CHAPTER FOUR

DC-DC CONVERSION:
DC CHOPPERS

4.1 INTRODUCTION

A dc-to-dc converter, also known as d.c. chopper, is a static device which is used to obtain a variable d.c. voltage from a constant d.c. voltage source. Also the dc-to-dc converter is defined in more general way as an electrical circuit that transfers energy from a d.c. voltage source to a load. The energy is first transferred via power electronic switches to energy storage devices and then subsequently switched from storage into the load. The switches used are GTO, IGBT, Power BJT, and Power MOSFET for low power application and thyristors or SCRs for high power application. The storage devices are inductors and capacitors. The power source is either a battery (d.c. volt) or a rectified a.c. volt. This process of energy transfer results in an output voltage that is related to the input voltage by the duty ratios of the switches.

The dc-to-dc converter products are used extensively for divers applications in the healthcare (bio-life science, dental, imaging, laboratory, medical), communications, computing, storage, business systems, test and measurement, instrumentation, and industrial equipment industries. They are used in electric motor drives, in switch mode power supplies (SMPS), trolley cars, battery operated vehicles, traction motor control, control of large number of d.c. motors, etc. They are also used as d.c. voltage regulators.

Types of dc-to-dc converters

There are mainly two types of dc-to-dc converters: (1) Step-down choppers, and (2) Step-up choppers. In step-down choppers, the output
voltage will be less than the input voltage, whereas in step-up choppers output voltage will be more than the input voltage. Hence a chopper is considered as d.c. equivalent of an a.c. transformer since it behaves in an identical manner. Also dc-to-dc converters can be categorized into two groups:

1) **DC-to-DC converters without isolation**
   These dc-to-dc converters do not have any isolation transformer between input and output stages. Some of the commonly used dc-to-dc converters without isolation are:
   a. Buck converter  
   b. Boost converter  
   c. Buck-boost converter  
   The buck converter is step-down converter (output voltage \(<\) input voltage) whereas Boost converter is a step-up converter (output voltage \(>\) input voltage). The buck-boost converter is derived from step-up and step-down converters. The buck-boost converter can be operated in step-up or step-down mode depends on the duty cycle of switch \((t_{on}/T)\), where \(t_{on}\) is the duration for which switch is on and \(T\) is the switching time period).

2) **DC-to-DC converters with isolation**
   A transformer is provided in between to isolate the input and output stages. The electrical isolation is an additional feature and is mainly useful in cases where the input voltage level \((V_{in})\) and output voltage level \((V_{out})\) differs significantly i.e. high or low values of \(V_{out}/V_{in}\). The dc-to-dc converters with isolation is again divided into two types based on polarity of transformer core excitation:
   - Unidirectional core excitation, core is excited with forward currents of only one direction. In these dc-to-dc converters the isolation transformer core is operated in only the positive part of B-H curve.  
   - Bidirectional core excitation, core is excited with currents in either direction. In these dc-to-dc converters the isolation transformer core is operated alternatively in positive and negative portions of B-H curve.

Some of the commonly used dc-to-dc converters with isolation are:
   a. Cuk converter (can be used in non-isolated mode also)  
   b. Fly-back converter  
   c. Forward converter  
   d. Full-bridge converter  
   e. Half-bridge converter  
   f. Push-pull converter
4.2 PRINCIPLE OF STEP-DOWN CHOPPER

The basic chopper circuit, often referred to as a class-A chopper, with resistive load is shown in Fig.4.1. It consist of a semiconductor switch S (often a thyristor) and its commutation circuit. The thyristor in the circuit acts as a switch. By the action of the switch, i.e. when thyristor is ON, supply voltage appears across the load and when thyristor is OFF, the voltage across the load will be zero. The output voltage waveform is as shown in Fig.4.2 which has an average value $V_{ave} = V_o$.

![Fig.4.1 Step-down simple chopper circuit.](image)

![Fig.4.2 Chopper output voltage waveform, case of R-load.](image)

The applied voltage is therefore “chopped” by the action of the switch. Since the average load voltage $V_o$ is less than the input voltage, this circuit is called “step-down chopper”. For all loads the current is unidirectional and the polarity of the load voltage is non-reversible. Operation takes place only in the positive voltage, positive current quadrant of the load voltage/load current plane so the circuit is referred to as a one-quadrant chopper.

Methods of Control

The output d.c. voltage can be varied by the following methods.

(a) Pulse width modulation control or constant frequency operation.
(b) Variable frequency control.
Pulse Width Modulation

- $t_{on}$ is varied keeping chopping frequency ‘$f$’ and chopping period ‘$T$’ constant.
- Output voltage is varied by varying the ON time $t_{on}$.

**Typical thyristor chopper circuit:**

Fig.4.3 shows one typical thyristor chopper circuit. Thyristor ($T_m$) is the main thyristor through which the flow of power is controlled. The capacitor $C$ and the four Thyristors ($T_1, T_2, T_3, T_4$) is the commutation circuit (this circuit had been already discussed in Chapter One).

![Fig.4.3 Typical thyristor d.c. chopper with forced commutation circuit.](image)

The function of the commutation circuit is to switch off the main thyristor at the end of each ON period. If the capacitor is charged such as plate $a$ is positive. To switch OFF ($T_m$), Thyristors ($T_1, T_3$) are triggered ON. This results in applying reverse polarity voltage across ($T_m$) and hence it will be switched OFF. Also the capacitor polarity will be reversed; i.e. plate $b$ will now be positive. Thyristor ($T_m$) is switched ON for the next ON period, and now to switch OFF ($T_m$), Thyristors ($T_2, T_4$) are switched ON, and so the cycle is repeated. However, the thyristor commutation circuits are discussed in details in Appendix-B.

The $L_i-C_i$ circuit in the input terminal is a filter circuit that reduces the harmonics injected into the power source. The inductance ($L_o$) in the output terminal is filter that smooth the current passing through the load. The diode ($D_f$) is a freewheeling diode, its function is to circulate the d.c. current through it when the main Thyristor ($T_m$) is OFF.

### 4.3 ANALYSIS OF A STEP-DOWN DC CHOPPER WITH RESISTIVE-LOAD

Referring to Fig.4.2, the average output voltage $V_o$ can be found as:

Let $T = $ control period = $t_{on} + t_{off}$

$\begin{align*}
\text{(4.1)}
\end{align*}$
and,

\[ f = \text{chopping frequency} = \left( \frac{1}{\text{control period} (T)} \right) = \frac{1}{T} \quad (4.2) \]

Typical chopping of the frequencies are usually in the range \(10 \, \text{Hz} < (f = \omega/2\pi) < 1000\,\text{Hz}\) for thyristor chopper and up to 10kHz for transistor choppers. In low power applications MOSFET switches can be used at frequencies in excess of 200 kHz.

The average value of the load voltage waveform shown in Fig. 4.2 may be expressed by the basic relationship,

\[ V_o = V_{av} = \frac{1}{T} \int_0^{t_{on}} V_d \, dt \quad (4.3) \]

Evaluating this integration gives,

\[ V_o = \frac{t_{on}}{T} V_d = \gamma V_d \quad (4.4) \]

where, \( \gamma = \frac{t_{on}}{T} = \text{Duty cycle} \) (4.5)

- Maximum value of \( \gamma = 1 \) when \( t_{on} = T \) (\( t_{off} = 0 \))
- Minimum value of \( \gamma = 0 \) when \( t_{on} = 0 \) (\( t_{off} = 0 \))

The output voltage is stepped-down by the factor \( \gamma \), \((0 \leq V_o \leq V_d)\).

Therefore, this form of chopper is a step-down chopper. Since the load is resistive the average load current is given by the basic equation

\[ I_o = \frac{V_o}{R} \quad (4.6) \]

\[ I_o = \frac{1}{R} (\gamma V_d) \quad (4.7) \]

The average power transferred to the load may be expressed by the basic relationship,

\[ P_o = \frac{1}{T} \int_0^T v_o i_o \, dt = V_o \times I_o \quad (4.8) \]

The \textit{rms} value of the load voltage waveform is given by,
The \(\text{rms}\) value of the load current,

\[
I_{\text{rms}} = \frac{V_{\text{rms}}}{R}
\]  \hspace{1cm} (4.10)

In the equivalent circuit of the Fig.4.1 the load power can be written, in a form more convenient for calculation as

\[
P_o = I_{\text{rms}}^2 \times R
\]  \hspace{1cm} (4.11)

Since the input voltage \(V_d\) is constant, average power is only transferred from the supply to the chopper by the combination of \(V_d\) with the average value of the input current \(I_s(\text{av})\),

\[
P_{\text{in}} = V_d \times I_s(\text{av})
\]  \hspace{1cm} (4.12)

The ripple factor, \(RF\) : It is a measure of the ripple content. Also, the ripple factor, defining the ratio of the a.c. components to the average value, is given by

\[
\text{ripple factor} = \frac{\sqrt{V_{\text{rms}}^2 - V_o^2}}{V_o}
\]  \hspace{1cm} (4.13)

\[
RF = \frac{V_d\sqrt{\gamma - \gamma^2}}{V_d \gamma} = \frac{1 - \gamma}{\gamma}
\]  \hspace{1cm} (4.14)

For full conduction \(\gamma = 1\) and \(RF = 0\).

Note1: In this type of chopper both the voltage and current are always positive, hence this chopper is called a single-quadrant Buck converter or class-A chopper.

Note2: The chopper switch can also be implemented by using a power BJT, power MOSFET, GTO, and IGBT transistor. The practical devices have a finite voltage drop ranging from
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0.5V to 2V, and for the sake of simplicity, the voltage drop of these power semiconductor devices are generally neglected.

Example 4.1

A class-A d.c. chopper circuit (Buck converter) is supplied with power form an ideal battery of 100 V. The load voltage waveform consists of rectangular pulses of duration 1 ms in an overall cycle time of 2.5 ms. For resistive load of \( R = 10 \, \Omega \), it is required to calculate:

(a) The duty cycle \( \gamma \).
(b) The average value of the output voltage \( V_o \).
(c) The \( \text{rms} \) value of the output voltage \( V_{\text{rms}} \).
(d) The ripple factor \( RF \).
(e) The output d.c. power.

Solution

(a) \( t_{on} = 1 \, \text{ms}, \quad T = 2.5 \, \text{ms} \)

\[
\gamma = \frac{t_{on}}{T} = \frac{1 \, \text{ms}}{2.5 \, \text{ms}} = 0.4
\]

(b) \( V_{av} = V_o = \gamma V_d = 0.4 \times 100 = 40 \, \text{V} \).

(c) \( V_{\text{rms}} = \sqrt{\gamma} V_t = \sqrt{0.4} \times 100 = 63.2 \, \text{V} \).

(d) \( RF = \sqrt{\frac{1 - \gamma}{\gamma}} = \sqrt{\frac{1 - 0.4}{0.4}} = 1.225 \)

(e) The load (output) current and the average power are

\[
I_o = \frac{V_o}{R} = \frac{40}{10} = 4 \, \text{A}
\]

\[
P_{av} = I_o V_o = 4 \times 40 = 160 \, \text{W}
\]

4.4 ANALYSIS OF A STEP-DOWN DC CHOPPER WITH \( R-L \) LOAD: EXACT ANALYSIS

Referring to Fig.4.5 which shows typical class-A chopper operating with series resistive-inductive load, the current \( i_o \) will alter between certain maximum and minimum values. These values depend on the parameters of the circuit and the value of the duty cycle (operation conditions). The waveforms of the output voltage and current are shown in Fig. 4.6. However, class-A chopper can operates in two modes with \( R-L \) loads:
Fig. 4.5 Class-A chopper operating with series resistive-inductive load.

Fig. 4.6 Load current and voltage waveforms with inductive load: (a) Load voltage waveform, (b) Continuous load current, (c) Discontinuous load current ($t_x$ is the instant when the current falls to zero).

- For high values of $\gamma$, the load current fluctuates in magnitude but it is more or less “continuous”, as shown in Fig. 4.6(b).
- For low values of $\gamma$, the load current may fall to zero during the OFF periods “discontinuous”, as shown in Fig. 4.6(c). However the load voltage waveform is the same for both cases as shown in Fig. 4.6(a).
Analytical properties of the load current waveform

With an $R$-$L$ load, during the ON-state, when switch $S$ closed, the equivalent circuit is represented by Fig.4.7. The current flows through the circuit is governed by the following equation:

$$V_d = Ri_o + L \frac{di_o}{dt} \quad \text{for} \quad 0 \leq t \leq t_{on} \quad (4.15)$$

![Fig.4.7 ON-state equivalent circuit.](image)

To solve this equation, we re-arrange it as,

$$(V_d - Ri_o)dt = L \, di_o \quad (4.16)$$

Integrating both sides and simplify,

$$\int \frac{di_o}{i_o - \frac{V_d}{R}} = \int \frac{-R}{L} \, dt$$

$$\ln \left( i_o - \frac{V_d}{R} \right) = \frac{-R}{L} \, t + K$$

$$i_o - \frac{V_d}{R} = e^{\frac{-R}{L}t+K} = e^K \frac{V_d}{R} e^{-\frac{R}{L}t} = K' e^{-\frac{R}{L}t}$$

where $K$ and $K'$ are constants.

- Special case : at $t = 0$, $i_o = 0$

$$0 - \frac{V_d}{R} = K' e^0 \quad \therefore \quad K' = -\frac{V_d}{R}$$

Hence, the equation for the current is,

$$i_o(t) = \frac{V_d}{R} - \frac{V_d}{R} e^{-\frac{R}{L}t} = \frac{V_d}{R} \left(1 - e^{-t/\tau}\right)$$

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Where $\tau = \text{time constant} = \frac{L}{R}$,

- The general case when: at $t = 0$, $i_o = I_o$

$$i_o(t) = \frac{V_d}{R} + (I_o - \frac{V_d}{R}) e^{-t/\tau}$$

$$(4.17)$$

This general equation of the current can be re-written in terms of $\omega t$ as,

$$i_o(\omega t) = \frac{V_d}{R} (1 - e^{-\omega t/\omega \tau}) + I_o e^{-\omega t/\omega \tau}$$

$$(4.18)$$

where $\omega \tau = \omega L/R$

Equation (4.18) can be considered as a general solution for the instantaneous current flowing through an $R$-$L$ load fed by d.c. chopper circuit, and when applied to the chopper circuit shown in Fig.4.5 yields,

At the end of the ON period $t = t_{on} = \gamma T$, $i_o = I_2$ (or $I_{max}$) and therefore,

$$I_2 = \frac{V_d}{R} (1 - e^{-\gamma T/\tau}) + I_1$$

$$(4.19)$$

On opening $S$, Fig.4.8, the current “$i_o$” is commutated to the freewheeling diode and falls exponentially from $I_2$ to $I_1$ (or $I_{min}$) according to

$$0 = R i_o + L \frac{di_o}{dt} \quad t_{on} \leq t \leq T$$

$$(4.20)$$

Thus

$$i_o = I_2 e^{-(t-t_{on})/\tau} \quad t_{on} \leq t \leq T$$

$$(4.21)$$

Fig.4.8 OFF-state equivalent circuit.

At the end of the OFF period, $t = T$, $I = I_1$ and therefore,

$$I_1 = I_2 e^{-(1-\gamma)T/\tau}$$

$$(4.22)$$
\[ I_2 = I_1 e^{(1-\gamma)T/\tau} \]  \hspace{1cm} (4.23)

The variation of the current is shown in Fig.4.9.

![Fig.4.9 Current waveform for R-L load.](image)

The average value of load current is defined by,

\[ I_{oav} = \frac{1}{T} \int_{t=0}^{T} i_o(t) \, dt \]  \hspace{1cm} (4.24)

The average value of the current drawn from the source is

\[ I_{sav} = \frac{1}{T} \int_{t=0}^{T} i_s(t) \, dt \]  \hspace{1cm} (4.25)

where \( i_s = i_o \) \hspace{1cm} 0 \leq t \leq t_{on}

\[ i_s = 0 \] \hspace{1cm} \hspace{1cm} t_{on} \leq t \leq T

The peak-to-peak ripple of the load current \( I_r \) is given by \( I_r = \Delta I = I_2 - I_1 \).

From above equations, with simple algebraic manipulation one can show that:

\[ \frac{I_r}{I_m} = \frac{1 - e^{-\gamma T/\tau} - e^{-(1-\gamma)T/\tau} + e^{-T/\tau}}{1 - e^{-T/\tau}} \]  \hspace{1cm} (4.26)

where \( I_m = V_d / R \) : Maximum ON-state load current (i.e., when \( \gamma = 1 \)).

Thus, for given circuit parameters (namely \( R \) and \( L \)) and the cycle time, \( \frac{I_r}{I_m} \) is function of \( \gamma \). The variation of \( \frac{I_r}{I_m} \) with \( \gamma \) is shown in Fig.4.10 for different values of \( (T/\tau) \). From this figure one can simply draw the following conclusions:
1. The maximum ripple occurs at \( \gamma = 0.5 \).
2. The ripple is very small when \( T < 0.1 \pi \), (or when \( f > 10 / \pi \)).

Fig. 4.10. The variation of \( \frac{I_r}{I_m} \) with \( \gamma \).

4.5 ANALYSIS OF A STEP-DOWN DC CHOPPER WITH LOAD CONSISTING BACK EMF (E)

When the load has a back emf \( E \) (such as battery or motor), Fig. 4.11, the governing equations during the ON and OFF periods are respectively:

\[ V_o = V_d = Ri_o + L \frac{di_o}{dt} + E \quad 0 \leq t \leq t_{on} \quad (4.27) \]

\[ 0 = Ri_o + L \frac{di_o}{dt} + E \quad t_{on} \leq t \leq T \quad (4.28) \]

Fig. 4.11 Operation with load consisting back emf.
In this case the current will be either continuous or discontinuous depending on the values of $E$ and the duty cycle $\gamma$. However, the output voltage waveform is assumed to be the same for both continuous and discontinuous current operation which is also the same for resistive load. It should be noted that the voltage forcing the current build up during the ON period is $(V_d - E)$, and during the OFF period, the decay of the current is aided by $E$. Thus a slower ON-state increase and faster OFF-state decrease in the current is expected.

4.5.1 Continuous Current Operation Condition with Back - emf

For analysis of class-A chopper circuit operation with back emf $E$, it is convenient to start by considering the case of continuous current conditions, when the back emf $E$ is small, and the waveform of the current alter between $I_{\text{max}}$ and $I_{\text{min}}$ as in Fig.4.6(b) as follows:

When switch S is ON (and $D_{FW}$ OFF) at $t = 0$, then at $t = 0^+$ (just after switching), $v_o = V_d$, $i_o = I_{\text{min}}$. From Eq.(4.17) and these initial conditions,

$$i_o(t) = \frac{V_d - E}{R} \left( 1 - e^{-t/\tau} \right) + I_{\text{min}} e^{-t/\tau} \quad 0 \leq t \leq t_{\text{on}} \quad (4.29)$$

At $t = t_{\text{on}}$, when S is opened, $i_o = I_{\text{max}}$, and:

$$i_o(t) = I_{\text{max}} = \frac{V_d - E}{R} \left( 1 - e^{-t_{\text{on}}/\tau} \right) + I_{\text{min}} e^{-t_{\text{on}}/\tau} \quad (4.30)$$

Equation (4.30) is not time dependent and remains true after S switches off.

When switch S is off (and D on), with $t = t_{\text{on}}^+$, in Fig.4.9, $v_o = 0$ and $i_o(t) = I_{\text{max}}$ and in the interval $t_{\text{on}} \leq t \leq T$, the load current is given by

$$i_o(t) = -\frac{E}{R} \left( 1 - e^{-(t-t_{\text{on}})/\tau} \right) + I_{\text{min}} e^{-(t-t_{\text{on}})/\tau} \quad (4.31)$$

But at $t = T$, $i_o(t) = I_{\text{min}}$, equation Eq.(4.31) may then be rewritten as

$$i_o(t) = I_{\text{min}} = -\frac{E}{R} \left( 1 - e^{-(T-t_{\text{on}})/\tau} \right) + I_{\text{max}} e^{-(T-t_{\text{on}})/\tau} \quad (4.32)$$

The simultaneous solution of Eq.(4.32) and Eq.(4.30) yields

$$I_{\text{max}} = \frac{V_d}{R} \left( \frac{1 - e^{-t_{\text{on}}/\tau}}{1 - e^{-T/\tau}} \right) - \frac{E}{R} \quad (4.33)$$
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For full conduction of switch S in Fig. 4.11, \( \gamma = 1 \), so that \( I_{\text{max}} = I_{\text{min}} = (V_d - E)/R \). The use of Eq. (4.33) and Eq. (4.34) permits solution of Eq. (4.29) and Eq. (4.30) to determine a value for \( i_o(t) \) at any instant of the cycle during continuous conduction.

4.5.2 Discontinuous Current Operation Condition

Under certain circuit conditions, depending on the duty cycle and the value of the back emf \( E \), the load current \( i_o(t) \) may fall to zero resulting in the discontinuous pulse pattern of Fig. 4.12; this waveform consists of two parts that may be classified according to the conduction of switch S, as in subsection (4.1.1) above. The maximum current is now, in general, different from the value \( I_{\text{max}} \) obtained with continuous operation and it occurs for a different value of \( t_{\text{on}} \). The minimum current for discontinuous operation is, by definition, zero.

Fig. 4.12 Load voltage and current waveforms for discontinuous current operation.

- With switch S on (and diode D off),
  In the interval \( 0 \leq t \leq t_{\text{on}} \) of Fig. 4.12 (c) the current equation Eq. (4.30) applies with the limits that \( v_o = V_d \), and \( i_o(t) = 0 \) at \( t = 0^+ \). This gives
\[ i_o(t) = \frac{V_d - E}{R} \left(1 - e^{-t/\tau}\right) \]  

(4.35)

Maximum \( i_o(t) \) occurs at \( t = t_{on} \) so that

\[ I_{max_d} = \frac{V_d - E}{R} \left(1 - e^{-t_{on}/\tau}\right) \]  

(4.36)

Where \( I_{max_d} \) is defined as the maximum current value for discontinuous current operation.

- With switch S off (and \( D_{FW} \) on)

When S switches off in Fig.4.11 the load voltage \( v_o(t) \) falls to zero due to the conduction through diode \( D_{FW} \). The circuit differential equation (4.28) is then modified to:

\[ \frac{di_o}{dt} + \frac{R}{L}i_L = \frac{-E}{L} \]  

(4.37)

This has the solution of Eq.(4.38) except that the maximum current is now given by Eq.(4.36) to result in

\[ i_o(t) = \frac{-E}{R} \left(1 - e^{-(t-t_{on})/\tau}\right) \]

\[ + \frac{V_d - E}{R} \left(1 - e^{-t_{on}/\tau}\right) e^{-(t-t_{on})/\tau} \]  

(4.38)

**The current extinction angle \( x \)**

To find the current extinction angle \( x \), let the repetition periodicity \( t_{on} + t_{off} \), Fig.4.2, be designated as \( 2\pi \) radians, hence

\[ t_{on} + t_{off} = 2\pi \]  

(4.39)

Since the independent variable is chosen as \( \omega t \) at the periodic time of the over all (ON + OFF) cycle is

\[ \text{Periodic time} = \frac{2\pi}{\omega} \]  

(4.40)

The frequency of the chopper operation is the inverse of the periodic time,

\[ \text{Chopping frequency} = f_c = \frac{\omega}{2\pi} \]  

(4.41)

The ON period of the chopper \( t_{on} \) is,

\[ t_{on} = \gamma(t_{on} + t_{off}) = 2\pi\gamma \]  

(4.42)

The ON time of the switch in seconds is therefore

\[ \text{Switch ON – time} = 2\pi\gamma/\omega \]  

(4.43)

The terms \( t_{on}, t_{off} \) now serve the double purpose of identifying the conduction state of switch S in Fig.4.11 and also defining its period of conduction or extinction in radians.

Let current extinction occur at \( t = t_x \) (\( \omega t = x \)) in Fig.4.12 (b), where \( x \) is
the current extinction angle which occurs at time $t_x$. Putting $i_o(x) = 0$ and $\omega t_x = x$ into Eq.(4.38) gives expression for $x$,

$$x = \omega \tau \ln \left[ e^{t_{on}/\omega \tau} \left\{ 1 + \frac{V_d - E}{E} \left( 1 - e^{-t_{on}/\omega \tau} \right) \right\} \right]$$

(4.44)

Or in terms of the duty cycle $\gamma$,

$$x = \omega \tau \ln \left[ e^{(2\pi \gamma)/\omega \tau} \left( 1 + \frac{V_d - E}{E} \left( 1 - e^{-2\pi \gamma/\omega \tau} \right) \right) \right]$$

(4.45)

In terms of time in seconds, we can re-write Eq.(4.44) above as,

$$t_x = \tau \ln \left\{ 1 + \frac{V_d - E}{E} \left( 1 - e^{-t_{on}/\tau} \right) \right\} \text{ seconds}$$

(4.46)

where $t_x = \frac{x}{\omega}$ and $\frac{2\pi \gamma}{\omega} = t_{on}$

If $t_{on}$ is decreased to the value $t_{on}^x$ at which $I_{min} = 0$ that represent the boundary between continuous and discontinuous operation. i.e. the converter is operating at the point of changeover from continuous current operation to discontinuous current operation. Then, from Eq. (4.5),

$$\gamma' = \frac{t_{on}^x}{T} = \frac{t_{on}^x}{t_{on}^x + t_{off}}$$

(4.47)

For this boundary condition, from Eq. (4.34)

$$\frac{E}{V_d} = \frac{e^{t_{on}^x/T} - 1}{e^{T/\tau} - 1}$$

(4.48)

or

$$m = \frac{e^{\gamma' \sigma} - 1}{e^{\sigma} - 1}$$

where $m = \frac{E}{V_d}$

and $\sigma = T/\tau$

The relationship between $m = \left( \frac{E}{V_d} \right)$ and $\gamma'$ is shown in Fig.4.13 with the factor $\sigma = (T/\tau)$ as parameter. If a circuit operate with a specified value of $\gamma$, then the criteria for continuous or discontinuous operation are,
\[ \begin{align*}
\gamma > \gamma', & \text{ continuous current} \\
\gamma < \gamma', & \text{ discontinuous current}
\end{align*} \]  \tag{4.49}

where \( \gamma' \) is defined by Eq.(4.47).

![Graph showing the relation between \( m \) and \( \gamma' \) for different values of \( \sigma \).](image)

Fig.4.13 Relation between \( m \) and \( \gamma' \) for different values of \( \sigma \).

If the circuit is passive, \( E = 0 \) and Eq.(4.38) can only be satisfied by \( \gamma' = 0 \) in other words, there is no finite value of \( t_{on} \) that will result in discontinuous operation. Although the current may become small it is finite and operation is, therefore, continuous.

### 4.6 Harmonic Analysis of the Load Voltage Waveform of Class-A DC Chopper

As it had been considered that the repetition periodicity \( t_{on} + t_{off} \), Fig.4.2, be designated as \( 2\pi \) radians to facilitate harmonics analysis, so that, \( t_{on} + t_{off} = 2\pi \). Hence, \( t_{on} = \gamma(t_{on} + t_{off}) = 2\pi \gamma \).

In many cases it had been found that the output voltage waveform \( v_o(\omega t) \) is a very close approximation to one of the ideal waveforms shown in Fig.4.2 for both continuous and discontinuous current operation. When this is so, the waveform may be represented as,
\[ v_0(\omega t) = V_d \bigg|_0^{2\pi y} + 0 \bigg|^{2\pi y}_{2\pi y} \quad (4.50) \]

Since this waveform is periodic function, the time variation of \( v_0 \) may be represented by Fourier series as,

\[ v_0(\omega t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t \]

\[ = V_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega t + \varphi_n) \]

where, the Fourier coefficients are:

\[ \frac{a_0}{2} = \frac{1}{2\pi} \int_0^{2\pi} v_0^2(\omega t) d\omega t \]

\[ = \frac{1}{2\pi} \int_0^{2\pi y} V_d d\omega t = \frac{V_d}{2\pi} \left[ \omega t \right]_0^{2\pi y} \]

\[ \therefore \frac{a_0}{2} = V_0 = \gamma V_d \quad (4.51) \]

This value is the average value of the output voltage waveform which represents the zero frequency components in the series or the d.c. value of \( v_0 \). The Fourier coefficients for the \( n \)th harmonic (higher order harmonics) of the load voltage waveform are given by,

\[ a_n = \frac{1}{\pi} \int_0^{2\pi} v_0(\omega t) \cos n\omega t \, d\omega t \]

\[ a_n = \frac{V_d}{\pi} \int_0^{2\pi y} \cos n\omega t \, d\omega t = \frac{V_d}{n\pi} \sin 2\pi ny \quad (4.52) \]

\[ b_n = \frac{1}{\pi} \int_0^{2\pi} v_0(\omega t) \sin n\omega t \, d\omega t \]

\[ b_n = \frac{V_d}{\pi} \int_0^{2\pi y} \sin n\omega t \, d\omega t = \frac{V_d}{n\pi} (1 - \cos 2\pi ny) \quad (4.53) \]

The peak amplitude \( \hat{V}_{on} \) and phase-angle \( \varphi_n \) of \( n \)th load voltage harmonic are therefore given by,
The fundamental component of the output voltage waveform is found for \( n = 1 \), thus:

\[
\hat{V}_{on} = c_n = \sqrt{a_n^2 + b_n^2} = \frac{V_d}{n\pi} \sqrt{[\sin^2 2\pi n\gamma + (1 - \cos 2\pi n\gamma)^2]}
\]

\[= \frac{2V_d}{n\pi} \sin n\pi\gamma \quad (4.54(a))\]

\[
\varphi_n = \tan^{-1}\left(\frac{a_1}{b_1}\right) = \tan^{-1}\left(\frac{\sin 2\pi n\gamma}{1 - \cos 2\pi n\gamma}\right) \quad (4.55(a))
\]

\[= \frac{\pi}{2} - n\pi\gamma \quad (4.55(b))\]

Example 4.2

In a class-A chopper circuit an ideal battery of terminal voltage 100 V supplies a series load of resistance 0.25 Ω, and a series inductor of 1 mH. The load consists of a series battery of 10 V. The control time \( T \) of the chopper is 2.5 ms and the ON time is 1 ms.

(a) Specify whether the current will be continuous or discontinuous?
(b) Calculate the average output voltage \( V_o \), and average output current \( I_o \).
(c) Calculate the maximum and minimum values of the output current.
(d) Sketch to scale compatible waveforms for \( v_o, i_o, i_D, \) and \( i_s \).
(e) Calculate the fundamental component of output harmonic voltage and the first order harmonic current.

**Solution**

(a) To find whether the current will be continuous or discontinuous, we have to calculate the parameters $\gamma$, $m$, $\sigma$, and $\gamma'$ first,

$$\gamma = \frac{t_{on}}{T} = \frac{1}{2.5} = 0.4$$

$$m = \frac{E}{V_d} = \frac{10}{100} = 0.1$$

$$\sigma = \frac{T}{\tau} = 2.5 \times 10^{-3} \times \frac{R}{L} = 2.5 \times 10^{-3} \times \frac{0.25}{1 \times 10^{-3}} = 0.625$$

But

$$m = \frac{e^{\gamma'\sigma} - 1}{\epsilon^\sigma - 1} = 0.1 = \frac{e^{0.625\gamma'} - 1}{e^{0.625} - 1}$$

$$e^{0.625\gamma'} = 1.0868$$

$$0.625 \gamma' = \ln 1.0868 \quad \rightarrow \quad \gamma' = 0.1332$$

$$\gamma' = 0.1332$$

Since $\gamma' < \gamma \quad \therefore$ the current is continuous.

(b) From Eq.(4.4), the average output voltage $V_o$ is,

$$V_o = 0.4 \times 100 = 40 \, V$$

$$I_o = \frac{V_o - E}{R} = \frac{40 - 10}{0.25} = 120 \, A$$

(c) Expressions for $I_{max}$ and $I_{min}$ are given in equations (4.33) and (4.34),

$$t_{on}/\tau = \frac{0.25 \times 10^{-3}}{1 \times 10^{-3}} = 0.25$$

$$I_{max} = \frac{100}{0.25} \left( \frac{1 - e^{-0.25}}{1 - e^{-0.625}} \right) - \frac{10}{0.25} = 150.4 \, A$$
(d) Waveforms for $v_o$, $i_o$, $i_D$, and $i_s$ are shown in Fig.4.14.

(e) The fundamental component of output harmonic voltage is calculated from Eq.(4.56),

\[ V_{o1} = \frac{c_1}{\sqrt{2}} = \frac{V_d}{\pi} \sqrt{[1 - \cos \omega t_{on}]} \]

\[ \omega = \frac{2\pi}{T} = \frac{2\pi}{2.5 \times 10^{-3}} = 2513 \text{ rad/s} \]

\[ = \frac{100}{\pi} \sqrt{[1 - \cos(2513 \times 10^{-3})]} = 42.18 \text{ V} \]

\[ I_{o1} = \frac{V_{o1}}{Z_{o1}} = \frac{V_{o1}}{\sqrt{R^2 + (\omega L)^2}} = \frac{42.18}{\sqrt{0.25^2 + (2.513)^2}} = 16.7 \text{ A} \]

Fig.4.14 Class-A chopper waveforms.
Example 4.3

Repeat Example 4.2 when \( V_d = 100 \text{ V}, L = 0.2 \text{ mH}, R = 0.25 \text{ } \Omega, E = 40 \text{ V}, T = 2.5 \text{ ms}, t_{on} = 1.25 \text{ ms} \).

Solution

(a) To find whether the current is continuous or discontinuous,
\[
m = \frac{E}{V_d} = \frac{40}{100} = 0.4
\]
\[
\sigma = \frac{T}{\tau} = \frac{2.5 \times 10^{-3} \times \frac{R}{L}}{2.5 \times 10^{-3} \times \frac{0.25}{0.2 \times 10^{-3}}} = 3.125
\]
\[
m = \frac{e^{\gamma \sigma} - 1}{e^{\sigma} - 1} = 0.4 = \frac{e^{3.125 \gamma'} - 1}{e^{3.125} - 1} \quad \rightarrow \gamma' = 0.692
\]
\[
\gamma = \frac{t_{on}}{T} = \frac{1.25}{2.5} = 0.5
\]

Since \( \gamma' > \gamma \) \quad \therefore \text{the current is discontinuous.}
\[
\tau = \frac{0.2 \times 10^{-3}}{0.25} = 0.8 \times 10^{-3} \text{ s}
\]
\[
\frac{t_{on}}{\tau} = \frac{1.25 \times 10^{-3}}{0.8 \times 10^{-3}} = 1.56
\]

From Eq. (4.46)
\[
t_x = \tau \ln \left[ e^{(t_{on}/\tau)} \left\{ 1 + \frac{V_d - E}{E} \left( 1 - e^{t_{on}/\tau} \right) \right\} \right]
\]
\[
t_x = 0.8 \times 10^{-3} \ln(e^{1.56} \left\{ 1 + \frac{100 - 40}{40} (1 - e^{-1.56}) \right\} ) = 1.87 \text{ ms}
\]
\[
V_o = \gamma V_d + \frac{T - t_x}{T} E = 0.5 \times 100 + \frac{(2.5 - 187) \times 10^{-3}}{2.5 \times 10^{-3}} \times 40 = 60 \text{ V}
\]

(b) Since the current is discontinuous, \( I_{min} = 0 \). From Eq. (4.36)
\[
I_{max_d} = \frac{V_d - E}{R} \left( 1 - e^{-t_{on}/\tau} \right) = \frac{100 - 40}{0.25} (1 - e^{-1.56}) = 189.5 \text{ A}
\]
(c) Waveforms for $v_o$, $i_o$, $i_D$, and $i_s$ are shown in Fig.4.15.

Fig.4.15 Waveforms.

4.7 ANALYSIS OF A STEP-DOWN DC CHOPPER WITH $R$-$L$ LOAD: APPROXIMATE ANALYSIS

Consider a class-A chopper circuit with $R$-$L$ load as shown in Fig.4.16(a). This is a step down chopper with one quadrant operation. If we use the simplified linear analysis by considering that $T \ll \tau$, where $(T = t_{on} + t_{off})$. In this case the current is continuous as shown in Fig.4.16(b).

Fig.4.16 Class-A chopper circuit with $R$-$L$ load: (a) Circuit, and (b) Voltage and current waveforms.
Referring to Fig.4.16:
- The current variation is almost linear and the current waveform becomes triangular.
- During the ON period, the equation govern the circuit is,

\[ V_d = Ri + L \frac{di}{dt} \]  \hspace{1cm} (4.57a)

Since \( \frac{di}{dt} \) = constant, hence during ON period:

\[ \frac{di}{dt} = \frac{l_2 - l_1}{t_{on}} = \frac{\Delta I}{t_{on}} \]  \hspace{1cm} (4.57b)

Where \( \Delta I \) is the peak-to-peak of the load current. Thus the equation of the current is given by:

\[ i_1 = l_1 + \frac{\Delta I}{\gamma T} t \hspace{1cm} 0 \leq t \leq t_{on} \]  \hspace{1cm} (4.58)

where \( \gamma = \frac{t_{on}}{T} \)

During the off period:

\[ \frac{di}{dt} = \frac{l_1 - l_2}{t_{off}} = - \frac{\Delta I}{t_{off}} = - \frac{\Delta I}{T - t_{on}} = - \frac{\Delta I}{(1 - \gamma)T} \]  \hspace{1cm} (4.59)

Hence, during the off the equation of the current is

\[ i_2 = l_2 - \frac{\Delta I}{(1 - \gamma)T} (t - t_{on}) \hspace{1cm} t_{on} \leq t \leq T \]  \hspace{1cm} (4.60)

The average value of the output current is

\[ I_{av} = \frac{1}{T} \left[ \frac{1}{2} t_{on}(l_2 - l_1) + \frac{1}{2} t_{off}(l_2 - l_1) + l_1 T \right] \]  \hspace{1cm} (4.61)

\[ I_{av} = \frac{1}{2} (l_2 + l_1) \]  \hspace{1cm} (4.62)

**Example 4.4**

A 140V battery supplies R-L load through a class-A d.c. chopper. The load has a freewheeling diode across it is composed of 0.4 H in series with 10 Ω resistor. Load current, due to improper selection of frequency of chopping, varies widely between 8 A and 9.2 A.

(a) Find the average load voltage, current and the duty cycle of the chopper.

(b) What is the operating frequency \( f \) ?

(c) Find the ripple current to maximum current ratio.
Solution
(a) The average load current is:
\[ I_{av} = \frac{1}{2} (I_2 + I_1) = \frac{9.2 + 8}{2} = 8.6 \text{A} \]

The average load voltage is
\[ V_{av} = V_o = \gamma V_d \]
\[ V_{av} = 0.614 \times 140 = 86 \text{ V}. \]

(b) To find the operating (chopping) frequency:
During the ON period,
\[ V_d = Ri + L \frac{di}{dt} \]
Assuming \( \frac{di}{dt} \equiv \text{constant} \)
\[ \frac{di}{dt} \approx \frac{\Delta I}{t_{on}} = \frac{9.2 - 8}{\gamma T} = \frac{1.2}{\gamma T} \]

From Eq. (4.5a)
\[ L \frac{di}{dt} \approx V_d - I_{av} R = 140 - 10 \times 8.6 = 54 \text{ V} \]

or \[ \frac{di}{dt} = \frac{54}{L} = \frac{54}{0.4} = 135 \text{ A.s} \]

but \[ \frac{di}{dt} = \frac{1.2}{\gamma T} = 135 = \frac{1.2}{0.614 T} \]

\[ \therefore T = \frac{1.2}{0.614 \times 135} = 14.47 \text{ ms} \]

hence \[ f = \frac{1}{T} = \frac{1}{14.47 \times 10^{-3}} = 69 \text{ Hz} \]

The maximum current \( I_m \) occurs at \( \gamma = 1 \),
\[ \therefore I_m = \frac{\gamma V_d}{R} = \frac{1 \times 140}{10} = 14 \text{ A} \]
Ripple current $I_r = \Delta I = 9.2 - 8 = 1.2$ A

\[
\therefore \quad \frac{I_r}{I_m} = \frac{1.2}{14} = 0.085 \quad \text{or} \quad 8.5\%.
\]

**Input Current $I_s$**

For the class-A chopper shown in Fig. 4.16(a), the ON-state and OFF-state equivalent circuits are as depicted in Fig. 4.17. When the thyristor is closed (during the ON period), the load current “$i$” rises from $I_1$ to $I_2$ and falls from $I_2$ to $I_1$ during the off period as shown in Fig. 4.18(a). The input current $i_s$ flows during the ON period only as shown in Fig. 4.18 (b).

![ON-State equivalent circuit](image1)

![OFF-State equivalent circuit](image2)

(a) ON-State equivalent circuit. \hspace{1cm} (b) OFF-State equivalent circuit.

**Fig. 4.17 ON and OFF-states equivalent circuits.**

![Load Current](image3)

![Input current](image4)

(a) \hspace{1cm} (b)

**Fig. 4.18 Class-A d.c. chopper : (a) Load Current, (b) Input current.**

The equation of the input current is

\[
i_s = i_1 = I_1 + \frac{\Delta I}{\gamma T} t \quad \quad \quad 0 \leq t \leq t_{on}
\]
The average value of the current drawn from the supply is simply found by,

\[
    i_s = 0 \quad t_{on} \leq t \leq T
\]

Approximate equations for the minimum and maximum load currents

To find approximate formulae for the minimum current \( I_1 \) and maximum current \( I_2 \) in a chopper circuit let us consider the equivalent circuit of the chopper during the OFF-state period \( t_{off} \) as depicted in Fig. 4.17 (b). During the OFF period, the only voltage source is the inductor voltage which its average value is given by

\[
    V_{L(\text{av})} = L \frac{di}{dt} \approx -L \frac{I_1 - I_2}{t_{off}}
\]

or \( V_{L(\text{av})} \frac{t_{off}}{L} = I_2 - I_1 \) \( (4.65) \)

But \( I_{av} = \frac{1}{2} (I_2 + I_1) \)

\[
    I_1 = 2 \; I_{av} - I_2 \quad (4.66)
\]

Substituting Eq. (4.66) into Eq. (4.65) yields

\[
    \frac{t_{off}}{L} V_{L(\text{av})} = I_2 - (2 \; I_{av} - I_2) = 2I_2 - 2 \; I_{av}
\]

For further simplification of the above equation yields

\[
    I_2 = I_{max} = \frac{V_o}{R} + \frac{t_{off}}{2L} V_o \quad (4.67)
\]
\[ I_1 = I_{min} = \frac{V_o}{R} - \frac{t_{off}}{2L} V_o \]  \hspace{1cm} (4.68)

where \( V_o = V_{L(\text{av})} = V_{(\text{av})} \) \hspace{1cm} and \hspace{1cm} \( I_{av} = \frac{v_o}{R} \)

**Example 4.5**

A d.c. Buck converter operates at frequency of 2000 Hz from 200 V d.c. source supplying a 5 \( \Omega \) resistive load. The inductive component of the load is 5 mH. For output average voltage of 100 V, find:

(a) The duty cycle.
(b) \( t_{on} \).
(c) The \( \text{rms} \) value of the output current.
(d) The average value of the output current.
(e) \( I_{max} \) and \( I_{min} \).
(f) The input power.
(g) The peak-to-peak ripple current.

**Solution**

(a) \( V_{av} = V_o = \gamma V_d \)

\[
\gamma = \frac{V_{av}}{V_d} = \frac{200}{100} = 0.5
\]

(b) \( T = 1/f = 1/2000 = 0.5 \text{ ms} \)

\[
\gamma = \frac{t_{on}}{T} \rightarrow t_{on} = \gamma T = 0.5 \times 0.5 = 0.25 \text{ ms}
\]

(c) \( V_{rms} = \sqrt{\gamma} \cdot V_d = \sqrt{0.5} \times 200 = 140.14 \text{ V} \)

\( I_{rms} = V_{rms}/R = 140.14/5 = 28 \text{ A} \)

(d) \( I_{av} = \frac{V_{av}}{R} = \frac{100}{5} = 20 \text{ A} \)

(e) \( I_{max} = \frac{V_{av}}{R} + \frac{t_{off}}{2L} V_{av} = \frac{100}{5} + \frac{(0.5 - 0.25) \times 10^{-3}}{2 \times 5 \times 10^{-3}} \times 100 \)

\[ = 20 + 2.5 = 22.5 \text{ A} \]
\[ I_{\text{min}} = \frac{V_{av}}{R} - \frac{t_{off}}{2L} V_{av} = \frac{100}{5} - \frac{(0.5 - 0.25) \times 10^{-3}}{2 \times 5 \times 10^{-3}} \times 100 \]

\[ = 20 - 2.5 = 17.5 \text{ A} \]

(f)

\[ I_{s(\text{av})} = \frac{\gamma}{2} (I_{\text{min}} + I_{\text{max}}) = \gamma I_{av} = 0.5 \times 20 = 10 \text{ A} \]

\[ P_{in} = I_{s(\text{av})} V_d = 10 \times 200 = 2000 \text{ W} \]

(g)

\[ I_{p-p} = \Delta I = I_{\text{max}} - I_{\text{min}} = 22.5 - 17.5 = 5 \text{ A} \]

4.8 **STEP-UP DC CHOPPER (BOOST CONVERTER)**

The boost converter, also called fly-back converter, is a dc-to-dc converter that steps up the dc voltage from low value to a desired high value. In contrast to the step-down chopper, the minimum average output voltage of the step-up chopper equals the input voltage \( V_d \). Again changing the duty cycle can change the average output voltage \( V_o \). The boost converter basic circuit is shown in Fig. 4.19.

![Boost converter](image)

Fig.4.19 Boost converter.

The principle of operation of this chopper is the storage of energy at low supply voltage (in an inductor) and its release at higher voltage. i.e. the energy transfer, from the source to the load, takes place only during the off period of the switch. This is why it is called fly-back converter. A thyristor or a transistor is usually used to act as a switch for transferring the energy in the storage component (the inductor) to the output circuit. Since the energy is supplied in pulses, a large capacitor at the output is required to smooth the output voltage. The essential parts of the circuit are shown, where “S” represents the main switching element.

Let us assume that the circuit has been operating for a long time and the inductor current varies between minimum value \( I_1 \) and maximum value \( I_2 \) as a function of time. We begin our analysis when the inductor current is at its minimum and the switch S is closed at \( t = 0 \), see
Fig. 4.20(a), the inductor current builds up during the period $0 \leq t \leq t_{on}$ according to the differential equation,

$$V_d - L \frac{di_s}{dt} = 0$$  \hspace{1cm} (4.69)

With high chopping frequency and assuming linear and small current variation, its solution is,

$$V_d = L \frac{I_2 - I_1}{t_{on}} = L \frac{\Delta I}{t_{on}}$$  \hspace{1cm} (4.70)

During the ON time the inductor current increases from its minimum value toward its maximum value. In other words, the stored energy in the inductor increases during the time the switch is in the closed position.

At the beginning of the OFF time, $S$ will be open (chopper becomes OFF) and the inductor current $i_s$ will start reducing from $I_2$ to $I_1$, Fig.4.20 (b). Due to the reversal of current change (it is reducing now, not increasing) the inductor voltage reverses polarity. Therefore, the voltage at point “A” in Fig.4.19 will be higher than that at point “B” thus, with the beginning of the OFF period the diode $D$ becomes forward biased and the current is directed toward the load via diode $D$. The inductor current, therefore, charges the capacitor and supplies the load current. The diode $D$ blocks not only the current flow toward the source when the switch is in the closed position but also stops the output voltage from appearing across the closed switch. The inductor also helps control the percent current ripple and determines whether or not the circuit is operating in the continuous conduction mode. The capacitor $C$ provides the filtering action by providing a path for the harmonic currents away from the load. In addition, its value is large enough so that the output voltage ripple is very small. Referring to Fig.4.20 (b), during the OFF period:
Using where: the ON time ratio. Therefore, can be controlled by controlling the ON and OFF time of the chopper.

Equation (4.73) states that the output voltage of the boost converter is indirectly proportional to \((1-\gamma)\) and directly proportional to the source voltage \(V_d\). Since the duty cycle is usually less than unity, the output voltage is greater than the applied voltage. This is the reason why a boost converter is commonly called the step-up converter. When the switch, the inductor, and the capacitor are treated as ideal elements, or low-loss chopper circuit, the average power dissipated by these components is zero. Consequently, the average power supplied by the source must be approximately equal to the average power delivered to the load. That is,

\[
P_{in} \approx P_{out}
\]

or

\[
I_S V_d \approx I_o V_o
\]

and from Eq. (4.73)

\[
I_o = \frac{V_d}{V_o} I_S = (1-\gamma)I_S
\]

or the average input (source) current can be expressed in terms of the average output (load) current as:

\[
I_S = \frac{I_o}{(1-\gamma)}
\]
Since the source current is exactly the same as the inductor current, the average inductor current is,

\[ I_{L,avg} = I_S = \frac{I_o}{(1 - \gamma)} \]  \hfill (4.79)

It is to be noted that the average current in the inductor for the boost converter is not the same as the average load current, which was true for the buck converter. The expressions for the maximum and minimum currents through the inductor may now be written as

\[ I_{L,max} = I_2 = I_{L,avg} + \frac{\Delta I_L}{2} = \frac{V_o}{R(1 - \gamma)} + \frac{V_o}{2fL} (1 - \gamma)\gamma \]  \hfill (4.80)

\[ I_{L,min} = I_1 = I_{L,avg} - \frac{\Delta I_L}{2} = \frac{V_o}{R(1 - \gamma)} - \frac{V_o}{2fL} (1 - \gamma)\gamma \]  \hfill (4.81)

The inductor current variation between its maximum and minimum values is as shown in Fig.4.21, keeping in mind that it also represents the source current.

![Fig.4.21 Inductor and source currents waveforms.](image)

The peak-to-peak current ripple can be expressed in terms of the output voltage, by subtracting Eq.(4.80) from Eq.(4.79) above, as

\[ \Delta I_L = \frac{V_o}{fL} (1 - \gamma)\gamma \]  \hfill (4.82)

The current through the diode is shown in Fig.4.22. Its average value is,

![Fig.4.22 The diode current waveform.](image)
Since the average current in the diode is equal to the average current through the load resistor $R$, the average current in the capacitor, as expected, is zero. When the switch $S$ is in its closed position, the capacitor supplies the load current. Hence, from $0 \leq t \leq t_{on}$, the capacitor current is

$$i_C(t) = -I_o = -\frac{V_o}{R}$$  \hspace{1cm} (4.84)

When the switch $S$ is opened, the inductor current supplies both the capacitor current and the load current. Thus, during the time interval $t_{on} \leq t' \leq T$, the capacitor current is

$$i_C(t) = i_L(t) - I_o$$  \hspace{1cm} (4.85)

The maximum and minimum values of the capacitor current when the switch $S$ is in its open position as

$$I_{C,\text{max}} = I_{L,\text{max}} - I_o = \frac{V_o\gamma}{R(1 - \gamma)} + \frac{V_o}{2fL} (1 - \gamma)\gamma$$  \hspace{1cm} (4.86)

$$I_{C,\text{min}} = I_{L,\text{min}} - I_o = \frac{V_o\gamma}{R(1 - \gamma)} - \frac{V_o}{2fL} (1 - \gamma)\gamma$$  \hspace{1cm} (4.87)

It is also must be noted that

$$\Delta I_L = I_{C,\text{max}} - I_{C,\text{min}}$$  \hspace{1cm} (4.88)

The capacitor current waveform is shown in Fig. 4.23.
The input resistance at the chopper terminal (the equivalent resistance appearing across the source) is

\[ R_{\text{in}} = \frac{V_d}{I_s} = \frac{(1 - \gamma)V_o}{I_o/(1 - \gamma)} = (1 - \gamma)^2 \frac{V_o}{I_o} \]

\[ R_{\text{in}} = (1 - \gamma)^2 R \quad (4.89) \]

The voltage regulation of practical chopper is very poor. A feedback regulator is therefore very essential for operation with variable load to stabilize the output voltage. This is depicted in Fig.4.24.

![Fig.4.24 Chopper voltage regulation.](image)

**Example 4.6**

A boost converter is used to convert 12 V input to supply 50 W at an output voltage of 48 V ± 0.5 %. The input current variation shall be no more than ± 1%. If the switching frequency is 10 kHz, specify the smallest inductance and capacitance to meet the given specifications.

**Solution**

\[ V_o = 48 \text{ V} \]
\[ \gamma = 1 - \frac{V_d}{V_o} = 1 - \frac{12}{48} = 0.75 \]
\[ T = \frac{1}{f} = \frac{1}{10 \text{ kHz}} = 100 \mu\text{s} \]
\[ t_{\text{on}} = \gamma T = 0.75 \times 100 = 75 \mu\text{s} \]
\[ t_{\text{off}} = (1 - \gamma)T = (1 - 0.75)100 \mu\text{s} = 25 \mu\text{s} \]
\[ P_{\text{out}} = I_o V_o \]
Assuming the converter is operating with low – loss, i.e. $P_{in} \approx P_{out}$, hence
\[ I_s = \frac{50}{12} = 4.16 \text{ A} \]

During the ON period of the chopper,
\[ V_d = L \frac{di_s}{dt} = L \frac{\Delta I_s}{t_{on}} \]

The maximum allowable variation in the input current is $\pm 1\%$, the peak-to-peak variation at the input current is therefore $2\%$. Hence,
\[ \Delta I_s = \frac{2}{100} I_s = \frac{2}{100} \times 4.16 = 0.08 \text{ A} \]

since from the relation: \[ V_d = L \frac{\Delta I_s}{t_{on}} \quad L > \frac{V_d t_{on}}{\Delta I_s} \]
\[ \therefore \quad L > 11.25 \text{ mH} \]

The capacitor has to supply 1.04 A for the entire ON-period ($t_{on}$) such that $V_o$ will not changed more than $\pm 0.5 \%$ of the rated value. Thus, the peak-to-peak variation of the output voltage is
\[ \Delta V_o = \frac{1}{100} \times 48 = 0.48 \text{ V} \]

The variation in the input voltage is approximately determined from,
\[ I_o = I_c = C \frac{dV_o}{dt} = C \frac{\Delta V_o}{t_{on}}, \quad \Delta V_o = \frac{I_o t_{on}}{C} \]
\[ C > \frac{I_o t_{on}}{\Delta V_o}, \quad \text{or} \quad C > \frac{1.04 \times 75 \times 10^{-6}}{0.48} \]
\[ \therefore \quad C > 162.5 \mu \text{F} \]

The required capacitor is 160 $\mu$F
4.9 OTHER CLASSES OF CHOPPERS

In the previous sections, we discuss the basic principle of class-A chopper (one-quadrant) operation in its two types, the step down chopper, also called Buck converter and the step up chopper which is called Boost converter. However, there are other types of choppers that are classified depending upon the directions of current flow and voltage. These choppers are:

1- Class-B chopper
2- Class-C chopper
3- Class-D chopper
4- Class-E chopper

4.9.1 Class-B Chopper Circuit (Two-Quadrant Operation)

If the supply voltage $V_d$ is not reversible, reversal of load current can be realized in the circuit of Fig.4.25(a). In this circuit, converter is capable of regeneration. When the thyristor $T_1$ switched on the supply voltage is

![Class-B Chopper Circuit](image)

Fig.4.25 Class-B, chopper : (a) Positive current mode,(b) Negative current mode.
clamped across the passive $R-L$ load and active load $E_b$ (which is may be a motor). Hence, positive or 'motoring' current flows causing diode $D_1$ to be reverse biased. When $T_1$ is switched off the potential of point p drops from $V_s$ to zero. The load current cannot change instantaneously and a return path is provided via diode $D_1$. When this current has been driven down to zero by the back $emf$, thyristor $T_2$ switches on to provide a path for negative armature current. Thyristor $T_2$ is then switched off and the instantaneous negative armature current $i_o$ transferred through diode $D_2$ to the supply, and constitutes a regenerative current pulse. The opposition of supply voltage $V_d$ reduces the negative current to zero and thyristor $T_1$ is switched on to restart the cycle of events.

The circuit of Fig.4.25 therefore operates in the two positive voltage quadrant of the load voltage / load current plane. Hence diode $D_2$ allows the current flow only from load to source and the back $emf$ $E_b$ is responsible for negative current flow.

4.9.2 Class-C Chopper

In some texts the circuit of class-B chopper shown in Fig.4.25(b) (only operation in Q2) is called class-B chopper, while the circuit shown in Fig.4.25(a) is called class-C chopper when class-B chopper operates in two-quadrant Q1 and Q2, i.e. as rectifier and as inverter. Fig.4.26 shows the quadrant of operation of this type of chopper.

![Two-quadrant operation of class-C chopper](image)

4.9.3 Class-D Chopper

The schematic circuit diagram of the chopper is shown in Fig.4.27(a). This type of chopper also operates in two quadrants as shown in Fig.4.27(b). The output $i_o$ is always positive, whereas the load voltage $v_o$ may be either positive or negative. Rectifying process occurs when $v_o$ and $i_o$ are both being positive, however, when the voltage is reversed inverting process takes place and the energy fed from load to the source. The operation of class-D chopper can be summarized as follows:

- When $T_1$ and $T_2$ conduct simultaneously, the chopper operates in the 1st quadrant. In this case $v_o$ and $i_o$ both are positive. The converter is rectifying and the inductor $L$ stored energy.
When \( T_1 \) and \( T_2 \) are switched off, the load inductor produces sufficient voltage across it to maintain the current \( i_o \) in the same direction. The inductor voltage forward biases \( D_1 \) and \( D_2 \) causing these diodes to conduct and supply energy from load to the source.

4.9.4 Class-E Chopper (Full-Bridge DC-DC Converter)

Fig.4.28(a) shows the circuit diagram of a full bridge dc-to-dc converter which is also called a class-E chopper. This chopper operates in the four quadrants of \( v_o-i_o \) diagram, i.e. the output voltage as well as the output current both can take positive or negative values as shown in Fig.4.28(b).

When \( T_1 \) and \( T_2 \) conduct simultaneously, the chopper operates in the 1\textsuperscript{st} quadrant. In this case \( v_o \) and \( i_o \) both are positive.

When \( T_3 \) and \( T_4 \) conduct simultaneously, the chopper operates in the 3\textsuperscript{rd} quadrant. In this case \( v_o \) and \( i_o \) both are negative.
If $T_4$ is turned off, the inductor of the $R-L$ load will force the current to flow through $D_1$ and $D_2$ and load power is fed to the source. In this case $i_o$ is negative and $v_o$ is positive, inverting operation, and the chopper is operate in the 2\textsuperscript{nd} quadrant.

Similarly, if $T_2$ is turned off, the inductor of the $R-L$ load will force the current to flow through $D_3$ and $D_4$ and load power is fed to the source. In this case $i_o$ is positive and $v_o$ is negative, inverting operation, and the chopper is operate in the 4\textsuperscript{th} quadrant.

Class-E chopper is, therefore, can operate in the four quadrants which is suitable for use in reversible d.c. drives to operate a d.c. motor in forward, reversing, breaking and regenerating modes. Hence the d.c. drives using class-E choppers are considered to be highly efficient and their dynamic response is also fast.

### 4.10 DC-DC SWITCH-MODE CONVERTERS

These types of converters are widely used in industry as regulated switch-mode power supplies. They are basically chopper circuits that may use BJT transistor, MOSFET, IGBT, thyristor or GTO for switching operation.

Power supplies in industrial applications have to meet the following specifications:

1. Isolation between the source and load.
2. High power density for reduction of size and weight.
3. Controlled direction of power flow.
4. High conversion efficiency.
5. Input and output waveforms with a low “THD” for small filters.
6. Controlled PF of the source in an a.c. voltage.

Power supplies can be categorized into two types:

1. DC power supplies.
2. AC power supplies.

In d.c. power supplies, usually two-stage converter (ac-dc) is used, however this type is known as a rectifier either controlled or uncontrolled, but the voltage output is always stepped down and often unregulated.

There is, however, another type of ac-dc converter which uses three-stage (ac-dc, dc-ac, and ac-dc) which can be accomplished by PWM or switching regulator or resonant oscillation switched mode power supplies SMPS. Typical block diagram of a three-stage switch-mode power supply is depicted in Fig.4.29.
The output d.c. voltage of stage-1 in Fig.4.29 which is obtained by rectifying the main supply line voltage will be greatly affected by the fluctuation of the line voltage magnitude. However, using switch-mode dc-to-dc converters in stage-2 and stage-3 will convert the unregulated d.c. input to these stages into a d.c. output voltage at a desired level.

Depending on requirement of application switched-mode dc-to-dc converter can be one of three basic converter types:

(a) Forward (Buck).
(b) Fly back.
(c) Balanced (Buck Boost).

4.10.1 Forward or (Buck) Regulator Using MOSFET

This converter is basically like a step-down d.c. chopper with low pass LC filter connected across the load as shown in Fig.4.30. The load is assumed to be pure resistance and the transistor is ideal. The converter operates in two modes:

Mode 1: When the transistor $Q$ is ON at $t = 0$, here the input current is flowing through $LC$ and load.
Mode 2: When the transistor $Q$ is OFF at $t = t_1$, $FW$ diode conducts and current flow through $LC$ and load.

Waveforms of the output voltage and current for the two modes of operation of the converter are depicted in Fig.4.31.
Fig. 4.30 Forward converter operation in two modes: (a) Regulator circuit, (b) Mode-1 equivalent circuit, (c) Mode-2 equivalent circuit.

Analytical properties of the forward Buck regulator

The voltage across inductor is $v_L = L \frac{di}{dt}$

Assuming that inductor current rises linearly from a minimum current $I_1$ to a maximum current $I_2$ in time $t_1$, then

$$V_s - V_a = L \frac{I_1 - I_2}{t_1} = L \frac{\Delta I}{t_1}$$

and falls linearly from $I_2$ to $I_1$ in time $t_2$
Fig. 4.31 Forward regulator waveforms.
Assuming a lossless transistor switch:

\[ V_a = L \frac{\Delta I}{t_2} \]

\[ \Delta I = \frac{(V_d - V_a)}{L} t_1 = \frac{V_a}{L} t_2 \]

\[ t_1 = \gamma T \quad \text{and} \quad t_2 = (1 - \gamma)T \quad \therefore V_a = \gamma V_d \]

Hence the average input current is \( I_s = \gamma I_a \)

The switching period \( T \),

\[ T = \frac{1}{f} = t_1 + t_2 = L \frac{\Delta I}{V_d - V_a} + L \frac{\Delta I}{V_a} = L \frac{\Delta I V_d}{V_a (V_d - V_a)} \]

Which gives a peak–peak ripple current as

\[ \Delta I = \frac{V_a (V_d - V_a)}{f L V_d} \quad \text{or} \quad \frac{V_d \gamma (1 - \gamma)}{f L} \quad \text{(4.90)} \]

Using KCL:

\[ i_L = i_c + i_a \]

If we assume load ripple is very small that can be neglected, thus

\[ \Delta i_L = \Delta i_c \]

The average capacitor current which flow for \( \frac{t_1}{2} + \frac{t_2}{2} = \frac{T}{2} \) is

\[ I_c = \frac{\Delta I}{4} \]

\[ v_c = \frac{1}{C} \int i_c \, dt + v_c \,(t = 0) \]

Peak-peak ripple voltage of \( C \) is thus:
\[
\Delta V_c = v_c(t = 0) - v_c = \frac{1}{c} \int_0^{T/2} \frac{\Delta I}{4} \, dt = \frac{\Delta I}{8 f C}
\]

\[
\Delta V_c = \frac{V_a(V_d - V_a)}{8fLCV_d}
\]  \hspace{1cm} (4.91)

Advantages:
1- Use one transistor.
2- High efficiency.
3- I/p current is continuous.
4- Step-up convertor without transformer.

Disadvantages:
1- High peak of current flows through the transistor.
2- Output voltages very sensitive to change.
3- Difficult to protect the transistor in case of short-circuit.
4- Average output current is less than average of inductor current.

Example 4.7
A forward buck regulator has \( V_d = 15 \) V. The average output voltage required is \( V_a = 6 \) V with peak-to-peak ripple input of 10 mV, the switching frequency is 20 kHz and the peak-to-peak ripple of current of the inductor \( L \) is 0.6 A. Calculate the duty cycle and the values of \( L \) and \( C \) required.

Solution

\[ V_a = \gamma V_d \quad \rightarrow \quad \gamma = \frac{V_a}{V_d} = \frac{6}{15} = 0.4 \]

\[ L = \frac{V_a(V_d - V_a)}{\Delta I V_d} \quad T = \frac{V_a(V_d - V_a)}{\Delta I V_d f} = \frac{6 \times (15 - 6)}{0.6 \times 15 \times 20 \times 10^3} = 300 \mu H \]

\[ C = \frac{\Delta I}{8 \Delta V_c f} = \frac{0.6}{8 \times 10 \times 10^{-3} \times 20 \times 10^3} = 375 \mu F \]

4.10.2 Fly Back Regulator (Boost Regulator)
The principle of operation of the fly back converter is that the energy is stored in an inductor and then transfers it to the load storage capacitor such that the output voltage is in excess of the input voltage. There are two types of fly back convertors:
(1) Step-up voltage called boost regulator.
(2) Step-up/step-down voltage fly back regulator called Buck-boost.
**Boost regulator (step-up chopper)**

The boost regulator provides output voltage which is higher than the input voltage. Hence it works like step-up d.c. voltage transformer. Fig.4.32 shows a typical circuit diagram of the boost regulator using MOSFET as a switching device. Waveforms of the output voltage and current for the operation of the converter are depicted in Fig.4.33.

Fig. 4.32 Boost converter operation in two modes: (a) Converter circuit, (b) Mode-1 equivalent circuit, (c) Mode-2 equivalent circuit.

The operation of the regulator can be illustrated as follows:

**Mode 1:** At $t = 0$ transistor $Q$ is ON, input current flows through the inductor $L$ and transistor $Q$. The diode $FW$ is reversed biased, thus isolating the output stage. The inductor $L$ stores energy from the supply.

**Mode 2:** At $t = t_1$ transistor $Q$ is OFF, current flows through $L, C$, load and the freewheeling diode $FW$. Both the input $V_d$ and inductor $L$ supply energy to the load.

Assuming the inductor current flows continuously and rises linearly from $I_1$ to $I_2$ in time $t_1$, then

$$V_d = L \frac{\Delta I}{t_1} \quad (4.92)$$

Inductor current falls linearly from $I_2$ to $I_1$ in $t_2$.  

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Fig. 4.33  Boost regulator waveforms.
Dividing both sides by $T$ and rearranging terms yield $t_1 = \gamma T$ (4.93) and $t_2 = (1 - \gamma)T$ (4.94)

\[ V_d \gamma T = (V_a - V_d)(1 - \gamma)T \]

Dividing both sides by $T$ and rearranging terms yields,

\[ \frac{V_a}{V_d} = \frac{1}{(1 - \gamma)} \rightarrow \frac{V_o}{V_d} = \frac{1}{(1 - \gamma)} \] (4.95)

so that $V_o > V_d$ for this converter. Theoretically, at least, $V_o \to \infty$ when $\gamma = 1$.

Assuming a lossless transistor switch:

\[ V_d I_s = V_a I_a = \frac{\gamma V_d I_a}{(1 - \gamma)} \]

Two useful equations can be obtained from the above equation

\[ \frac{V_a}{V_d} = \frac{\gamma}{(1 - \gamma)} \rightarrow \frac{V_o}{V_d} = \frac{\gamma}{(1 - \gamma)} \] (4.96)

Hence the average input current is

\[ I_s = \frac{1}{1 - \gamma} I_a \rightarrow \frac{I_a}{I_s} = 1 - \gamma \rightarrow \frac{I_o}{I_s} = 1 - \gamma \] (4.97)

\[ T = \frac{1}{f} = t_1 + t_2 = L \frac{\Delta I}{V_a} + L \frac{\Delta I}{V_a - V_d} = L \frac{\Delta I V_d}{V_d(V_a - V_d)} \]

which gives a peak-to-peak inductor ripple current as

\[ \Delta I = \frac{V_d(V_a - V_d)}{f L V_a} \text{ or } \Delta I = \frac{V_d \gamma}{f L} \] (4.98)

The peak-to-peak capacitor ripple voltage can be obtained by considering that the capacitor supplies current to the load during the period when
transistor $Q$ is ON for $t = t_1$. Under this condition, the average capacitor current is $I_a$, and hence $\Delta V_c$ is

$$\Delta V_c = v_c - v_c(t = 0) = \frac{1}{c} \int_0^{t_1} I_c \, dt = \frac{I_a}{C} t_1$$

Using Eq.(4.99) and Eq.(4.96) one can get

$$\Delta V_c = \frac{I_a(V_a - V_s)}{fC V_a}$$

Advantages:
1. Uses one transistor.
2. High efficiency.
3. Input current is always continuous.
4. Step-up converter without using external transformer.

Disadvantages
1. High peak of current flows through the transistor.
2. The output voltage is very sensitive to the change in duty cycle $\gamma$.
3. Difficult to protect the transistor in case of short circuit.
4. Average output voltage is less than average inductor current by a factor of $(1-\gamma)$ and a much higher rms current flow through filter capacitor resulting in the use of a large capacitor and inductor (filters) than those of Buck regulator.

4.10.3 Buck-Boost Regulator

In this type of regulators, the output voltage may be less than or greater than the input voltage $V_d$. The output voltage is always opposite to the input voltage, hence the Buck-boost regulator is also called as inverting regulator. The circuit diagram of the regulator is typically shown in Fig.4.34. The $L-C$ and $FW$ diode are the filtering components whereas $Q$ is the transistor switch. Waveforms of the output voltage and current for the operation of the converter are depicted in Fig.4.35.

Two modes are exist in operation of Buck-boost regulator:

**Mode-1:**
Here when the transistor $Q$ is ON and diode $FW$ is reversed biased, the input current rises through inductor $L$ and transistor $Q$, hence inductor $L$ stores energy from $t = 0$ to $t_1 (= \gamma T)$. The capacitor discharges and supply current to the load. The output voltage takes the form of the capacitor voltage exactly.
Mode-2:
At $t = t_1$, transistor $Q$ is turned off. The equivalent circuit will be as shown in Fig.4.34 (c) and current through $L$ will through $L$, $C$, $FW$ diode and load.

Fig. 4.34 Buck-Boost regulator operation in two modes: (a) Converter circuit, (b) Mode 1 equivalent circuit, (c) Mode 2 equivalent circuit.

The inductor generates voltage $v_L = V_d = L \frac{di}{dt}$ in the direction shown which forward biased $FW$ diode. Thus inductor $L$ supplies energy to the load from $t = t_1$ to $t = T$. Assuming that the load current is continuous and ripple-free, and that the inductor current rises linearly from $I_1$ to $I_2$ in time $t_1$, thus

$$V_d = L \frac{I_2 - I_1}{t_1 - 0} = L \frac{\Delta I}{t_1}$$

$$\therefore \quad t_1 = \frac{L \Delta I}{V_d}$$

where $\Delta I$ is the peak-peak ripple current of $L$.

Inductor current falls linearly from $I_2$ to $I_1$ in time $t_2$, thus
\[ V_a = L \frac{\Delta I}{t_2} \]

\[ \Delta I = \frac{V_a t_1}{L} = \frac{-V_a t_2}{L} \]

\[ V_o = \frac{-V_d \gamma}{1 - \gamma} \]  \hspace{1cm} (4.101)

---

Fig. 4.35 Buck-Boost regulator waveforms.
Assuming a lossless transistor switch:

\[ V_d I_s = V_o I_o \]

Hence, the average input current related to the average current by

\[ I_s = \frac{I_o \gamma}{1 - \gamma}. \quad (4.102) \]

To find the peak-to-peak inductor ripple current, the control period \( T \) can be obtained from

\[ T = \frac{1}{f} = t_1 + t_2 = L \frac{\Delta I}{V_d} - L \frac{\Delta I}{V_o} = L \frac{\Delta I (V_o - V_d)}{V_d V_o} \quad (4.103) \]

From this equation, the peak-to-peak ripple current can be obtained

\[ \Delta I = \frac{V_d V_o}{f \cdot L (V_o - V_d)} \quad (4.104) \]

or \[ \Delta I = \frac{V_d \gamma}{f \cdot L} \quad (4.105) \]

The average inductor current, assuming continuous current, is therefore given by

\[ I_L = I_s + I_o = \frac{I_o \gamma}{1 - \gamma} + I_o = \frac{I_o}{1 - \gamma} \quad (4.106) \]

To find the peak-to-peak capacitor ripple voltage consider the circuit when transistor \( Q \) is ON. At this condition, the capacitor \( C \) supply current to the load for the period from \( t = 0 \) to \( t = t_1 \) in average discharging current \( I_c = -I_o \). Therefore, the peak-to-peak ripple voltage of the capacitor is

\[ \Delta V_c = \frac{1}{c} \int_0^{t_1} I_c \, dt = \frac{1}{c} \int_0^{t_1} I_o \, dt = \frac{I_o t_1}{c}. \]

\[ t_1 = \frac{V_o}{(V_o - V_d) f} \]

\[ \therefore \Delta V_c = \frac{I_o V_o}{(V_o - V_d) f c} \]

or \[ \Delta V_c = \frac{I_o \gamma}{f c} \quad (4.107) \]
Advantages:
1- Provide reversal output voltage without using transformer.
2- High efficiency.
3- Under fault, \( \frac{di}{dt} \) of fault current would be \( \frac{V_d}{L} \) and limited by \( L \).
4- Output short circuit protection would be easy implemented.

Disadvantages:
1- Output current is discontinuous.
2- High peak current flows through transistor \( Q \).

PROBLEMS

4.1 A class-A transistor chopper circuit shown in Fig.4.36 supplied with power from an ideal battery of terminal voltage 150 V. The load voltage waveforms consists of rectangular pulses of duration 1 ms in an overall cycle of 3 ms.

(a) Sketch the waveforms of \( v_L \) and \( i_L \).
(b) Calculate the duty cycle \( \gamma \).
(c) Calculate the average and \textit{rms} values of the load voltage.
(d) Find the average value of the load current if \( R = 10 \Omega \).
(e) Calculate the input power and the ripple factor \( RF \).

Fig.4.36

[Ans : (b) 0.333, (c) 50 V, 86.6V, (d) 2.5 A, (e) 0.833 A, 125 W, 1.414]
4.2 A class-A thyristor d.c. chopper shown in Fig.4.37 is operating at a frequency of 2 kHz from 96 V d.c. source to supply a resistive load of 8 Ω. The load time constant is 6 ms. If the mean load voltage is 57.6 V, find duty cycle (mark to space ratio), the mean load current, and the magnitude of the current ripple. Derive any formula used.

Fig.4.37

[Ans: \( \gamma = 0.6 \), \( I_{av} = 7.2 \) A, \( \Delta I = 0.24 \) A]

4.3 A d.c. Buck converter (class-A chopper) shown in Fig.4.37 supplies power to a load having 6 Ω resistance and 20 mH inductance. The source voltage is 100 V d.c. and the output load voltage is 60 V. If the ON time is 1.5 ms, find:

(a) Chopper switching frequency.
(b) \( I_{max} \) and \( I_{min} \) (\( I_2 \) and \( I_1 \)).
(c) The average diode current.
(d) The average input current.
(e) Peak-to-peak ripple current.

[Ans: (a) \( f_c = 40 \) Hz , (b) \( I_{max} = 1.5 \) A, \( I_{min} = 8.5 \) A, (c) \( I_{avD} = I_{av} = 10 \) A , (d) \( \Delta I = 3 \) A]
4.5 In a class-A chopper circuit an ideal battery of terminal voltage 220 V supplies a load of resistance 10 Ω. The chopping frequency is \( f = 1 \) kHz and the duty cycle is set to be 0.5. Determine:

(a) The average output voltage.
(b) The *rms* output voltage.
(c) The chopper efficiency.
(d) The ripple factor.
(e) The fundamental component of output harmonic voltage.

[Ans: (a) \( V_{av} = 110 \) V, (b) \( V_{L, r.m.s} = 155.56 \) V, (c) \( \eta = 100\% \), (d) \( RF = 1.0 \), (e) \( V_{L, r.m.s} = 99 \) V]

4.6 In the transistor chopper circuit shown in Fig.4.36 (problem 4.1) \( V_d = 300 \) V, \( L = 3 \) mH, \( R = 1.0 \) Ω and it operating with \( T = 3 \) ms, and \( t_{on} = 1.5 \) ms. Using the approximate solution to:

(a) Determine the minimum, maximum and average values of load current.
(b) Express the load current variation in during the of ON and OFF periods.

[Ans: (a) \( I_{min} = 112.5 \) A, \( I_{max} = 187.5 \) A, \( I_{av} = 150 \) A, (b) \( i_1 = 112.5 + 50000t \), \( i_2 = 212 - 50000t \)]

4.7 Repeat problem 4.6 , determine the minimum, maximum and average values of load current using exact solution and compare your results.

[Ans: \( I_{min} = 113.15A, I_{max} = 186.7A, I_{av} = 149.925A \)]

4.8 Repeat problem 4.7 assuming that \( E_b = 40 \) V.

[Ans: \( I_{min} = 78.18 \) A, \( I_{max} = 146.7 \) A, \( I_{av} = 112.44 \) A]

4.9 In the thyristor chopper circuit shown in Fig.4.37, \( V_d = 220 \) V, \( L = 30 \) mH, \( R = 0.5 \) Ω and it operating with \( T = 3 \) ms, \( t_{on} = 1.5 \) ms and \( E_b = 30 \) V. State clearly any approximation you may make to simplify the solution. Find the minimum, maximum, average load current and average value of the current drawn from the 220V source. Draw the equivalent circuit appearing across the source terminals.

[Ans:157.25 A, 162.75 A, 160 A, 80 A. Equivalent circuit parameters are: 60 V source in series with 2 Ω resistance.]
4.10 A dc-dc converter is connected to a 100 V d.c. source to supply an inductive load having 40 mH in series with 5 Ω resistance. FWD is placed across the load. Load current varies between two limits of 10 A and 12 A.

(a) Find the time ratio (duty cycle) and the frequency of the chopper.
(b) Find the equivalent Thevenin resistance across the source and determine the average source current.

[Ans: (a) 0.55, 309 pps; (b) 16.53 Ω, 6.05 A]

4.11 Sketch the power circuit diagram for a class-A thyristor chopper. Show waveforms of the load voltages for the two duty cycle conditions (a) $\gamma = 0.25$, and (b) $\gamma = 0.8$. For both conditions calculate the average value, $rms$ value and ripple factor of the load voltage waveform for $V_d = 100$ V.

[Ans : (a) 25 V, 50 V, 1.224, (b) 80 V, 89.44 V, 0.5]

4.12 A class-A d.c. chopper circuit is supplied with power from an ideal battery of 120 V. The load voltage waveform consists of rectangular pulses of duration 1ms in an overall cycle time of 2.5 ms. Calculate the $rms$ values of the fundamental component and the third harmonic component of the load voltage waveform.

{Ans: 51.36 V, 12.73 V]

4.13 The output d.c. voltage $V_o$ of a step-down (Buck) converter is maintained at 5 V by controlling the duty cycle $\gamma$. The input d.c. voltage $V_d$ to the converter varies in the range of 10 - 40 V. The chopping frequency $f_c$ of the converter is 20 kHz. The minimum load current, $I_o$, of the converter is 1A. Assuming ideal devices and components, calculate the minimum inductance required for operation of the converter in continuous conduction mode.

[Ans : $L = 108.9 \mu$H]

4.14 The output d.c. voltage $V_o$ of a boost (step-up) converter is maintained at 24V by controlling the duty cycle $\gamma$. The input d.c. voltage $V_d$ to the converter varies in the range of 8 - 16 V. The switching frequency $f_c$ of the converter is 20 kHz. The minimum load of the converter is 5 W. Assuming ideal devices and components, calculate the minimum inductance $L$ required for operation of the converter in continuous conduction mode.

[Ans: $L = 427 \mu$H]