11.1 INTRODUCTION

In DC drives d.c. motors are employed in a very large power range, from few watts to several hundreds of kilowatts. Many applications required precise speed control as in textile and spinning and weaving industries, also some applications required very precise position adjustments (as in robotics). Electric trains require smooth speed control as well as the electric cars. In all these industrial applications d.c. motors are used since they provide smooth and precise speed control as compared with a.c. motors. Therefore, optimum performance and efficiency are the main concern in these applications. Variable speed drive (VSD) systems help in optimisation of process so as to reduce operational and maintenance costs. In order to study the DC drives it is important that the d.c. motor and its characteristics must be studied first and reviewed. This will be given in the following sections.

11.2 DC MOTORS

The structure of a direct current (d.c.) motor has two basic components, the field winding and the armature winding. The field winding in a d.c. machine is generally mounted on the frame or stator, while the armature winding invariably mounted on the rotor. Both are supplied from direct current electrical source as shown in Fig.11.1.
11.3 TYPES OF DC MOTORS

The armature circuit and the field circuit may be interconnected in the three basic ways shown in Fig.11.2. Each connection results in particular motor performance and each is suited to particular load applications. These basic types are:

- Separately-excited d.c. motor
- Self-excited d.c. motors
  - (a) Shunt d.c. motor
  - (b) Series d.c. motor
  - (c) Compound d.c. motor

In the separately-excited d.c. motor, (Fig.11.1), the field winding is excited from separate source. In the self-excited d.c. motor, the field winding can be connected in two different ways, the field winding may be connected across the armature (i.e. in shunt), resulting in a shunt motor (Fig.11.2(a)) or the field winding may be connected in series with the armature (Fig.11.2(b)) resulting in a series d.c. motor.

![Fig.11.2 Basic types of self-excited d.c. machines: (a) shunt excited machine, (b) series excited machine, and (c) short-shunt compound machine.](image)
shunt and series fields as shown in Fig.11.2(c). If the shunt winding is connected across the armature, it is known as short-shunt motor (Fig.11.2(c)). In an alternative connection, the shunt winding is connected across the series connection of armature and series winding, and the machine is known as long-shunt motor (Fig.11.3). Both shunt field winding and series field winding are practically wound on the same pole as shown in Fig.11.4(a).

![Compound d.c. motor: Long-shunt type.](image)

In a compound wound d.c. motor the shunt field is normally stronger than the series field (i.e., has more ampere-turns). Compound motor are also be of two types namely, cumulative compound wound motor and differential compound wound motor. Cumulative compound motor is one in which the field windings are connected in such a way that the shunt winding flux $\varphi_{sh}$ and the series winding flux $\varphi_s$ are produced in the same direction so that they add together as shown in Fig.11.4(b).

![Compound motor field windings](image)

(a)  (b)  (c)

Fig.11.4 (a) Compound motor field windings, (b) Differential compound, (c) Cumulative compound.

On the other hand, the differential compound wound motor is one in which the field windings are connected in such a way that the series winding flux $\varphi_s$ opposes the shunt winding flux $\varphi_{sh}$ and weakens it, as illustrated in Fig.11.4(c).
11.4 PRINCIPLES OF DC MOTORS

11.4.1 The Equivalent Circuit of a Separately-Excited DC Motor

A separately-excited d.c. motor is a motor whose field circuit is supplied from a separate constant voltage power supply. Fig. 11.5 shows the electrical equivalent circuits of a separately-excited d.c. motor. In this figure, the armature circuit is represented by an ideal voltage source $E_a$ and a resistor $R_a$ in series with armature inductance $L_a$. This representation is really the Thevenin equivalent of the entire rotor structure, including rotor coils, interpoles and compensating windings, if present.

![Electrical equivalent circuit of a separately-excited d.c. motor](image)

Fig.11.5 Electrical equivalent circuit of a separately-excited d.c. motor.

The field coils, which produce the magnetic flux in the motor, are represented by inductor $L_f$ and resistor $R_f$. This circuit is approximate because we made some of the few simplifications:

i- The brush drop voltage is often only a very tiny fraction of the generated voltage in the machine. Thus, in cases where it is not too critical, the brush drop voltage may be left out or included in the $R_a$.

ii- The internal resistance of the field coils is sometimes lumped together with the variable resistor and the total is called $R_f$.

The principal equations of d.c. machine are:
The internal induced (back) emf $E_a$ is given by:

$$E_a = K_e \phi n \quad (11.1)$$
and the electromagnetic (developed) torque $T_e$ is

$$T_e = K_T \Phi I_a$$  \hspace{1cm} (11.2)

The relation between the terminal voltage $V_t$ and the induced \textit{emf} is

$$V_t = E_a + I_a R_a$$  \hspace{1cm} (11.3)

Input electrical power to the armature circuit

$$P_{in} = V_t I_a$$  \hspace{1cm} (11.4)

Developed power

$$P_d = T_e \omega$$  \hspace{1cm} (11.5)

Output mechanical power

$$P_{out} = T_L \omega$$  \hspace{1cm} (11.6)

where

- $n$ = speed of the motor in revolution per minute (rpm),
- $\Phi$ = flux per pole in Weber (Wb),
- $K_e =$ machine constant $= \frac{pZ}{60} a$,
- $P =$ number of poles, $Z =$ total number of conductors in the armature circuit , $a =$ number of parallel paths ( $a = p$ for lap winding, $a = 2$ for wave winding).
- $K_T =$ torque constant $= 9.55 K_e$,
- $I_a =$ armature current (A).
- $\omega =$ angular speed $= \frac{2\pi n}{60}$ (rad/s)

### 11.4.2 Speed and Torque Equations

The output characteristic (speed-torque relationship) of a separately-excited d.c. motor can be derived from the induced voltage equation (11.1) and torque equation (11.2) of the motor plus the motor general equation (11.3) as follows:

From Eq.(11.2) current $I_a$ can be expressed as:

$$I_a = \frac{T_e}{K_T \Phi}$$  \hspace{1cm} (11.7)

Combining the $V_t$, $E_a$ and $I_a$ equations:

$$V_t = K_e \Phi n + \frac{T_e}{K_T \Phi} R_a$$  \hspace{1cm} (11.8)

Finally, solving for the motor speed:
The later is called the fundamental equation of the speed of d.c. motor.
The no load speed \( n_o \) is found when \( T_e = 0 \), hence the no load speed is,

\[
n_o = \frac{V_t}{K_e \phi} \tag{11.10}
\]

**Representation of the speed equation of the d.c. motor in terms of the angular velocity \( \omega \)**

Referring to the two basic equations of d.c. motor Eqs.(11.1) and (11.2), in which the constants \( K_e \) and \( K_T \) are defined previously as the machine constant and the torque constant respectively. The relation between these two constant is, \( K_T = 9.55 K_e \) for all types of d.c. machine. In the SI system of units the constants \( K_e \) and \( K_T \) are identical \((K_e = K_T = K)\) and have the dimensions Newton metres per Weber ampere or Volt seconds per Weber radian. Since the angular velocity \( \omega = \frac{2\pi n}{60} \) rad /s, equations (11.1) and (11.2) can be re-written as,

\[
E_a = K_e \phi n = K_e \phi \frac{60}{2\pi} \omega = 9.55 K_e \phi \omega \tag{11.11}
\]

or \( E_a = K_T \phi \omega = K \phi \omega \) \tag{11.12}

and \( T_e = K \phi I_a \) \tag{11.13}

From which,

\[
I_a = \frac{T_e}{K \phi} \tag{11.14}
\]

Substituting Eq.(11.1) and Eq.(11.2) in Eq.(11.3) yields,

\[
V_t = K \phi \omega + \frac{T_e}{K \phi} R_a \tag{11.15}
\]

Finally, solving for the motor speed:

\[
\omega = \frac{V_t}{K \phi} - \frac{R_a}{K^2 \phi^2} T_e \tag{11.16}
\]
Equation (11.16) represents the general equation of the speed of d.c. motors in terms of the angular velocity $\omega$ in rad per second which is used instead of $n$ in rpm in many text books.

The no load speed is when $T_e = 0$, hence the no load speed $\omega_0$:

$$\omega_0 = \frac{V_t}{K \phi} \quad (11.17)$$

During transient periods where $n \neq$ constant

$$V_t = E_a + R_a I_a + L_a \frac{dI_a}{dt} \quad (11.18)$$

For steady-state, $n =$ constant and $\frac{dI_a}{dt} = 0$.

11.5 MECHANICAL CHARACTERISTICS OF DC MOTORS IN DRIVING CONDITIONS

When a d.c. motor is used in driving system, its basic operational characteristics are determined by both the values of the resisting torque $T_L$ created by the load, and the electromechanical properties of the motor itself. In the steady-state operation of the drive, it has been shown in Chapter Ten, Section (10.4), that the value of the torque $T_m$ developed by the motor should equal to the load torque, i.e. $T_m = T_L$. As it is seen from Eq.(11.2) that the electromagnetic torque $T_e = T_m$ is proportional to the armature current $I_a$ and the effective machine flux per pole $\phi$. Thus the variation of the load torque $T_L$ should result in variation of the motor torque $T_m$ as well, i.e. variation of armature current $I_a$ and the magnetic flux $\phi$. The relation between the motor torque and the armature current, $T_m = f(I_a)$, is called the electrical characteristic or internal characteristic of the motor.

In drive systems, the internal characteristic of the motor is not very important, since we are interested in the motor shaft speed and not the current. Therefore, to find out the motor’s actual speed corresponding to specific value of the motor torque, one should know the actual relation between the speed and torque $n = f(T_m)$, which is called the external or mechanical characteristic of the motor. In general, the mechanical characteristic of a d.c. motor depends on its type whether it is separately, shunt, series or compound. Each type has its own mechanical characteristic which is different from the others as it will be explained hereinafter.

11.5.1 Mechanical Characteristics of a Separately-Excited d.c. Motor

The output characteristic of a separately excited d.c. motors which is the relation between the torque and speed is given in Eq.(11.9) or
Eq.(11.16). This equation is called the d.c. motor speed equation. Now since \( V_t, \phi, K_e, K_f \) and \( R_a \) are all assumed constants, Eq.(11.9) can be expressed analytically as,

\[
n = \alpha - \frac{1}{\beta} T_e = \alpha - m T_e
\]  

(11.19)

which is just a straight line with a negative slope \( m = \frac{1}{\beta} \). However, In this equation

\[
\alpha = \frac{V_t}{K_e \phi} = n_o
\]  

(11.20)

where \( n_o \) = no load speed, i.e. when \( T_e = 0 \).

\[
\beta = \frac{K_f K_e \phi^2}{R_a}
\]  

(11.21)

\( \beta \) is a constant called the coefficient of hardness of the motor.

The resulting mechanical characteristics (speed-torque characteristics) of the separately-excited motor is shown in Fig.11.6 (Curve-1).

![Fig.11.6 Speed-torque characteristic of separately-excited d.c. motor (Curve-1) and shunt (self-excited) d.c. motor (Curve-2).](image)

**11.5.2 Mechanical Characteristics of Shunt d.c. Motor**

The equivalent circuit of a shunt d.c. motor is shown in Fig.11.7(a). The output characteristic of a shunt and separately-excited d.c. motors are approximately the same. As it has been mentioned that the separately-excited d.c. motor is a motor whose field circuit is supplied from a separate constant-voltage d.c. source, whereas a shunt d.c. motor is a motor whose field circuit gets its power directly across the armature terminals of the motor as shown in Fig.11.7(a). This means that, when the
supply voltage $V_t$ to a motor is assumed constant, there is no practical difference in behaviour between these two machines. Unless otherwise specified, whenever the behaviour of a shunt motor is described, the separately-excited motor is included too. Hence for shunt motor:

The KVL equation for the armature circuit is

$$V_t = E_a + I_a R_a$$  \hspace{1cm} (11.22)

The currents relations is

$$I_L = I_a + I_f$$  \hspace{1cm} (11.23)

The power flow diagram in shunt motor is shown in Fig.11.7(b).

$$P_d = T_e \omega = I_L V_t - (I_f V_t + I_a^2 R_a)$$ \hspace{1cm} (11.24)

Note that: $P_c =$ mechanical loss + iron loss and both are speed dependent.

The speed equation of the shunt motor is the same equation of the separately-excited motor (Eqs.(11.9) and (11.16)). These equations are just a straight line with a negative slope. The resulting torque-speed characteristic of a shunt d.c. motor is also shown in Fig.11.6 (Curve-2).
To explain how does a shunt d.c. motor respond to a load, suppose that the load on the shaft of a shunt motor is increased, then the load torque $T_L$ will exceed the developed torque $T_e$ in the machine, and the motor will start to slow down. When the motor slows down, its internal generated voltage drops ($E_a = K_e n \phi$), so the armature current in the motor $I_a = (V_t - E_a) / R_a$ increases.

As the armature current increases, the developed torque in the motor increases ($T_e = K_T I_a \phi$) and finally the developed torque will equal the load torque at a lower mechanical speed of rotation.

### 11.5.3 Mechanical Characteristics of Series d.c. Motor

A series d.c. motor is a d.c. motor whose field windings consist of relatively few turns connected in series with the armature circuit. The equivalent circuit of this type of motor is shown in Fig.11.8. A distinct feature of a series motor is that the field current $I_S$ is equal to the armature current $I_a$ and the current drawn from the supply $I_L$, i.e.

$$I_S = I_a = I_L$$  \hspace{1cm} (11.25)
Induced Torque in a Series d.c. Motor

The basic behaviour of a series d.c. motor is due to the fact that the flux is directly proportional to the armature current, at least until saturation is reached, (Fig.11.9). As the load on the motor increases, its flux increases too. As seen earlier, an increase in flux in the motor causes a decrease in its speed. The result is that a series d.c. motor has a sharply drooping speed-torque characteristic. The developed torque is

$$T_e = K_T \phi I_a$$  \hspace{1cm} (11.26)

The flux in this machine is directly proportional to its armature current (at least until iron saturates). Therefore, the flux in the machine can be given by

$$\phi = K_f I_a$$ \hspace{1cm} (11.27)

![Magnetization curve of a series motor.](image)

where $K_f$ is a constant. Thus,

$$T_e = K_T \phi I_a = K_T K_f I_a^2$$ \hspace{1cm} (11.28)

Series d.c. motors are therefore used in applications requiring very high torques. Example: starter motors in cars, elevator motors, tractor motors etc.
The Terminal Characteristic of a Series d.c. Motor

The assumption of a linear magnetization curve implies that the flux in the motor will be given by Eq.(11.27). This equation will be used to derive the speed-torque characteristic curve for the series motor.

Derivation of the speed-torque characteristic:

Referring to Fig.11.8, the KVL for this motor is

\[ V_t = E_a + I_a(R_a + R_s) \]  \hspace{1cm} (11.29)

The armature current \( I_a \) is given by,

\[ I_a = \frac{T_e}{\sqrt{K_T K_f}} \]  \hspace{1cm} (11.30)

Also, \( E_a = K_e \phi n \), thus substituting this and Eq.(11.30) in Eq. (11.29) yields

\[ V_t = K_e \phi n + \sqrt{\frac{T_e}{K_T K_f}} (R_a + R_s) \]  \hspace{1cm} (11.31)

If the flux can be eliminated from this expression, it will directly relate the torque of a motor to its speed \( n \). Notice that \( I_a = \phi^2/K_f \) and \( T_e = (K_T/K_f) \phi^2 \). Thus,

\[ \phi = \frac{K_f}{K_T} \sqrt{T_e} \]  \hspace{1cm} (11.32)

Substituting Eq. (11.32) into Eq. (11.31), and solve for \( n \) results in:

\[ n = \frac{V_t}{K_e} \times \frac{1}{\sqrt{T_e}} \frac{1}{K_f} \frac{1}{K_T} (R_a + R_s) \]  \hspace{1cm} (11.33)

and the torque equation is

\[ T_e = T_m = K_T K_f I_a^2 \]  \hspace{1cm} (11.34)

The speed-torque curve of series motor will vary, according to Eq.(11.33) as shown in Fig.11.10. It can be easily, from Eq. (11.33), being noted that the speed-torque characteristic of series motor is a hyperbola.
Fig. 11.10 The speed-torque characteristic of a series d.c. motor.

with asymptote at the speed axis. To plot this hyperbola, substituting $T_e = 0$ in Eq.(11.33) yields, $n = n_o = \infty$. i.e., the no load speed at the ideal no load running of the series motor is infinite (excessive speed). Therefore, at very small load torques are likely to cause rapid increase in the motor speed $n$ which is dangerous to the motor mechanical construction (armature winding, bearings, and commutator structure).

Series motors can usually protected against the danger of excessive speeds by a positive connection to their load before starting. However, for small motors, below 200W, the mechanical losses in the motor may be sufficient to prevent excessive no load speed. In high power motors, even a threefold increase, $n = 3n_n$, where $n_n$ is the normal (rated) speed may result in mechanical over stresses dangerous to the armature. Therefore, these machines should not be run at no load condition.

On the other hand, the starting torque $T_{st}$ can be calculated by setting $n = 0$ in Eq. (11.33), thus

$$T_{st} = \frac{V_t^2}{(R_a + R_s)^2} K_f K_f$$

(11.35)

Since the values of $R_a$ and $R_s$ are usually very small, hence the starting torque of the series motor is considerably large compared with that of separately-excited and shunt motors. This feature makes it preferable to be used to start heavy loads such as electric vehicles, electric trains and elevators.
11.5.4 Mechanical Characteristics of Compound d.c. Motor

The connection diagram of a compound motor is shown in Fig.11.11. The mechanical characteristics of the compound motor occupy intermediate position between the characteristics of the shunt and the series motors due to the fact that the compound motors contains both the shunt and the series windings.

Fig.11.12 depicts the speed-torque characteristics of the three types of d.c. motors namely, shunt, series and compound. As is clear from these characteristics, the compound motor at ideal idle running has an ultimate no load speed $n_o$. Beside, when compared with shunt motor, the compound machine develop stronger starting electromagnetic torque $T_{st}$, and when compared with series motor, it exhibits more “rigid” mechanical characteristics.

![Compound motor connection diagram](image1)

**Fig.11.11:** Compound motor connection diagram.

![Compound motor mechanical characteristics](image2)

**Fig.11.12** Compound motor mechanical characteristics as compared with shunt and series motors.
11.6 DC MOTORS SPEED CONTROL

The speed control of d.c. motors is very important subject in the consideration of the application of these motors. The various methods of speed control follow directly from the fundamental equation of the d.c. motor speed equation (11.9), from which one can predict the ways to control \( n \) if we write down it in the following simple form,

\[
n = \frac{V_t - I_a R_a}{K_e \phi}
\]  

(11.36)

This equation shows that the speed is directly proportional to the applied voltage \( V_t \), inversely proportional to the flux per pole \( \phi \) and changing the armature resistance \( R_a \) by adding an external resistance in series with it. Therefore, it is clear that the motor speed \( n \) can be varied by the following methods:

(i) Varying the terminal voltage \( V_t \), hence varying the applied voltage to the armature \( V_a \).
(ii) Adjusting the field resistance \( R_f \) (and thus varying the field flux \( \phi \)).
(iii) Inserting a resistor in series with the armature circuit ( \( R_a + R_{add} = \sum R_a \) ), (rheostat control).

The first method is the most common method to decrease or increase the speed, while the third one is rarely used now days since it results in excessive losses.

11.6.1 Motor Speed Control of Shunt and Separately Excited d.c. Motors

(A) Changing the Armature Voltage

This method involves changing the voltage applied to the armature of the motor without changing the voltage applied to the field. If the voltage \( V_a \) is increased, then \( I_a \) rises, since \( I_a = (V_a \uparrow - E_a) / R_a \). As \( I_a \) increases, the developed torque \( T_e = K_f \phi I_a \) increases, making \( T_e > T_L \), and the speed of the motor increases. Motor accelerated to new speed: \( 1 \rightarrow 2 \rightarrow 3 \) as shown in Fig.11.13. Now, as the speed increases, \( E_a (=K_e \phi n \uparrow) \) increases, causing the armature current to decrease. This decrease in \( I_a \) decreases the developed torque, causing \( T_e = T_L \) at a higher rotational speed.

(B) Changing the Field Resistance (Field weakening method)

If the field resistance increases, then the field current decreases \( (I_f \downarrow = V_t / R_f \uparrow) \), and as the field current decreases, the flux decreases as well. A decrease in flux causes an instantaneous decrease in the internal
Fig. 11.13 The effect of armature voltage variation on speed control of d.c. separately-excited and shunt motor.

generated voltage $E_a \downarrow (=Ke \phi \downarrow n)$, which causes a large increase in the machine’s armature current since, The developed torque in a motor is given by $T_e = K_T \phi I_a$. Since the flux in this machine decreases while the current $I_a$ increases, so which way does the developed torque change?

To understand what is happening, the following example will illustrate the sequence of events for the motor shown in Fig. 11.14:

$$I_a \uparrow = \frac{V_t - E_a}{R_a}$$

Fig. 11.14 Shunt motor with added resistance in the field circuit.

Figure 11.14 shows a shunt d.c. motor with an armature resistance of 0.25 Ω. It is currently operating with a terminal voltage of 240V and an internal generated voltage $E_a$ of 235V. Therefore, the armature current flow is: $I_a = (240V - 235V) / 0.25\Omega = 20A$.

Now, what happens in this motor if there is a 1% decrease in flux?

If the flux decrease by 1%, then $E_a$ must decrease by 1% too, because $E_a = Ke \phi \downarrow n$. Therefore, $E_a$ will drop to:
\( E_{a2} = 0.99 \ E_{a1} = 0.99 \ (235) = 232.65\text{V} \)

The armature current must then rise to:
\[ I_a = \frac{(240-232.65)}{0.25} = 29.4 \text{ A} \]

Thus, a 1% decrease in flux produced a 47% increase in armature current.

So, to get back to the original discussion, the increase in current predominates over the decrease in flux, \( T_e > T_L \) and the motor speeds up. However, as the motor speeds up, \( E_a \) rises, causing \( I_a \) to fall. Thus, developed torque \( T_e \) drops too, and finally \( T_e \) equals \( T_L \) at a higher steady-state speed than originally \((1 \to 2 \to 3)\), see Fig.11.15.

![Graph showing speed-torque characteristics](image)

Fig.11.15 The effect of field resistance \( R_f \) variation on speed control of a shunt motor’s speed-torque characteristics.

The effect of increasing the \( R_f \) is depicted in Fig.11.15. Notice that as the flux in the machine decreases, the no-load speed of the motor increases, while the slope of the speed-torque curve becomes steeper.

**(C) Inserting a Resistor in Series with the Armature Circuit**

If a resistor is inserted in series with the armature circuit, see Fig.11.16, the effect is to drastically increase the slope of the motor’s torque-speed characteristic, making it operates more slowly if loaded. This fact can be seen from the speed equation (11.9) which can be re-written as:

\[ n = \frac{V_t}{K_e \phi} - \frac{R_a + R_{add}}{K_e K_r \phi^2} T_e \]  

(11.37)

If \( R_{add} \downarrow I_a \) and \( T_e \uparrow \), hence motor accelerated \((n \text{ increased from point 1 to point 3})\), see Fig.11.17.
If $R_{add} \uparrow I_a$ and $T_e \downarrow$, hence motor decelerated ($n$ reduced from point 4 to point 6).

![Diagram of shunt motor control](image)

Fig.11.16 Rheostat speed control of shunt motor.

The insertion of a resistor is a very wasteful method of speed control, since the losses in the inserted resistor are very large. For this reason, it is rarely used.

![Graph showing speed variation](image)

Fig.11.17 Speed variation by insertion additional resistance $R_{add}$ in the armature circuit.

### 11.6.2 Safe Ranges of Operation for the Two Common Methods

**Field Resistance Control**

- The lower the field current in a shunt (or separately-excited) d.c. motor, the faster it turns; and the higher the field current, the
slower it turns. Since an increase in field current causes decrease in speed, there is always a minimum achievable speed by field circuit control. This minimum speed occurs when the motor’s field circuit has the maximum permissible current flowing through it.

- If a motor is operating at its rated terminal voltage, power and field current, then it will be running at rated speed, also known as base speed. Field resistance control can control the speed of the motor for speeds above base speed but not for speeds below base speed. To achieve a speed slower than base speed by field circuit control would require excessive field current, possibly burning up the field windings.

Armature Voltage Control

- The lower the armature voltage on a separately-excited d.c. motor, the slower it turns, and the higher the armature voltage, the faster it turns. Since an increase in armature voltage causes an increase in speed, there is always a maximum achievable speed by armature voltage control. This maximum speed occurs when the motor’s armature voltage reaches its maximum permissible level.

- If a motor is operating at its rated terminal voltage, power and field current, then it will be running at rated speed, also known as base speed. Armature voltage control can control the speed of the motor for speeds below base speed but not for speeds above base speed. To achieve a speed faster than base speed by armature voltage control would require excessive armature voltage, possibly damaging the armature circuit.

These two techniques of speed control are obviously complementary. Armature voltage control works well for speeds below base speed, and field resistance control works well for speeds above base speed.

- There is a significant difference in the torque and power limits on the machine under these two types of speed control. The limiting factor in either case is the heating of the armature conductors, which places an upper limit on the magnitude of the armature current $I_a$.

For armature voltage control, the flux in the motor is constant, so the maximum torque in the motor is $T_{\text{max}} = K_T \phi I_{a,\text{max}}$. This maximum torque is constant regardless of the speed of the rotation of the motor. Since the power out of the motor is given by $P = T\omega$, ($\omega = 2\pi n / 60$ is the angular velocity of the motor shaft), the maximum power is $P_{\text{max}} = T_{\text{max}} \omega$. Thus, the maximum power
out is directly proportional to its operating speed under armature voltage control.

- On the other hand, when field resistance control is used, the flux does change. In this form of control, a speed increase is caused by a decrease in the machine’s flux. In order for the armature current limit is not exceeded, the developed torque limit must decrease as the speed of the motor increases. Since the power out of the motor is given by \( P = T \omega \) and the torque limit decreases as the speed of the motor increases, the maximum power out of a d.c. motor under field current control is constant, while the maximum torque varies as the reciprocal of the motor’s speed, see Fig.11.18.

\[
T_m \quad P_m \quad n
\]

Armature voltage control (Constant torque control).
Field current control (Constant power control).

Fig.11.18 Torque and power limits for a d.c. motor.

**Example 11.1**

A separately-excited d.c. motor used to drive a fan whose torque is proportional to the square of the speed. When the armature circuit of the motor is connected across 200 V, it takes armature current of 16 A and the motor runs at speed of 1000 rpm. If the speed of the motor is to be reduced to 750 rpm, calculate the required terminal voltage and the current drawn by the motor at the new speed. Assume the armature resistance is 0.5 \( \Omega \) and neglect brush voltage drop.

**Solution**

Armature current \( I_{a1} = 16 \) A
Back emf \[ E_{a1} = V_1 - I_{a1}R_a \]

\[ 200 - 16 \times 0.5 = 192 \text{ V} \]

Speed \( n_1 = 1000 \text{ rpm} \)

Torque \( T_1 \propto n_1^2 \) \[ \Rightarrow T_1 = Cn_1^2 \]

where \( C \) is constant

\[ T_1 = K_r \phi I_{a1} \]

Let the voltage required to lower the speed to 750 rpm be \( V_2 \)

\[ V_2 = E_{a2} - I_{a2}R_a \]

To find \( V_2 \) we must find \( E_{a2} \) and \( I_{a2} \).

To find \( I_{a2} \): since for a separately-excited motor, the flux \( \phi \) is constant,

\[ T_2 = K_r \phi I_{a2} \]

Also \( T_2 = Cn_2^2 \)

\[ \therefore \frac{T_1}{T_2} = \frac{Cn_1^2}{Cn_2^2} = \frac{n_1^2}{n_2^2} = \left( \frac{1000}{750} \right)^2 = 1.778 \]

\[ T_2 = \frac{T_1}{1.778} = 0.562 T_1 \]

\[ K_r \phi I_{a2} = 0.562 (K_r \phi I_{a1}) \]

\[ I_{a2} = 0.562 \times 16 = 9 \text{ A} \]

To find \( E_{a2} \):

To find \( E_{a2} \):

\[ E_{a1} = K_e \phi n_1 \quad \text{and} \quad E_{a2} = K_e \phi n_2 \]

\[ \therefore \frac{E_{a1}}{E_{a2}} = \frac{K_e \phi n_1}{K_e \phi n_2} = \frac{n_1}{n_2} = \frac{1000}{750} = 1.33 \]

\[ E_{a2} = \frac{E_{a1}}{1.33} = \frac{192}{1.33} = 144.36 \text{ V} \]

Hence \( V_2 = 144.36 + 9 \times 0.5 = 148.86 \text{ V} \approx 150 \text{ V} \)
Example 11.2

A 240 V d.c. shunt motor has an armature resistance of 0.2 Ω. When the armature current is 40 A, the speed is 1000 rpm. (a) Find additional resistance $R_x$ to be connected in series with armature to reduce the speed to 600 rpm. Assume the armature current remains the same. (b) If the current decreases to 20 A (with resistance $R_x$ connected) find the new speed of the motor.

Solution

(a)

$$E_{a1} = V_1 - I_{a1}R_a$$

$$= 240 - 40 \times 0.2 = 232 V$$

$$E_{a2} = V_1 - I_{a1}(R_a + R_x)$$

$$= 240 - 40 \times (0.2 + R_x) = 232 - 40R_x$$

But

$$\frac{n_2}{n_1} = \frac{E_{a2}}{E_{a1}}$$

$$\frac{600}{1000} = \frac{232 - 40R_x}{232}$$

$$600 \times 232 = 1000 \times (232 - 40R_x)$$

$$R_x = \frac{232 - 0.6 \times 232}{40}$$

From which:

$$R_x = 2.325 \Omega$$

(b)

$$\frac{n_2}{1000} = \frac{240 - 20(2.325 + 0.2)}{240 - 20 \times 0.2}$$

From which:

$$n_2 = 803 \text{ rpm}$$
Example 11.3

A 300 V d.c. shunt motor runs at 1600 rpm when taking an armature current of 40 A. The armature resistance is 0.5 Ω. It is required to:

(a) Calculate the speed when a resistance is inserted in the field circuit as to reduce the flux to 80% of its nominal value (flux weakening),

(b) Calculate the speed when the field resistance is decreased to a value such that the flux is increased to 120% of its nominal value.

Assume that the armature current remains constant in both cases.

Solution

(a) In speed control of d.c. motor using flux variation method, the terminal voltage and armature resistance are kept constant, while the flux $\phi$ is varied. Therefore,

$$n_1 = \frac{V_t - R_a I_{a1}}{K_e \phi_1}$$  \hspace{1cm} (a)

$$n_2 = \frac{V_t - R_a I_{a2}}{K_e \phi_2}$$  \hspace{1cm} (b)

By dividing equation (a) by (b) yields,

$$\therefore \frac{n_2}{n_1} = \frac{V_t - R_a I_{a2}}{V_t - R_a I_{a1}} \times \frac{\phi_1}{\phi_2}$$

Since $I_{a2} = I_{a1}$, hence

$$\frac{n_2}{n_1} = \frac{\phi_1}{\phi_2}$$

$$n_2 = n_1 \times \frac{\phi_1}{\phi_2} = 1600 \times \frac{100}{80} = 2000 \text{ rpm}$$

(b) When the flux is increased by 120%

$$n_2 = n_1 \times \frac{\phi_1}{\phi_2} = 1600 \times \frac{100}{120} = 1333.3 \text{ rpm}$$

Note: Higher speeds can be obtained with field weakening control method.
11.6.3 Speed Control of Series d.c. Motors

Speed control of series d.c. motor may be achieved through either field control, or armature control method:

(A) Field control methods: The speed of a series motor can be controlled by varying the flux in any one of the following ways.

(i) Using field diverting resistor: The field current in a series motor winding can be reduced by connecting a shunt resistance across the field winding so that a small portion of the field current is diverted to the shunt resistance thus reducing the excitation mmf and weakening the flux, Fig.11.19 (a).

As the field current $I_s$ is reduced, $\phi$ will be reduced accordingly since $\phi = K_s I_s$ and the speed $n$ will increases according to Eq.(11.8) and Eq.(11.9). To find an equation for the speed as a function of the diverting resistance:

By KVL:

$$E_a = V_t - I_L R_a - \frac{R_d R_s}{(R_d + R_s)} I_L$$

$$E_a = K_e \phi n = K_e K_s I_s n = K_{es} I_s n$$

$$K_{es} I_s n = V_t - I_L R_a - \frac{R_d R_s}{(R_d + R_s)} I_L$$

But

$$I_s = \frac{R_d}{(R_d + R_s)} I_L$$
that result in:

\[
K_{es} n \frac{R_d}{(R_d + R_s)} I_L = V_T - I_L R_a - \frac{R_d R_s}{(R_d + R_s)} I_L
\]

This method gives speeds above normal because the reduction of the flux. As the field current is decreased, the speed-torque characteristics are shifted upward parallel to each other as shown in Fig.11.19(b). The diverting resistor should be highly inductive so that any change in the load current will not immediately affect the field winding current.

(ii) **Using tapped-field winding:** To increase the speed of the series motor, the flux is reduced by reducing the number of turns of the field winding. This is achieved by using field winding with a number of tappings brought outside, as shown in Fig.11.20.

![Fig.11.20 Speed control of series motor using tapped-field method.](image)

**(B) Armature control methods:** speed control of series motor by armature control may be accomplished by one of the following methods,

(i) **Armature terminal voltage control:** Unlike with the shunt d.c. motor, there is only one efficient way to change the speed of a series d.c. motor. That method is to change the terminal voltage of the motor. If terminal voltage is increased, the speed will increase for any given torque.

From Eq. (11.38), the speed of a series motor can be controlled by variation of the terminal voltage \(V_t\) using variable d.c. supplies. Although this method was very expensive to be achieved in the past, it becomes the most common in use now a days due to the advent of the
powerful power semiconductor devices which provide a cheap, small size and reliable variable d.c. supplies such as the d.c. choppers and controlled rectifiers.

(ii) Armature resistance control: This method is obtained by the same way as for d.c. shunt motor with the exception that the control resistance is connected in series with the supply voltage such that the total armature resistance seen by the supply (Thevenin equivalent resistance) is \( \sum R_a = R_a + R_s + R_{add} \), see Fig.11.21(a).

\[
\begin{align*}
\sum R_a &= R_a + R_s + R_{add} \\
&= \frac{V_T}{\phi} \times \frac{1}{\sqrt{T_e}} \frac{(R_a + R_s + R_{add})}{K_eK_f} \\
\end{align*}
\]

\( A \) high torque is obtained at low speed and a low torque is obtained at high speed. Series motors are therefore used where large starting torques are required as in hoist, cranes, etc.

Example 11.4

A d.c. series motor draws 22 A from a 240 V line while running at 840 rpm. It has an armature resistance of 0.6 Ω and a series field resistance of 0.5 Ω. A diverter is to be added to the circuit so that the speed increases to
1200 rpm while the line current increases to 28 A. Find the value of the diverter resistance $R_d$.

**Solution**

\[
I_L R_s = 22 \times 0.5 = 11 \text{ V}
\]

\[
I_L R_A = 22 \times 0.6 = 13.2 \text{ V}
\]

\[
n = \frac{V_T - I_L (R_A + R_s)}{K_e \varphi} = \frac{V_T - I_L (R_A + R_s)}{K_e I_L}
\]

\[
840 = \frac{240 - 11 - 13.2}{K_e \times 22}
\]

Therefore, $K_e = 0.0116$

Now using Eq.(11.38)

\[
n = \frac{V_T (R_d + R_s)}{K_e I_L R_d} - \frac{R_s}{K_e} - \frac{(R_d + R_s) R_A}{K_e R_d}
\]

\[
1200 = \frac{240 (R_d + 0.5)}{28 \times 0.0116 \times R_d} - \frac{0.5}{0.0116} - \frac{(R_d + 0.5) \times 0.6}{0.0116 R_d}
\]

From which

\[
556.42 = \frac{343.59}{R_d} \quad \rightarrow \quad R_d = 0.62 \Omega
\]

**11.7 FOUR-QUADRANT OPERATION OF A DRIVE SYSTEM AND MOTOR BRAKING**

**11.7.1 FOUR-QUADRANT OPERATION OF DC MACHINE**

A d.c. machine can operate as a motor, as a generator or as a brake as illustrated in the following diagram (Fig.11.22). It has been assumed in this diagram that the field current is fixed (in magnitude and direction) and the armature reaction is negligible such that $K_\varphi$ is constant. In this case the $\omega - T$ equation is linear.

\[
\omega = \frac{V_T}{K \varphi} - \frac{R_a}{K^2 \varphi^2} T_e
\]  \hspace{1cm} (11.40)
Assumptions:

- The positive or forward speed is arbitrarily chosen in counterclockwise direction (it can also be chosen as clockwise). The positive torque is in the direction that will produce acceleration in forward speed, as shown in Fig.11.22.

- The plane of Fig.11.22 is divided into four quadrants, thus four modes of operation. The quadrants are marked as: I, II, III and IV.

![Four-quadrant operation of a d.c. motor.](image)

**Quadrant I**
Both torque and speed are positive – the motor rotates in forward direction, which is in the same direction as the motor torque. The power of the motor is the product of the speed and torque \( P = T_c \omega \), therefore the power of the motor is positive. Energy is converted from electrical form to mechanical form, which is used to rotate the motor. The mode of operation is known as **forward motoring**.

**Quadrant II**
The speed is in forward direction but the motor torque is in opposite direction or negative value. The torque produced by the motor is used to ‘brake’ the forward rotation of the motor. The mechanical energy during the braking is converted to electrical energy – thus the flow of energy is from the mechanical system to the electrical system. The product of the torque and speed is negative thus the power is negative, implying that the
motor operates in \textit{braking mode}. The mode of operation is known as \textit{forward braking}.

\textbf{Quadrant III}

The speed and the torque of the motor are in the same direction but are both negative. The reverse electrical torque is used to rotate the motor in reverse direction. The power, i.e. the product of the torque and speed, is positive implying that the motor operates in \textbf{motoring mode}. The energy is converted from electrical form to mechanical form. This mode of operation is known as \textit{reverse motoring}.

\textbf{Quadrant IV}

The speed is in reverse direction but the torque is positive. The motor torque is used to ‘brake’ the reverse rotation of the motor. The mechanical energy gained during the braking is converted to electrical form, thus power flow from the mechanical system to the electrical system. The product of the speed and torque is negative implying that the motor operates in \textit{braking mode}. This mode of operation is known as \textit{reverse braking}.

\subsection*{11.7.2 ELECTRICAL BRAKING OF DC MOTORS}

A motor is said to be in braking mode when $T_e$ and $\omega_m$ (motor speed) are in opposite direction (Fig.11.23). If $E_a$ becomes $> V_T$ for any reasons, then $I_a$ will become negative (reverse) and $T_e$ will become in the same direction of $T_L$; which opposes rotation. Hence the speed will reduced (since negative dynamic torque is acting on the motor shaft) as described in the following equations.

\begin{align*}
T_e &= T_L + T_{\text{loss}} + T_j \\
T_j &= J \frac{d\omega_m}{dt} = (T_e - T_L) \\
T_e \text{ remain negative until } T_j &= 0.
\end{align*}
If the motor supply is disconnected while motor speed is $\omega_1$ then it takes some time, $t_{\text{stop}}$, until it reaches zero speed. In this case the developed motor speed is zero and the accelerated torque $T_j$ is negative (acting to decelerate the motor).

The stop time, $t_{\text{stop}}$, can be determined from

$$
t_{\text{stop}} = \int_{\omega = \omega_1}^{0} -\frac{Jd\omega}{T_j} = \int_{\omega = \omega_1}^{0} \frac{-Jd\omega}{T_L + T_{\text{loss}}} = \int_{0}^{\omega_1} \frac{Jd\omega}{T_L + T_{\text{loss}}} \quad (11.43)
$$

During the deceleration period, the stored energy is completely consumed in supplying rotational losses and in supplying the coupled load by the required mechanical energy as input. Finally, the speed of the rotating part (the rotor of the motor and its coupled load) attains zero whereby the stored kinetic energy is zero. Note that $T_L$ and $T_{\text{loss}}$ are, in general, rising functions of speed and being very small at low speeds, and therefore $t_{\text{stop}}$ is high.

11.7.3 Types of d.c. Motor Electric Braking

There are three types of electrical braking applied to the d.c. motors, namely, regenerative braking, plugging braking and dynamic breaking.

1. Regenerative Braking
   - This type of braking happen when the motor speed increases above the no-load speed $\omega_{\text{no}}$ (for example, lowering of a load by electrically operated winch and when an electric train goes downhill). The mechanical energy in this type of breaking is converting into electrical energy, part of which is return to the supply and the rest of the energy is lost in the machine.
   - Most of the motors pass smoothly from motoring to generating operation if the induced emf $E_a$ exceeds the source voltage $V_t$ (due to increase of motor speed from $\omega_{\text{mp}}$ to $\omega_{\text{mr}}$). In this case the current $I_a$ becomes negative and the machine will act as a generator pumping power back into the source. This regeneration created by the negative $T_i$ which accelerates the machine from point 1 to point 2, picking up the speed in excess of the no load speed $\omega_{\text{no}}$ at point 3 as shown in Fig.11.24. Under this condition $E_a = K \phi \omega_{\text{mr}}$ ($\omega_{\text{mr}} > \omega_{\text{no}}$) becomes greater than the supply voltage $V_t$. 

Consider now a separately-excited d.c. motor in regenerative mode. Fig.11.25(a) shows the motor working at its normal state at point 2. Now for the transition from 1 to 2 in general case, for Fig.11.25(b),

\[ I_a = \frac{V_t - E_a}{R_a} \]  

(11.44)

The equivalent diagram for this transition is depicted in Fig.11.25. Since in regenerative braking: \( \omega_{mr} > \omega_{mo} \), then at point 2

\[ E_{a2} = K\phi \omega_{mr} > V_t \]

Then

\[ I_{a2} = \frac{V_t - K\phi \omega_{mr}}{R_a} < 0 \]  

(11.45)

Fig.11.25 Separately-excited d.c. motor in regenerative mode.
To find the value of the electromagnetic torque developed by the motor in the regenerative braking condition, substitute Eq.(11.45) into the following torque equation yields

\[ T_{e2} = T_{m2} = K\phi I_{a2} = K\phi \frac{V_t - K\phi \omega_{mr}}{R_a} < 0 \quad (11.46) \]

Equation (11.46) indicates that, in regenerative breaking condition, the motor torque has negative value.

To find an analytical expression for the mechanical characteristic in regenerative condition, equation (11.46) must be solved with respect to \( \omega_{mr} \), that is

\[ \omega_{mr} = \omega_{mo} - \frac{R_a}{K^2\phi^2} T_{m2} \quad (11.47) \]

where \( \omega_{mo} = \frac{V_T}{K\phi} \) is the no-load ideal speed.

Notes:

- To maintain the current below the maximum permissible value, an external resistance \( R_x \) may be needed for this purpose.
- The series motor cannot be used in the regenerative breaking condition, i.e at negative load torque (-\( T_m \)).

2. Plugging

This type of breaking is applicable for all types of d.c. motors, namely separately-excited, shunt, compound and series motors. In this method the polarity of the applied terminal voltage of the motor is reversed. As a result the motor torque \( T_m \) reverses its direction and acts as a break to the motor shaft by reducing its speed to zero. At this instant, i.e. when \( \omega_m = 0 \), the supply must be switched off otherwise the motor will run in reverse direction with negative speed. Fig.11.26 shows braking of separately-excited motor by plugging method.

It is important that, during voltage reversal an external resistance \( R_x \) should be inserted with the armature circuit to limit the braking current. Referring to Fig.11.27, the sequences of events during plugging for separately excited motor are:

From point 1 to point 2 : Current and torque reversal

\[ i_{a2} = \frac{-V_t - E_a}{R_a + R_x} < 0 \quad (11.48) \]
Fig. 11.26 Motor braking by plugging.

Fig. 11.27 Plot of mechanical characteristics of separately-excited motor in plugging mode.
At point 3:

\[ \omega_{m3} = 0 \quad \text{and} \quad T_L = \text{negative} \]

The motor starts to run in opposite direction toward point 4.

### 3. Dynamic Braking

This method of braking the motor is disconnected from the supply and operated as a generator by the kinetic energy of rotor. The kinetic energy is then dissipated in an external resistance connected across the motor. With this technique, the energy required from the supply to brake the motor is eliminated as compared to the previous plugging method. This method of braking can be applied to brake d.c. motors, synchronous a.c. motors as well as a.c. induction motors and generally referred to as **Rheostatic Braking**. See Fig.11.28.

![Fig.11.28 Dynamic breaking of d.c. separately-excited motor.](image)

The sequences of events during plugging are:

From 1 to 2: Current reversal

At point 2:

\[ I_a = \frac{E_a}{R_x + R_a} \]

At point 3:

\[ \omega_{m3} = 0 \quad \text{and} \quad I_a = 0 \]

Note:

- Short circuiting the motor makes \( V_t = 0 \).
- \( R_x \) is used to limit the current and to dissipate the stored kinetic energy.
Example 11.5

A 30 kW, 415 V d.c. shunt motor is braked by plugging. Calculate the value of the external resistance $R_x$ to be placed in series with the armature circuit to limit braking current to 164 A. The armature resistance of the motor is 0.1 $\Omega$ and the full load armature current is 100 A at full load speed of 600 rpm. What is the braking torque obtained from the motor?

Solution

The *emf* induced in the motor

$$E_a = V_t - I_a R_a = 415 - 100 \times 0.1 = 405 \text{ V}$$

Voltage across the armature at the instant of braking

$$= 405 + 415 = 820 \text{ V}$$

The total resistance required to limit the current to 164 A

$$R = \frac{820}{164} = 5 \Omega = R_x + R_a$$

$$R_x = 5 - 0.1 = 4.9 \Omega$$

Full load torque:

$$T_{mf} = \frac{P}{\omega} = \frac{30000}{2\pi n} \times 60 = \frac{30000}{2\pi \times 600} \times 60 = 483.7 \text{ Nm}$$

Since the flux in the shunt motor is constant, hence the torque is proportional to the armature current, hence the initial braking torque is

$$T_b = T_{mf} \times \frac{I_b}{I_{afl}} = 483.7 \times \frac{164}{100} = 793.26 \text{ Nm}$$

Example 11.6

A 600 V d.c. shunt motor having an efficiency of 80% operates a hoist having an efficiency of 75%. Determine the current taken from the supply when the hoist raise the load of 450 kg at speed of 3 m/s. If rheostat braking is used to brake the motor, what is the value of the external resistance must be put in series with the armature circuit in order to lower the same load at the same speed?
Solution

Work done when raising the load

\[ W = F \cdot v = m \cdot g \cdot v = 450 \times 9.81 \times 3 \]

\[ = 13243.5 \text{ Nm/s (Watts)} \]

\[
\text{power input} = \frac{\text{output}}{\eta_m \times \eta_{hoist}} = \frac{13243.5}{0.8 \times 0.75} = 22072.5 \text{ W}
\]

Current drawn from the supply

\[ I_m = \frac{\text{power input}}{V} = \frac{22072.5}{600} = 36.78 \text{ A} \]

When the load is lowered, the motor will operate as a generator. The output of the generator will be dissipated in the resistance. Assume the generator has efficiency of 85%, hence,

\[ \text{power output of generator} = 600 \times 9.81 \times 3 \times 0.85 \times 0.75 \]

\[ = 11256.975 \text{W} \]

If we neglect the armature resistance, the inserted external resistance is

\[ \text{Power dissipated in } (R_x) = \frac{V^2}{R_x} \]

\[ \therefore R_x = \frac{600^2}{11256.975} = 31.98 \Omega \]
PROBLEMS

11.1 A d.c. drive employs a d.c. series motor to raise and lower a constant weight $W$ which hangs from a cable that wound on a drum of constant radius $r$. The velocity of the weight when rising is 6 m/s. The motor has an armature resistance of 0.4 Ω and series field resistance of 0.08 Ω. The supply voltage to the motor is 460 V. Calculate the value of the external resistance that must be inserted in series with the armature circuit to permit the weight $W$ to be lowered at a speed of 4 m/s.

[Ans: 3.67 Ω]

11.2 A variable d.c. drive system is used to drive a fan which has the load characteristic $T_L \propto n^2$, where $T_L$ is the load torque and $n$ is the mechanical speed in rpm. When the motor runs at 600 rpm the back emf generated is 120 V and the current drawn by the armature is 28 A. Calculate the external resistance that must be added to the armature circuit to reduce the speed of the motor to 400 rpm. Neglect all electrical and mechanical losses of the motor including the armature resistance loss.

[Ans: 3.21 Ω]

11.3 A 100 V, 1120 W (1.5 hp), 2900 rpm, permanent magnet d.c. motor has an armature resistance of 0.5 Ω, takes armature current of 14 A at rated load. Calculate the value of the external resistance placed in series with the armature circuit to reduce the speed of the motor to 1800 rpm. Assuming the torque to be constant and neglect brush voltage drop.

[Ans: 2.5 Ω]

11.4 A 240 V, 1000 rpm 100 A, separately-excited d.c. motor has an armature resistance of 0.04 Ω. The motor is braked by plugging from an initial speed of 1100 rpm. Calculate:

(a) The value of the external resistance $R_x$ to be placed in series with the armature circuit to limit braking current to twice the full load value.

(b) The braking torque obtained from the motor.

(c) Torque when the speed has fallen to zero.

[Ans: (a) 2.458 Ω, (b) 450.7 Nm, (c) 216 Nm]

11.5 A 220 V, 20 kW d.c. shunt motor running at its rated speed of 1200 rpm is to be braked by plugging. The armature resistance is 0.1 Ω and the rated efficiency of the motor is 88%. Determine:

(a) The resistance to be connected in series with the armature to limit the initial braking current to twice the rated current.
(b) The initial braking torque.
(c) The torque when the speed of the motor falls to 400 rpm.

[Ans: (a) 1.98 Ω, (b) 318.31 Nm, (c) 214.73 Nm]

11.6 A 40 kW, 500 V d.c. shunt motor having an efficiency of 80% operates a hoist having an efficiency of 70% . Determine the current taken from the supply in order to raise a load of 500 kg at 4 m/s. If the rheostatic braking is employed, what resistance must be put in the armature circuit in order to lower the same load at the same speed?

[Ans: 70 A, 22.75 Ω]

11.7 A 50 hp, 440V d.c. shunt motor is braked by plugging. Calculate the value of the resistance to be connected in series with the armature circuit to limit the initial braking current to 150A . Calculate the braking torque so obtained. Assume armature resistance as 0.1Ω, full-load armature current is 100A and full-load speed is 600 rpm.

[Ans: 5.8 Ω, 878 Nm ]

11.8 A 200 V d.c. series motor takes line current of 25 A when runs at 500 rpm. The armature resistance is 0.5 Ω and the series field resistance is 0.3 Ω. If the load torque remains constant, find the value of the additional resistance to be inserted in series with the armature to reduce the speed to 250 rpm.

[Ans: 3.6 Ω]

11.9 A variable-speed drive system uses a d.c. motor which is supplied from a variable voltage source. The torque and power profiles are shown in Fig, 11.7. The drive speed is varied from 0 to 1750 rpm (base-speed) by varying the terminal voltage (from 0 to 600V) with the field current maintained constant,

(a) Determine the motor armature current if the torque is held constant at 250 Nm up to the base-speed.
(b) The speed beyond the base-speed is obtained by field weakening while the armature voltage is held constant at 600 V . Determine the torque available at a speed of 2500 rpm if the armature current is held constant at the value obtained in part (a). Neglect all losses.

[Ans: (a) 76.35 A, (b) 175 Nm]
11.10 A 250 V, 10 hp series motor is mechanically coupled to a fan and draws 30 A and runs at 500 rpm when connected to a 220 V supply with no external resistance connected to the armature circuit (i.e., $R_{add} = 0$). The torque required by the fan is proportional to the square of the speed. $R_a = 0.5 \, \Omega$ and $R_s = 0.3 \, \Omega$. By neglecting the armature reaction and rotational loss, it is required to:

(a) Determine the power delivered to the fan and the torque developed by the machine,

(b) The speed is to be reduced to 300 rpm by inserting a resistance ($R_{add}$) in the armature circuit. Determine the value of this resistance and the power delivered to the fan.

[Ans: (a) 129.5 Nm, (b) 10.9 Ω, 1464.5 W]

11.11 A 300 V, d.c. shunt motor runs at a speed of 800 rpm when driving a constant-torque load. The armature resistance is 0.5 Ω and the field resistance is adjusted to give 3A in the field circuit. Under certain load condition the motor draws a line current of 20 A.

(a) Calculate the resistance that must be added to the armature circuit to reduce the speed of the motor to 400 rpm.

(b) What is the value of the additional resistance that must be inserted in the armature circuit to make the motor operates in the stalling condition?

[Ans:(a ) 8.57 Ω, (b) 17.14 Ω]