14. DC – to – AC Converters

Single-phase inverters:

14.1 Single-phase half-bridge inverter

This type of inverter is very simple in construction. It does not need output transformer like parallel inverter. It sometimes called center-tapped source inverter. The basic configuration of this inverter is shown in Fig 14.1.

- The top and bottom switch has to be “complementary” i.e. If the top switch is closed (ON), the bottom must be off, and vice-versa.
- The output voltage waveform is a square wave as shown in Fig.14.2.
In practice, a dead time between ON and OFF (td) for Q₁ & Q₂ is required to avoid short circuit or “shoot-through” faults. This leads to produce quasi-square wave output voltage (Fig.14.2).

![Fig.14.3](image)

**Quasi-squared output voltage waveform**

\[ td = \text{Dead time} \]

**Performance of Half-Bridge Inverter with Resistive - Inductive loads:**

- If the load is resistive, the output current waveform will be a copy of the voltage waveform as shown in Fig.14.4 (a). The output voltage is a square (or quasi-square) wave.

- However with an inductive load Fig.14.4 (b), the load current \( i \) is delayed although the output voltage wave is still a square. The current will grow exponentially during the positive half-cycle from \(-I_n\) to \( I_p\) according to the following equation:

\[
\left( \frac{V_d}{2} = Ri + L \frac{di}{dt} \right).
\]

- Through D1 [load returning power to the upper half of the source].
- Through Q1 [load absorbing power from the upper half of the source] until \( t = \frac{T}{2} \), whereby \( i = I_p \).
Negative half cycle starts by conduction of Q2. Current start its change from \( I_p \) to zero through D2 and then to \(-I_n\) through Q2, according to: 
\[
\left(-\frac{V_d}{2}\right) = Ri + L \frac{di}{dt}.
\]

RMS value of the Load voltage:
\[
V_{r.m.s} = \sqrt{\frac{1}{T} \int_0^T V_o^2 \, dt} = \left[ \frac{1}{T} \left( \frac{V_d}{2} \right)^2 T \right]^{\frac{1}{2}} = \frac{V_d}{2}
\]
- Load voltage \( v_o (\omega t) \) can be expressed in terms of harmonics by Fourier series as:

\[
v_o (\omega t) = \sum_{n=1,3,5}^{\infty} \frac{4}{n\pi} \left( \frac{V_d}{2} \right) \sin n \omega t , \quad v_o (\omega t) = 0 \text{ for } n=2,4,6...
\]

Where: \( \omega = 2\pi f_0 \) is the frequency of the output voltage in (rad/sec).

- The fundamental component of the load voltage had a peak value \( V_{o1p} = C_1 = \frac{4}{\pi} \frac{V_d}{2} \), and it has r.m.s value

\[
V_{o1rms} = \frac{C_1}{\sqrt{2}} = \frac{\sqrt{2} V_d}{\pi} = 0.45 \ V_d = V_{L1}
\]

- For an R-L load, the instantaneous load current \( i \) can be found by dividing the instantaneous output voltage by the load impedance \( Z = R + jn\omega L \), or \( Z = \sqrt{R^2 + (n\omega L)^2} \) , thus

\[
i(\omega t) = \frac{v_o (\omega t)}{Z}
\]

\[
i(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{2V_d}{n\pi \sqrt{R^2 + (n\omega L)^2}} \sin (n \omega t - \Psi n)
\]

where \( \Psi n = \tan^{-1} \frac{n\omega L}{R} \)

- If \( I_{o1} \) is the r.m.s fundamental load current, the fundamental output power (for \( n=1 \) is):

\[
P_1 = V_{o1} I_{o1} \cos \Psi_1 = I_{o1}^2 R
\]

\[
= \left[ \frac{2V_d}{\sqrt{2\pi \sqrt{R^2 + (\omega L)^2}}} \right]^2 \cdot R
\]

Note: In most applications (e.g. electric motor drives) the output power due to the fundamental current is generally the useful power,
and the power due to harmonic currents is dissipated as heat and increases the load temperature.

Example 1: The single-phase half-bridge inverter in Fig. 14.1 has a resistive load of $R = 2.4 \Omega$ and the d.c. input voltage $V_d = 48$ V. Determine:

(a) The r.m.s value of the load voltage, $V_{\text{rms}}$.

(b) The r.m.s value of the load voltage at the fundamental frequency $V_{L1}$.

(c) The output power $P_o$.

(d) The average and peak currents of each transistor.

(e) The peak reverse blocking voltage $V_{BR}$ of each transistor.

(f) The THD factor.

Solution:

(a) $V_{\text{rms}} = \frac{V_d}{\sqrt{2}} = \frac{48}{\sqrt{2}} = 24$ V.

(b) $V_{L1} = 0.45 V_d = 21.6$ V.

(c) $P_o = \frac{V_{\text{rms}}^2}{R} = \frac{(24)^2}{2.4} = 240$ W.

(d) The peak transistor current $I_T = \frac{V_{\text{rms}}}{R} = \frac{24}{2.4} = 10$ A.

Because each transistor conducts for a 50% duty cycle, the average current of each transistor is $I_Q = 0.5 \times 10 = 5$ A.

(e) The peak reverse blocking voltage $V_{BR} = 2 	imes 24 = 48$ V.

(f) $\text{THD} = \frac{V_h}{V_{L1}} = \frac{\text{Harmonic Voltage}}{\text{Fundamental Voltage}}$

$V_h = \sqrt{\sum_{n=2,3,5} V_{n\text{r.m.s.}}^2} = \sqrt{V_{L1\text{r.m.s.}}^2 - V_{L1}^2}$

$\therefore \text{THD} = \frac{\sqrt{V_{L1\text{r.m.s.}}^2 - V_{L1}^2}}{V_{L1}^2} = \sqrt{\frac{(24)^2 - (21.6)^2}{(21.6)^2}}$

$= 48.34\%$.
14.2 Single-phase full-bridge inverter

The single-phase bridge – Inverter using BJT transistors is shown in Fig.14.5. This inverter is constructed from two half – bridge inverter using single DC source Vd and the load is connected between the centers of the two legs.

![Fig.14.5 Single-phase full-bridge inverter circuit](image)

- **SQUARE-WAVE OUTPUT**:

Q1 and Q2 are triggered simultaneously and so are Q3 and Q4. Each device is made to conduct for half time of the output cycle, the load voltage waveform with the transistor base currents are shown in Fig 14.6.

![Fig.14.6](image)
Performance with R - L load:

- With resistive-inductive load, the current $i_0$ lags the square wave output voltage $v_0$ as shown in Fig.14.7.

- Triggering Q1 & Q2 connects the load to $Vd$. For steady load condition, $i_0$ grows exponentially through D1 D2 and then through Q1 Q2 from $-I_n$ to $I_p$ according to $(Vd = Ri + L\frac{di}{dt})$.

- Negative half cycle starts by triggering Q3 & Q4 at $t_1 = \frac{T}{2}$, and Q1 Q2 goes off when $i_{b1} = i_{b2} = 0$ (base current blocking).

Fig.14.7 current for single-phase full-bridge inverter.

- Load voltage reverses to $-Vd$ and $i_o$ will flow through D3D4 and then through Q3Q4 according to the equation $(-Vd = Ri + L\frac{di}{dt})$.

At the end of negative half cycle $i_o = -I_n$.

Note that: $t_1 = \frac{T}{2} = \frac{1}{2f} = \frac{1}{2} \left(\frac{2\pi}{\omega}\right) = \frac{\pi}{\omega}$

Load r.m.s voltage $= \left[\frac{1}{t_1} \int_{0}^{t_1} V_d^2 dt\right]^{\frac{1}{2}} = V_d$. 


Load voltage $V_0(t)$ may be expressed as:

$$V_0(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{4}{n\pi} Vd \sin n\omega t.$$  

The peak value of the fundamental component ($n=1$) of the load voltage is,

$$V_{01p} = \frac{4Vd}{\pi} \quad \text{(maximum or peak value)}$$

The RMS value of the fundamental component is

$$V_{01rms} = \frac{4Vd}{\sqrt{2}} = \frac{4Vd}{\sqrt{2}\pi} = 0.9 Vd \quad \text{(r.m.s value).}$$

Load r.m.s current and power can be determine from

$$I_{rms} = \left[ \frac{2}{T} \int_{0}^{T/2} i_o^2 \, dt \right]^{1/2}$$

And

$$P = \frac{2}{T} \int_{0}^{T/2} v_o \cdot i_o \, dt.$$  

where the instantaneous value of the load current $i_o$ for an R-L load is

$$i_o(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{4Vd}{n\pi \sqrt{R^2 + (n\omega L)^2}} \sin(n\omega t - \Psi n)$$

The angle by which the load current lags the load voltage is.

$$\Psi n = \tan^{-1} \frac{n\omega L}{R}.$$  

The total harmonic distortion factor is,

$$\text{THD} = \sqrt{\frac{(V_{Lrms}^2 - V_{L1}^2)_{V_{L1}^2}}{V_{L1}^2}}$$
• When diodes D1 and D2 are conducting, the energy fed back to the source, thus they are known as feedback diodes.

Example: For the single-phase full-bridge, transistor inverter shown in Fig.1, $V_{dc}=100\,\text{V}$, load resistance $R=10\,\text{Ω}$, load inductance $L=25\,\text{mH}$ and the output frequency is 60 Hz.

It is required to analyse the circuit by determining: (a) The amplitudes of the Fourier series terms for the output voltage wave up to the 9th order harmonics, (b) The amplitudes of the Fourier series terms for the load current wave up to the 9th order harmonics, (c) The power absorbed by the load in terms of harmonics, (d) compute the total harmonic distortion factor (THD).

![Diagram](image)

**Solution:**

(a) The output voltage wave in terms of Fourier series is:

$$V_o(\omega t) = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_{dc}}{n\pi} \sin nw t$$

The amplitude $C_n$ of the $n$th order harmonic is:

$$C_n = \frac{4V_{dc}}{n\pi} = \frac{4 \times 100}{n\pi} = \frac{127.3}{n} = V_n$$

$$V_o(\omega t) = \frac{127.3}{1} \sin wt + \frac{127.3}{3} \sin 3wt + \frac{127.3}{5} \sin 5wt$$

$$+ \frac{127.3}{7} \sin 7wt + \frac{127.3}{9} \sin 9wt$$

$$= 127.3 \sin wt + 42.4 \sin 3wt + 25.5 \sin 5wt$$

$$+ 18.2 \sin 7wt + 14.1 \sin 9wt$$

(b) The amplitude of the $n$th harmonic component of the load current is:

$$I_n = \frac{V_n}{Z_n}$$

where $Z_n = \sqrt{R^2 + (n\omega L)^2}$

In terms of Fourier Series:

$$I_o(\omega t) = \sum_{n=1,3,5,\ldots}^{\infty} \frac{V_n}{Z_n} \sin (n\omega t - \theta_n)$$

where $\theta_n = \tan^{-1} \frac{n \omega L}{R}$.

(c) $P_n = I_n^2 \cdot R = \left( \frac{I_n}{\sqrt{2}} \right)^2 \cdot R$. 

9
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The total power is

$$P = \sum P_n = 443 \text{ W}.$$  

(d) \[ \text{THD} = \sqrt{\frac{V_{o,\text{rms}}^2 - V_{1,\text{rms}}^2}{V_{1,\text{rms}}^2}} = \frac{\sqrt{V_{o,\text{rms}}^2 - V_{1,\text{rms}}^2}}{V_{1,\text{rms}}} \]

\[ V_{o,\text{rms}} = V_d = 100 \text{ V} \]

\[ V_{1,\text{rms}} = \frac{\sqrt{2}}{\sqrt{2}} \cdot \frac{127.3}{\sqrt{2}} = 90 \text{ V} \]

\[ \text{THD} = \frac{\sqrt{(100)^2 - (90)^2}}{(90)} = \frac{\sqrt{1900}}{90} = 0.4843 \]

or \[ 0.4843 \times \frac{1900}{1900} = 48.4\% \]
14.3 Inverter Output Voltage Control

Many inverter applications require a means of output voltage control. In most of these applications this control is usually required in order to provide stepless adjustment of the inverter output voltage.

The methods of control can be grouped into three broad categories:
1. Control of voltage supplies to the inverter
2. Control of voltage delivered by the inverter
3. Control of voltage within the inverter.

- There are a number of well-known methods of controlling the d.c. voltage supplies to an inverter or the a.c. voltage delivered by an inverter. These include the use of d.c. choppers, and phase-controlled rectifiers. The principal disadvantage of these methods is that the power delivered by the inverter is handed twice, once by the d.c. or a.c. voltage control and once by the inverter.

- Control of the inverter output voltage may be achieved by incorporating time-ratio controls within the inverter circuit.

- A method of controlling the voltage within an inverter involves the use of pulse wide modulation techniques. With this technique the inverter output voltage is controlled by varying the duration of the output voltage pulses.
**Pulse width voltage and frequency control**

A method of controlling the output voltage and frequency within an inverter involves the use of pulse wide modulation techniques. With this technique the inverter output voltage is controlled by varying the duration of the output voltage pulses as shown in Fig.14.8.

PWM is obtained by comparing a reference signal, $A_r$ with a triangular carrier wave, $A_c$. By varying $A_r$ from 0 to $A_c$, the pulse width $\delta$ can be varied from $0^\circ$ to $180^\circ$. The modulation index is defined as,

$$\text{Modulation Index} = M = \frac{A_r}{A_c}$$

The output voltage may be given by,

$$V_{ov,\text{rms}} = \left[ \frac{2}{2\pi} \int_{\pi/2}^{\pi/2} v_{dc}^2 \, d\omega t \right]^{1/2} = V_{dc} \sqrt{\frac{\delta}{\pi}}$$

The harmonic content can be reduced by using severed pulses in each half-cycle of output voltage. The frequency of reference signal sets the output frequency $f_o$, and the carrier frequency $f_c$ determines the number of pulses per half-cycle $P$.

$$P = \frac{f_c}{2 \cdot f_o}$$

As $M$ varies from 0 to 1, $\delta$ varies from 0 to $\pi / P$ and the output voltage from 0 to $v_{dc}$. 
Since the waveform is half-wave symmetry, the Fourier series expansion for the output voltage is

\[ V_o(t) = \sum_{n=1,3,5,...}^{\infty} B_n \sin n\omega t; \quad A_0 = A_n = 0 \]