1 - l_2}{t_{off}} = - \frac{\Delta I}{t_{off}} = - \frac{\Delta I}{T-t_{on}} = - \frac{\Delta I}{T-\gamma T} = - \frac{\Delta I}{(1-\gamma)T} \]

Hence, during the off the equation of the current is

\[ i_2 = I_2 - \frac{\Delta I}{(1-\gamma)T} (t-t_{on}) \quad t_{on} \leq t \leq T \]

The average value of the output current is

\[ I_{av} = \frac{1}{T} \left[ \frac{1}{2} t_{on}(l_2 - l_1) + \frac{1}{2} t_{off}(l_2 - l_1) + l_1 T \right] \]

\[ I_{av} = \frac{1}{2} (l_2 + l_1) \]
Example 2; An 80 V battery supplies RL load through a DC chopper. The load has a freewheeling diode across it is composed of 0.4 H in series with 5Ω resistor. Load current, due to improper selection of frequency of chopping, varies widely between 9A and 10.2.

(a) Find the average load voltage, current and the duty cycle of the chopper.

(b) What is the operating frequency $f$?

(c) Find the ripple current to maximum current ratio.

Solution:

(a) The average load voltage and current are:

\[
V_{av} = V_o = \gamma V_d
\]

\[
I_{av} = \frac{1}{2} (I_2 + I_1) = \frac{9 + 10.2}{2} = 9.6A
\]

\[
I_{av} = \frac{V_{av}}{R} = \frac{\gamma V_d}{R} \quad \text{or} \quad \gamma = \frac{I_{av} R}{V_i} = \frac{9.6 \times 5}{80} = 0.6
\]

\[V_{av} = 0.6 \times 80 = 48 \text{ V.}\]

(b) To find the operating (chopping) frequency:

During the ON period,

\[V_d = Ri + L \frac{di}{dt} \quad \ldots \ldots \ldots \ldots \ldots (1)\]

Assuming \[\frac{di}{dt} \approx \text{constant}\]

\[
\frac{di}{dt} \cong \frac{\Delta I}{t_{on}} = \frac{10.2 - 9}{\gamma T} = \frac{1.2}{\gamma T}
\]

From eq.(1)

\[L \frac{di}{dt} \cong V_d - I_{av} R = 80 - 5 \times 9.6 = 32 \text{V}\]

or \[\frac{di}{dt} = \frac{32}{L} = \frac{32}{0.4} = 80 \text{ A.s}\]
but \[ \frac{di}{dt} = \frac{1.2}{\gamma T} = 80 = \frac{1.2}{0.6T} \]

\[ \therefore T = \frac{1.2}{0.6 \times 80} = 25 \text{ ms} \]

Hence \[ f = \frac{1}{T} = \frac{1}{25 \times 10^{-3}} = 40 \text{ Hz} \]

The maximum current \( I_m \) occurs at \( \gamma = 1 \),

\[ \therefore I_m = \frac{V V_d}{R} = \frac{1 \times 80}{5} = 16 \text{ A} \]

Ripple current \( I_r = \Delta I = 10.2 - 9 = 1.2 \text{ A} \)

\[ \therefore \frac{I_r}{I_m} = \frac{1.2}{16} = 0.075 \text{ or } 7.5\% \]

**Input Current \( I_s \)**

For the class-A chopper had shown in Fig.9.4, the On-state and OFF-state equivalent circuits are as depicted in Fig.9.6. When the thyristor is closed (during the ON period), the load current “\( i \)” rises from \( I_1 \) to \( I_2 \) and falls from \( I_2 \) to \( I_1 \) during the off period as shown in Fig.9.7(a). The input current \( I_s \) flows during the ON period only as shown in Fig.9.7(b).
The equation of the input current is

\[ i_s = i_1 = I_1 + \frac{\Delta I}{\gamma T} t \quad 0 \leq t \leq t_{on} \]

\[ i_s = 0 \quad t_{on} \leq t \leq T \]

The average value of the current drawn from the supply is simply found by,

\[ I_{s(av)} = \frac{1}{T} \left[ \frac{1}{2} t_{on} (l_2 - l_1) + \frac{t_{on} I_1}{T} \right] \]

\[ I_{s(av)} = \frac{1}{T} \left[ \frac{1}{2} t_{on} (l_2 + l_1) \right] = \frac{t_{on}}{2T} (l_2 + l_1) = \gamma I_{av} \]
**Minimum and Maximum Load Currents**

The minimum current $I_1$ and maximum current $I_2$ can be found from the following two equations:

$$I_1 = I_{min} = \frac{V_o}{R} - \frac{t_{off}}{2L} V_o$$

$$I_2 = I_{max} = \frac{V_o}{R} + \frac{t_{off}}{2L} V_o$$

Where $V_o = V_{av}$

**Note: The proof of these two equations is not needed**

**Example 3:** A DC Buck converter operates at frequency of 1 kHz from 100V DC source supplying a 10 Ω resistive load. The inductive component of the load is 50mH. For output average voltage of 50V volts, find:

(a) The duty cycle

(b) $t_{on}$

(c) The rms value of the output current

(d) The average value of the output current

(e) $I_{max}$ and $I_{min}$

(f) The input power

(g) The peak-to-peak ripple current.
Solution:

(a) \[ V_{av} = V_o = \gamma V_d \]
\[ \gamma = \frac{V_{av}}{V_d} = \frac{50}{100} = 0.5 \]

(b) \[ T = \frac{1}{f} = \frac{1}{1000} = 1ms \]
\[ \gamma = \frac{t_{on}}{T} \]
\[ t_{on} = \gamma T = 0.5 \times 1ms = 0.5 \text{ ms} \]

(c) \[ V_{rms} = \sqrt{\gamma} V_i = \sqrt{0.5} \times 100 = 70.07 \text{ V} \]

(d) \[ I_{av} = \frac{V_{av}}{R} = \frac{50}{10} = 5 \text{ A} \]

(e) \[ I_{max} = \frac{V_{av}}{R} + \frac{t_{off}}{2L} V_{av} = \frac{50}{10} + \frac{(1-0.5) \times 10^{-3}}{2 \times 50 \times 10^{-3}} \times 50 \]
\[ = 5 + 0.25 = 5.25 \text{ A} \]
\[ I_{min} = \frac{V_{av}}{R} - \frac{t_{off}}{2L} V_{av} = \frac{50}{10} - \frac{(1-0.5) \times 10^{-3}}{2 \times 50 \times 10^{-3}} \times 50 \]
\[ = 5 - 0.25 = 4.75 \text{ A} \]

(f) \[ I_{s(av)} = \frac{\gamma}{2} (I_{min} + I_{max}) = \gamma I_{av} = 0.5 \times 5 = 2.5 A \]
\[ P_{in} = I_{s(av)} V_d = 2.5 \times 100 = 250 \text{ W} \]

(g) \[ I_{p-p} = \Delta I = I_{max} - I_{min} = 5.25 - 4.75 = 0.5 A \]
9.4 Application to DC Drives

**Chopper-Fed DC Motor Drive**

- **D.C motor** is considered a SISO (Single Input and Single Output) system having torque/speed characteristics compatible with most mechanical loads. This makes a **DC motor** controllable over a wide range of speeds by proper adjustment of the terminal voltage.
- A simple chopper – fed DC motor drive is shown in Fig.9.8. The basic principle behind DC motor speed control is that the output speed of DC motor can be varied by controlling armature voltage for speed below and up to rated speed keeping field voltage constant. The armature voltage can be controlled by controlling the duty cycle of the converter (here the converter used is a DC chopper).

![Diagram of Chopper-Fed DC Motor Drive](image)

**Fig.9.8: Chopper-Fed DC Motor Drive.**

- In Fig. 9.8, the converter output gives the dc output voltage $V_a$ required to drive the motor at the desired speed. In this diagram, the DC motor is represented by its equivalent circuit consisting of inductor $L_a$ and resistor $R_a$ in series with the back emf $(E_b)$. The thyristor Th1 is triggered by a pulse width modulated (PWM) signal to control the average motor voltage. Theoretical waveforms illustrating the chopper operation are shown in Fig.9.9.
Fig. 9.9

The average armature voltage is a direct function of the chopper duty cycle \( \gamma \),

\[ V_{av} = \gamma \ V_d \]

Note that this relation is valid only when the armature current is continuous. In steady state, the armature average current is equal to

\[ I_{av} = \frac{V_{av} - E_b}{R_a} \]

Where \( I_{av} \) = average armature current (A).

\( E_b \) is the internal generated voltage (back emf) is given by:

\[ E_b = K_e \varphi \ n \]

Finally, solving for the motor's speed:

\[ n = \frac{V_{av}}{K_e \varphi} - \frac{R_a}{K_T K_e \varphi^2} T_d \]
or 

\[ n = \frac{\gamma V_d}{K_e \phi} - \frac{R_a}{K_T K_e \phi^2} T_d \]

Where \( n \) = speed in rpm.

and the motor torque is 

\[ T_d = K_T I_A \phi \]

\( K_T \) = Torque constant = 9.55 \( K_e \) \( \) (See Lecture No.8).

At starting, \( n=0 \). The starting torque \( T_{st} \) may be found as:

\[ n = 0 = \frac{\gamma V_d}{K_e \phi} - \frac{R_a}{K_T K_e \phi^2} T_{st} \]

\[ \therefore T_{st} = \frac{9.55 \gamma V_d}{R_a} K_e \phi \]

Fig.9.10 Speed – torque characteristics.
Example 1: A separately excited d.c. motor with $R_a = 0.3 \Omega$, and $L_a = 15 \, \text{mH}$ is to be speed controlled over a range 0-2000 rpm. The d.c. supply is 220V. The load torque is constant and requires an average armature current of 25A.

(a) Calculate the range of the duty cycle $\gamma$ required if the motor design constant $K_e \Phi = 0.1002 \, \text{V/rpm}$.

Solution: In the steady-state, the armature inductance has no effect. The required motor terminal voltages are:

At $n = 0$, $E_b = 0$, so that

$$V_{dc} = E_b + I_a R_a = I_a R_a = 25 \times 0.3 = 7.5 \, \text{V}.$$

At $n = 2000 \, \text{rpm}$,

$$E_b = K_e \Phi n = 0.1002 \times 2000 = 200.4 \, \text{V}.$$

$$\therefore V_{dc} = E_b + I_a R_a = 200.4 + 25 \times 0.3 = 207.9 \, \text{V}.$$

$$V_o = \gamma V_d$$

To give $V_o = 7.5 \, \text{V}$, \(\gamma \times 200.4 = 7.5\) or $\gamma = \frac{7.5}{200.4} = 0.0374$

To give $V_o = 207.9 \, \text{V}$, \(\gamma \times 200.4 = 207.9\) or $\gamma = \frac{207.9}{200.4} = 0.943$.

Range of $\gamma$: $0.0374 < \gamma < 0.943$.

(b) If the chopper was to be switched fully on, what is the speed of the motor when $\gamma = 1$?

Sol. when $\gamma = 1$, $V_o = 220$.

$$n = \frac{E_b}{K_e \Phi} \gamma \quad \therefore E_b = V_o - I_a R_a = 220 - 25 \times 0.3 = 212.5 \, \text{V}$$

$$n = \frac{212.5}{0.1002} = 2121 \, \text{rpm}$$
Example:

An electrically-driven automobile is powered by a d.c. series motor rated at 160 V, 200 A. The motor resistance and inductance are respectively 0.65 Ω and 6 mH. Power is supplied from ideal battery of 180 V via class-A d.c. chopper having a fixed frequency of 100 Hz. The machine constant \( K_e = 0.00025 \) and the motor speed is 2500 rpm. Determine the maximum and minimum currents, the mean torque and the mean power produced by the motor when running at 2500 rpm with duty cycle \( D \) of 3/5.

Solution:

Chopping period \( T = \frac{1}{F} = \frac{1}{100} = 10 \text{ ms} \)

\[ t_{on} = DT = \frac{3}{5} \times 10 = 6 \text{ ms} \]

\[ t_{off} = 10 - 6 = 4 \text{ ms} \]

\[ I_{max} = \frac{V_{av}}{R_a} + \frac{t_{off}}{2L_a} \frac{V_{av}}{R_a} = \frac{V_{av}}{R_a} + \frac{t_{off}}{2L_a} \frac{V_{av}}{R_a} \]

\[ = \frac{3}{5} \times 120}{0.65} + \frac{4 \times 10^{-3}}{2 \times 6 \times 10^{-3}} \left( \frac{3}{5} \times 120 \right) \]

\[ = 110.7 + 24 = 134.7 \text{ A} \]

\[ I_{min} = \frac{V_{av}}{R_a} - \frac{t_{off}}{2L_a} \frac{V_{av}}{R_a} \]

\[ = 110.7 - 24 = 86.76 \text{ A} \]

For series motor:

Mean torque: \( T_e = K_e \phi I_{av} = 9.55 \ K_e \phi \left( \frac{I_{max} + I_{min}}{2} \right)^2 \)

\[ = 9.55 \times 0.00025 \left( \frac{139.7 + 86.76}{2} \right)^2 \]

\[ = 30 \text{ N.m} \]

Mean power: \( P_e = \omega T_e = \frac{2 \pi}{60} \times 2500 \times 30 = 7.85 \text{ kw} \)

\[ = 10.5 \text{ hp} \]